



## WP 37\_11

**Luciano Fanti**

Department of Economics, University of Pisa, Italy

**Nicola Meccheri**

Department of Economics, University of Pisa, Italy  
The Rimini Centre for Economic Analysis (RCEA), Italy

# ON PRODUCT DIFFERENTIATION AND PROFITS IN UNIONIZED DUOPOLIES

Copyright belongs to the author. Small sections of the text, not exceeding three paragraphs, can be used provided proper acknowledgement is given.

The *Rimini Centre for Economic Analysis* (RCEA) was established in March 2007. RCEA is a private, nonprofit organization dedicated to independent research in Applied and Theoretical Economics and related fields. RCEA organizes seminars and workshops, sponsors a general interest journal *The Review of Economic Analysis*, and organizes a biennial conference: *The Rimini Conference in Economics and Finance* (RCEF). The RCEA has a Canadian branch: *The Rimini Centre for Economic Analysis in Canada* (RCEA-Canada). Scientific work contributed by the RCEA Scholars is published in the RCEA Working Papers and Professional Report series.

The views expressed in this paper are those of the authors. No responsibility for them should be attributed to the Rimini Centre for Economic Analysis.

# On product differentiation and profits in unionized duopolies

Luciano Fanti and Nicola Meccheri\*

*Department of Economics, University of Pisa, Italy*

July 5, 2011

## Abstract

This work aims to investigate if the conventional wisdom, that a decrease in the degree of product differentiation *always* reduces firms' profits, remains true in a differentiated duopoly model with decentralized, or firm-specific, monopoly unions. It is shown that, provided that unions are sufficiently wage-oriented, that is, they sufficiently prefer wages to employment, the conventional result can actually be reversed under both Cournot and Bertrand competition, implying that incentives for firms towards *less* differentiation may arise. Moreover, the range of product differentiation values, for which the "reversal result" applies, is larger when firms compete in quantities than in prices.

Keywords: unionized duopoly, product differentiation, profits

JEL Codes: J43, J50, L13

---

\*E-mail addresses: lfanti@ec.unipi.it (L. Fanti) and meccheri@ec.unipi.it (N. Meccheri)

# 1 Introduction

A conventional wisdom in industrial economics suggests that a decrease in the degree of product differentiation always reduces firms' profits by increasing the intensity of product market competition, irrespective of the fact that firms compete *à la* Cournot or *à la* Bertrand in the product market (e.g. Shy 1995, pp. 138-140). The theoretical reason behind this result can be understood by referring to the standard differentiated duopoly model, due to Singh and Vives (1984), in which a decrease in the degree of product market differentiation diminishes total demand and induces firms to compete more aggressively. Under both quantity and price competition, this unambiguously leads to lower firms' profits.

Whilst in the standard Singh and Vives's (1984) model firms' marginal production costs are assumed to be exogenously given, the growing literature on unionized oligopolies (see, e.g., the seminal works by Horn and Wolinsky (1988) and Dowrick (1989)) relaxes such assumption by admitting that (labor) costs are the outcome of a strategic game played between firms and unions before the former compete in the product market.

In this work, we investigate if the conventional wisdom, that a decrease in product differentiation *always* reduces firms' profits, remains true or can be reversed in a unionized duopoly model with decentralized, or firm-specific, monopoly unions.

Our main results can be summarized as follows. When firm-specific unions endogenously fix wages and product differentiation decreases, another important effect, that we will term *endogenous* or *union wage effect*, affects firms' profits, together with the standard "pure" *competition effect*. Whilst the standard effect acts in reducing profits, the former always operates in the opposite direction, irrespective of the mode of competition. We highlight that, provided that unions are sufficiently wage-oriented (that is, they sufficiently prefer wages to employment), the union wage effect outweighs the competition effect for some degree of product differentiation, hence actually reversing the conventional result. This also implies that incentives for

firms towards *less* differentiation may arise. Furthermore, we also show that the range of product differentiation values, for which the “reversal result” applies, is larger when firms compete in quantities than in prices.

