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Real-time Optimal Monetary Policy with Undistinguishable Model Parameters and Shock Processes Uncertainty

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Abstract

This paper studies optimal real time monetary policy when the central bank takes the volatility of the output gap and inflation as proxy of the undistinguishable uncertainty on the exogenous disturbances and the parameters of its model. The paper shows that when the uncertainty surrounding a specific state variable increases, the optimal policy response to that variable should increase too while the optimal response to the remaining state variables should attenuate or be unaffected. In this way the central bank moves preemptively to reduce the risk of large deviations of the economy from the steady state that would deteriorate the distribution forecasts. When an empirical test is carried out on the US economy the model predictions tend to be consistent with the data.

JEL: C51; C52; E52; E58

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1. Introduction

A pervasive feature of monetary policymaking is the uncertainty about the state of the economy, the economy's structure and the inferences that the public will draw from policy actions or economic developments (Bernanke 2007). Policymakers have long recognized that coping with these forms of uncertainty poses complicate issues to real-time monetary decisions. Uncertainty on the output gap, for example, corresponds to uncertainty on where the economy is located with respect to the business cycle. Thus real-time decisions, which are necessarily based on output gap estimates, can turn out to be wrong from an ex-post perspective. This problem is well documented by Orphanides and van Norden (2002) who show that the reliability of the output gap measure is quite low. Uncertainty on the structure of the economy is another key cause of policy errors. This uncertainty stems from the limited knowledge of the critical forces that govern the economic system at any point in time. Clearly, the more these forces are missed in modeling the economy, the larger the policy errors. The problem raised by the uncertainty on the state and structure of the economy is further aggravated by the difficulty to relate observable changes in the volatility of key state variables --as inflation and output gap-- to these sources of uncertainty. As noted by Bernanke (2007), "Apart from issues of measurement, policymakers face enormous challenges in determining the sources of variation in the data."

This paper studies the optimal monetary policy under three realistic assumptions. The first is that in real-time policymakers do not know with certainty the exogenous disturbances to the economy and the parameters of their model. Furthermore, they are unable to distinguish the impact of these sources of uncertainty on the inflation and output gap processes. Under these assumptions some natural questions arise. Given the importance that modern monetary policy attributes to the distribution forecast of inflation and output gap for policy actions, should the difficulty to tell in real time the impact of the various sources of uncertainty on these variables be considered by policymakers? And to what extent, if any, and how perceived changes in the level of undistinguishable uncertainty should be considered? Given the current economic conditions, these are timely questions. The US, for instance, witnessed average real-time output gap volatility rising from 2.1% per annum over the 1997-2006 period to 2.6% per annum afterwards, and average GDP price inflation volatility rising from 0.3% per annum in 1997-2006 to 0.5% per annum afterwards.

To address these questions, we introduce a final assumption. We let policymakers use their judgment to form an opinion on the level of volatility of the inflation and output gap processes and take this volatility as proxy for the level of undistinguishable uncertainty in the exogenous disturbances and the model parameters.

These assumptions get reflected in a stylized two-step representation of the central bank decision process. First, central banks estimate at a certain time frequency the state and model of the economy being aware that the estimates are surrounded by uncertainty. Second, at a higher frequency, policymakers assess the amount of uncertainty of the estimates using as

proxies the volatility of the inflation and output gap processes and take well-informed policy decisions.

Methodologically, the analysis is performed by means of a New-Keynesian model that conveniently accounts for undistinguishable uncertainty in the state and structure of the economy. We estimate the model and use the Markov jump linear quadratic approach developed by Svensson and Williams (2007) to study how optimal monetary policy responds to changes in the uncertainty surrounding the output gap and inflation as proxies of uncertainty about the state and structure of the economy.

Within this Markov jump linear quadratic framework, the paper shows that when the uncertainty surrounding a state variable increases, the optimal policy response to that variable should increase too, while the optimal response to the remaining state variables should attenuate or be unaffected. This result matters for policy in that when policymakers have real-time limited information on the sources of uncertainty, nonetheless perceive shifts in the relative volatility of inflation and output gap, they may exploit this information to fine-tune the policy response to the state variables.

The remaining of the paper is structured as follows. Section 2 introduces the theoretical model. In section 3 we estimate the aggregate demand and supply, (henceforth AD and AS) using US data. In section 4 we use the estimates of the AD and AS along with a standard calibration for the central bank preferences to find the optimal monetary policy when central bank decisions consider the volatility level on the inflation and output gap process. Our theoretical predictions are then tested on US monetary policy data. Section 5 provides some robustness analysis and Section 6 summarizes our findings and offers some conclusions.

2. Theoretical model

The behavior of the private sector is captured by a New-Keynesian model with realistic monetary policy transmission lags. The model can be derived from microfoundations as shown in Flamini (2007) and is similar in spirit to Boivin and Giannoni (2006). The AD and AS are respectively described by the following relations

$$y_{t+1} = [\alpha_y y_t + (1 - \alpha_y) y_{t+2|t} - \alpha_r (i_{t+1|t} - \pi_{t+2|t})] \phi_{t+1}^y \quad (1)$$

$$\pi_{t+2} = [\beta_\pi \pi_{t+1} + (1 - \beta_\pi) \pi_{t+3|t} + \beta_y y_{t+2|t}] \phi_{t+2}^\pi \quad (2)$$

where for any variable x , the expression $x_{t+\tau|t}$ denotes the rational expectation of x in period $t + \tau$ conditional on the information available in period t , all the variables are in terms

of log deviations from a constant steady state, and y_t , i_t , π_t denote output gap, nominal short term interest rate, and inflation rate, respectively.

The presence of the factors ϕ_t^y and ϕ_t^π is an innovation with respect to the previous literature. ϕ_t^y and ϕ_t^π are random variables capturing the uncertainty on the state and structure of the economy assuming that in real time it is impossible for the central bank to distinguish the impact of these sources of uncertainty on the volatility of the output gap and inflation processes. This assumption is motivated by the fact that in real time policymakers face difficulties in observing specific disturbances hitting the economy and have limited knowledge on their properties too. Furthermore, the true model of the economy is not known with certainty so that even if there were full knowledge of the disturbances, difficulties would arise in nowcasting and forecasting their impact on the economy.

To model the assumption of undistinguishable uncertainties, we let ϕ_t^y and ϕ_t^π have a symmetric distribution, expected value equal to one and variance proportional to the volatility of the inflation and output gap processes as we will discuss below. At any point in time, a value of the factors different from one can capture exogenous disturbances hitting the economy and/or errors in the central bank estimates of the parameters. Regarding the former, an example can be a preference or a technology shock which occurs between period t and $t+1$ but is not in the policymakers' information set at $t+1$. On the other hand, errors in the estimates can be caused by a time varying nature of some structural parameters, which is missed by the policymakers¹. In this way, ϕ_t^y and ϕ_t^π record general model uncertainty, that is, uncertainty that stems from the structure of the model and that is impossible to attribute to specific parameters in real time.

