THE RIMINI CENTRE FOR ECONOMIC ANALYSIS

WP 36_13

Eleftherios Zacharias
Athens University of Economics and Bussiness, Greece
The Rimini Centre for Economic Analysis (RCEA), Italy

STRATEGIC POSITIONING OF GOODS IN A MARKET WITH A NICHE

The Rimini Centre for Economic Analysis
Legal address: Via Angherà, 22 – Head office: Via Patara, 3 - 47900 Rimini (RN) – Italy
www.rcfea.org - secretary@rcfea.org

Strategic positioning of goods in a market with a niche

Eleftherios Zacharias

Athens University of Economics and Bussiness*

Rimini Centre for Economic Analysis

June 27, 2013

Abstract

We use the Hotelling's model allowing for a "gap" in the consumers' preferences. As a result, the characteristics space is divided in two separate intervals. The largest one represents the main market, and the smallest represents a niche. We find that in this set up the principle of maximum differentiation may not hold. We also, examine the incentives of a firm to adopt a niche marketing strategy. That is, to relocate and price its product so that to maximize its profits from the niche market only. We show that, as the reservation value of the consumers for the product increases, it is more profitable for a firm to adopt a nich marketing strategy.

JEL Classification: M31, M21, L13

Keywords: Hotelling model, niche marketing, market segmentation.

1 Introduction

In this paper we use the standard Hotelling model to examine the positioning of a product in a market which is characterized by a main part and a niche:[1] Usually, different groups of consumers have different preferences for specific products. The term niche refers to a small group of customers with preferences that are distinct from the preferences of the majority of consumers in the market. We further examine the incentives of firms to adopt niche marketing strategies. That is, to position

*Department of Economics, Athens University of Economics and Bussiness, 76 Patission Str., GR104 34 Athens, Greece, Tel: +30 2108203388, Fax: +30 2108238249, Email: ezachar@aueb.gr

[1] Ansari et al (1994) use the Hotelling model to examine the positioning of multi-attribute products and Serfes and Zacharias (2012) use it to examine the position of firms in the presence of network effects.

their products into small profitable segments which have been ignored by other firms.[2] To do that we modify the standard Hotelling interval $[0,1]$ by allowing a "gap" (α, b) with $0 < \alpha < b < 1$ with no consumers at all. The interval $[0,a]$ denotes the main market and the interval $[b,1]$ denotes the niche market. By adopting this approach, we preserve both that (a) the two parts belong to the same market as the mass of consumers is one and (b) the two parts have certain differences: The interval (α, b) between the main and the niche markets is a measure of how distant the preferences of the consumers at the two sub-markets are.

In the standard Hotelling model we assume a continum of consumers who locate in the $[0,1]$ line. In doing so, we implicitly assume that there are no "crossroads" in this street. That is, we do not allow for intervals in the line with no consumers at all. Similarly, when the $[0,1]$ interval refers to the characteristics space, we assume that preferences change in a continuous way. This may be a reasonable assumption in a number of cases. For example, when preferences are associated with the age of the consumers, then it is reasonable to assume that these are represented by the interval $[y,z]$, where y denotes the youngest consumer and z the oldest consumer for whom the characteristic occurs. However, this may not be the case when the market consists of different groups of consumers whose preferences may belong to disjoint intervals of the characteristics space: It may be the case that the intensity for shopping is described by the intervals $[0,a]$ for men and by $[b,1]$ for women with $a < b$.

Niche marketing is a common practice in many sectors: Small tourist firms may focus on "rular" or "eco-turism" (Roberts and Hall (2004)) to attract enviromentally consious consumers. Adopting niche marketing strategies may help firms to remain profitable when the competition increases. Parrish et at (2004a, b) discuss how such strategies will allow US firms to compete against firms from low cost countries in the textile and apparel industries. Tamagnini and Tregear (1998) discuss niche marketing strategies in the delicatessen meat sector in Britain etc.

