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PRODUCTIVITY, COMMODITY PRICES AND THE REAL EXCHANGE RATE: THE LONG-RUN BEHAVIOR OF THE CANADA-US EXCHANGE RATE

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Productivity, Commodity Prices and the Real Exchange Rate:

The Long-Run Behavior of the Canada-US Exchange Rate

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Abstract

The paper examines the Canada-U.S. real exchange rate since the early 1970's to test two popular explanations of the long-run real exchange rate based on the influence of sectoral productivities and commodity prices. The empirical analysis finds that both variables exert a significant long-run effect. However, the relation for the real exchange rate has shifted as the effect of each variable has become stronger and a positive trend is present since 1990. The effect of productivity, moreover, is opposite to that predicted by the standard Balassa-Samuelson theory. An explanation of these findings is suggested based on a general-equilibrium model that includes differentiated traded manufactures and homogeneous commodities.

JEL Classification: F41; F31

Key Words: Real exchange rates; Productivity; Commodity prices; Balassa-Samuelson model.

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1. Introduction

There has been much interest in identifying the long-run determinants of the real exchange rate. A basic hypothesis based on the purchasing power parity asserts that the real exchange rate is constant in the long run. For many trading partners, however, the long-run real exchange rate appears to be time varying. Two explanations of this behavior have emerged. One explanation links the real exchange rate to the relative price of commodities (e.g., Chen and Rogoff, 2003). The other explanation based on the well-known Balassa-Samuelson hypothesis relates the real exchange rate to home-foreign ratio of productivity in traded relative to nontraded goods.

As Canada is an important producer of commodities and its productivity performance is significantly different from that of the U.S., the evidence for the Canada-U.S. real exchange rate provides a useful test of the two explanations. Relevant previous research includes Helliwell, Issa and Lafrance (2006); they find that both real non-energy commodity prices and Canada-U.S. productivity ratio for manufacturing relative to the ratio for the whole economy are significant long-run determinants of the real exchange rate.[1] Contrary to the Balassa-Samuelson hypothesis, however, higher productivity in manufacturing (a proxy for the non-commodity traded-goods sector) in Canada causes an appreciation of the real value of the U.S. dollar in their model. A possible explanation of these results is suggested by Benigno and Thoenissen (2003), who use a DGE model where (unlike the Balassa-Samuelson model) traded goods are differentiated and productivity improvement in the home traded goods sector worsens the terms of trade by increasing the relative supply of these goods. The negative effect on the real value of the home currency via this channel can potentially offset the positive Balassa-Samuelson effect arising

[1] For earlier work linking the Canada-US real exchange rate to commodity prices, see Amano and van Norden (1993, 1995). Also see Issa, Lafrance and Murray (2008) who explore the influence of energy prices on the Canadian dollar.

from an increase in the relative price of nontraded goods.[2] Their model, however, does not include homogeneous traded goods such as commodities, and it is not clear how the presence of such goods would affect the terms of trade adjustment.

The present paper extends this literature on both empirical and theoretical fronts. Our empirical analysis examines the robustness of a long-run relation between the real exchange rate and indexes of real commodity prices and productivity ratios since early 1970's. The empirical evidence suggests that a long-run relation between these variables does not exist for the whole period unless some important shifts are incorporated in the relation since 1990. The effects of both the commodity-price and productivity indexes are stronger in the period after than before 1990. Moreover, a significant positive trend is present in the post-1990 but not the pre-1990 period. There are also other differences between the two periods. Energy prices, for example, are an important component of the appropriate commodity price index after (but not before) 1990.[3] The Canada-U.S. manufacturing productivity ratio has a significant effect on the real exchange rate in each period, but the productivity ratio in manufacturing relative to all sectors (an index suggested by the Balassa-Samuelson model) does not exert a significant effect, especially during the second period. The estimated relation for both periods confirms the paradoxical result that higher manufacturing productivity in Canada is associated with a stronger real value of the U.S. dollar.

Our theoretical analysis explores whether the empirical evidence discussed above can be coherently explained using a well-specified general-equilibrium model. To introduce a role for

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[2] Indeed, calibrating their model to UK-Euro area, Benigno and Thoenissen (2003) find that an increase in the UK traded-goods productivity leads to a sufficient deterioration in the UK terms of trade to bring about a decrease in the real value of Sterling. The terms of trade response can be reversed under certain conditions. See, for example, Corsetti, Dedola and Leduc (2004), who develop a model (calibrated to the US economy) in which higher productivity in the traded goods sector in fact improves the terms of trade.

[3] This result is consistent with the findings of Issa, Lafrance and Murray (2008) that the influence of energy prices on the exchange rate has become stronger in the recent period.

both commodity prices and the terms of trade, we modify the Balassa-Samuelson framework to allow traded goods to consist of differentiated manufactured goods and homogeneous commodities, and the production of commodities to depend on natural resources. A 3-country framework is used to let conditions in the rest of the world (that is, excluding United States and Canada) determine the real price of commodities. We calibrate the model to Canadian and U.S. data and examine whether it is capable of explaining key facts about the long-run behavior of the real exchange rate.

In the model, the elasticity of substitution between Canadian and U.S. manufacturing products is a key determinant of the effect of the manufacturing productivity ratio on the real exchange rate.[4] We show that values of this elasticity within the range suggested in the literature can explain the signs and the magnitudes of the estimated effects of the productivity ratio in both periods. The effect of the commodity price index on the real exchange rate in the model is also shown to depend on the substitution elasticity. An interesting implication of the model is that a decrease in the elasticity strengthens the impact of both the productivity and commodity-price indexes on the real exchange rate. This result suggests that changes in the coefficients of the two indexes in the real exchange rate in the recent period could be due to a decrease in the substitutability between Canadian and U.S. manufactures. Arguably, such a decrease may have resulted from specialization induced by trade liberalization and technological change in the 1990's.

We also examine the effects of changes in nontraded goods productivity in the model, although the empirical analysis does not adequately explore these effects due to data limitations. An interesting result of the model's analysis is that given the asymmetries between Canada and

[4] See Choudhri and Schembri (2010) for a discussion of the role of the elasticity of substitution in determining the effect of traded-goods productivity improvement on the real exchange rate.

the United States, the effect of a productivity improvement in nontraded goods on the real exchange rate is stronger for Canada than for the United States. This result implies that similar productivity growth in the nontraded goods sector in the two countries would lead to an appreciating Canadian real exchange rate. This implication suggests a potential explanation of the presence of a positive trend in the real exchange relation that omits productivity indexes for nontraded goods.

Empirical evidence is presented in section 2. Section 3 summarizes the model, and undertakes quantitative analysis of the model to examine how well it explains the evidence. Section 4 concludes the paper.

2. Empirical Evidence

The behavior of the real exchange rate since 1970 and its relation to indexes of productivity differentials and commodity prices is explored in Figures 1 through 5. Correlations between the different series are shown in Table 1. Figure 1 examines the relation between the log of Canada-U.S. labor productivity ratio in Manufacturing (lpm) and the log of real exchange rate (rer), defined as the real value of U.S. dollar using GDP deflators.[5] In the standard Balassa-Samuelson model, higher traded goods productivity in Canada would lead to a depreciation of the real value of U.S. dollar in rer, and thus the two variables would be negatively related. These variables would, however, be positively related if traded goods are differentiated and the terms of trade effect is strong enough to more than offset the conventional Balassa-Samuelson effect (via the relative price of nontraded goods). No clear pattern emerges from the figure, which suggests a positive association between the two variables from mid-1970's to 1990 and

[5] We follow the previous literature, which considers the GDP-deflator-based real exchange rate (in which the price indexes is based on the production mix) a more useful measure than the consumption-based real exchange rate (in which the price indexes are based on representative consumption baskets).

after 2000, but a negative association during the 1990's (for the total period, the correlation coefficient for these series in Table 1 is small and negative). The figure also shows that the Manufacturing productivity ratio exhibits a falling trend since the early 1990's. To illustrate the influence of this trend, Figure 2 exhibits an adjusted series (lpm_adj), in which actual values have been replaced by detrended values from the first quarter of 1990.[6] Interestingly, the productivity ratio after this adjustment tends to move in the same direction as the real exchange rate even during the 1990's. Indeed, except for early 1970's, the figure exhibits a positive association between the two series throughout the sample period (the correlation coefficient for the series equals 0.63).

One possible explanation of the stronger association of the real exchange rate to the adjusted series is that the trend is picking up long-term changes in the productivity ratio in nontraded sectors, and detrending controls for the effect of these changes. As an alternative way to account for the effect of the nontraded goods productivity ratio, Figure 3 examines the behavior of the log of Canada-U.S. labor productivity ratio in Manufacturing relative to the Business sector ($rlpm$).[7] This series does show a less pronounced negative trend after 1990, it does not seem to be as strongly related to the real exchange rate as the detrended Manufacturing productivity ratio (the correlation coefficient between this series and the real exchange rate series is also small and negative). We explore an explanation of this result in our theoretical analysis below.

Figures 4 and 5 relate movements of real exchange rate in logs to the log of real commodity price index (rpc) and the log of the index excluding energy prices ($rpne$). The real

[6] Here we calculate the detrended values simply by removing a deterministic trend (estimated for the post-1990 period). As there is no significant trend in the real exchange rate during this period, the rer series has not been adjusted in the figure.
[7] The business sector represents most private sectors in the economy, and thus this sector's productivity is a proxy for aggregate productivity.

indexes are defined as the commodity prices in U.S. dollars divided by U.S. GDP deflator. The figures suggest the expected negative relation between the real exchange rate and real commodity prices (the correlation coefficients for these series with respect to the real exchange rate series are -0.63 and -0.68). The two commodity price indexes show a similar pattern, but note that *rpc* series (that includes energy prices) shows a more pronounced rise than *rpne* series in recent years. Our empirical analysis below further compares the performance of these indexes in explaining real exchange rate movements.