While the coefficients of the AS and AD can be estimated at a fixed frequency, the information flow accessible to central bank and relevant for policy decisions is continuous and sometimes not apt for a direct use in the estimation process. Nevertheless, this information flow can be useful for policymakers to form an opinion on the variance of ϕ_t^y and ϕ_t^π . To model this idea we assume that policymakers use all the available information and their judgment to form an opinion on the volatility of the inflation and output gap processes. Then, they consider this volatility as proxy for the variance of ϕ_t^y and ϕ_t^π in the determination of the optimal monetary policy.

Within this framework, the central bank optimization problem consists of finding the interest rate path that maximizes its preferences subject to the AD and AS and to the opinion on the

¹ See Rubio-Ramirez and Villaverde (2007) for empirical evidence in favour of parameter drifting in DSGE models and the literature therein for empirical evidence on time varying parameters in dynamic models.

level of uncertainty proxied by the volatility of the inflation and output gap processes. Turning to central bank preferences, they are described by the following standard loss function

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\mu \pi_{t+\tau}^2 + \lambda y_{t+\tau}^2 + \nu (i_{t+\tau} - i_{t+\tau-1})^2 \right],$$

where μ , λ and ν are weights that express the preferences of the central bank for the inflation and output gap stabilization targets, and the instrument smoothing target, respectively.

2.1. Optimal monetary policy with Non-certainty Equivalence

In our analysis we let the central bank consider undistinguishable uncertainty via multiplicative shocks. The use of a multiplicative rather than additive shock provides a convenient way of investigating optimal monetary policy in presence of uncertainty. It is well known, in fact, that the linear-quadratic setup features certainty equivalence, which implies that optimal monetary policy does not depend on additive uncertainty. Instead, considering ϕ_t^y and ϕ_t^{π} as multiplicative shocks we can relax the certainty equivalence assumption and study how uncertainty affects the optimal policy. We follow this route by adopting a Markov jump-linear-quadratic setup which is implemented with a discrete support for ϕ_t^y and ϕ_t^{π} and assuming that in any period t these shocks can take n_j different values corresponding to n_j exogenous modes drawn by nature and indexed by $j_t \in \{1, 2, \dots, n\}$. Thus ϕ_t^y and ϕ_t^{π} correspond to $\phi_{j_t}^y$ and $\phi_{j_t}^{\pi}$, respectively.

It is worth noting that by relaxing the certainty equivalence assumption, this framework allows introducing an important realistic aspect of monetary policy, namely the policymakers' focus on the *distribution forecasts* of the target variables rather than the mean forecasts.

We first consider the case in which the central bank has a flat prior belief on the distribution of the multiplicative shock and then in section 5 we explore an alternative assumption to check for the robustness of the results. The central bank flat prior is modeled by letting the modes follow a Markov process with constant transition probabilities given by

$$P_{jk} \equiv \Pr\{j_{t+1} = k | j_t = j\} = \frac{1}{n}, j, k \in \{1, 2, \dots, n\}.$$

and the associated stationary distribution for the shock $\phi_{j_t}^h$ for $h = y, \pi$ is given by

$\phi_{j_t}^h$	$(1 - 2\delta_{\phi^h})$	$(1 - \delta_{\phi^h})$	1	$(1 + \delta_{\phi^h})$	$(1 + 2\delta_{\phi^h})$
$\Pr(\phi_{j_t}^h)$	0.2	0.2	0.2	0.2	0.2

with the parameter δ_{ϕ^h} allowing to change the variance of the distribution.

As to the central bank knowledge before choosing the instrument-plan $\{i_{t+\tau|t}\}_{\tau=0}^{\infty}$ at the beginning of period t , the information set consists of the transition matrix P along with its stationary distribution, and the n_j different values that each of the shocks can take in any mode. Therefore, the central bank cannot observe the modes yet takes into account the volatility of the shocks to find the optimal monetary policy

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\rho_{\pi} \pi_{t+1|t} + \rho_y y_t) \quad (3)$$

conditional on the distributions of the multiplicative shocks $\phi_{j_t}^y$ and $\phi_{j_t}^{\pi}$.

3. Model estimation

To choose the coefficients of the AD and AS equations (1) and (2), we jointly estimate these and the monetary policy equation in (3) using US data for the period 1969Q4-2009Q2. We use the Effective Federal Funds rate as the nominal interest rate. Inflation is the annual proportional change in the GDP price index. For inflation, we (i) use real-time median forecasts of GDP price inflation obtained from the Survey of Professional Forecasters (SPF) database maintained by the Federal Reserve Bank of Philadelphia, and (ii) replace GDP price inflation forecasts with their actual values. The output gap is proxied by (i) real-time output detrended by a quadratic trend, (ii) real-time output detrended by a Hodrick-Prescott (1997) trend, and (iii) final output detrended by the Congressional Budget Office (CBO) measure of potential GDP. To construct the real-time output gap data, we estimate for each quarter both a quadratic and a Hodrick-Prescott trend from 1965Q3 to that quarter using real-time output data available in that quarter. The output gap for the quarter is the end-of-sample residual from that quadratic trend and Hodrick-Prescott (HP) trend regressions, respectively. This means, for example, that in constructing the output gap data for the period 1969Q4 to 2009Q2 (159 quarters), we re-estimated 159 regressions.² Figure 1 plots the federal funds rate and the different measures of inflation and the output gap. The federal funds rate is higher during the 1970s and early 1980s

² Preliminary unit root analysis (results are available on request) indicated that the output gap is stationary whereas the order of integration of the interest rate and inflation is more ambiguous; we assume that all variables are stationary (see e.g. Clarida et al, 2000, for a discussion of similar issues).

and reaches its lowest level following the 2007-2009 financial crisis. Inflation is higher during the 1970s and early 1980s; it also rises towards the end of our sample. Compared to the other output gap measures, real-time output detrended by a quadratic trend filter suggests a more severe downturn in the mid-1970s and following the 2007-2009 financial crisis.