In this paper we use the Hotelling duopoly model allowing for an interval with no consumers, to examine the price and location decisions of two horizontally differentiated firms when the market consists of two parts: The main market and a niche. We assume that the niche market is served

[2] Sometimes, authors use the similar term "segment" to describe a niche and "market segmentation" for niche marketing. For a discussion of the special characteristics of a niche market, its differences from a segment and the characteristics of a successful "niche marketing" strategy see Dalgic and Leeuw (1994), Kotler (2003) and Parrish et al (2006).

only by one firm. We first allow both firms to compete in the main market. We find that the principle of maximum differentiation may not hold. We then examine the incentives of the second firm to adopt a niche marketing strategy, that is to focus on the niche market only. In this setup, the two firms form two local monopolies and target the two different sub-markets. We show that in general, as the reservation value of the consumers for the product increases, it is more profitable for a firm to adopt a nich marketing strategy.

1.1 The Hotelling line with a main and a niche market

We use the standard Hotelling's duopoly model assuming that consumers have a finite reservation price for the differentiated product. Consumers are located at two different disjoint intervals in the characteristics space (the main and the niche markets respectively) with different densities. For simplicity, we normalize the total number of consumers to one. More specifically, we assume that consumers are located uniformly at the main market (the interaval $[0, a]$) with density $f_1(x) = \frac{1}{m}$, and the niche market (the interval $[b, 1]$), with density $f_2(x) = \frac{1}{n}$. The mass of consumers in the main market is $\frac{a}{m}$, and in the niche market is $\frac{1-b}{n}$.[3] Thus, we have:

$$\frac{a}{m} + \frac{1-b}{n} = 1 \Rightarrow m = \frac{an}{-1+b+n}. \tag{1}$$

Furthermore we assume $a < b$. The distance $b-a$ is the lowest bound of the difference in preferences between the main market and the niche. The only assumption that we make is that the mass of consumers in the main market is greater than one half of the population. That is, $\frac{a}{m} > \frac{1}{2}$.[4] Each consumer buys one unit of the good.

In the analysis we assume quadratic transportation cost and zero marginal cost for both firms. Let the two firms be A and B. Firm A is located at x_A and B at x_B with $x_A < x_B$. The price each firm sets is P_i, where $i = A, B$. We examine (a) the pricing and location decisions of firms in this setting when they compete and (b) the incentives of A to serve the main market only and of B to serve the niche market only. The utility that a consumer, who is located at point x in the

[3] A number of papers exmine the price and location decisions when the distribution of the preferences is not uniform and/or various types of transportation cost. These include Neven (1986), Tabuchi and Thisse (1995), Anderson et al (1997) and Dearmon and Kosmopoulou (2008).

[4] This specification is very general: We allow α to be less than one half, and even α to be less than $1-b$. Of course in such case the density of the consumers in the main market is very high.

line, gets from buying the product from firm A, is:

$$U_A(x) = V - t(x - x_A)^2 - P_A,$$

and when he buys from firm B is:

$$U_B(x) = V - t(x - x_B)^2 - P_B.$$

A consumer located at x solves:

$$\max\{V - t(x - x_A)^2 - P_A, V - t(x - x_B)^2 - P_B, 0\}. \quad (2)$$

where t is a positive real number which shows the unit transport cost. This specification implies that strong preference for one firm results in strong aversion for the other firm by a factor t. V is the reservation price, that is the maximum price that a consumer who is located either at x_A or x_B, is willing to pay for the good. Here we assume that all consumers have the same V. However, it would be interesting to examine what happens when consumers in the two segments of the market have different Vs. We assume that the two firms cover the whole market and their locations can not be outside the $[0, 1]$ interval. We initially assume that the two firms compete in the main market (the marginal consumer is located at $x < a$). We then assume that we have two local monopolies with firm A covering the main market and firm B covering the niche market.[5] Finally, the analysis focuses on pure strategy equilibria.

1.2 Location choice

We first examine the pricing and location decisions of A and B when the two firms compete in the main market. This is a two period game. In the first period the two firms choose their locations. In the second they compete by setting prices. In our analysis we assume that the main market is covered by both firms, whereas the niche market is covered by B only. Here, B chooses the price P_B so that to maximize its profits by selling its product to both sub-markets. As a result, the decision of B to position its product in the niche market, that is when $b \leq x_B \leq 1$ does not imply that B adopts a niche marketing strategy. We make the following definition:

[5] In such case the two parts of the market can be considered as two different markets. Each firm will locate at the mid point of each part: $x_A = \frac{a}{2}$ and $x_B = \frac{1+b}{2}$.