We next undertake formal tests to examine whether a long-run relation exists between the real exchange rate and indexes of relative productivity and commodity prices. We use a bounds testing procedure suggested by Pesaran, Shin and Smith (2001), to test for a level relation between the variables regardless of whether they are *I(0)* or *I(1)*. This approach is appealing because it avoids pre-testing variables to see if they have a unit root or not. One limitation of unit root tests is that they have low power against an alternative of highly persistent stationary process, especially for limited time series data. The results of these tests, moreover, can differ across variations that include or exclude a trend or a change in the sample period. Our procedure avoids these problems associated with the unit root tests.

To apply the bounds testing procedure, we estimate the following conditional error correction model:

$$\Delta rer_t = \delta_0 + \delta_1 t + \pi_1 rer_{t-1} + \pi_2 x_{1,t-1} + \pi_3 x_{2,t-1} + \sum_{s=1}^{n} \Delta rer_{t-s} + \sum_{s=0}^{n} \Delta x_{1,t-s} + \sum_{s=0}^{n} \Delta x_{2,t-s} + u_t \text{ ,(1)}$$

where $x_1 (= lpm \text{ or } rlpm)$ is the productivity index, $x_2 (= rpc \text{ or } rpne)$ is the index of commodity prices, t represents the trend variable and u_t is the error term. As the time series are not too long, especially for the subperiods, the above model uses a parsimonious specification that relates the real exchange rate to two variables at a time. We assume that rer_t has no long-run impact on x_{1t} and x_{2t}.[8] Given this assumption, the null hypothesis of no level relationship is tested by a Wald-type test of the restriction that $\pi_1 = \pi_2 = \pi_3 = 0$. Pesaran, Shin and Smith provide critical bounds for the test statistics. The null hypothesis is accepted if the test statistic is below the lower bound, rejected if it is above the upper bound, and the test is inconclusive if the statistics is within the bounds. A similar bounds procedure is used to test the null hypothesis that $\pi_1 = 0$.

We estimated equation (1) by OLS with and without the trend for the whole sample period.[9] To choose the lag order (letting n vary from 1 through 4), we used Akaike Information Criterion (AIC) as well as Schwarz Bayesian Criterion (SBC).[10] The results for testing the null hypothesis of no level relation for different variations of model (1) are shown in Table 2. The table shows that if all observations are used to estimate model (1), the null hypotheses is not rejected (at the 5% level) for all variations. As diagnostic tests indicate parameter instability, we split the sample into two roughly equal subperiods (one ending 1989Q4 and the other beginning 1990Q1), and estimated relation (1) separately for each subperiod.[11] . The subperiod results are very different and indicate that *rer* is related in the long-run with *lpm* and *rpne* in the pre-1990

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[8] In the terminology of Pesaran, Shin and Smith, x_{1t} and x_{2t} are long-run forcing variables for rer_t. This assumption does not rule out short-run endogeneity of x_{1t} and x_{2t}, and is consistent with our model where these variables are determined exogenously in the long-run.

[9] The sample (dictated by the availability of different indexes) includes observations from 1972Q1 to 2007Q1 for regressions with *lpm* , and from 1976Q1 to 2007Q1 for regressions with *rlpm* .

[10] For the choice of n in a particular regression, the sample was adjusted to have the same number of observations for all estimations.

[11] We used a variety of procedures based on breakpoint tests and recursive estimates. These tests did not indicate a clear date for shifts in the relation. For example, according to the Quandt-Andrews test for unknown structural breakpoints, the most likely breakpoint location depends on which variation of the relation is used and lies between 1992 and 2002. The CUSUM of squares test suggests a break in the late 1980's. Also see Chen, Rogoff and Rossi (2010), who find that structural breaks are a serious concern for relations investigating causality between exchange rates and commodity prices. Using Andrews QLR test, they find a structural break in 2002Q3 in the exchange rate-commodity price relation for Canada.

period, and with lpm and rpc in the post-1990 period. A long-run relation is indicated for pre-1990 period both with and without trend, and for the post-1990 period only if trend is included. These results do not hold if $rlpm$ replaces lpm as the productivity index.[12]

For cases that indicate the existence of a long-run relation, Table 3 examines whether the coefficients of level regressors (i.e., π_1, π_2 and π_3) are significantly different from zero individually.[13] In both periods, the effects of commodity prices (indexed by $rpne$ in the first and rpc in the second) as well as productivity (measured by lpm in each period) are significant. The coefficient of the commodity-price index has the expected negative sign while the productivity index has a positive sign, which is inconsistent with the prediction of the standard Balassa-Samuelson model. To estimate the long-run effects of commodity prices and productivity on the real exchange rate, we use the following level relation implied by model (1):

$$rer_t = \mu_0 + \mu_1 t + \mu_2 x_{1,t} + \mu_3 x_{2,t} + u'_t, \tag{2}$$

where $\mu_1 = \delta_1/(1-\pi_1)$, $\mu_2 = \pi_2/(1-\pi_1)$ and $\mu_3 = \pi_3/(1-\pi_1)$. Table 4 presents OLS estimates of (2) for cases where a level relation is indicted (i.e. $x_1 = lpm, x_2 = rpne$ in pre-1990 period, and $x_1 = lpm, x_2 = rpc$ in the post-1990 period).[14] This table summarizes the facts that we seek to explain in our theoretical analysis below. In the first period, the long-run elasticity of the real exchange rate with respect to the productivity index (μ_2) is 0.49, and that with respect to the commodity prices index (μ_3) is -0.31. Both of these effects have become stronger in the second period: the productivity elasticity has increased to 0.94 and the absolute value of the commodity

[12] With $rlpm$ as the productivity index, the no-level-relation hypotheses is not rejected in all cases except one (which represents a regression that is based on pre-1990 observations, includes $rpne$ and trend, and uses lag order determined by SBC)

[13] For the pre-1990 period, a level relation is also indicated for regressions with $rlpm$ and $rpne$ according to SBC, but not AIC. These cases are not shown in Table 3.

[14] If a level relation exists, OLS regression provides consistent estimates.

price elasticity has increased to has increased to 0.51 (higher $\bar{R}^2$ indicates that the explanatory power of the regression also increases in the second period). Another important difference between the two periods is that while there is no significant trend in the relation in the first period, there is a significant positive trend in the second period.

3. Explaining the Evidence

3.1 Model

To explain the facts discussed above, we consider a model that extends the one-factor Balassa-Samuelson framework to include two types of traded goods: differentiated manufactured goods and homogeneous commodities. We use a three-country framework (Canada, United States and the Rest of the World) to allow conditions outside Canada and the United States to determine commodity prices. In this setup, changes in commodity prices as well as productivity indexes can be viewed as exogenous and the model can be used to examine the effects of these changes on the Canada-US real exchange rate. Because the paper is concerned only with the long-run effects, we do not explicitly model the dynamics that would arise from nominal rigidities or international borrowing or lending, but focus instead on steady-state equilibrium with zero net foreign assets (since the time subscripts are not needed, they are omitted).[15]

In this section we only discuss relations that describe the basic structure of the model. Additional relations (implied by household and firm optimization) are discussed in Appendix B. We focus on the equations for the Canadian economy. Analogous equations are assumed for the

[15] Following the standard approach, a unique steady state with zero net foreign assets can be assured by the presence of a debt-dependent transaction cost or a risk premium. Note that conditions for such steady- state equilibrium (in a model with sticky prices and international financial flows) are the same as those for short-run equilibrium in a model with flexible prices and financial autarky.

US economy with a single asterisk used to denote US variables and parameters. Two asterisks are used to denote Rest of the World (RW) variables and parameters.

The single-period utility is assumed to be of the form:

$$u(C,L)=\left(\frac{C^{1-\theta}}{1-\theta}-\frac{\psi L^{1+\phi}}{1+\phi}\right), \tag{3}$$

where L represents labor supply and C is aggregate consumption defined by the following CES index:

$$C=\left[\chi_N^{1/\nu}Z_N^{(\nu-1)/\nu}+\chi_M^{1/\nu}Z_M^{(\nu-1)/\nu}+\chi_C^{1/\nu}Z_C^{(\nu-1)/\nu}\right]^{\nu/(\nu-1)}, \quad \chi_N+\chi_M+\chi_C=1, \tag{4}$$

where Z_N and Z_M represent consumption baskets of differentiated nontraded and traded (manufactured) goods, Z_C is the consumption of homogeneous traded goods (commodities), and ν is the elasticity of substitution between these aggregates. A two-tier CES function is used to define the manufactured goods basket as

$$Z_M=\left[\chi_{MA}^{1/\tilde{\eta}}Z_{MA}^{(\tilde{\eta}-1)/\tilde{\eta}}+\chi_{MR}^{1/\tilde{\eta}}Z_{MR}^{(\tilde{\eta}-1)/\tilde{\eta}}\right]^{\tilde{\eta}/(\tilde{\eta}-1)}, \quad Z_{MA}=\left[\chi_{MC}^{1/\eta}Z_{MC}^{(\eta-1)/\eta}+\chi_{MU}^{1/\eta}Z_{MU}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{5}$$

where $\chi_{MA}+\chi_{MR}=1$, $\chi_{MC}+\chi_{MU}=1$ and Z_{MC}, Z_{MU} and Z_{MR} are consumption bundles of Canadian, US and RW varieties of the manufactured products (Z_{MA} is an American aggregate of the Canadian and U.S. bundles). This specification allows the elasticity of substitution between the Canadian and U.S. bundles (η) to be different than that between the American and RW bundles ($\tilde{\eta}$).