Using GMM (with lagged values as instruments), column (i) of Table 1 reports real-time estimates with output detrended by a quadratic trend and with SPF inflation forecasts; column (ii) reports real-time estimates with output detrended by a Hodrick-Prescott trend and with SPF inflation forecasts, and column (iii) reports final estimates with output detrended by the CBO measure of potential output and with inflation forecasts replaced by actual inflation values. We estimate that the weight on inflation ranges from 1.53 to 2.12; the weight on the output gap ranges from 1.07 to 1.94 and the persistence parameter ranges from 0.83 to 0.90. Our estimates indicate a more aggressive response of policymakers to inflation and the output gap using real-time as opposed to final data. The inflation estimates are in line with other results in the literature (e.g. Judd and Rudebusch, 1998, Clarida et al, 2000, Castelnuovo, 2003) and satisfy the Taylor (1993) principle that excessive inflation should trigger increases in the real interest rate.³

Our estimates suggest that the weight on past output in the aggregate demand equation ranges between 0.50 and 0.54 whereas the weight on past inflation in the aggregate supply equation ranges between 0.51 and 0.91. The estimates of the AS imply that backward looking inflation effects are more important than forward looking ones. This is in line with Rudd and Whelan (2005) and Linde (2005) but contradicts e.g. Gali and Gertler (1999), Gali et al (2005) and Kim and Kim (2008). The model with final data fits monetary policy and the AD and AS equations best as it delivers the lowest regression standard error and the highest adjusted R^2 .

4. Optimal monetary policy response to output gap and inflation uncertainty

This section starts by presenting the theoretical predictions of the model considering the separate and joint impact of inflation and output gap volatility on the optimal monetary policy. It then tests the model empirically using US data.

4.1. Theoretical predictions of the model

We choose the parameter values for the AD and AS equations based on the final-time estimates reported in column (iii) of Table 1 (which fit the data best). We then compute the

³ Gerberding et al (2005) and Gerdesmeier and Roffia (2005) find that the use of real-time output data as opposed to final output data increases the output effect in the Taylor rule for the Bundesbank and the EU area, respectively. A possible explanation is that the magnitude of the response using revised data could suffer from downward bias owing to the errors-in-variables problem.

optimal monetary policy response to changes in the volatility of the inflation and output gap processes under the assumption, common in the literature, that the central bank pursues flexible inflation targeting and has small preferences on interest rate smoothing (that is, $\mu = 1$, $\lambda = 0.1$ and $\nu = 0.2$)⁴.

Figure 2 reports the coefficients of the optimal monetary policy for increasing values of the standard deviations of $\phi_{j_t}^y$ and $\phi_{j_t}^\pi$, i.e. σ_y and σ_π . The range of the standard deviations has been chosen in order to obtain realistic measure of uncertainty in current output and inflation. As to the former, we assumed that the upper bound of the standard deviation of the shock to the output gap is 0.4. It follows that, in any period t , due to the presence of uncertainty captured by the factor ϕ_t^y , actual output gap growth may differ from the one-period ahead forecast of, at the most, 56 percentage points of its value⁵. For example, if the expected value of the output gap growth based on the period t information set is 1%, then its actual value in period $t+1$ may shift at the most to 0.43% or 1.56%. This range is in line with the low reliability of output gap estimates in real time discussed for example by Orphanides and van Norden (2002) who find that ex-post revisions of the real-time output gap estimates can be of the same order of magnitude as the output gap itself. Turning to inflation, we let $\sigma_\pi \in [0, 0.175]$ so that in any period t actual inflation may differ from the one-period ahead forecast of, at the most, 25 percentage points of its value. This level of the variability of inflation is in line with the central bank experience.

Describing our findings, first Figure 2 (a, b) shows that the optimal response to a state variable increases in the level of uncertainty surrounding that variable. Since the admissible range of output gap uncertainty is more than two times as large as the one of inflation uncertainty, this increase is remarkably large for the optimal response to the output gap in presence of output gap uncertainty.

To gain the intuition for this result is useful to recall that relaxing the certainty equivalence assumption the central bank objective is to have the distribution forecasts of inflation and the output gap that “look good”. In this model, the quality of the distribution forecasts, measured for instance by their volatility, is negatively affected by two factors: the perceived volatility in the inflation and output gap processes and the distance of the economy from its long-run equilibrium. While the role played by the former is evident, to see the role played by the latter consider that due to the multiplicative nature of the shock capturing the volatility in the inflation and output gap processes, the further away is the economy from the long run equilibrium, the

⁴ We focus on flexible inflation targeting as, in practice, no inflation targeting central bank pursues strict inflation targeting.

⁵ Given the uniform distribution for ϕ_j^y reported before, when $\sigma_y \in [0, 0.4]$ it follows that $\phi_j^y \in [0.43, 1.56]$.

larger is the impact of the shock for a given level of its volatility. Now, when there is an exogenous increase in the volatility of the shocks, the length of the potential deviations of the economy from its steady state increases and may combine with the second factor in a vicious cycle that quickly deteriorates the distribution forecasts. To take this contingency into account monetary policy needs to move preemptively. It thus gets more reactive in presence of an increase in the volatility of the inflation and output gap.

Figure 2 also shows that the optimal response to inflation is inversely related to output gap uncertainty, whereas the optimal response to the output gap tends to be unrelated to inflation uncertainty; see panels (a) and (b) respectively. This symmetry breaking occurs as output gap uncertainty introduces a trade-off between the quality of the distribution forecasts for inflation and the output gap. In contrast, the presence of inflation uncertainty does not introduce a similar trade-off. To see the mechanism at work, let us consider the behavior of the central bank in two different scenarios. In the first one, inflation deviates from its long-run value, i.e. $\pi \neq 0$, in presence of output gap uncertainty; in the second one, output gap deviates from its long-run value, i.e. $y \neq 0$, in presence of inflation uncertainty. In the first case, the policymakers' attempt to stabilize inflation requires a deviation of the output gap from its long-run value. This implies a loss for the central bank. However, the presence of output gap uncertainty now adds a further loss as it makes potentially harder to take the output gap back to the equilibrium in the subsequent periods. Thus, policymakers will trade-off slower inflation stabilization for smaller deterioration of the output gap distribution forecast. This gets reflected in an attenuation of the policy response to inflation. Conversely, when the output gap deviates from its long-run value and the central bank faces inflation uncertainty, the attempt to stabilize the output gap will not require a deviation of inflation from its long-run value. On the contrary, by stabilizing the output gap, policymakers prevent output gap deviations from perturbing inflation via the Phillips curve. This explains why the policy response to the output gap is not affected by inflation uncertainty.

The result that accounting for uncertainty can lead to a more aggressive policy has precedents in the literature. Söderström (2002), with a backward-looking model found that the policy responsiveness to inflation increases with the uncertainty on the persistence of this variable. Using a microfounded forward-looking model, Kimura and Kurozumi (2007) confirmed this result. The current paper, abstracting from specific sources of uncertainty considered in previous contributions, shows that a more aggressive policy response to a state variable should occur when the *overall uncertainty* surrounding that variable increases.

It is worth noting that, in real time, information on changes in specific sources of uncertainty seems more difficult to gather than general information on variations in the uncertainty of the output gap and inflation processes. Thus, the previous result matters for policy design in that it unveils the utility of limited information on the general uncertainty in the output gap and inflation processes when central banks cannot rely on detailed information on the uncertainty sources.

