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# FEEDBACK EQUILIBRIA IN A DYNAMIC RENEWABLE RESOURCE OLIGOPOLY: PRE-EMPTION, VORACITY AND EXHAUSTION



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# Feedback equilibria in a dynamic renewable resource oligopoly: pre-emption, voracity and exhaustion*

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**Abstract**

We revisit Benchekroun (2008) to describe a differential oligopoly game of resource extraction under static, linear feedback and nonlinear feedback strategies, to show that (i) feedback rules entail resource exhaustion for a finite number of firms; and (ii) feedback strategies are more aggressive than static ones as long as the resource stock is large enough, in accordance with the acquired view based on the traditional pre-emption argument associated with feedback information.

**JEL codes:** C73, L13, Q2

**Keywords:** dynamic oligopoly, renewable resources, feedback strategies

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# 1 Introduction

The joint exploitation of common pool resources and the associated tragedy has been extensively debated ever since Gordon (1954) and Hardin (1968) in uncountably many contributions, of which one can hardly offer a comprehensive overview. Given the inherently dynamic nature of the issue, resource extraction has been consistently treated using the tools of optimal control and differential game theory.[1]

The analysis of dynamic market interplay through differential games has revealed - among other things - that feedback information boosts strategic interaction among firms as compared to open-loop information, triggering a pre-emption mechanism leading firms to expand production (for an overview, see Dockner *et al.*, 2000, ch. 10). Here, we intend to investigate the bearings of this implication of feedback information on the issue of resource (or species) preservation/extinction relying on a differential oligopoly game of resource extraction proposed by Benchekroun (2003, 2008; see also Fujiwara, 2008), in which Cournot firms harvest a renewable resource under alternative information structures.

As in Benchekroun (2008), we allow for the presence of $n$ unregulated firms,[2] illustrating the static Cournot equilibrium and the linear and nonlinear feedback strategy equilibria as well. Relaxing the key assumption present in Benchekroun (2008) concerning the perpetual positivity of the resource

[1] See Clemhout and Wan (1985), Clark (1990), Dockner and Sorger (1996), Dawid and Kopel (1997), Sorger (1998) and Benchekroun and Long (2002), among many others. All of these use continuous time models. For setups where instead time is discrete, see Levhari and Mirman (1980), Benhabib and Radner (1992) and Dutta and Sundaram (1993a,b), *inter alia*.

[2] The extension to the case of $n$ firms is mentioned in Fujiwara (2008, fn. 8, p. 219) while it is properly investigated in Benchekroun (2008) and Colombo and Labrecciosa (2013). The latter paper, in particular, focuses on the consequences of an *ex ante* resource parcelization among a population of firms. Fujiwara (2011) investigates the welfare effects of increasing the number of firms when these are characterised by different levels of technological efficiency.

stock, there neatly emerges that under either type of feedback strategies, the pressure exerted by firms' profit-seeking incentives causes the exhaustion of the resource at the steady state for a finite number of firms (in correspondence of which, equilibrium profits also drop to zero). This is to be indeed imputed to the intensification of strategic interplay resulting from feedback information, as is usually the case in any differential oligopoly games we have been acquainted with thus far, at least since Fershtman and Kamien (1987) and Reynolds (1987). In particular, under feedback rules, exhaustion is attained in correspondence of a lower number of firms if feedback strategies are linear rather than nonlinear. Instead, the resource certainly survives ad infinitum if firms play the static Cournot-Nash equilibrium (or, equivalently, they follow open-loop rules) forever. Additionally, we also illustrate the presence of a *voracity effect* operating for sufficiently high levels of the resource growth rate, whereby higher growth rates lead to lower steady state resource stocks. The bottom line of our analysis is therefore the following. We are accustomed to think that in static or open-loop equilibria, firms decide by the clock disregarding the stock (of the state variable, whatever it is), while under feedback rules they design strategies accounting explicitly for the stock, the latter behaviour being obviously smarter than the former. Here however, it turns out that a smarter attitude based on feedback rules certainly jeopardises the survival or preservation of natural resources for a finite and therefore admissible industry structure, while conversely some more naivety would be welcome. It is worth stressing that it would be so even from the firms' standpoint as an infinite replication of the static Cournot-Nash equilibrium grants positive profits all along, for any finite number of firms. In a nutshell, if the issue is that firms do not internalise the external consequences of profit-seeking behaviour, the ensuing analysis says that we should be happy they don't (at least insofar as they remain unregulated across information structures).