The basket of nontraded goods and the bundles of manufactured products are given by

$$Z_N=\left[\int_0^1 Z_N(i)^{(\sigma-1/\sigma}di\right]^{\sigma/(\sigma-1)}, \quad Z_{MC}=\left[\int_0^1 Z_{MC}(j)^{(\sigma-1/\sigma}dj\right]^{\sigma/(\sigma-1)}, \tag{6}$$

$$Z_{MU} = \left[\int_0^1 Z_{MU}(j^*)^{(\sigma-1/\sigma} dj^* \right]^{\sigma/(\sigma-1)}, Z_{MR} = \left[\int_0^1 Z_{MR}(j^{**})^{(\sigma-1/\sigma} d\tilde{j} \right]^{\sigma/(\sigma-1)}, \qquad (7)$$

where varieties of nontraded goods and manufactured products produced in Canada are indexed by $i \in [0,1]$ and $j \in [0,1]$; manufactured varieties produced in the US and RW are indexed by $j^* \in [0,1]$ and $j^{**} \in [0,1]$; and σ is the elasticity of substitution between varieties within each aggregate.[16]

The production functions for the nontraded and manufactured goods are

$$Y_N = A_N L_N, \quad Y_M = A_M L_M, \qquad (8)$$

where Y_N, L_N and A_N represent the output, labor input and labor productivity for nontraded goods, and Y_M, L_M and A_M are the corresponding variables for manufactured products. The production of commodities also requires the use of natural resources, and we assume the following Cobb-Douglas production function:

$$Y_C = L_C^{\ \alpha} N^{1-\alpha}, \qquad (9)$$

where Y_C is the output while L_C, and N are the labor and natural-resource inputs for commodities. The total supply of natural resources is fixed.[17] The market-clearing conditions for labor and the three sectors are

$$L = L_N + L_M + L_C, \qquad (10)$$

$$Y_N = Z_N, \quad Y_M = Z_{MC} + Z_{MC}^* + Z_{MC}^{**}, \quad Y_C = Z_C + X_C, \qquad (11)$$

where X_C represents exports (imports) of commodities if its value is positive (negative).

[16] We assume, for simplicity, that the number of varieties is fixed. Choudhri and Schembri (2010) explore the implications of endogenously-determined number of varieties for the Balassa-Samuelson effect, and show that the quantitative impact of this extension is not too large.
[17] As our focus is on productivity changes in manufactured products and nontraded goods, a productivity index is not included in the production function for commodities. However, a technological improvement in this production function could be represented by an appropriate change in *N*.

. Let $p_N, p_M, p_C, p_{MA}, p_{MR}, p_{MC},$ and p_{MU} represent the equilibrium real prices of aggregates, $Z_N, Z_M, Z_C, Z_{MA}, Z_{MR}, Z_{MC},$ and Z_{MU} in terms of the Canadian consumption index, C. The real prices for corresponding U.S. aggregates are defined in terms of the U.S. consumption index, C^* and those for RW aggregates in terms of the RW index, C^{**}. Define q as the relative price of C^* in terms of C, and q^* as that of C^{**} in terms of C^*. The variable q represents the consumption-based Canada-U.S. real exchange rate (the real value of US dollar) while q^* is the U.S.-RW real exchange rate.

To relate to our empirical measure of the GDP-deflator based real exchange rate, we define the real price deflator as

$$p_D = p_N(Y_N / Y) + p_{MC}(Y_M / Y) + p_C(Y_C / Y), \tag{12}$$

where $Y = Y_N + Y_M + Y_C$.[18] The real exchange rate based on domestic product deflators then equals

$$q_D = qp_D^* / p_D \tag{13}$$

where p_D^* is the U.S. real price deflator (defined analogously as p_D).

The real prices of manufactured products and commodities in Canada, U.S. and RW are related as

$$p_{MC} = qp_{MC}^* = qq^* p_{MC}^{**},\ p_{MU} = qp_{MU}^* = qq^* p_{MU}^{**},\ p_{MR} = qp_{MR}^* = qq^* p_{MR}^{**}, \tag{14}$$

$$p_C = qp_C^* = qq^* p_C^{**}. \tag{15}$$

Finally, conditions for balanced trade for Canada and the United States are

$$p_{MU} Z_{MU} + p_{MR} Z_{MR} = p_{MC}(Z_{MC}^* + Z_{MC}^{**}) + p_C X_C, \tag{16}$$

$$p_{MC}^* Z_{MC}^* + p_{MR}^* Z_{MR}^* = p_{MU}^*(Z_{MU} + Z_{MU}^{**}) + p_C^* X_C^*. \tag{17}$$

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[18] Initial steady state can be considered as the base period for p_D since all prices in the initial steady state are normalized to equal one.

These conditions ensure that RW also has balanced trade.

In the present model, productivity affects the real exchange rate through two new channels: the terms of trade and the relative commodity price. To explore the contribution of these channels relative to the conventional Balassa-Samuelson channel (operating via the relative price of the nontraded good), we decompose the real exchange rate change as follows. We first express the real exchange rate in (13) as $q_D = (1/\tau)(p_D^* / p_{MU}^*)/(p_D / p_{MC})$, where $\tau \equiv p_{MC} / qp_{MU}^*$ represents the bilateral terms of trade for the differentiated traded good (manufacturing). Next, we use this expression, (12) and its foreign counterpart to linearize the real exchange rate around its initial steady state value. Letting a bar over a variable denote its steady state value and a hat over a variable denote the log deviation from the steady state value, we can express the linearized value of the real exchange rate as

$$\hat{q}_D = \hat{q}_D^{BS} + \hat{q}_D^{TT} + \hat{q}_D^{COM} + res, \qquad (17)$$

where $\hat{q}_D^{BS}, \hat{q}_D^{TT}$ and $\hat{q}_D^{COM}$ are, respectively, the components representing the Balassa-Samuelson, terms of trade and commodity price channels defined as:

$\hat{q}_D^{BS} = (\bar{Y}_N^* / \bar{Y}^*)(\hat{p}_N^* - \hat{p}_{MU}^*) - (\bar{Y}_N / \bar{Y})(\hat{p}_N - \hat{p}_{MC})$, $\hat{q}_D^{TT} = -\hat{\tau} = \hat{q} + \hat{p}_{MU}^* - \hat{p}_{MC}$,

$\hat{q}_D^{COM} = (\bar{Y}_C^* / \bar{Y}^*)(\hat{p}_C^* - \hat{p}_{MU}^*) - (\bar{Y}_C / \bar{Y})(\hat{p}_C - \hat{p}_{MC})$, and res is a residual representing the effect of changes in output shares.[19] Note that the Balassa-Samuelson component shows the adjustment via the price of the nontraded to the differentiated traded good in each country. As the analytical solution of the model is complex because of the 3-good, 3-country setup of the present model, we use numerical analysis to examine the effects of changes in productivity and world

[19] This residual arises because the weights used in the price index (12) depend on current period shares and are variable. This residual is found to be very small in the numerical analysis discussed below.

commodity price through each component.[20] In this analysis, we highlight the role of the substitution elasticity between home and foreign manufacturing in determining these effects.

2.2 Calibration and Results

The Canadian and U.S. data used to calibrate the model are summarized in Table 5. Because the empirical analysis suggests structural shifts between pre-1990 and post-1990 periods, we calibrate the parameters of the model to data for each period to capture these changes in parameter values across the two periods. The share of manufactured products (identified with non-commodity tradable goods and services) in GDP is smaller in Canada than the United States and this difference has narrowed in the second period. The Canadian share for commodities, on the other hand, is larger than the United States and this difference has risen in the second period. There are also important asymmetries in trade flows between the two countries. Canada is a net exporter of commodities while the United States is a net importer. Both the Canadian net commodity exports and the U.S. net commodity imports have increased in the second period. Finally, the Canadian imports of manufactured products from the United States account for a much larger proportion of GDP in both periods than U.S. imports of these goods from Canada.[21]

The data in Table 5 (and certain conditions implied by the model) were used to determine the values for a set of model parameters. The values of these data-determined parameters for the two periods in the baseline case are shown in Table 6, and the calibration procedure is explained in Appendix C. For the remaining parameters, we choose values that are generally assumed in the literature. If there is disagreement about the value of a parameter, we explore the effect of variations within a plausible range of values in our sensitivity analysis. The assumed baseline

[20] Choudhri and Schembri (2010) derive an analytical solution for a 2-good, 2-country model (without commodites) and show that the substitution elasticity between home and foreign goods plays an important role in determining the productivity effect via the terms of trade channel.
[21] This difference is consistent with a larger number of varieties produced in the United States due to larger size.

values (as well as the range of values considered for certain parameters) are also shown in Table 6.

The elasticity of substitution between Canadian and US manufactured goods (η) influences the adjustment in the terms of trade for manufactures, and thus plays an important role in determining the effects of changes in productivity and commodity prices on the real exchange rate. There is little consensus about the value of this elasticity. A value for this elasticity between 0.5 and 1.5 is often assumed in calibrating new open economy macroeconomic models. Estimation of such models, however, typically yields an estimate of the elasticity close to the lower half of this range (e.g., see Bergin, 2004, Lubik and Schorfheide, 2005). In contrast, studies at a more disaggregated or multi-sectoral level (e.g., Hertel et al, 2004) report much higher estimates. We consider a range of values from 0.5 to 2.0 for this parameter, which is wider than that considered in macroeconomic models.