The present work brings together some existing contributions and also extends the literature by providing a novel result. In particular, Correa-López and Naylor (2004) study in detail as wages behave and affect profits in a unionized duopoly model with product differentiation, in order to show that a reversal can apply in relation to the ranking of Cournot and Bertrand equilibrium profits. Although the mechanism highlighted by Correa-López and Naylor (2004) is relevant in our context, they do not investigate the issue of how changes in product differentiation affect wages in the first place. Hence, they do not consider the possibility that firms’ profits can increase with decreasing product differentiation, which instead represents the goal of our work.

Dhillon and Petrakis (2002) analyze the relationship between product differentiation and wages. Even if their main analysis aims to show that with *centralized* (or *coordinated*) unions, wages turn out to be independent of product market characteristics, including the degree of product differentiation, and bargaining institutional features (the so-called “wage rigidity result”), they also provide a case with decentralized unions where the wage is decreasing in the degree of product substitutability. Our work delves into this finding, by showing that unions’ preferences non-monotonically affect the (negative) behavior of wages with respect to product differentiation, and finding out the conditions under which this can lead to a “reversal” of the standard relationship between product differentiation and firms’ profits.

To the best of our knowledge, the only paper that specifically deals with the possibility of a reversal result between product differentiation and profits is Zanchettin (2006, Section 4).<sup>1</sup> In a (*non-unionized*) differentiated duopoly with a wide range of cost and demand asymmetry between firms, he shows

---

<sup>1</sup>Zanchettin’s (2006) main purpose, however, is to compare, in a differentiated duopoly with asymmetric firms, Cournot and Bertrand equilibria.

that, under both modes of competition, the *efficient* firm’s profit and industry profits as a whole can decrease with the degree of product differentiation. Our paper distinguishes from Zanchettin (2006) because we introduce the role of unions in determining wages into the analysis, instead of the presence of asymmetries between firms. Hence, findings, as well as the economic mechanisms behind them, differ. Most notably, in our framework, when the conditions for the reversal result apply, *both* firms’ profits increase with decreasing product differentiation, while in Zanchettin (2006) this only may occur for the most efficient firm.<sup>2</sup>

The remaining part of the paper is organized as follows. Section 2 describes the basic framework. The main results on the relationship between the degree of product differentiation and profits, as well as the mechanisms producing the “reversal result” under different modes of competition, are analyzed and discussed in Section 3. Finally, Section 4 concludes, while more technical analysis is relegated to the final Appendix.

## 2 Model

### 2.1 Some preliminaries

We consider a model of differentiated duopoly, in which the product market demand for the representative firm  $i$  is linear and given by:

---

<sup>2</sup>Notice that decreasing product differentiation also implies increasing competition between firms. The possibility of a non-standard relationship between competition and (industry) profits in a unionized framework is studied by Naylor (2002), which, however, differs from our work in various aspects. Firstly, Naylor (2002) considers a homogeneous oligopoly, instead of a differentiated duopoly. Second, and most importantly, in Naylor (2002) increasing competition means increasing the number of firms competing in the product market, instead of decreasing the degree of product differentiation. As a consequence, although in both cases the possibility of the reversal result necessarily hinges on a negative relationship between “increased competition” and wages, the underlying mechanisms are very different.

$$p_i = 1 - q_i - \gamma q_j \quad (1)$$

where  $q_i$  and  $q_j$  are outputs by firm  $i$  and  $j$ , respectively, and  $\gamma \in (0, 1)$  denotes the extent of product differentiation, with goods assumed to be imperfect substitutes. In particular, when  $\gamma \rightarrow 1$ , the products of the two firms tend to be completely undifferentiated, hence firms compete *de facto* in the same market. Instead, when  $\gamma \rightarrow 0$ , a monopoly tends to affirm in each market.