4.1.1. Optimal policy responses with both inflation and output gap uncertainty

So far we have considered the case in which the uncertainty surrounding one process (either the output gap or inflation) increases as the uncertainty surrounding the other process is constant. While investigating this scenario has been instructive to see the specific contributions of σ_y and σ_π on the optimal policy, a more realistic one occurs when both processes are surrounded by uncertainty and, in real-time, policymakers perceive shifts in the volatility of one process relative to the volatility of the other process.

To study this case, we introduce an output gap relative volatility ratio and investigate how the optimal policy reacts to changes in this ratio. We define the output gap relative uncertainty ratio as $\sigma_y / (\sigma_y + \sigma_\pi)$, and impose the following inverse relation between σ_y and σ_π

$$\sigma_y = \bar{\sigma}_y - \frac{\bar{\sigma}_y}{\bar{\sigma}_\pi} \sigma_\pi,$$

where $\bar{\sigma}_y = 0.4$ and $\bar{\sigma}_\pi = 0.175$ refer to the upper bound for the volatility of the output gap and inflation processes, respectively. Thus

$$\frac{\sigma_y}{\sigma_y + \sigma_\pi} = \frac{1}{\left(1 - \frac{\bar{\sigma}_y - \bar{\sigma}_\pi}{\bar{\sigma}_y \bar{\sigma}_\pi} \sigma_\pi\right)} - \frac{1}{\bar{\sigma}_\pi} \frac{1}{\left(1 - \frac{\bar{\sigma}_y - \bar{\sigma}_\pi}{\bar{\sigma}_y \bar{\sigma}_\pi} \sigma_\pi\right)} \sigma_\pi.$$

It is worth noting that the ratio is equal to zero if and only if there is maximum uncertainty in the inflation processes and no uncertainty in the output gap process; on the other hand, it is equal to one if and only if there is maximum uncertainty in the output gap process and no uncertainty in the inflation process, that is

$$\frac{\sigma_y}{\sigma_y + \sigma_\pi} \begin{cases} = 0 & \text{iff } \sigma_\pi = \bar{\sigma}_\pi, \sigma_y = 0 \\ = 1 & \text{iff } \sigma_\pi = 0, \sigma_y = \bar{\sigma}_y \end{cases}$$

Figure 3 plots the optimal policy coefficients for the output gap and inflation versus the output gap relative volatility ratio. Figure 3 shows that when the uncertainty in the output gap is inversely related to the uncertainty in inflation, movements in the volatilities of y and π should be associated with changes in the same direction of the policy response to these variables. Furthermore, by equally splitting the y and π volatility ranges we notice that with more uncertainty in the inflation process than the output gap process (region A), the optimal monetary policy tends to react more to inflation than to the output gap in a fashion similar to what predicted by the Taylor rule. Instead, in the opposite case (region B), the policy response to the

output gap exceeds the one to inflation. Finally, in region B, the spread between the policy responses is remarkably larger than in region A and the response to the less volatile state variable tends to become negligible, while it remains important in region A. This latter response is in line with the breaking symmetry effect previously described, which is caused by the different impact of the y and π uncertainty on the optimal policy.

Summing up, these findings show that also in the more realistic case in which policymakers face uncertainty in both processes, there is (i) a positive relation between the volatility of a state variable and reactivity of the associated policy response, and (ii) an asymmetric policy behavior associated with movements in the relative uncertainty.

4.2. Empirical application on US monetary policy

The theoretical predictions reported in Figure 3 are now tested on US monetary policy over the 1969Q4-2009Q2 period. We construct the uncertainty measures σ_{yt} , $\sigma_{\pi t}$, and the relative

uncertainty ratio $\left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t$, respectively, by taking the 8-quarter moving standard deviation of

inflation and the output gap (results using a 16-quarter moving standard deviation are qualitatively similar). Our measures of uncertainty are reported in Figure 4. Inflation uncertainty is greatest in the 1970s, and towards the end of our sample. Output gap uncertainty declines throughout the 1980s with resurgences in the early 1990s, after the 9/11 terrorist attacks, and following the financial crisis at the end of our sample.

To allow for asymmetric uncertainty effects, we express the monetary policy rule as:

$$\begin{aligned}
i_t = & \rho_i i_{t-1} + (1 - \rho_i) I_t^{\sigma_y / (\sigma_y + \sigma_\pi)} \left[\rho_{\pi,0}^+ + \rho_{\pi,1}^+ \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t \right] \pi_{t+1|t} \\
& + (1 - \rho_i) I_t^{\sigma_y / (\sigma_y + \sigma_\pi)} \left[\rho_{y,0}^+ + \rho_{y,1}^+ \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t \right] y_t \\
& + (1 - \rho_i) [1 - I_t^{\sigma_y / (\sigma_y + \sigma_\pi)}] \left[\rho_{\pi,0}^- + \rho_{\pi,1}^- \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t \right] \pi_{t+1|t} \\
& + (1 - \rho_i) [1 - I_t^{\sigma_y / (\sigma_y + \sigma_\pi)}] \left[\rho_{y,0}^- + \rho_{y,1}^- \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t \right] y_t
\end{aligned} \tag{4}$$

where $I_t^{\sigma_y / (\sigma_y + \sigma_\pi)}$ is the indicator function:

$$I_t^{\sigma_y/(\sigma_y+\sigma_\pi)} = \begin{cases} 1, & \text{if } \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t \geq \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_{t-1} \\ 0, & \text{if } \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t < \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_{t-1} \end{cases} \quad (5)$$

Model (4) using the indicator function (5) differs from the linear Taylor rule model (3) in that it allows for a regime-switching relationship between the interest rate, inflation and the output gap depending on whether there is higher relative uncertainty in the output gap process (in which case increasing values of $\left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t$ are observed) against higher relative uncertainty in the inflation process (in which case decreasing values of $\left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t$ are observed). The response to inflation switches from $\rho^+_{\pi,0} + \rho^+_{\pi,1} \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t$ when there is higher relative uncertainty in the output gap process to $\rho^-_{\pi,0} + \rho^-_{\pi,1} \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t$ when there is higher relative uncertainty in the inflation process. Similarly, the response to the output gap switches from $\rho^+_{y,0} + \rho^+_{y,1} \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t$ to $\rho^-_{y,0} + \rho^-_{y,1} \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t$. We expect higher relative uncertainty in the output gap process to raise the response of monetary policy to the output gap ($\rho^+_{y,1} > 0$) and lower the response of monetary policy to inflation ($\rho^+_{\pi,1} < 0$). On the other hand, higher relative uncertainty in the inflation process should lower the response of monetary policy to the output gap ($\rho^-_{y,1} < 0$) and increase the response to inflation ($\rho^-_{\pi,1} > 0$). The model in (4) simplifies to a model with symmetric uncertainty effects if $\rho^+_{y,0} = \rho^-_{y,0}$, $\rho^+_{y,1} = -\rho^-_{y,1}$, $\rho^+_{\pi,0} = \rho^-_{\pi,0}$, and $\rho^+_{\pi,1} = -\rho^-_{\pi,1}$.