Definition 1 *A firm adopts a niche marketing strategy, when it chooses both its price and its location to maximize its profits from the niche market only.*

Let the marginal consumer who is indifferent between buying from A or B to be located at $x < a$. The profits of A are:

$$\pi_A = \frac{P_A \cdot x}{m}.$$

As B sells to both the main and the niche markets, its profits are:

$$\pi_B = \frac{P_B \cdot (a - x)}{m} + \frac{P_B \cdot (1 - b)}{n}.$$

We first derive the profits of each firm. We have the following Proposition.

Proposition 2 *The profits of the firm who serves the main market only are:*

$$\pi_A = \frac{(x_B - x_A)(-2(-1+b)m + (2a + x_A + x_B)n)^2 t}{18mn^2}. \tag{3}$$

The profits of the firm who covers both the main and the niche markets are:

$$\pi_B = \frac{(x_B - x_A)(4(-1+b)m + (-4a + x_A + x_B)n)^2 t}{18mn^2}. \tag{4}$$

And the indifferent consumer is located at

$$x = \frac{x_A + x_B}{6} + \frac{an}{3(-1+b+n)} = \frac{x_A + x_B}{6} + \frac{m}{3}. \tag{5}$$

The proof of the above Proposition and all the remaining proofs are in the Appendix.

We now examine the location choice of the two firms. We find that the location of the two firms in not affected directly by the "gap" $b - a$ in the preferences of the two groups of consumers. We show that as competition takes place only in the main market, what matters is the density of the main market.[6] More particular, A will always locate at $x = 0$ (the standard result holds as A competes with B in the main market and transportation cost is quadratic). However, B may not locate at $x = 1$ as the standard result in this case suggests. The second firm locates at the other endpoint of the market, that is, $x_B = 1$ only when the density $\frac{1}{m}$ in the main market is low. Under these parameter values, the existence of the niche does not influence the decision of B:

[6] Obviously, high density in the main market implies low density in the niche market $\frac{1}{n}$ if α and b are constant. In general, from (1) we have that the variables α,b,m,n are not independent.

The benefit when B moves closer to A is small and the standard result of maximum differentiation holds. However, as the density of the main market increases, we may have that $b < x_B < 1$ or even $a < x_B < b$. Indeed, as the density of the main market increases, B has an incentive to move closer to A, as by doing that it can attract a large number of consumers. In such case the demand effect outweights the decrease in prices due to the decrease in product differentiation (when B moves closer to A). Finally, the second firm will never locate at the main market (that is, $x_B > a$) as the standard result when we have quadratic transportation cost implies. Recall that in such case, the optimal locations for the two firms are outside the unit interval of the market in which they compete, which here is the interval $[0, \alpha]$. We have the following Proposition:

Proposition 3 *In the Hotelling's model when the total market consists of the main market and a niche market we have:*

When $\frac{(1+2m)}{6} < \alpha$ *and* $\frac{1}{m} \leq \frac{4}{3}$ *the two firms compete in the main market and* $x_A = 0$, $x_B = 1$.

When $\frac{5}{9} < \frac{\alpha}{m}$ *and* $\frac{1}{m} > \frac{4}{3}$ *the two firms compete in the main market and* $x_A = 0$, $x_B = \frac{4m}{3} < 1$.

We also have that:

Firm B *locates at* $b \leq x_B < 1$ *if and only if*

$$\frac{4}{3} < \frac{1}{m} \leq \frac{4}{3b},$$

and at $x_B < b$ *if and only if:*

$$\frac{4}{3b} < \frac{1}{m}.$$

Furthermore, we can not have $x_B \leq a$.

We further derive the profits of the two firms. When the density of the main market is high, although B has an incentive to move closer to A (we have shown that $x_B < 1$), the profits of A are always greater than the profits of B. This happens as (a) the profits of B in the niche market are relatively small and as (b) B has less profits due to the "gap" in the preferences. The profits of A are still higher than the profits of B when the density decreases: I.e. when $1 < \frac{1}{m} < \frac{4}{3}$: In such case B locates at the other endpoint of the product space ($x_B = 1$). Although the increase in differentiation allows B to increase its profits, the existence of the "gap" in the preferences of the consumers makes its profits smaller than the profits of A. However, the opposite occurs for $1 > \frac{1}{m}$.