Following standard assumptions in the literature, let assume that only labor input is used for production and exhibits constant returns, that is,  $q_i = l_i$ , where  $l_i$  represents the total amount of input employed by the firm  $i$  to produce  $q_i$  output units of the variety  $i$ . Hence, the firm  $i$ 's profit can be written as:

$$\pi_i = (p_i - w_i)q_i \quad (2)$$

where  $w_i$  is the per-worker wage paid by the firm  $i$ , with  $w_i < 1$ .

With an exogenously given wage (i.e.  $w_i = w_j = w$ ), it is straightforward to verify that, in symmetric equilibrium (with  $\pi_i = \pi_j = \pi$ ), firms' profits under Cournot and Bertrand competition are given by, respectively:

$$\pi_C = \frac{(1-w)^2}{(2+\gamma)^2}, \quad \pi_B = \frac{(1-\gamma)(1-w)^2}{(1+\gamma)(2-\gamma)^2} \quad (3)$$

which both imply that, according to the conventional wisdom, (equilibrium) profits always decrease when  $\gamma$  increases, i.e. profits are positively correlated with the degree of product differentiation. Furthermore, for following analysis, it is worth remarking that the magnitude of the reduction in profits due to increasing  $\gamma$  negatively depends on the wage level. Indeed, according to the mode of product competition, we get:

$$\frac{\partial (|\partial\pi_C/\partial\gamma|)}{\partial w} = -\frac{4(1-w)}{(2+\gamma)^3} < 0, \quad \forall \gamma \in (0, 1) \quad (4)$$

$$\frac{\partial (|\partial\pi_B/\partial\gamma|)}{\partial w} = -\frac{4(\gamma^2 - \gamma + 1)(1 - w)}{(1 + \gamma)^2(2 - \gamma)^3} < 0, \quad \forall \gamma \in (0, 1). \quad (5)$$

This is because, as the wage increases, equilibrium output diminishes, hence the room for revenue reduction linked to a decrease of product differentiation tends to reduce.

## 2.2 Unionized duopoly

Now, we join the literature on unionized oligopolies (see, e.g., Horn and Wolinsky 1988; Dowrick 1989; Naylor 1998, 1999; Correa-López and Naylor 2004; Lommerud et al. 2005; Correa-López 2007) by admitting that labor cost is no longer exogenously given, but it is the outcome of a two-stage strategic game played between firms and labor unions.

Following the backward induction logic, at stage 2, each firm decides, according to the mode of product market competition, its optimal (profit-maximizing) output or price, which also (directly or indirectly) implies its labor demand. At stage 1, instead, we assume that wages are unilaterally set by firm-specific monopoly unions that are characterized by identical Stone-Geary utility functions, given by:

$$V_i = (w_i - \bar{w})^\theta l_i^{1-\theta} \quad (6)$$

where  $\bar{w}$  is the reservation wage, while  $\theta \in (0, 1)$  captures the relative importance of wages and employment to the unions. This functional form is quite general and encompasses common assumptions such as rent-maximization, arising when  $\theta = 0.5$ , and total wage bill maximization, when  $\theta = 0.5$  and  $\bar{w} = 0$ .<sup>3</sup>

Since both firms are unionized, unions' choices about wages take place simultaneously across firms, taking the other firm's wage as given. Standard

---

<sup>3</sup>Also notice that, in this latter case, the maximization problem for unions is equivalent to the one facing profit maximizing upstream firms that are allowed to set the prices of the input they deliver to downstream firms. Furthermore, the special case in which  $\theta \rightarrow 0$  (i.e. unions tend to only care about employment) approximates the situation one would get if the input suppliers are price-takers at a competitively given price.