The model in (4) is estimated jointly with the AD-AS equations (1) and (2). To save space, we only report estimates of (4) in Table 2 (estimates of (1)-(2) are very similar to those reported in Table 1 and are available on request). We estimate that $\rho^+_{y,1} > 0$, $\rho^+_{\pi,1} < 0$, $\rho^-_{y,1} < 0$, and $\rho^-_{\pi,1} > 0$. These estimates are statistically significant, suggesting that higher (lower) relative uncertainty in the output gap process raises (lowers) the response of monetary policy to the output gap and lowers (raises) the response to inflation. All three models estimated in columns (i)-(iii) of Table 2 fit the data better than the corresponding models in Table 1. Therefore, both

inflation and output gap uncertainty matter for US monetary policy. Amongst all estimated models, the model with final data (reported in column (iii) of Table 2) delivers the best fit. For this model, the average output gap response drops from 1.61 when there is higher relative uncertainty in the output gap to 1.48 when there is higher relative uncertainty in inflation. On the other hand, the average inflation response increases from 1.49 when there is higher relative uncertainty in the output gap to 1.56 when there is higher relative uncertainty in inflation. Consistent with Figure 3, the average output gap response is higher (lower) than the average inflation response when there is higher (lower) relative uncertainty in the output gap process. These average estimates provide some evidence of asymmetries in the response to inflation and the output gap when uncertainty is considered.

Estimates of the real-time models (in columns (i) and (ii) of Table 2) also suggest that the average inflation response increases when there is higher relative uncertainty in inflation. On the other hand, the output gap response increases when there is higher relative output gap uncertainty for the model in column (i) but not for the one in column (ii); the latter finding is arguably due to the $\rho_{y,0}^+$ coefficient being imprecisely estimated. Overall, our empirical findings suggest that the response of US monetary policy to the output gap and inflation depends on output gap and inflation uncertainty and there is also some evidence that these uncertainty effects are asymmetric.

5. Robustness analysis

At this point, some natural questions inspire a battery of experiments to check for the robustness of our results. We first consider to what extent, if any, changes in the central bank preferences may affect these results. This question matters in that policymakers' preferences are not known with certainty and might also change due to special facts or contingencies. Thus we consider the case where output gap stabilization is as important as inflation stabilization⁶. The result of this experiment is reported in Figure 5 and shows that policymakers' preferences do not affect the monotonicity or the curvature type of the ρ_y and ρ_π paths; they only affect the degree of concavity or convexity of these paths.

We then ask what happens to the case where the multiplicative shocks exhibit some persistence. Arguably, this could be due to some exogenous disturbance and/or change in the structure of the economy whose medium or long-lived nature is not known yet by the policymakers.

To address this question, we introduce some inertia into the Markov chain. Thus, a new Markov matrix, P , is constructed such that: (i) the probability that the shock keeps the same value over two periods is equal to 0.5, (ii) the probability that the shock jumps to adjacent values

⁶ These alternative preferences are captured by setting in the loss function $\mu = 1$, $\lambda = 1$, $\nu = 0.2$.

is equal to 0.3, and (iii) by skipping the adjacent values, the probability that the shock jumps to the closer remaining values is equal to 0.1. Therefore, P takes the form

$$P = \begin{bmatrix} 0.5 & 0.3 & 0.1 & 0.1 & 0 \\ 0.15 & 0.5 & 0.15 & 0.1 & 0.1 \\ 0.1 & 0.15 & 0.5 & 0.15 & 0.1 \\ 0.1 & 0.1 & 0.15 & 0.5 & 0.15 \\ 0 & 0.1 & .1 & 0.3 & 0.5 \end{bmatrix}.$$

Given the Markov matrix above, its associated stationary distribution for the shock $\phi_{j_t}^h$ for $h = y, \pi$ is given by

$\phi_{j_t}^h$	$(1 - 2\delta_{\phi^h})$	$(1 - \delta_{\phi^h})$	1	$(1 + \delta_{\phi^h})$	$(1 + 2\delta_{\phi^h})$
$\Pr(\phi_{j_t}^h)$	0.1598	0.2371	0.2062	0.2371	0.1598

Results in this case are very similar to the previous ones based on a flat prior belief on the Markov process for the modes⁷.

Finally, we consider to what extent, if any, the realistic policy transmission lags embedded in the model affect the relation between inflation and output gap uncertainty and optimal policy. We then relax the assumption of a one-period lag between policy action and output gap response and of a further one-period lag between output gap and inflation changes. Accordingly, the AD and AS take the conventional form

$$y_{t+1} = [\alpha_y y_t + (1 - \alpha_y) y_{t+2|t+1} - \alpha_r (i_{t+1} - \pi_{t+2|t+1})] \phi_{t+1}^y$$

$$\pi_{t+1} = [\beta_\pi \pi_t + (1 - \beta_\pi) \pi_{t+2|t+1} + \beta_y y_{t+1}] \phi_{t+1}^\pi$$

The results of the experiment are reported in Figures 6-7. Comparing these findings with the ones in Figures 2-3 shows that abstracting from transmission lags leads only to minor quantitative changes.

⁷ Results are available upon request.

6. Conclusions

This paper investigates optimal real-time monetary policy when policymakers consider the presence of undistinguishable uncertainty about the state and structure of the economy proxied by the volatility of the inflation and output gap processes.

First, the paper shows that in presence of undistinguishable uncertainty either on inflation or the output gap, considering this uncertainty in the policy decisions results in a more aggressive response to the uncertain state variable. Furthermore, the optimal response to inflation is inversely related to output gap uncertainty while the optimal response to the output gap tends to be unrelated to inflation uncertainty.

Second, in presence of uncertainty in both the inflation and output gap processes, when there is relatively more uncertainty on inflation than on the output gap, optimal monetary policy resembles the Taylor rule. On the other hand, when the uncertainty on the output gap exceeds the one on inflation, optimal monetary policy tends to respond more strongly to the output gap and to ignore inflation. Finally, in intermediate cases, the policy response to the state variables tends to be similar.

These results are based on a preemptive behavior of the central bank aiming to reduce the risk of large deviations of the economy from its long-run equilibrium, which would deteriorate the distribution forecasts for inflation and the output gap. In an empirical test carried out on the US economy, we find that the model predictions tend to be consistent with the data.

The model discussed in the current paper can be extended to allow for the effects of other types of uncertainty such as exchange rate uncertainty. We intend to address these issues in future research.