The remainder of the paper is organised as follows. Section 2 illustrates the setup. Feedback Nash equilibria are illustrated in section 3. The voracity effect is dealt with in section 4. Section 5 concludes.

## 2 The model

The setup is the same as in Benchekroun (2008). We consider a differential oligopoly game of resource extraction over time $t \in [0, \infty)$. The industry consists of an $n$ firms producing a homogeneous good, whose inverse demand function is $p = a - Q$ at any time $t$, with $Q = \sum_{i=1}^{n} q_i$. Marginal cost $c \in (0, a)$ is constant and common to all firms, which operate without any fixed costs. During production, each firm exploits a renewable natural resource, whose accumulation is governed by the following dynamics:

$$\dot{S} = F(S) - Q \tag{1}$$

with

$$F(S) = \begin{cases} \delta S \,\forall\, S \in (0, S_y] \\ \delta S_y \left( \dfrac{S_{\max} - S}{S_{\max} - S_y} \right) \forall\, S \in (S_y, S_{\max}] \end{cases} \tag{2}$$

where $S$ is the resource stock, $\delta > 0$ is its *implicit* growth rate when the stock is at most equal to $S_y$ and $\delta S_y$ is the maximum sustainable yield. Taken together, (1-2) imply that (i) if the resource stock is sufficiently small the population grows at an exponential rate; and (ii) beyond $S_y$, the asset grows at a decreasing rate. Moreover, $S_{\max}$ is the *carrying capacity* of the habitat, beyond which the growth rate of the resource is negative, being limited by available amounts of food and space. In the remainder, we will confine our attention to the case in which $F(S) = \delta S$.

As in Fujiwara (2008, p. 218), we adopt the assumption $\delta \geq 5r/2$, which amounts to requiring that the rate of reproduction of the natural resource be high enough to ensure the non negativity of steady state equilibrium magnitudes with $n = 2$. In general, to ensure non-negativity *for any number of firms*, one should assume $\delta > (n^2 + 1) r/2$, as in Benchekroun (2008, p. 240, Assumption 1). The more restrictive assumption adopted by Benchekroun ensures the preservation of a positive resource stock in steady state for any industry structure; we are relaxing it for the explicit purpose of showing that industry fragmentation may indeed drive the resource itslef towards extinction.

If firms don't internalise the consequences of their behaviour at any time and play the individual (static) Cournot-Nash output $q^{CN} = (a-c)/(n+1)$ at all times, then the residual amount of the natural resource in steady state is $S^{CN} = n(a-c)/[\delta(n+1)] = Q^{CN}/\delta$. For future reference, it is worth noting that the static solution corresponds to the open-loop steady state one, which in this game is unstable (see Figure 1 in Fujiwara, 2008, p. 218; and Lambertini, 2013, p. 240). The initial condition is $S(0) = S_0 > n(a-c)/[\delta(n+1)]$, which suffices to guarantees $S > 0$ at all times under the static Cournot-Nash strategies.[3]

# 3 Feedback Nash equilibria

Following Fujiwara (2008), we consider both linear feedback strategies *à la* Benchekroun (2003) and non linear strategies *à la* Tsutsui and Mino (1990) and Shimomura (1991). We restrict our attention to symmetric equilibria. The Hamilton-Jacobi-Bellman equation writes as:

$$rV_i(S) = \max_{q_i}\{[a-c-Q]q_i + V'(S)[\delta S - Q]q_i\} \tag{3}$$

where $r > 0$ is the discount rate, common to all firms and constant over time; $V_i(S)$ is the firm $i$'s value function; and $V'(S) = \partial V(S)/\partial S$. The first order condition (FOC) on $q_i$ is

$$a - c - 2q_i - \sum_{j\neq i} q_j - V'(S) = 0 \tag{4}$$

In view of the *ex ante* symmetry across firms, we impose $q_j = q_i = q(S)$ and solve the FOC (4) to obtain $V'(S) = a - c - (n+1)q(S)$. Substituting this into (3) yields an identity in $S$. Differentiating both sides with respect