analysis (e.g. Correa-López and Naylor 2004) leads to the result that, according to type of competition in the product market, symmetric sub-game perfect equilibrium wages are given by, respectively:

$$w_C = \bar{w} + \frac{(2 - \gamma)\theta(1 - \bar{w})}{2 - \gamma\theta}, \quad w_B = \bar{w} + \frac{(2 + \gamma)(1 - \gamma)\theta(1 - \bar{w})}{2 - \gamma^2 - \gamma\theta}. \quad (7)$$

### 3 Product differentiation and firms' profits

In this section we will establish our results. In order to analyze the relationship between product differentiation and profits, consider first that when, e.g., the degree of product market differentiation decreases (that is, product market competition becomes fiercer), two distinct effects affect firms' profits. On the one hand, the *direct effect* (that can be labeled as “competition effect”) of increasing market competition for a given labor input price, which is always profit-reducing.<sup>4</sup> On the other hand, when wages are endogenously determined by unions, there is also an *indirect effect* (that we term “endogenous” or “union wage effect”) operating via changes in wages. Formally:

$$\frac{\partial \Pi}{\partial \gamma} = \underbrace{\frac{\partial \pi}{\partial \gamma}}_{\text{competition effect}} + \underbrace{\frac{\partial \pi}{\partial w} \cdot \frac{\partial w}{\partial \gamma}}_{\text{endogenous/union wage effect}} \quad (8)$$

where, in general,  $\Pi(\gamma, \theta, \bar{w}) = \pi[\gamma, w(\gamma, \theta, \bar{w})]$ .

In particular, when wages are exogenously given, the derivative of  $w$  with respect to  $\gamma$  is obviously zero, hence the endogenous wage effect is null. In such a case, only the competition effect operates and we get the standard result that, regardless of the mode of competition in the product market, profits always decrease with decreasing product differentiation. Instead, when wages are endogenously determined,  $\partial w/\partial \gamma$  may not be longer zero. In this

---

<sup>4</sup>It should be noted that changes in the degree of product differentiation affect total demand, hence profits, in a different way under Cournot and Bertrand competition (see, e.g., Shaked and Sutton 1990). Nevertheless, in both competition regimes, the sign of the “competition effect” is always negative.



regard, a cornerstone result, also known as “wage rigidity result”, has been affirmed by Dhillon and Petrakis (2002). They show that, under quite general conditions, with centralized (or coordinated) unions wages turn out to be same independently of the degree of product differentiation (as well as of other product market and bargaining institutional features), implying that, in such a case, the conventional result can never be reversed. But when, instead, wages are fixed by firm-specific (decentralized) unions,  $\partial w/\partial\gamma$  is not actually null. Indeed, by differentiating expressions in (7) with respect to  $\gamma$ , we get, respectively:

$$\frac{\partial w_C}{\partial\gamma} = -\frac{2\theta(1-\theta)(1-\bar{w})}{(2-\gamma\theta)^2} < 0, \quad \forall\gamma \in (0,1), \theta \in (0,1) \quad (9)$$

$$\frac{\partial w_B}{\partial\gamma} = -\frac{(2+\gamma^2)\theta(1-\theta)(1-\bar{w})}{(2-\gamma^2-\gamma\theta)^2} < 0, \quad \forall\gamma \in (0,1), \theta \in (0,1). \quad (10)$$

Such findings make sense and, intuitively, can be explained by the fact that, since unions are firm-specific, an increase of inter-firm competition in the product market, due to increased product substitutability, also translates into an increase of inter-union competition. More exactly, when  $\gamma$  increases, employment at a firm level becomes more sensitive with respect to wages and this drives firm-specific unions to undercut each other in wage setting in order to sufficiently preserve employment. Moreover, since with  $\partial w/\partial\gamma < 0$  the union wage effect is positive,<sup>5</sup> if the latter dominates the competition effect, the conventional relationship between *equilibrium* profits and the degree of product market competition is reversed.