To illustrate the long-run adjustment of the real exchange rate, we first examine the steady-state effects of changes in the productivity and commodity-price indexes on the real exchange rate for different values of η. Table 7 shows the percentage response of the real exchange rate to a one percent change in each index (i.e., the elasticity of q_D with respect to each index) for selected values of η within the 0.5-2.0 range.[22] To explore the role of the substitution elasticity in influencing the adjustment through different channels, the table also shows the percentage response of the Balassa-Samuelson, terms of trade and commodity price components

[22] For the commodity price index, we examine the effect of a 1% increase in the real US dollar price, p_C^*. Since the exogenous commodity price index is in terms of the real world price, p_C^{**}, we assume a change in p_C^{**} that leads to a 1% increase in p_C^*.

of the real exchange rate.[23] Model predictions shown in the table are based on the calibration of the model to data for the second period.[24]

An increase in A_M influences the real exchange rate by decreasing the price of Canadian manufactured products relative to the price of nontraded goods in Canada (the Balassa-Samuelson effect), the price of U.S. manufactured products (the terms of trade effect) and the price of commodities (the commodity price effect). As Table 7 shows, an increase in η does not influence the (negative) Balassa-Samuelson component, but it decreases the (positive) terms of trade component (it also increases the negative commodity component, which somewhat offsets the terms of trade component). Thus the net effect of the A_M increase on q_D, decreases in η, and it remains positive for values of η as high as 1.75. The signs of the Balassa-Samuelson, terms of trade and commodity price effects are all reversed in the case of an increase in A_M^*. Note, however, that absolute values of these effects for the A_M^* increase are different than for the A_M increase because of asymmetries between Canada and the United States.

An increase in A_N generates positive Balassa-Samuelson and commodity price effects and a negative terms of trade effect. Changes in η influence the terms of trade component and (to a lesser extent) the commodity price component, but do not affect the Balassa-Samuelson component. Because the increase in A_N does not directly affect the price of Canadian manufactures, the terms of trade effect in this case is weak and is dominated by the Balassa-Samuelson effect. Thus the elasticity of q_D with respect to A_N is positive over the entire range

[23] To measure these components, we use the decomposition given in (18), where the log deviation (denoted by a hat) of the price indexes and the real exchange rate is calculated as the percentage change between the new steady state (resulting from a 1% change in the specified index) and the initial state.
[24] The general pattern of the results is similar for predictions based on the calibration for the first period. Selected results of the calibrations for the two periods are compared in Figure 6 below.

of values for η. Note that for η in the 0.5-1.75 range, the elasticity of q_D with respect to A_M is also positive. A uniform productivity improvement in the manufacturing and nontraded goods sectors will unambiguously increase the real exchange rate in this range. This result is in sharp contrast to the standard Balassa-Samuelson theory, which predicts that a uniform productivity increase does not affect the real exchange rate.

The effect of an increase in A_N^* on the real exchange rate is opposite to that of A_N and the elasticity of q_D with respect to A_N^* is negative. Because of asymmetries between Canada and the United States, however, the absolute value of this elasticity is smaller than the elasticity for A_N, and this difference is more pronounced at lower values of η. This result has the interesting implication that equi-proportionate increases in A_N and A_N^* would cause an increase in q_D, and this effect would be stronger at lower values of η.

An increase in p_C^* lowers q_D because the United States is an importer and Canada is an exporter of commodities. The commodity-price increase does not affect the relative price of nontraded to manufactured goods (since the productivity for these goods is unchanged) and does not generate a Balassa-Samuelson effect. Interestingly, however, the commodity-price increase involves an adjustment in the terms of trade. An increase in Canadian commodity exports (induced by the increase in p_C^*) leads to an improvement in the Canadian terms of trade for manufactures (to keep trade balanced). A higher value of η causes a smaller terms of trade improvement and thus weakens the commodity-price effect on the real exchange rate. Therefore, the elasticity of q_D with respect to p_C^* increases in η.

We next examine how well the baseline model explains the empirical results. Parameters μ_2 and μ_3 in regression equation (2) represent the long-run elasticities of the (GDP-deflator based) real exchange rate with respect to the Canada-US manufacturing productivity ratio and the real commodity price index, respectively. The model prediction of μ_3 is provided by the elasticity of q_D with respect to p_C^*. As the elasticities for A_M and A_M^* differ in absolute values, we use the average absolute value of these elasticities as the model prediction of μ_2.[25] Figure 6 shows the relationship of the predicted values of μ_2 and μ_3 to η in the 0.5-1.5 interval for the pre-1990 period in panel (a) and the post-1990 period in panel (b). Comparison of panels (a) and (b) of the figure shows that although the relations for μ_2 and μ_3 differ across the two periods (because of shifts in calibrated parameters), they show a similar pattern in each period. Remarkably, the values of η needed to match model predictions of either μ_2 or μ_3 with their estimates in both pre-1990 and post-1990 periods lie within the narrow interval shown in the figure. In each period, however, the value of η that provides an exact match for μ_2 is smaller than that for μ_3, and thus there is a discrepancy between the predicted and estimated values, for at least one of these parameters. However, the discrepancy for each parameter is not too large for a certain range of values of η. For example, values of η between 1.0 and 1.2 in the pre-1990 period, and between 0.65 and 0.75 in the post-1990 period yield predictions which differ from the estimated values by less than less 2 standard deviations.[26]

The model also suggests an explanation of why the estimated value of μ_2 has increased while that of μ_3 has decreased since 1990. As illustrated by Figure 6, this shift could have been

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[25] In the model, the elasticity of q_D with respect to A_M / A_M^* depends on how this ratio is changed. Our measure of this elasticity assumes that the change in the ratio is brought about by changes in A_M and A_M^* that are equal but opposite in sign. Note that alternative assumptions about the source of the change in the A_M / A_M^* ratio would not affect the measure much as the difference between the absolute values of the two elasticities is relatively small.

[26] The estimates for μ_2 are less precise than those for μ_3. For example, the standard deviation for μ_2 is 0.16 in the pre-1990 and 0.14 in the post-1990 period while the standard deviations for μ_2 is 0.04 in both periods.

caused by a decrease in the value of η. Canadian manufacturing bundle may have become less substitutable for the US bundle because of technological changes or increased specialization induced by trade liberalization after 1990.

The above explanation of the shift in the exchange rate relation is based on a decrease, albeit small, in the elasticity of substitution between Canadian and US manufactured goods after 1990. One possible reason for such a change (at the aggregate level) is that recent technological advances have shifted production towards skill-based sectors that are less substitutable (at the disaggregate level) between the two countries. Another potential reason is increased specialization caused by the Canada-US Free Trade Agreement and the North American Free Trade Agreement negotiated during the 1990's. Trade liberalization, for example, could have increased the share of goods (at least in Canada) that are subject to external or internal scale economies. If such goods tend to be more differentiated, a greater share of these goods could account for a decrease in the aggregate substitution elasticity.[27]

A testable implication of the above explanation is suggested by the decomposition of the real exchange rate change into different effects. A decrease in η would strengthen the terms of trade effect and thus increase the elasticity of the terms of trade for differentiated traded goods with respect to the productivity differential (i.e., the elasticity of p_{MC} / p_{MU} with respect to A_M / A_M^*).[28] As discussed above, a decrease in η from a value in the 1.0-1.2 range in the first period to a value in the 0.65-0.75 range in the second period is needed to account for the shifts in

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[27] Although these explanation are arguments for a lower elasticity of substitution between goods in the aggregate production function, they are applicable as well as for a lower elasticity of substitution in the utility function, as specified in our model, because one can interpret these goods as intermediate goods that are transformed into final consumption goods.

[28] Note that Table 7 shows that the terms of trade component (which equals minus the percentage change in the terms of trade) for a 1% increase in A_M increases in η, and that for a 1% increase in A_M^* decreases in η. The table also shows that changes in η do not affect the relative price of the nontraded good and thus do not change the Balassa-Samuelson component.

μ_2 and μ_3. Model simulations (based on calibrated parameters for each period) indicate that such variation in η would lead to a change in the terms of trade elasticity from a range between -0.73 and -0.82 in the first period to a range between -1.17 and -1.32 in the second period.

To test this implication, we measured Canada-U.S. terms of trade as the ratio of the price index for Canadian manufactures to the price index for U.S. manufactures converted to Canadian dollars. A major limitation of this measure is that it does not adequately reflect the prices of Canadian manufacturing exports to and imports from the United States.[29] Nevertheless, we used these data and the data on Canada-U.S. productivity differential to estimate the terms of trade elasticity for the two periods.[30] The estimate of the elasticity is 0.088 (with Newey-West standard error equal to 0.603) for the first period and -1.959 (with Newey-West standard error equal to 0.435) for the second period. The absolute value of the terms of trade elasticity has significantly increased from the first to the second period, which is consistent with the explanation that the substitution elasticity has decreased across these periods. However, the difference in point estimates of the terms of trade elasticity is much larger than the difference predicted by the model.

Another puzzling fact is the presence of a positive trend in real exchange rate relation after 1990. One explanation of the positive trend could be that recent technological developments have also lead to significant productivity improvements in services, which are an important component of nontraded goods. As discussed above, the model implies that an increase in A_N

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[29] Data on unit values of exports and imports is sometimes used to measure the terms of trade. However, the unit value data are not only poor proxies for export and import prices, but also were not available at the appropriate classification level (i.e., Canadian exports and imports of manufactures to and from the United States).