Table 1: Simple Taylor rule model estimates using GMM
Sample: 1969Q4-2009Q2

	(i)	(ii)	(iii)
Interest rate equation			
ρ_i	0.83 (0.03)	0.90 (0.03)	0.89 (0.02)
ρ_π	2.12 (0.31)	2.04 (0.49)	1.53 (0.30)
ρ_y	1.16 (0.26)	1.94 (0.98)	1.07 (0.47)
Adjusted R ²	0.92	0.91	0.93
Regression standard error	0.94	0.95	0.89
J stat	[0.35]	[0.34]	[0.35]
Output gap equation			
α_y	0.53 (0.03)	0.50 (0.01)	0.54 (0.02)
α_r	0.05 (0.02)	0.04 (0.01)	0.06 (0.02)
Adjusted R ²	0.95	0.85	0.96
Regression Standard error	0.53	0.69	0.52
Inflation equation			
β_π	0.91 (0.04)	0.86 (0.03)	0.51 (0.02)
β_y	0.11 (0.03)	0.05 (0.01)	0.03 (0.01)
Adjusted R ²	0.97	0.95	0.98
Regression Standard error	0.42	0.36	0.20

Notes: All models include an intercept term; estimates of this are not reported.

(i): Real-time estimates. These use output detrended by a quadratic trend.

(ii): Real-time estimates. These use output detrended by a Hodrick-Prescott trend.

(iii) Final estimates. These use output detrended by the CBO measure of potential output.

Numbers in parentheses are the standard errors of the estimates. J stat is the p-values of a chi-square test of the system's overidentifying restrictions (Hansen, 1982). The instruments are a constant, four lags of the interest rate, inflation and the output gap.

Table 2: Taylor rule model estimates with uncertainty effects using GMM
Sample: 1969Q4-2009Q2

	(i)	(ii)	(iii)
Interest rate equation			
ρ_i	0.84 (0.03)	0.86 (0.03)	0.89 (0.03)
$\left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t \geq \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_{t-1}$ regime			
$\rho^+_{\pi,0}$	2.19 (0.30)	2.38 (0.39)	2.16 (0.45)
$\rho^+_{\pi,1}$	-0.83 (0.37)	-1.48 (0.47)	-1.85 (0.64)
Average inflation effect	1.88	1.85	1.49
$\rho^+_{y,0}$	0.44 (0.20)	0.41 (0.24)	1.04 (0.40)
$\rho^+_{y,1}$	1.23 (0.50)	2.72 (1.01)	1.57 (0.60)
Average output gap effect	0.90	1.40	1.61
$\left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_t < \left(\frac{\sigma_y}{\sigma_y + \sigma_\pi} \right)_{t-1}$ regime			
$\rho^-_{\pi,0}$	1.84 (0.31)	1.75 (0.38)	1.00 (0.31)
$\rho^-_{\pi,1}$	0.84 (0.40)	1.74 (0.71)	1.88 (0.61)
Average inflation effect	2.12	2.21	1.56
$\rho^-_{y,0}$	1.19 (0.32)	3.00 (1.03)	1.95 (0.65)
$\rho^-_{y,1}$	-0.99 (0.41)	-4.24 (1.65)	-1.53 (0.70)
Average output gap effect	0.85	1.86	1.48
Adjusted R ²	0.96	0.86	0.97
Regression standard error	0.51	0.67	0.47
J stat	[0.29]	[0.31]	[0.32]

Notes: All models include an intercept term; estimates of this are not reported. Numbers in parentheses are the standard errors of the estimates. J stat is the p-values of a chi-square test of the system's overidentifying restrictions (Hansen, 1982). The instruments are a constant, four lags of the interest rate, inflation and the output gap and one lag of inflation uncertainty and output gap uncertainty.

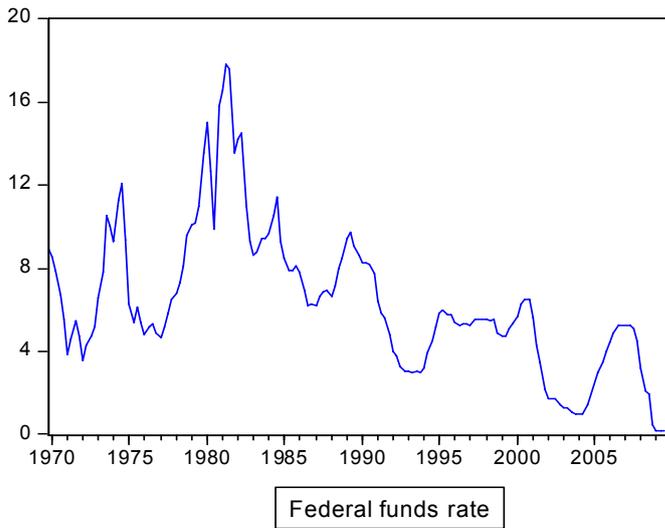
(i): Real-time estimates. These use output detrended by a quadratic trend.

(ii): Real-time estimates. These use output detrended by a Hodrick-Prescott trend.

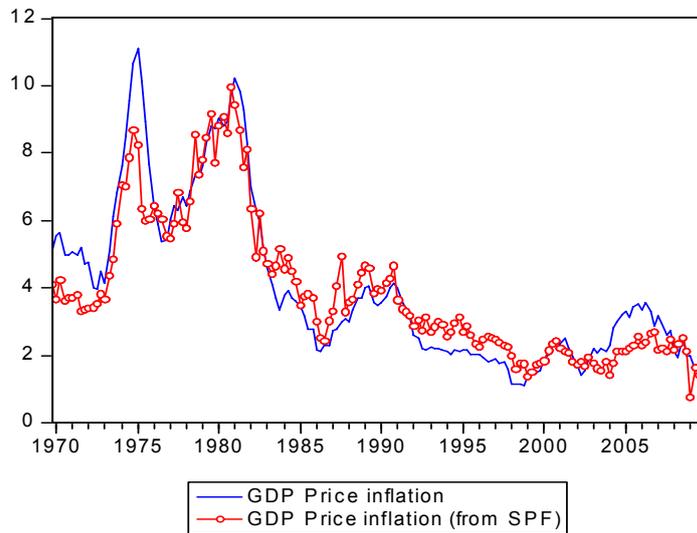
(iii) Final estimates. These use output detrended by the CBO measure of potential output.