**Result 1** *In a duopoly with firm-specific monopoly unions, the following statements concerning the relationship between firms’ equilibrium profits and the degree of product differentiation, apply:*

---

<sup>5</sup>Indeed, it is trivial to check from (3) that, as expected, profits are negatively correlated with wages (i.e.  $\partial\pi/\partial w < 0$ ) under both competition regimes (see Correa-López and Naylor (2004) for more details).

- **[Cournot]** Under Cournot competition, if unions are more wage- than employment-oriented, that is  $\theta > 0.5$ , there is always some degree of product differentiation  $\gamma$  sufficiently large, for which profits increase for decreasing product differentiation. Moreover (provided that  $\theta > 0.5$ ), the higher  $\theta$ , the larger the range for  $\gamma$ 's values, for which profits increase with  $\gamma$ .
- **[Bertrand]** Under Bertrand competition, profits increase for decreasing product differentiation, provided that unions are sufficiently wage-oriented, that is  $\theta > 0.71$ , and the degree of product differentiation  $\gamma$  is neither too much small nor too much large. Moreover, (provided that  $\theta > 0.71$ ) the higher  $\theta$ , the larger the range for  $\gamma$ 's values, for which profits increase with  $\gamma$ .

**Proof.** See the Appendix. ■

The content of the above result is graphically represented by Figure 1. In particular, while under Cournot competition the cutoff point is  $\theta = 0.5$ , under Bertrand competition, there is a cutoff point for  $\theta \approx 0.71$ , that is, for any  $\theta > 0.71$ , there is some  $\gamma \in (0, 1)$ , for which firms' (equilibrium) profits increase with  $\gamma$ . Furthermore, from the figure, it is straightforward to check that: a) there exist some  $\theta$ 's values ( $0.5 < \theta < 0.71$ ), for which the reversal result applies, for some  $\gamma$ , under Cournot, but it does not under Bertrand competition; and b) when  $\theta$  is sufficiently high ( $\theta > 0.71$ ), such that the reversal result applies under both competition regimes, the reversal of the relationship between product differentiation and profits occurs for a larger range of  $\gamma$ 's values under Cournot than under Bertrand. More exactly, the range of  $\gamma$ 's values that meet the reversal result under Bertrand is always a subset of those that satisfy the reversal under Cournot competition.<sup>6</sup>

---

<sup>6</sup>Notice that, by reading the figure in a different (inverse) way, i.e. fixing a given  $\gamma$  on the horizontal axis (instead of a given  $\theta$  on the vertical axis), it clearly arises that the cutoff for  $\theta$  is always lower under Cournot than under Bertrand, confirming that the reversal result is more likely to apply under the former regime.

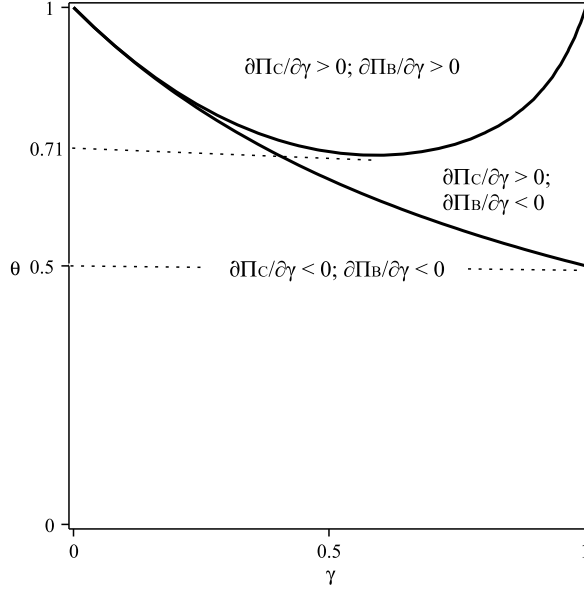


Figure 1: Profits' behavior with respect to product differentiation in  $\{\gamma, \theta\}$ -space