[30] The elasticity is measured by the OLS estimate of a_2 in the following regression equation: $tot_t = a_0 + a_1 t + a_2 lpm_t + e_t$, where tot_t is the log of the terms of trade and lpm_t is (as before) the log of the Canada-U.S. labor productivity ratio in manufacturing, t represents trend (which, as discussed earlier, could control for productivity changes in nontraded goods and other omitted changes) and e_t is the error term.

has a stronger effect on q_D than A_N^*, especially at lower values of η (see Table 6). Thus nontraded goods productivity growth in Canada during the recent period, which is similar or not much smaller than US growth, would tend to appreciate the real exchange rate over time, and this effect would be reflected in a positive trend term in regressions (1) and (2).

The empirical analysis attempted to account for the effect of productivity growth in nontraded goods by using the Canada-US productivity ratio in manufacturing relative to all sectors [a proxy for the productivity ratio in traded relative to nontraded goods, $(A_M / A_M^*)/(A_N / A_N^*)$] as the productivity index. The index was found to be not significantly related to the real exchange rate, especially in the recent period. This result is consistent with the present model. Unlike the standard Balassa-Samuelson analysis, both A_M / A_M^* and A_N / A_N^* ratios exert a positive effect on the real exchange rate in the model (within the relevant range of values of η).[31] Thus the effect of a productivity index that divides the first ratio by the second would be biased towards zero.

We also examined the sensitivity of our results to variations in the assumed parameters, α, θ and ϕ (within the range shown in Table 5). Our sensitivity analysis shows that an increase in α or θ tends to decrease the values of both $\mathrm{E}(A_M)$ and $\mathrm{E}(p_C^*)$ for a given value of η. A decrease in ϕ has similar effects.[32] The variations considered, however, do not make much difference to the values of these elasticities to significantly change the results of the baseline case.

[31] Moreover, the ratio A_N / A_N^* does not adequately reflect the net effect of changes in A_N and A_N^* (because the magnitude of the effect of each index is different). Note, however, that as the magnitudes of the effects of A_M and A_M^* are not too different, the ratio A_M / A_M^* is still a useful measure of the net effect of changes in A_M and A_M^*.

[32] Thus these variations shift both MU2 and MU3 relations in Figure 6 in the same direction.

4. Conclusions

The paper examines the evidence for the Canada-U.S. real exchange rate since early 1970's to explore the role of productivity in home relative to foreign manufacturing as well as real commodity prices in determining the long-run behavior of the real exchange rate. The paper makes two contributions to this literature. First, its empirical analysis finds that although both productivity and commodity-price indexes exert a significant long-run effect on the real exchange rate, the long-run relation for the real exchange rate has shifted in that the effect of each variable has become stronger and a positive trend is present since 1990. The effect of the productivity variable, moreover, is (as indicated by previous research) opposite to that predicted by the standard Balassa-Samuelson theory.

Second, the paper examines whether a simple extension of the Balassa-Samuelson model that incorporates both differentiated and homogeneous traded goods (i.e, manufactured products and commodities) is capable of explaining the empirical results. We show that a quantitative version of such a model calibrated to the Canadian and U.S. economies in the periods before and after 1990 is able to explain the signs and roughly account for the size of the coefficients of the two variables in each period if a moderate decrease in the elasticity of substitution between Canadian and U.S. manufactured products (well within the range suggested in the literature) is allowed for in the second period.

Another challenge is to explain the presence of the trend term in the long-run relation after 1990. It may be that the trend reflects the effect of some factors that are not considered in the model.[33] We do, however suggest an explanation based on the model. Given the asymmetry

[33] For example the model assumes balanced trade (for all three countries) in the long run. It could be argued that the desire by the rest of the world to increase holdings of U.S. securities over time could have lead to a rising real value of US dollar. Note, however, that significant growth in U.S. dollar international reserves of developing countries (notably China) occurred after the currency crises in the late 1990's while the trend in the real exchange rate shows

between Canada and the United States regarding the production and trade of commodities, the model implies that the effect of an improvement in the productivity of nontraded goods is stronger for Canada than the United States. Productivity growth in nontraded goods is not as well measured as for manufacturing, and is not included in the estimated empirical relation. Arguably, recent technological improvements may have also increased productivity in nontraded goods. If Canada experienced significant productivity growth in nontraded goods since 1990, which is similar to (or even less than) the United States, such a change could have caused an appreciation of the real exchange rate because of asymmetries and may have been picked up by the trend term.

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up in the early 1990's. Recent literature suggests, moreover, that portfolio considerations (influencing desired holdings of US securities) are not important at least for commodity countries (Chaban, 2009; Ferreira Filipe, 2012). In particular, Chaban (2009) shows that under flexible commodity prices, international shocks are transmitted to Canada without affecting equity flows.

Appendix A: Variable Definitions and Data Sources

1. *RER = ER*PDUS/PDCA*

   ER = Nominal (C$/US$) exchange rate, quarterly average of daily rates (source: Bank of Canada).

   PDUS = U.S. GDP deflator (source: U.S. Department of Commerce, Bureau of Economic Analysis).

   PDCA = Canadian GDP deflator (source: Statistics Canada).

2. *LPM = LPMCA/LPMUS*

   LPMCA = Canadian labor productivity (output per hour) in manufacturing (source: Bank of Canada, calculations based on data series from Statistics Canada)

   LPMUS = US labor productivity (output per hour) in manufacturing (source: U.S. Department of Labor, Bureau of Labor Statistics)

3. *RLPM = LPM/(LPBCA/LPNUS)*

   LPBCA = Canadian labor productivity (output per hour) in manufacturing (source: Bank of Canada, calculations based on data series from Statistics Canada)

   LPBUS = U.S. labor productivity (output per hour) in manufacturing (source: U.S. Department of Labor, Bureau of Labor Statistics)

4. *RPC = PC/PDUS*

   PC = Commodity price index in US dollars (source: Bank of Canada, calculations based on data series from Statistics Canada).

5. *RPNE = PNE/PDUS*

   PNE = Non-energy commodity price index in US dollars (source: Bank of Canada, calculations based on data series from Statistics Canada).

Appendix B: Additional Model Equations

This appendix briefly discusses additional model equations implied by household and firm optimization, which are needed for the model solution. We first discuss the equations for the Canadian economy. Optimization by households leads to the following standard conditions:

$$w = \psi L^{\mu} / C^{-\theta}, \tag{B1}$$

$$Z_N = \chi_N Z(p_N)^{-\nu}, \quad Z_M = \chi_M Z(p_M)^{-\nu}, \quad Z_C = \chi_C Z(p_C)^{-\nu}, \tag{B2}$$

$$\begin{aligned} Z_{MA} &= \chi_{MA} Z_M (p_{MA} / p_M)^{-\tilde{\eta}}, \quad Z_{MR} = \chi_{MR} Z_M (p_{MR} / p_M)^{-\tilde{\eta}}, \\ Z_{MC} &= \chi_{MC} Z_{MA} (p_{MC} / p_{MA})^{-\eta}, \quad Z_{MU} = \chi_{MU} Z_{MA} (p_{MU} / p_{MA})^{-\eta}, \end{aligned} \tag{B3}$$

$$\begin{aligned} Z_N(i) &= Z_N (p_N(i) / p_N)^{-\sigma}, \quad Z_{MC}(j) = Z_{MC}(p_{MC}(j) / p_{MC})^{-\sigma}, \\ Z_{MU}(j^*) &= Z_{MU}(p_{MU}(j^*) / p_{MU})^{-\sigma}, \quad Z_{MR}(j^{**}) = Z_{MR}(p_{MR}(j^{**}) / p_{MR})^{-\sigma}. \end{aligned} \tag{B4}$$

The price indexes satisfy the following conditions implied by the minimization of unit costs of different consumption indexes:

$$1 = \left[\chi_N p_N^{1-\nu} + \chi_M p_M^{1-\nu} + \chi_C p_C^{1-\nu} \right]^{1/(1-\nu)}, \tag{B5}$$

$$p_M = \left[\chi_{MA} p_{MA}^{1-\tilde{\eta}} + \chi_{MR} p_{MR}^{1-\tilde{\eta}} \right]^{1/(1-\tilde{\eta})}, \quad p_{MA} = \left[\chi_{MC} p_{MC}^{1-\eta} + \chi_{MU} p_{MU}^{1-\eta} \right]^{1/(1-\eta)}, \tag{B6}$$

$$\begin{aligned} p_N &= \left[\int_0^1 p_N(i)^{1-\sigma} di \right]^{1/(1-\sigma)}, \quad p_{MC} = \left[\int_0^1 p_{MC}(j)^{1-\sigma} dj \right]^{1/(1-\sigma)} \\ p_{MU} &= \left[\int_0^1 p_{MU}(j^*)^{1-\sigma} dj^* \right]^{1/(1-\sigma)}, \quad p_{MR} = \left[\int_0^1 p_{MR}(j^{**})^{1-\sigma} dj^{**} \right]^{1/(1-\sigma)}. \end{aligned} \tag{B7}$$

Optimal price setting by home firms in the nontraded and manufacturing sectors implies that

$$p_N(i) = (\sigma / (\sigma - 1)) w / A_N, \quad p_{MC}(j) = (\sigma / (\sigma - 1)) w / A_M. \tag{B8}$$