[3] To see this, just observe that if firms always play à *la* Cournot, the stock at a generic $t$ is

$$S(t) = \frac{n(a-c) + e^{\delta t}[\delta(n+1)S_0 - n(a-c)]}{\delta(n+1)}$$

which is surely positive if the above condition holds.

to $S$ and rearranging terms, any feedback strategy is implicitly given by the following differential equation:

$$q'(S) = \frac{(\delta - r)\left[(n+1)q(S) - (a-c)\right]}{2n^2 q(S) - \delta(n+1)S - (n-1)(a-c)}, \tag{5}$$

which must hold together with terminal condition $\lim_{t\to\infty} e^{-rt}V(s)$.

## 3.1 Linear feedback strategy

If the strategy is linear in $S$, so that $q(S) = \alpha S + \beta$, equation (5) becomes:

$$\alpha = \frac{(\delta - r)\left[(n+1)(\alpha S + \beta) - (a-c)\right]}{2n^2(\alpha S + \beta) - (n+1)\delta S - (n-1)(a-c)} \tag{6}$$

which is satisfied iff

$$\begin{aligned} &(\delta - r)\left[a - c - \beta\left(n+1\right)\right] + \alpha\left[2\beta n^2 - \left(a-c\right)\left(n-1\right)\right] \\ &\quad + \alpha\left[r\left(n+1\right) - 2\left(\delta\left(n+1\right) - \alpha n^2\right)\right] S = 0. \end{aligned} \tag{7}$$

The above equation gives rise to the following system of two equations

$$\begin{aligned} \alpha\left[r\left(n+1\right) - 2\left(\delta\left(n+1\right) - \alpha n^2\right)\right] &= 0 \\ (\delta - r)\left[a - c - \beta\left(n+1\right)\right] + \alpha\left[2\beta n^2 - \left(a-c\right)\left(n-1\right)\right] &= 0 \end{aligned} \tag{8}$$

to be solved w.r.t. the unknown parameters $\{\alpha, \beta\}$. The pairs solving (8) are $(\alpha = 0; \beta = (a-c)/(n+1))$, which replicates the static Cournot-Nash solution $q^{CN}$, and

$$\alpha = \frac{(n+1)(2\delta - r)}{2n^2}\,;\ \beta = -\frac{(a-c)\left[2\delta - r\left(n^2+1\right)\right]}{2\delta\left(n+1\right)n^2}. \tag{9}$$

In correspondence of (9), the individual output is

$$q^N_{LF}(S) = \frac{\delta\left(2\delta - r\right)\left(n+1\right)^2 S - \left(a-c\right)\left[2\delta - r\left(n^2+1\right)\right]}{2\delta\left(n+1\right)n^2} \tag{10}$$

where superscript $N$ stands for *Nash equilibrium* while subscript $LF$ stands for *linear feedback*. Leaving aside for brevity the replication of the stability analysis carried out by Fujiwara (2008, p. 218), we focus on (10). If $Q^*_{LF} = nq^*_{LF}$, the steady state amount of resource solving $\dot{S} = 0$ is (henceforth, starred values indicate steady state equilibrium magnitudes):

$$S^*_{LF} = \frac{nq^*_{LF}}{\delta} = \frac{(a-c)\left[2\delta - r\left(n^2+1\right)\right]}{\delta\left[2\delta - r\left(n+1\right)\right](n+1)} \tag{11}$$

which is non-negative for all $\delta > r\left(n^2+1\right)/2$ (the latter condition coinciding with the assumption $\delta > 5r/2$ if $n = 2$) It is then easily verified that

$$\frac{\partial Q^*_{LF}}{\partial n} = -\frac{2(a-c)(\delta - r)\left[2\delta + r\left(n^2-1\right)\right]}{(n+1)^2\left[2\delta - r(n+1)\right]^2} < 0 \tag{12}$$

for all $n \geq 1$. However, it is also true that $S^*_{LF} = Q^*_{LF} = 0$ for all $n \geq \sqrt{(2\delta - r)/r} > 2$ (under the above assumption).