Before concluding, a further discussion is useful in order to explain the rationale behind our findings. Indeed, since the unions' preferences parameter  $\theta$  plays a key role in determining the reversal result, it is worth investigating in some more detail the nexus between the union wage effect and  $\theta$ . In this regard, what is crucial is the behavior of  $|\partial w / \partial \gamma|$  with respect to  $\theta$ ,<sup>7</sup> which, under different mode of competition, is elucidated by the following:

$$\frac{\partial (|\partial w_C^U / \partial \gamma|)}{\partial \theta} = \frac{2(2 + \gamma\theta - 4\theta)(1 - \bar{w})}{(2 - \gamma\theta)^3} \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \Leftrightarrow \theta \begin{matrix} \leq \\ \geq \end{matrix} \frac{2}{4 - \gamma} \quad (11)$$

$$\begin{aligned} \frac{\partial (|\partial w_B^U / \partial \gamma|)}{\partial \theta} &= \frac{(4 - \gamma^4 + 2\gamma^4\theta + \gamma^3\theta + 2\gamma\theta - 8\theta)(1 - \bar{w})}{(2 - \gamma^2 - \gamma\theta)^3} \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \Leftrightarrow \\ &\Leftrightarrow \theta \begin{matrix} \leq \\ \geq \end{matrix} \frac{2 - \gamma^2}{4 - 2\gamma^2 - \gamma}. \end{aligned} \quad (12)$$

Hence, there is a “hump-shaped” relationship between  $|\partial w / \partial \gamma|$  and  $\theta$ . This holds true under both Cournot and Bertrand competition and can be

<sup>7</sup>Indeed, the (non-monotone) behavior of the union wage effect as a whole with respect to  $\theta$  parallels that of  $|\partial w / \partial \gamma|$ . Instead, it is easy to check that  $|\partial \pi / \partial w|$  is always decreasing in  $\theta$ .

explained by the fact that a change in wages takes place if changing the degree of product differentiation modifies the trade-off between wages and employment. This actually occurs especially when both wages and employment matter for unions, that is, for medium values of  $\theta$ .<sup>8</sup> Instead, for very low  $\theta$ 's values, wages are close to the reservation value  $\bar{w}$  and there is not much room for wage reductions. Thus, as  $\theta$  and the (equilibrium) wage increase, also the wage reduction due to increased product substitutability, i.e.  $|\partial w/\partial \gamma|$ , becomes more sizable. On the other side, for very high  $\theta$ 's values, unions have a strong preference for high wages and, even though there is considerable room for wage reductions, a change in  $\gamma$  will only trigger small wage adjustments. This could rise the question of why the reversal result applies when  $\theta$  is very high (e.g. close to one). This is because, as discussed at the end of Section 2.1, when  $\theta$ , hence the equilibrium wage, is very high, also the magnitude of the competition effect (i.e.  $|\partial \pi/\partial \gamma|$ ) tends to become negligible. In other words, when  $\theta$  is very high, even a marginal reduction in wages owing to decreased product differentiation can actually increase profits.

Finally, notice that, whilst the above argument generally holds true under both Cournot and Bertrand competition, when products tend to become close substitutes, the dynamics undergoes a radical change in the latter case. This is because, when *price* competition becomes extremely fierce and transfers to unions in the labor market, the unionized wage approaches the reservation level (from (7), it is possible to verify that, differently from  $w_C$ ,  $w_B \rightarrow \bar{w}$  when  $\gamma \rightarrow 1$ ).<sup>9</sup> Hence, as products become close substitutes, the union wage effect tends to vanish under price competition and, as a consequence, the conditions for the “reversal result” to apply become more and more difficult to be met.

---

<sup>8</sup>The mechanism underlying such result is partly similar to that produced by a merger, as discussed, e.g., in Lommerud et al. (2005).