Finally, profit maximization in the commodities sector leads to

$$w / p_C = \alpha Y_C / L_C. \tag{B9}$$

Analogous equations are assumed for the U.S. economy with an asterisk denoting U.S. variables and parameters. Analogous equations are also assumed for RW, but we assume that

p_C^{**}, p_{MR}^{**}, and Z_M^{**} are exogenous variables and use only the following relations for RW (representing the demand functions for Canadian and U.S. manufacturing products in the RW economy):

$$Z_{MC}^{**} = \chi_{MC}^{**} Z_{MA}^{**} (p_{MC}^{**} / p_{MA}^{**})^{-\eta}, \quad Z_{MU}^{**} = \chi_{MU}^{**} Z_{MA}^{**} (p_{MU}^{**} / p_{MA}^{**})^{-\eta}, \qquad \text{(B10)}$$

$$Z_{MA}^{**} = \chi_{MA}^{**} Z_M^{**} (p_{MA}^{**} / p_M^{**})^{-\tilde{\eta}}. \qquad \text{(B11)}$$

Appendix C: Model Calibration

Data for Calibration (shown in Table 5)

1. Share of the tradable sector excluding commodities (value added) in GDP for Canada *(SHMYCA*).

2. Share of the commodities sector (value added) in GDP for Canada (*SHCYCA)*.

3. Share of the tradable sector excluding commodities (value added) in GDP for the United States (*SHMYUS*).

4. Share of the commodities sector (value added) in GDP for the United States (*SHCYUS*).

5. The ratio of net exports of commodities to GDP for Canada (*SHCNXCA*).

6. The ratio of net exports of commodities to GDP for Canada (*HCSNXUS*).

7. The ratio of Canadian non-commodity imports from the United States to Canadian GDP (*SHUSIMCA*).

8. The ratio of U.S. non-commodity imports from Canada to U.S. GDP (*SHCAIMUS*).

Source: Bank of Canada, calculations based on value-added data (using North-American industrial classification) from Statistics Canada and US Department of Commerce, Bureau of Economic Analysis and data on trade flows from UN COMTRAD data base. The data for variables 1 through 4 represents average values for 1983-1986 for the first and average values for 2003-2006 for the second period. For variables 5 through 8, data for 1989 and 2006 are used for the two periods.[34]

Calibration of Model Parameters

Letting a bar denote initial steady-state values, we set Canadian real prices, $\bar{p}_N, \bar{p}_M, \bar{p}_C, \bar{p}_{MA}, \bar{p}_{MR}, \bar{p}_{MC}, \bar{p}_{MU}$ and the corresponding U.S. and RW prices equal to one, by

[34] The series used for estimating trade shares were not available before 1989.

normalization. The following model variables can then be matched with the above data as follows:[35]

$$\bar{Y}_M / \bar{C} = SHMYCA, \quad \bar{Y}_C / \bar{C}) = SHCYCA, \\ \bar{X}_C / \bar{C} = SHCNXCA, \quad \bar{Z}_{MU} / \bar{C} = SHUSIMCA, \tag{C1}$$

$$\bar{Y}_M^* / \bar{C}^* = SHMYUS, \quad \bar{Y}_C^* / \bar{C}^* = SHCYUS, \\ \bar{X}_C^* / \bar{C}^* = SHCNXUS, \quad \bar{Z}_{MC}^* / \bar{C}^* = SHCAIMUS. \tag{C2}$$

Using (C1), we determine $\chi_M, \chi_C, \chi_N, \chi_{MR,} \chi_{MA}, \chi_{MU}, \chi_{MC}$ by the following equations implied by the model:

$$\chi_M = \bar{Y}_M / \bar{C} + \bar{X}_C / \bar{C}, \quad \chi_C = \bar{Y}_C / \bar{C} - \bar{X}_C / \bar{C}, \quad \chi_N = 1 - \chi_M - \chi_C, \\ \chi_{MR} = (\bar{Z}_{MR} / \bar{C}) / \chi_M, \chi_{MA} = 1 - \chi_{MR}, \chi_{MU} = (\bar{Z}_{MU} / \bar{C}) / (\chi_{MA} \chi_M), \chi_{MC} = 1 - \chi_{MU}, \tag{C3}$$

Similarly, the corresponding U.S. parameters are determined by using (C2) and the U.S. counterpart to (C3).

To determine $\tilde{\chi}_{MC}^{**} = \chi_{MC}^{**} \chi_{MA}^{**} Z_M^{**}$, $\tilde{\chi}_{MC}^{**} = \chi_{MC}^{**} \chi_{MA}^{**} Z_M^{**}$, $\bar{Z}_{MR} / \bar{C}$, and $\bar{Z}_{MR}^* / \bar{C}^*$, we use the following relations derived from the model:[36]

$$\tilde{\chi}_{MC}^{**} = (\bar{Y}_M / \bar{C})\bar{C} - \chi_{MC} \chi_{MA} \chi_M \bar{C} - \chi_{MC}^* \chi_{MA}^* \chi_M^* \bar{C}^*, \\ \tilde{\chi}_{MU}^{**} = (\bar{Y}_M^* / \bar{C}^*)\bar{C}^* - \chi_{MU} \chi_{MA} \chi_M \bar{C} - \chi_{MU}^* \chi_{MA}^* \chi_M^* \bar{C}^*, \tag{C4}$$

$$\bar{Z}_{MR} / \bar{C} = \chi_{MC}^* \chi_{MA}^* \chi_M^* (\bar{C}^* / \bar{C}) + \tilde{\chi}_{MC}^{**} / \bar{C} + \bar{Y}_C / \bar{C} - \chi_C - \chi_{MU} \chi_{MA} \chi_M, \\ \bar{Z}_{MR}^* / \bar{C}^* = \chi_{MU} \chi_{MA} \chi_M (\bar{C} / \bar{C}^*) + \tilde{\chi}_{MU}^{**} / \bar{C}^* + \bar{Y}_C^* / \bar{C}^* - \chi_C^* - \chi_{MC}^* \chi_{MA}^* \chi_M^*, \tag{C5}$$

where we normalize $\bar{C} = 1$ and set $\bar{C}^*$ equal to 11.0 in the first and 11.5 in the second period (based on the ratio of U.S. to Canadian GDP in each period). Finally, given the assumed values of $\alpha, \sigma, \varepsilon$ and μ (see Table 5), N, L and ψ are determined by the following model based relations:

$$\bar{L}_C = \alpha(\bar{Y}_C / \bar{C})\bar{C}\sigma / (\sigma - 1), \quad N = [(\bar{Y}_C / \bar{C})\bar{C} / \bar{L}_C^{\alpha}]^{1/(1-\alpha)}, \\ L = L_C + (\bar{Y}_M / \bar{C})\bar{C} + (1 - \bar{Y}_M / \bar{C} - \bar{Y}_C / \bar{C})\bar{C}, \psi = [(\sigma - 1)(\varepsilon - 1) / \sigma\varepsilon] / L^{\mu}. \tag{C6}$$

[35] As commodities are assumed to be a homogeneous good, there is no intraindustry trade for this sector in the model. For calibration purposes, net exports of commodities in the data are identified with exports in the model if positive and imports if negative.

[36] The RW variable Z_M^{**} is treated as an exogenous variable. The shares, $\bar{Z}_{MR} / \bar{C}$ and $\bar{Z}_{MR}^* / \bar{C}^*$ are determined by model equations (rather than by data) to satisfy the balanced trade condition.

The corresponding U.S. relations are similarly determined using the U.S. counterpart of (C6).

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Figure 1. The Canadian Real Exchange Rate and the Canada-U.S. Productivity Ratio in Manufacturing

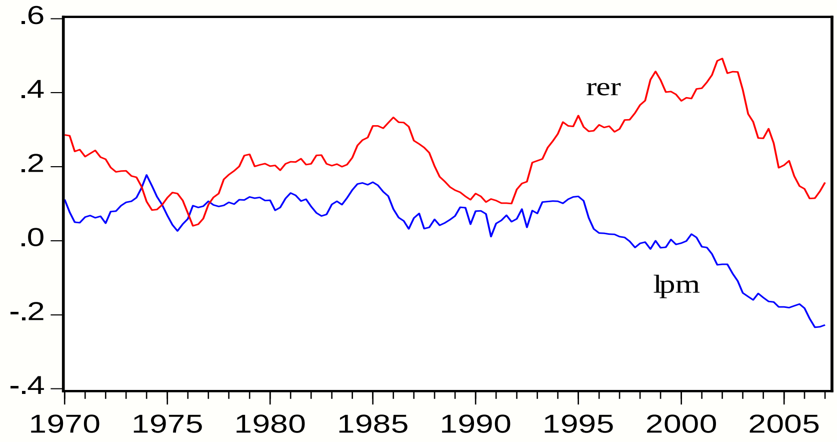


Figure 2. The Canadian Real Exchange Rate and the Adjusted Canada-U.S. Productivity Ratio in Manufacturing

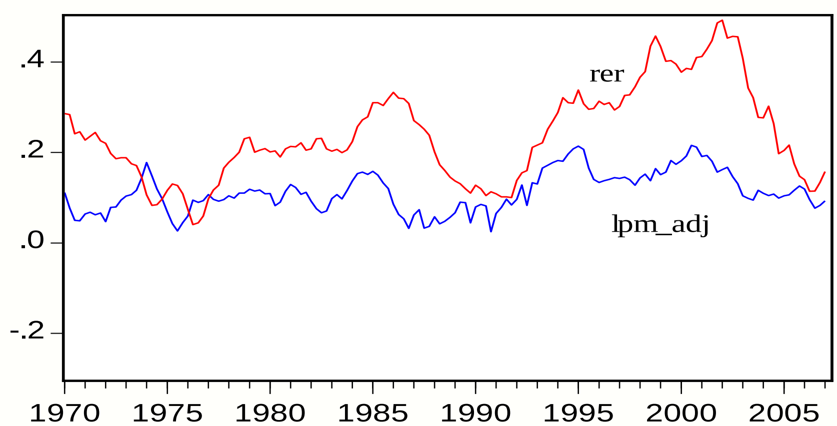


Figure 3. The Canadian Real Exchange Rate and the Canada-U.S. Productivity Ratio in Manufacturing Relative to the Business Sector