Figure 1: Federal funds rate, inflation and output gap data

(a) Federal funds rate



(b) Inflation measures



(c) Output gap data

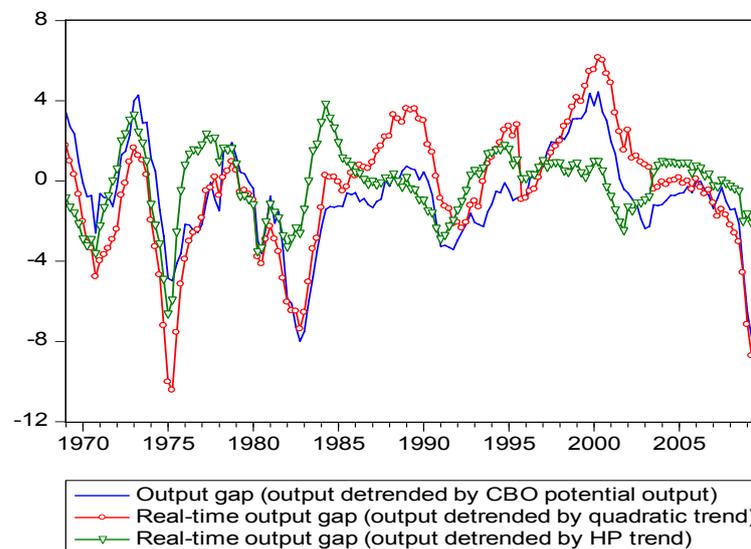


Figure 2: Optimal policy response to y and π in presence of uncertainty on the output gap and inflation processes. Central bank preferences: $\mu=1, \lambda=0.1, \nu=0.2$.

(a) output gap uncertainty

(b) inflation uncertainty

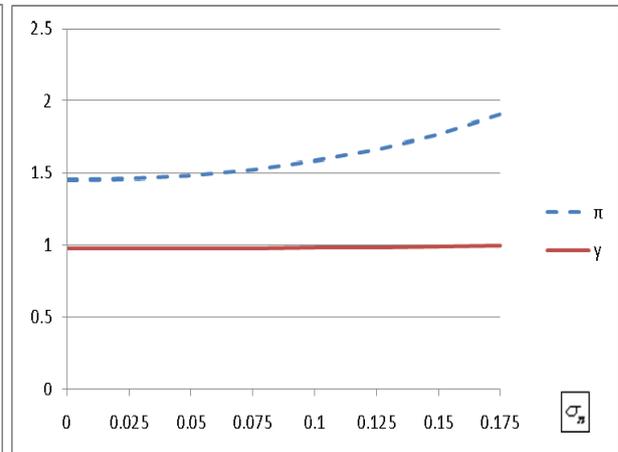
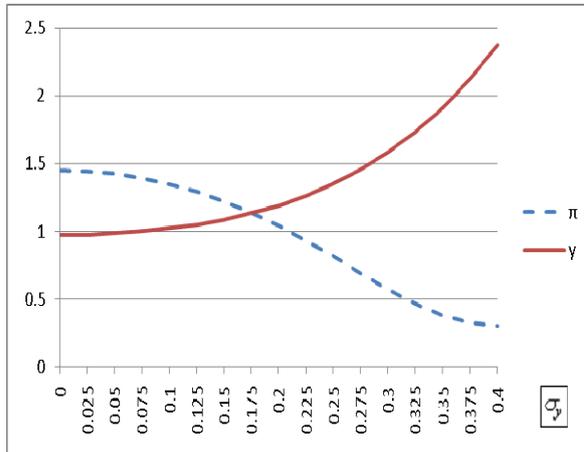


Figure 3: Optimal policy response to y and π in presence of relative output gap uncertainty. Central bank preferences: $\mu=1, \lambda=0.1, \nu=0.2$.

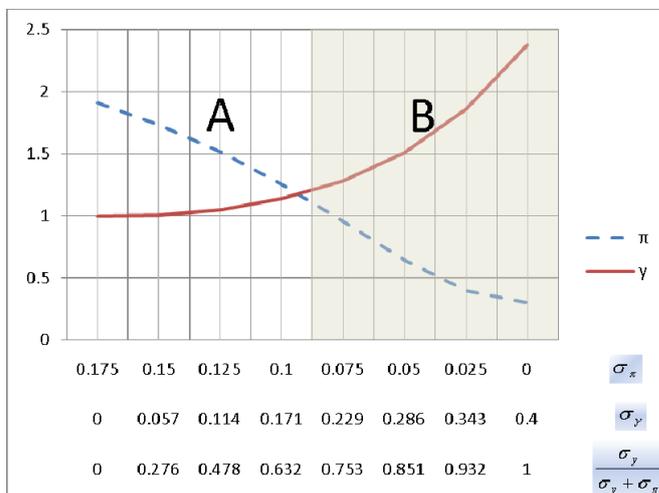
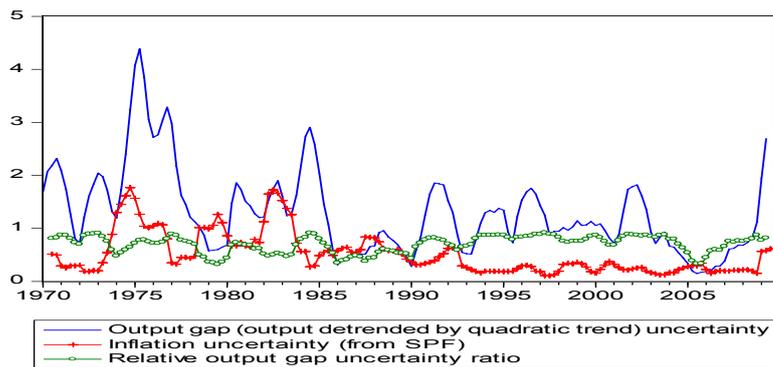
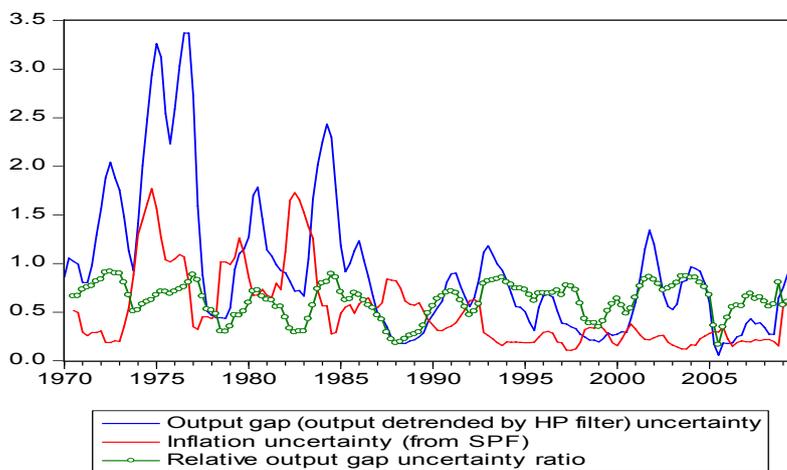


Figure 4: Output gap uncertainty, inflation uncertainty and relative output gap uncertainty ratio