To appreciate the bearings of the linear feedback solution on the residual resource stock, it is worth observing that, by construction, the game is a linear state one (this being the reason why the open-loop solution is subgame perfect, although unstable). Therefore, the adoption of linear feedback strategies is formally equivalent to introducing a square term for the state variable which is not present in the initial definition of the setup, as is clear from section 2. Once firms behave *as if* the game were indeed quadratic in the state, they boost strategic interaction and therefore also exploitation, much the same way as they would if, paradoxically enough, they received a subsidy to their extraction activities.

## 3.2 On the interpretation of linear feedback strategies

dimostrazione dell'esistenza di un sussidio applicato al quadrato dello stock che porta le imprese a replicare l'equilibrio linear feedback. Ovvero: nel linear feedback è "come se" le imprese ricevessero un incentivo a intensificare l'estrazione, che rende effettivamente lineare-quadratico un gioco che per costruzione è stato lineare.

### 3.3 Nonlinear feedback strategy

The case of nonlinear feedback strategies can be quickly dealt with. One imposes stationarity on the state equation, obtaining $q = \delta S/n$, whereby (5) becomes:

$$\frac{\delta}{n} = \frac{(\delta - r)\left[\delta(n+1)S + n(a-c)\right]}{n\,(n-1)\,(a-c-\delta S)}, \tag{13}$$

from which one obtains

$$S^*_{NLF} = \frac{(a-c)\,(\delta - nr)}{\delta\left[2\delta - r\,(n+1)\right]} = \frac{nq^*_{NLF}}{\delta} \tag{14}$$

with $\partial S^*_{NLF}/\partial n \propto \partial Q^*_{NLF}/\partial n < 0$ for all $n \geq 1$, and $S^*_{NLF} = Q^*_{NLF} = 0$ for all $n \geq \delta/r > \sqrt{(2\delta - r)/r}$.[4]

The foregoing analysis can be summarised in

**Lemma 1** *Under both linear and nonlinear feedback strategies, the steady state industry output is everywhere decreasing in the number of firms. However, so is also the steady state equilibrium resource stock, and both magnitudes drop to zero in correspondence of a finite number of firms, which is increasing in the resource growth rate and decreasing in the discount rate.*

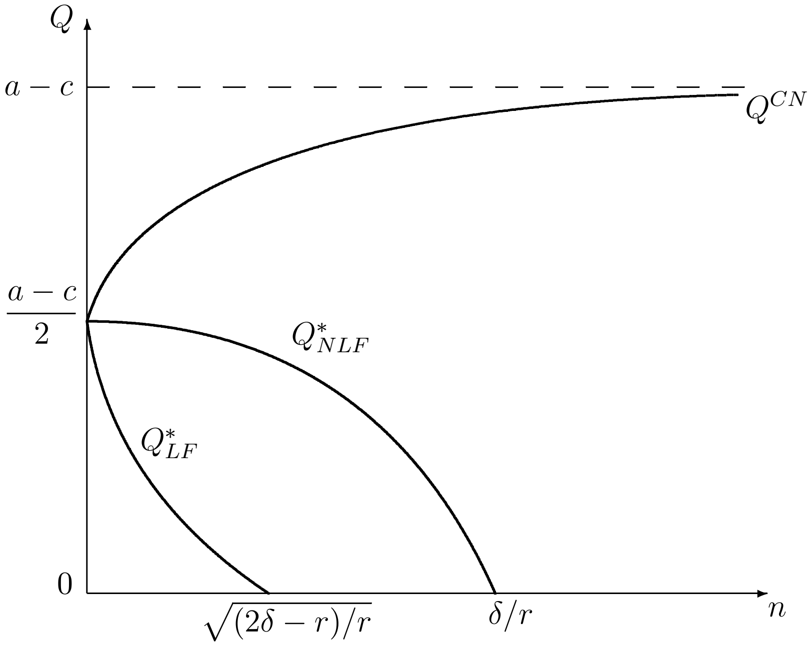
This generalises Fujiwara's conclusion to the general case of an oligopoly with $n$ firms, making explicit an observation that can be found in Fujiwara (2008, fn. 8, p. 219) as to the fact that increasing the number of firms reduces aggregate extraction and output. However, may we really draw the implication that under feedback rules oligopolistic interaction is indeed less competitive than monopoly or static oligopoly? This question, which is a tricky one in connection with the exhaustion issue, is addressed in the next section.