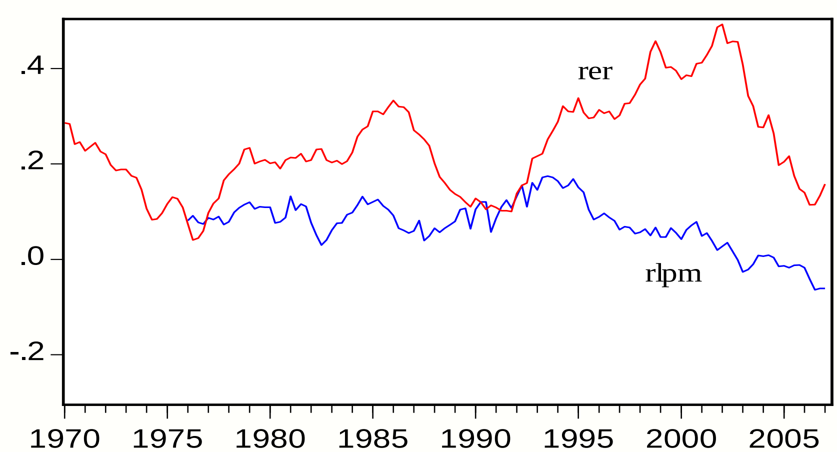


Figure 4. The Canadian Real Exchange Rate and the Real Commodity Price Index

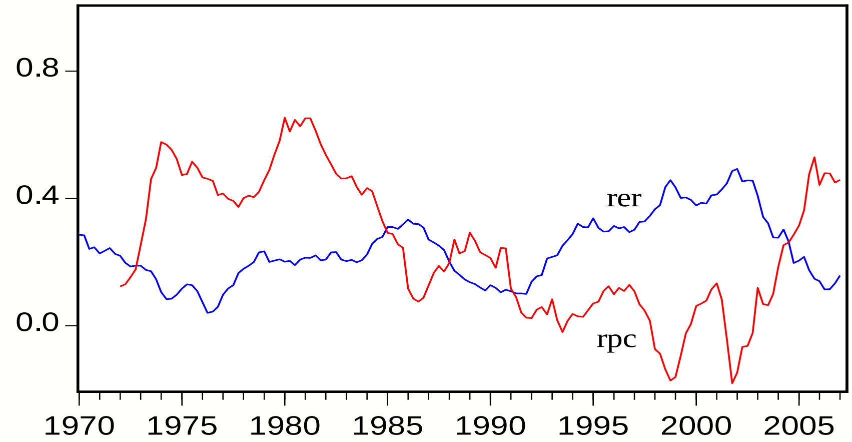


Figure 5. The Canadian Real Exchange Rate and the Real Commodity Price Index Excluding Energy

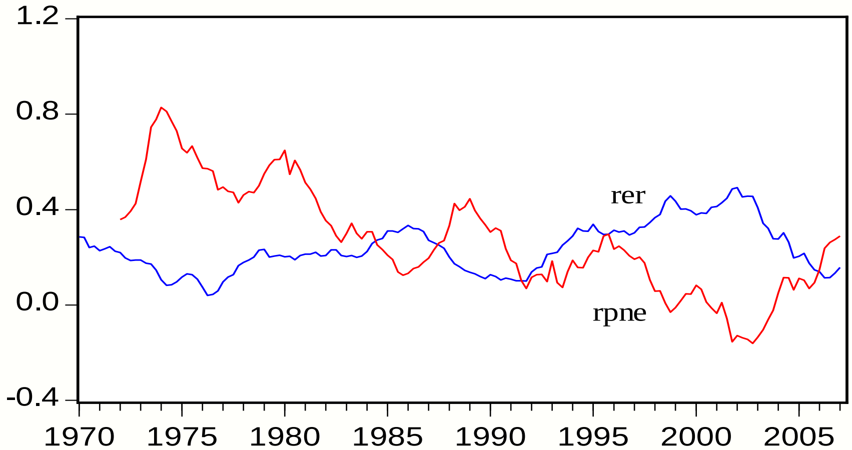
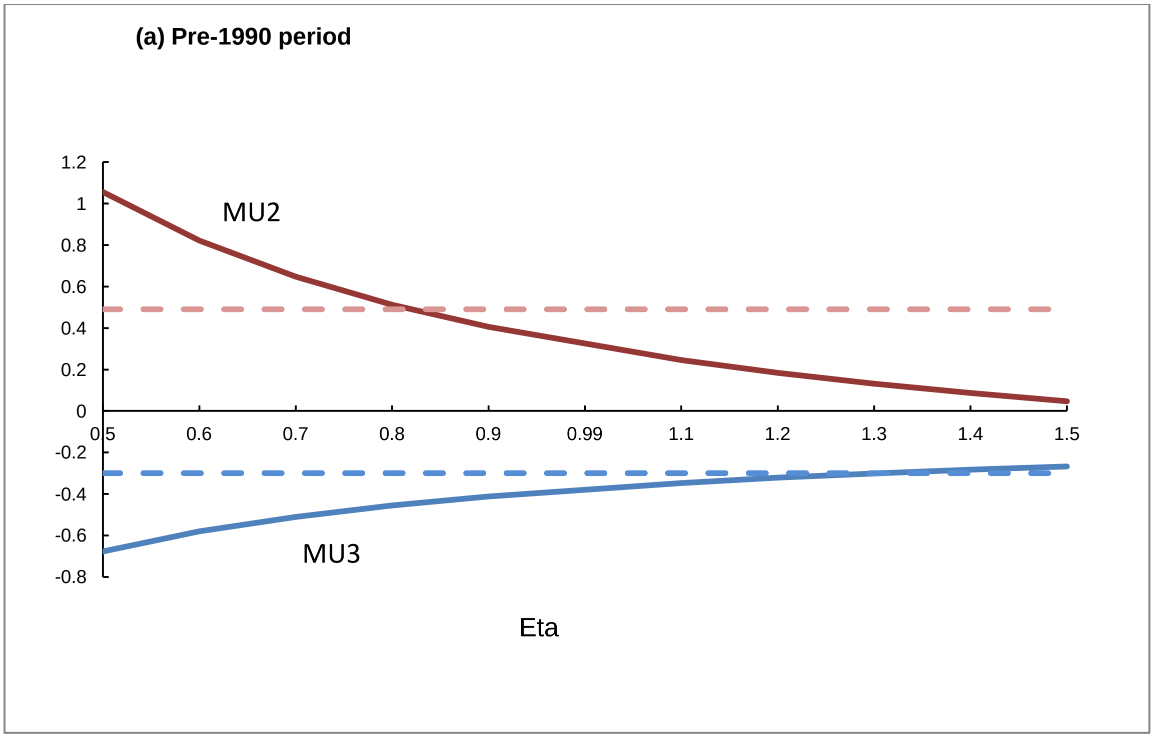
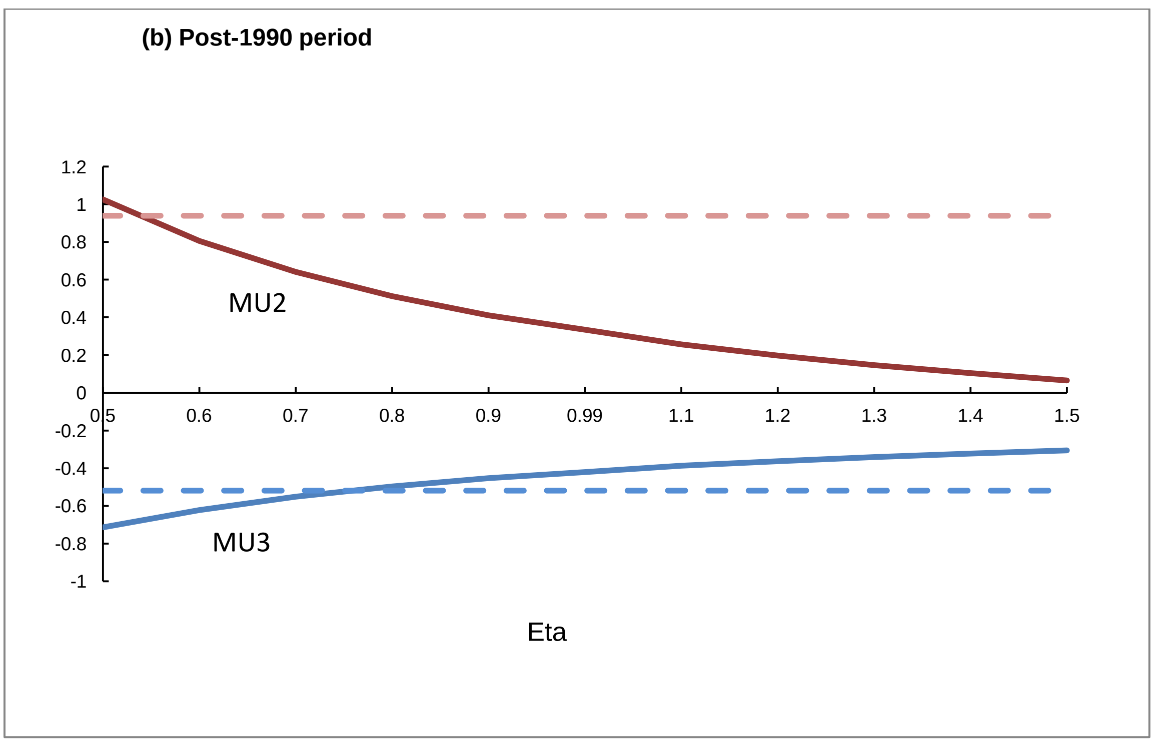


Figure 6. Real Exchange Rate Elasticities Related to the Substitution Elasticity





Note: MU2 and MU3 curves represent, respectively, the elasticities of the real exchange rate with respect to the productivity ratio and the real commodity price based on model simulations. Broken lines represent estimates of these elasticities for the two periods.