<sup>9</sup>Moreover, since firms' profits collapse to zero when  $\gamma \rightarrow 1$ , also  $|\partial \pi/\partial w| \rightarrow 0$ .

## 4 Conclusion

In this paper we have investigated if the conventional wisdom, that a decrease in the degree of product differentiation always diminishes firms' profits, holds true in a duopoly model with both Cournot and Bertrand competition and decentralized (firm-specific) monopoly unions. When product differentiation decreases, further than the standard competition effect (that always tends to reduce profits), another effect, which indirectly operates via changes in wages, affects profits. In particular, an increase of inter-firm competition in the product market, owing to a decrease of product differentiation, also translates into an increase of inter-union competition and this drives unions to undercut each other in wage setting in order to sufficiently preserve employment. If this "union wage effect" dominates the pure competition effect, the conventional profits' behavior with respect to the degree of product substitutability can be reversed. We have shown that, under both modes of competition, this event actually occurs for some degree of product differentiation, provided that unions are sufficiently wage-oriented.

By concluding, notice that the monopoly unions model, that we have followed to maintain our analysis as simple as possible, can be viewed as a limiting case of the wage-bargaining union, where the union has all the bargaining power. At the other extreme, with firms with all the bargaining power, we would get the competitive result where the wage equalizes the *exogenous* reservation level, hence, as discussed, the reversal result would never apply. This logically suggests that our reversal result would survive also in a more general Right-to-Manage model, provided that unions' bargaining power *vis-à-vis* firms is sufficiently high.

# Appendix

## Proof of Result 1

**Proof.**

[Cournot] Using (3), (7) and (8) with reference to Cournot competition, we get:

$$\begin{aligned} \frac{\partial \Pi_C}{\partial \gamma} \Big|_{w=w_C} &= \frac{\partial \pi_C}{\partial \gamma} \Big|_{w=w_C} + \frac{\partial \pi_C}{\partial w} \Big|_{w=w_C} \cdot \frac{\partial w_C}{\partial \gamma} = \\ &= -\frac{16(1-\theta-\gamma\theta)(1-\theta)^2(1-\bar{w})^2}{(2+\gamma)^3(2-\gamma\theta)^3} \geq 0 \Leftrightarrow 1-\theta-\gamma\theta \leq 0 \Leftrightarrow \theta \geq \bar{\theta}_C = \frac{1}{1+\gamma}. \end{aligned}$$

Firstly, notice that  $\bar{\theta}_C$  is monotonically decreasing in  $\gamma$ . Secondly, when  $\gamma \rightarrow 0$ ,  $\bar{\theta}_C \rightarrow 1$  and, when  $\gamma \rightarrow 1$ ,  $\bar{\theta}_C \rightarrow 0.5$ . Both imply that: i)  $\theta > \bar{\theta}_C$  (hence, profits increase with decreasing product differentiation) for some  $\gamma$ , iff  $\theta > 0.5$ , and ii) provided that  $\theta > 0.5$ , the higher  $\theta$ , the larger the range for  $\gamma \in (0, 1)$  such that  $\theta > \bar{\theta}_C$ .

[Bertrand] Using (3), (7) and (8) with reference to Bertrand competition, we get:

$$\begin{aligned} \frac{\partial \Pi_B}{\partial \gamma} \Big|_{w=w_B} &= \frac{\partial \pi_B}{\partial \gamma} \Big|_{w=w_B} + \frac{\partial \pi_B}{\partial w} \Big|_{w=w_B} \cdot \frac{\partial w_B}{\partial \gamma} = \\ &= -\frac{2H(2-\gamma^2)(1-\theta)^2(1-\bar{w})^2}{(1+\gamma)^2(2-\gamma)^3(2-\gamma^2-\gamma\theta)^3} \geq 0 \Leftrightarrow H \leq 0 \end{aligned}$$

where  $H = 4 - 4\theta - 4\gamma + 4\gamma^2\theta + 4\gamma^3 - 2\gamma^3\theta + \gamma^4\theta - 3\gamma^4 - \gamma^5 + \gamma^6$ . It follows that:

$$\frac{\partial \Pi_B}{\partial \gamma} \Big|_{w=w_B} \geq 0 \Leftrightarrow \theta \geq \bar{\theta}_B = \frac{\gamma^6 - \gamma^5 - 3\gamma^4 + 4\gamma^3 - 4\gamma + 4}{-\gamma^4 + 2\gamma^3 - 4\gamma^2 + 4}.$$