[4] The initial amount of resource must be lower than $S^*_{NLF}$ in order for $q^*_{NLF}$ to be an equilibrium strategy (see Itaya and Shimomura, 2001; Rubio and Casino, 2002).

## 3.4 Comparing equilibria

We are now in a position to comparatively assess firms' behaviour and its consequences across the three equilibria considered above. This exercise can be carried out graphically as in Figure 1, in the space $\{n, Q\}$, in which the curves representing the three possible aggregate outputs depart from the monopoly quantity $q_M = (a - c)/2$.

**Figure 1** Equilibrium industry output and structure




Given the fixed proportion between $S$ and $Q$, immediate implications can be drawn on the resource stock. Since the assumption $\delta > 5r/2$ is equivalent to $\sqrt{(2\delta - r)/r} > 2$, Figure 1 illustrates the following:[5]

**Proposition 2** *Feedback information leads to the exhaustion of the resource at the steady state for a finite number of firms, increasing in $\delta$ and decreasing*

[5] Fujiwara (2008) uses the duopoly version of this model to claim that 'duopoly can be more anti-competitive than monopoly' This, if true, would be puzzling as a less aggressive behavior would goes along with exhaustion. The explanation lies in the fact that Fujiwara's appraisal is valid in steady state, but not at any generic instant during the game.

*in $r$. Conversely, the residual resource stock at the static Cournot equilibrium is positive and increasing in $n$.*

It is worth noting that resource exhaustion, being accompanied by nil output levels, implies that steady state profits are also zero in correspondence of a finite number of firms, while the annihilation of profits at the static equilibrium takes place only in the limit as $n$ tends to infinity and the industry becomes perfectly competitive.

From the existing literature (see Fershtman and Kamien, 1987; Reynolds, 1987, 1991; and Cellini and Lambertini, 2004, *inter alia*), we are accustomed to think that feedback information intensifies strategic interaction among firms, which translates into larger outputs due to the incentive to pre-empt rivals generated by feedback rules themselves. How can we reconcile this acquired wisdom with the seemingly opposite picture emerging from Proposition 2? That is, to what extent it is true that feedback strategies are less competitive than monopolistic behaviour and, *a fortiori*, static Cournot-Nash strategies?

To answer these questions, observe that the difference

$$q^{N}_{LF}(S) - q^{CN} = \frac{(2\delta - r)\left[\delta(n+1)^2 S - (a-c)(n^2+1)\right]}{2\delta(n+1)n^2} > 0 \tag{15}$$

for all

$$S > \frac{(a-c)(n^2+1)}{\delta(n+1)^2} \equiv \overline{S}, \tag{16}$$

with $\overline{S} > S^{*}_{LF}$ for all $\delta > (n+1)r/2$, which simplifies to $\delta > 5r/2$ if $n = 2$. This reveals that, as long as the resource stock is larger than the threshold $\overline{S}$, linear feedback strategies are indeed more aggressive than static Cournot ones. As soon as $S$ drops below $\overline{S}$, the opposite applies throughout the continuation of the game, up to the steady state, where indeed the result portrayed in Proposition 2 and Figure 1 appears.