Table 1. Correlations Matrix for Different Series

	rer	lpm _adj	lpm	rlpm	rpc	rpne
rer	1.00					
lpm	-0.19	1.00				
lpm_adj	0.63	0.05	1.00			
rlpm	-0.13	0.88	0.27	1.00		
rpc	-0.63	0.18	-0.40	-0.05	1.00	
rpne	-0.68	0.54	-0.42	0.33	0.78	1.00

Table 2. Bounds Tests for the Level Relation

	F-Statistics ($\pi_1 = \pi_2 = \pi_3 = 0$)			
	$x_1 = lpm$ $x_2 = rpc$	$x_1 = lpm$ $x_2 = rpne$	$x_1 = rlpm$ $x_2 = rpc$	$x_1 = rlpm$ $x_2 = rpne$
All Observations				
With Trend	1.861 (3.228)	2.881 (4.699$^+$)	1.892 (2.670)	2.288 (3.086)
Without Trend	0.964 (2.082)	2.177 (4.053$^+$)	0.811 (2.572)	1.537 (3.397)
Pre-1990 Observations				
With Trend	2.770 (4.154)	6.441* (6.038*)	2.327	4.353 (6.361*)
Without Trend	3.272 (2.863)	6.951* (6.169*)	1.315	1.622 (5.021*)
Post-1990 Observations				
With Trend	7.047*	2.485	3.205	1.209
Without Trend	0.972 (2.112)	2.738	1.306 (2.083)	2.299

Note: The table shows F-statistics for regressions with lag order (n) chosen by AIC. If the lag order was different under SBC, the F-statistics for regressions based on this criterion is shown in brackets. The critical value bounds at the 5% level are 4.87 and 5.85 for regressions with trend, and 3.79 and 4.85 for regression without trend. An asterisk indicates a value above the upper bound and a plus sign a value within the bounds.

Table 3. Coefficients of Level Regressors

Period	Trend	Lag Order	π_1	π_2	π_3
Pre-1990	Yes	AIC	-0.172 (-3.347)	0.323* (4.030)	-0.059* (-2.622)
	Yes	SBC	-0.160$^+$ (-3.517)	0.234* (3.314)	-0.066* (-3.058)
	No	AIC	-0.174$^+$ (-3.450)	0.327* (4.348)	-0.057* (-3.097)
	No	SBC	-0.166* (-3.693)	0.248* (3.667)	0.057* (-3.310)
Post-1990	Yes	AIC/ SBC	-0.350* (-4.444)	0.471* (4.046)	-0.182* (-3.712)

Note: The table shows estimates of coefficients for level regressors in (1) [t-statistics is shown in brackets]. $x_2 = rpne$ in pre-1990 period, $x_2 = rpc$ in post-1990 period, $x_1 = lpm$ in both periods. An asterisk indicates significance at the 5% level. For π_1, the distribution of t- statistics is non-standard, and the critical value bounds at the 5% level are -3.41 and -3.95 for regressions with trend, and -2.86 and -3.53 for regression without trend. For this parameter, a plus sign indicates a value within the bounds.

Table 4. The Long-Run Effects

Period	Trend	μ_1	μ_2	μ_3	$\bar{R}^2$
Pre-1990	Yes	-0.000 (-0.141)	0.491* (3.017)	-0.308* (-7.137)	0.569
	No		0.494* (3.073)	-0.303* (-9.802)	0.575
Post-1990	Yes	0.008* (11.902)	0.943* (6.725)	-0.515* (-13.239)	0.908

Note: The table shows estimates of coefficients of regressors in (2) [t-statistics is shown in brackets]. $x_2 = rpne$ in pre-1990 period, $x_2 = rpc$ in post-1990 period, $x_1 = lpm$ in both periods. An asterisk indicates significance at the 5% level.

Table 5. Recent GDP and Trade Shares: Canada and the US

	Period 1		Period2	
	Canada	US	Canada	US
Proportion of GDP (%)				
Tradable Sector excluding commodities	38.0	43.3	40.0	43.0
Commodities Sector	6.7	4.2	9.0	3.0
Net exports of Commodities	4.9	-0.6	7.0	-1.4
Non-commodity imports from US/Canada	11.6	1.2	17.0	1.4

Source: See Appendix C

Table 6. Parameter Values in the Baseline Model

Data-based parameters and steady-state values

Consumption index parameters:	$\chi_N = 0.55, 0.51; \chi_M = 0.43, 0.47; \chi_C = 0.02, 0.02;$ $\chi_{MR} = 0.19, 0.19; \chi_{MA} = 0.81, 0.81;$ $\chi_{MC} = 0.67, 0.56; \chi_{MU} = 0.33, 0.44;$ ' $\chi_N^* = 0.53, 0.54; \chi_M^* = 0.43, 0.42; \chi_C^* = 0.04, 0.04;$ $\chi_{MR}^* = 0.17, 0.15; \chi_{MA}^* = 0.83, 0.85;$ $\chi_{MC}^* = 0.04, 0.04; \chi_{MU}^* = 0.96, 0.96;$ $\tilde{\chi}_{MC}^{**} = 0.02, 0.03; \tilde{\chi}_{MU}^{**} = 0.88, 0.89;$, $\psi = 0.88, 0.94, \psi^* = 4.7e-007, 3.5e-007$
Natural resource endowments:	$N = 0.11, 0.15;\ N^* = 0.78, 0.58$
Relative consumption:	$\bar{C}^* / \bar{C} = 11.0, 11.5$
Assumed Parameters	
Utility and technology parameters	$\theta = 2$ (1-4), $\phi = 4$ (1-4), α = 0.4 (.33-.66)
Substitution elasticities	$\sigma = 8$, $\nu = 0.99$, $\tilde{\eta} = 0.99\ (0.5-2.0)$, $\eta = 0.99\ (0.5-2.0)$

Note:For data-based parameters, the first and second numbers represent calibrated values for the pre-1990 and post-1990 periods, respectively. For the assumed parameters, values in brackets indicates the range of variation explored in the sensitivity analysis

Table 7. Model Predictions

η	Real Exchange Rate	Percentage Response of Balassa-Samuelson Component	Terms of Trade Component	Commodity Component
(a) Effect of a 1% increase in A_M				
0.5	1.120	-0.505	1.764	-0.158
0.75	0.640	-0.506	1.248	-0.112
≈ 1	0.378	-0.506	0.966	-0.086
1.25	0.213	-0.506	0.786	-0.070
1.5	0.100	-0.506	0.663	-0.059
1.75	0.018	-0.507	0.574	-0.051
2.0	-0.045	-0.507	0.505	-0.045
(b) Effect of a 1% increase in A_M^*				
0.5	-0.925	0.536	-1.577	0.100
0.75	-0.503	0.536	-1.106	0.059
≈ 1	-0.276	0.536	0.853	0.037
1.25	-0.131	0.537	-0.693	0.023
1.5	-0.033	0.537	-0.584	0.014
1.75	0.039	0.537	-0.505	0.007
2.0	0.094	0.537	-0.445	0.001
(c) Effect of a 1% increase in A_N				
0.5	0.393	0.508	-0.128	0.011
0.75	0.427	0.508	-0.090	0.008
≈ 1	0.446	0.508	-0.070	0.006
1.25	0.458	0.508	-0.057	0.005
1.5	0.467	0.508	-0.048	0.004
1.75	0.473	0.508	-0.041	0.004
2.0	0.477	0.508	-0.036	0.003

Table 6. Model Predictions (Continued)

η	Real Exchange Rate	Percentage Response of Balassa-Samuelson Component	Terms of Trade Component	Commodity Component
(d) Effect of a 1% increase in A_N^*				
0.5	-0.056	-0.538	0.492	-0.011
0.75	-0.189	-0.538	0.347	0.002
≈ 1	-0.261	-0.538	0.268	0.009
1.25	-0.307	-0.538	0.218	0.013
1.5	-0.338	-0.538	0.184	0.016
1.75	-0.360	-0.538	0.159	0.018
2.0	-0.378	-0.538	0.140	0.020
(e) Effect of a 1% increase in p_C^*				
0.5	-0.714	0.000	-0.723	0.004
0.75	-0.522	0.000	-0.510	-0.014
≈ 1	-0.417	0.000	-0.394	-0.025
1.25	-0.351	0.000	-0.321	-0.031
1.5	-0.306	0.000	-0.271	-0.036
1.75	-0.273	0.000	-0.234	-0.039
2.0	-0.247	0.000	-0.206	-0.041

Note: The sum of the three components does not exactly equal the real exchange rate change because of the presence of a small residual.