Firstly, notice that  $\bar{\theta}_B \rightarrow 1$  when both  $\gamma \rightarrow 0$  and  $\gamma \rightarrow 1$ . Secondly, by differentiating, we get  $\frac{d\bar{\theta}_B}{d\gamma} = -\frac{2\gamma^9 - 7\gamma^8 + 20\gamma^7 - 48\gamma^5 - 10\gamma^6 + 48\gamma^4 + 16\gamma^3 - 8\gamma^2 - 32\gamma + 16}{(\gamma^4 - 2\gamma^3 + 4\gamma^2 - 4)^2}$ , which, on the interval  $\gamma \in (0, 1)$ , has only one root in  $\gamma \approx 0.59$ . Moreover,

we also have that  $\frac{d^2\bar{\theta}_B}{d\gamma^2}|_{\gamma \approx 0.59} > 0$ . Finally, by substituting  $\gamma \approx 0.59$  in  $\bar{\theta}_B$ , we get  $\bar{\theta}_B|_{\gamma \approx 0.59} \approx 0.71$ .

All that implies that, in  $\gamma \in (0, 1)$ ,  $\bar{\theta}_B$  is “U-shaped” with a minimum value of  $\approx 0.71$ , which, in turn, leads to Result 1 (see also Figure 1). ■

## References

- [1] Correa-López, M. (2007), Price and Quantity Competition in a Differentiated Duopoly with Upstream Suppliers, *Journal of Economics & Management Strategy* 169: 469-505.
- [2] Correa-López, M. and Naylor, R.A. (2004), The Cournot-Bertrand Profit Differential: A Reversal Result in a Differentiated Duopoly with Wage Bargaining, *European Economic Review* 48: 681- 696.
- [3] Dhillon, A. and Petrakis, E. (2002), A Generalised Wage Rigidity Result, *International Journal of Industrial Organization* 20: 285-311.
- [4] Dowrick S.J. (1989), Union-Oligopoly Bargaining, *Economic Journal* 99: 1123-1142.
- [5] Horn, H. and Wolinsky, A. (1988), Bilateral Monopolies and Incentives for Merger, *RAND Journal of Economics* 19: 408-419.
- [6] Lommerud, K.E., Straume, O.R. and Sørgard, L. (2005), Downstream Merger with Upstream Market Power, *European Economic Review* 49: 717-743.
- [7] Naylor, R.A. (1998), International Trade and Economic Integration when Labour Markets Are Generally Unionised, *European Economic Review* 42: 1251-1267.
- [8] Naylor R. (1999), Union Wage Strategies and International Trade, *Economic Journal* 109: 102-125.

- [9] Naylor R. (2002), Industry Profits and Competition under Bilateral Oligopoly, *Economics Letters* 77: 169-175.
- [10] Shaked, A. and Sutton, J. (1990), Multiproduct Firms and Market Structure, *RAND Journal of Economics* 21: 45-62.
- [11] Shy, O. (1995), *Industrial Organization: Theory and Applications*, Cambridge MA: MIT Press.
- [12] Singh, N. and Vives, X. (1984), Price and Quantity Competition in a Differentiated Duopoly, *RAND Journal of Economics* 15: 546-554.
- [13] Zanchettin, P. (2006), Differentiated Duopoly with Asymmetric Costs, *Journal of Economics & Management Strategy* 15: 999-1015.