The last step consists of verifying whether, during the game, $Q^{N}_{LF}(S) = nq^{N}_{LF}(S) > q_M$ in an admissible range of $S$. It turns out that this holds true for all

$$S > \widehat{S} \equiv \frac{(a-c)\left[\delta\left(2+n\left(n+1\right)\right)-\left(n^{2}+1\right)r\right]}{\delta\left(2\delta-r\right)\left(n+1\right)^{2}} \tag{17}$$

with $\widehat{S} \in \left(S^{*}_{LF}, \overline{S}\right)$ for all admissible values of parameters. Hence, at any instant in which $S > \widehat{S}$, following linear feedback rules the oligopoly extracts and sells more than a monopolist. Thus, our analysis can be summarised in

**Proposition 3** *Consider a generic instant* $t \in [0, \infty)$ *. If, at time* $t$, $S > \overline{S}$, *then* $Q^{N}_{LF}(S) > nq^{CN}$ *for all* $n \geq 1$*. If instead* $S \in \left(\widehat{S}, \overline{S}\right)$ *, then* $Q^{N}_{LF}(S) \in \left(q_{M}, nq^{CN}\right)$ *. Finally, if* $S < \widehat{S}$*, then* $Q^{N}_{LF}(S) < q_{M}$*.*

Proposition 3 tells that the intensity of aggregate production (or resource extraction) at a generic point in time before the steady state is reached is decreasing in the existing stock of resource, falling below the monopoly level if the stock falls below a well defined threshold. Put it differently, the steady state picture does not encompass the behaviour of the industry while the game is still unraveling.

# 4 Voracity effect

Our exercise is also connected with the so-called *voracity effect* first explored in Lane and Tornell (1996) and Tornell and Lane (1999) and then investigated by Benchekroun (2008, pp. 245-48) using the same resource extraction game we have adopted here. In a nutshell, the voracity effect says that the *a priori* intuition suggesting that the higher is the resource growth rate, the higher should be the steady state volume of that resource, in fact may not be correct. This happens because a higher reproduction rate drives firms to hasten extraction, as indeed illustrated by (15-16) above. In this regard, we briefly complement the above analysis by looking at the comparative statics properties of the steady state levels of $S$ in the three cases under examination:

$$\frac{\partial S^{CN}}{\partial \delta} = -\frac{n(a-c)}{(n+1)\delta^2} < 0 \text{ everywhere}$$
$$\frac{\partial S^*_{LF}}{\partial \delta} = -\frac{(a-c)\left[(n+1)(n^2+1)r^2 + 4\delta(\delta-(n^2+1)r)\right]}{(n+1)\left[2\delta - r(n+1)\right]^2\delta^2} < 0 \forall \delta > \widetilde{\delta}$$
$$\frac{\partial S^*_{NLF}}{\partial \delta} = -\frac{(a-c)\left[n(n+1)r^2 + 2\delta(\delta-2nr)\right]}{\left[2\delta-(n+1)r\right]^2\delta^2} < 0 \forall \delta > \widehat{\delta} \tag{18}$$

with

$$\widetilde{\delta} = \frac{r}{2}\left[n^2+1+\sqrt{(n^2+1)(2+n(n+1))}\right]$$
$$\widehat{\delta} = r\left[n+\sqrt{\frac{n(3n+1)}{2}}\right] \tag{19}$$

and $\widetilde{\delta} > \max\left\{\widehat{\delta}, (n^2+1)r/2\right\}$. It is also easily ascertained that $\widetilde{\delta}$ and $\widehat{\delta}$ are increasing and convex in $n$. This allows us to formulate our final result:

**Proposition 4** *$\delta > \widetilde{\delta}$ suffices to ensure that the steady state resource stock be decreasing in the growth rate, irrespective of the structure of information underlying firms' equilibrium strategies. Under feedback rules, increasing the number of firms makes the appearance of voracity progressively less likely.*

# 5 Concluding remarks

Revisiting the dynamic game of renewable resource extraction by Benchekroun (2008), we have singled out a feature that has been previously overlooked, namely, that feedback strategies, although appearing less aggressive than static ones in steady state, indeed imply a higher pressure on the resource on the part of firms, whereby the steady state stock may indeed be driven to zero at equilibrium for a finite number of firms. This can be explained on the basis of a pre-emtpion incentive operating during the game, accompanied by a voracity effect if the growth rate of the resource is high enough.

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