(a) Real-time data, output detrended by quadratic trend



(b) Real-time data, output detrended by Hodrick-Prescott filter



(c) Final data, output detrended by CBO measure of potential output

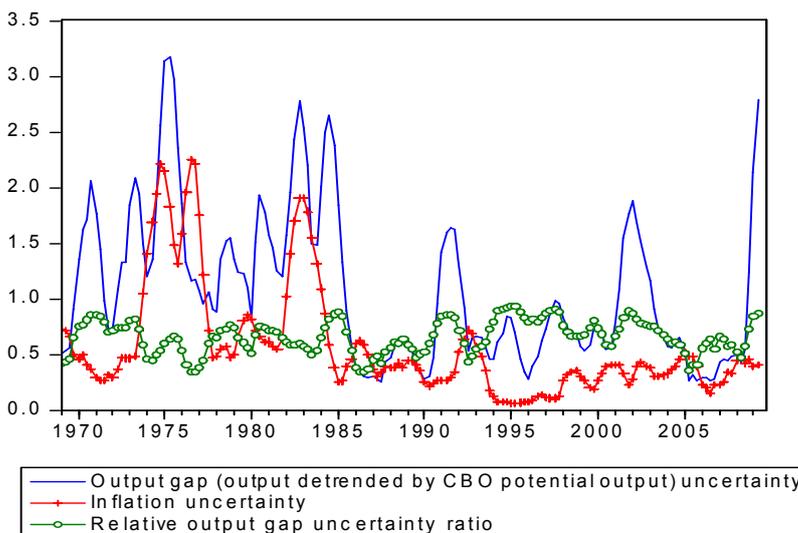


Figure 5: Optimal policy response to y and π in presence of uncertainty on the output gap and inflation processes. Central bank preferences: $\mu=1, \lambda=1, \nu=0.2$.

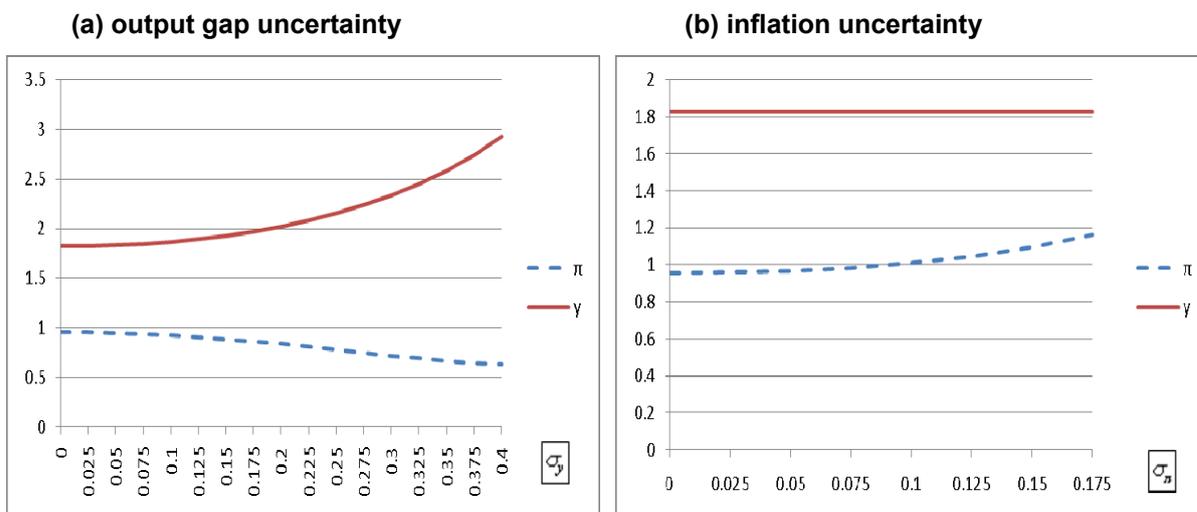


Figure 6: Optimal policy response to y and π with no policy transmission lags and in presence of uncertainty on the output gap and inflation processes. Central bank preferences: $\mu=1, \lambda=0.1, \nu=0.2$.

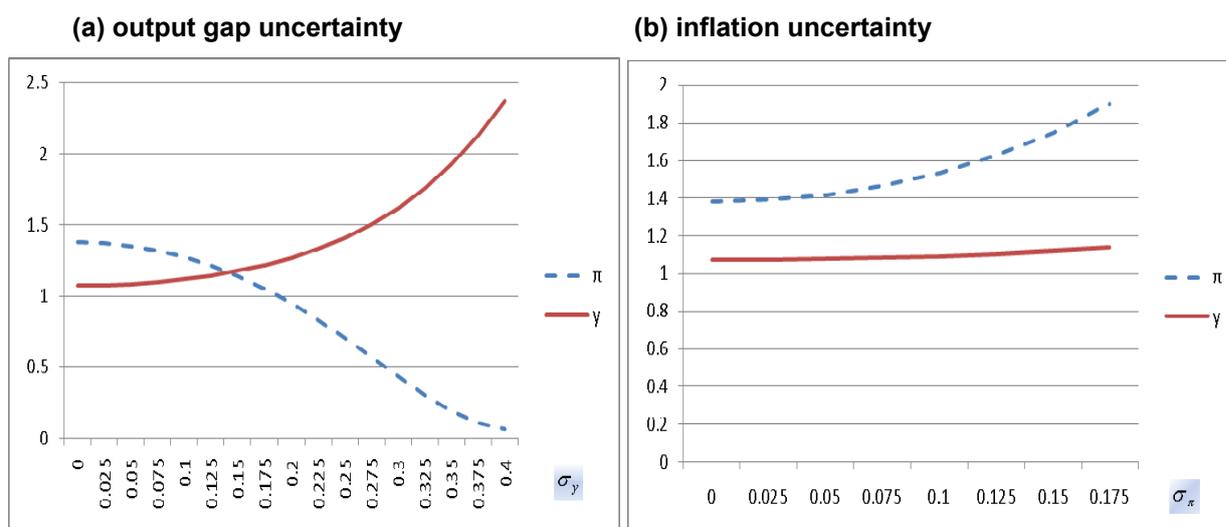
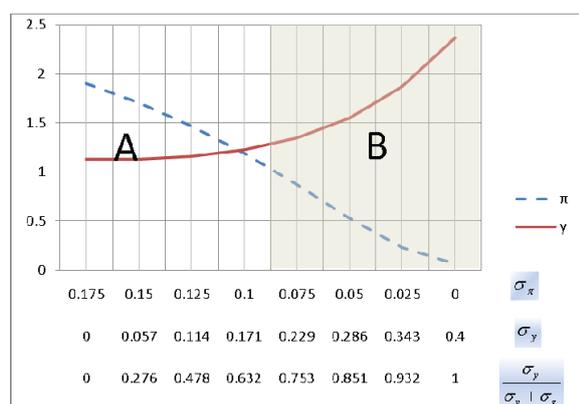


Figure 7: Optimal policy responses to π with no policy transmission lags and in presence of relative output gap uncertainty. Central bank preferences: $\mu=1, \lambda=0.1, \nu=0.2$.



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