In such case, α is relatively large (as we have assumed that $\frac{\alpha}{m} > \frac{1}{2}$) and firm A should decrease its price to attract its "distant" customers. At the same time if (a) α is relatively closer to b, B need not decrease its price by much to attract consumers from the main market, if (b) b is closer to 1 (with $b-a$ to be nondecreasing), the niche market is concentrated around a small interval (as $1-b$ decreases) and B can obtain higher profits from the niche market. As a result, B can have higher profits than A. We can write the profits of both firms as:

Proposition 4 *When $x_B < 1$, and for $V_c \geq \frac{145m^2t}{81}$ the profits of A are: $\pi_A = \frac{200m^2t}{243}$ and the profits of B are: $\pi_B = \frac{128m^2t}{243}$.*

When $x_B = 1$, and for $V_o \geq \frac{(1+2m)(13+2m)t}{36}$ the profits of A are $\pi_{Ac} = \frac{(1+2m)^2t}{18m}$ and the profits of B are: $\pi_{Bo} = \frac{(1-4m)^2t}{18m}$.

We now examine the pricing decisions of the two firms when B adopts a niche marketing strategy. In such case, B relocates and prices its product to maximize its profits by selling only to the consumers in the interval $[b, 1]$. As in our analysis we assume that there are only two firms in the market, in such case we have two local monopolies. A will be the only firm in the main market and B the only firm in the niche.

1.3 Serving both or the "niche" market only

We now examine the incentives of B to adopt a "niche marketing" strategy. That is, we examine the conditions under which firm B serves only the niche market and (as in this specification there are only two firms in the market) firm A serves only the main market. As a result, the total market is consisted of two local monopolies. In the analysis we still assume that the whole market is covered. In such case firm A is located at the center of the main market at $\frac{\alpha}{2}$ and firm B is located at the center of the niche market at $\frac{1+b}{2}$.[7] We first derive the profits of firm A (which are denoted by π_{Am}) and the profits of B (which are denoted by π_{Bm}). We have the following Proposition

Proposition 5 *When we have two local monopolies, A covers the main market when $V \geq V_c =$*

[7] Here, we assume that both firms reposition their products. Someone can assume that the locations of both firms do not change. In such case, both firms change their prices only. Obviously, without repositioning, their profits will be smaller if A and B still cover the whole market.

$\frac{\alpha^2 t}{4}$ *and its profits are:*

$$\pi_{Am} = \frac{4\alpha V - \alpha^3 t}{4m},$$

Similarly, firm B covers the niche market when $V \geq V_m = \frac{(1-b)^2 t}{4}$ *and its profits are:*

$$\pi_{Bm} = \frac{(1-b)(4V - (1-b)^2 t)}{4n}.$$

The above analysis allow us to derive the conditions under which firm B decides to serve the niche market only.

It is easy to show that the profits of A increase, as it becomes a monopolist in the main market. As in such case it locates at the center of the main market, most consumers suffer less transportation cost when they buy from A. As a result, A can increase its profits by setting a higher price. Also, its market share inceases as it serves all the consumers in the main market. The result holds both when we consider A's profits when B is located inside the characteristics space, and at $x_B = 1$. We have the following Proposition for the profits of A.

Proposition 6 *The profits of A increase if A is the only firm that serves the main market.*

We now examine the profits for B as V increases. From Proposition 4, when B competes with A and the two firms cover the whole market the profits of B do not depend on V. Competition forces both firms to adopt more aggressive pricing strategies and as a result the marginal consumer may have positive utility. The profits of B when it adopts a nich marketing strategy and forms a local monopoly increase with V: The monopoly can increase its profits up to the level that makes the utility of the marginal consumer equal to zero. As a result, for sufficiently high V it is profitable for B to adopt a nich marketing strategy. We first derive the V for which the profits of B are the same when B is a local monopoly and (a) when it competes and $x_B < 1$ and (b) when it competes and $x_B = 1$: We denote by V_{eq} the value of V for which $\pi_{Bm} = \pi_B$ and by V_{eqo} the value of V for which $\pi_{Bm} = \pi_{Bo}$. From above, the profits of B when it adoprs a nich marketing strategy are higher when $V > V_{eq}$ or $V > V_{eqo}$.

The comparison of profits is possible for the maximum of the acceptable values of V that we have with and without competition. From above, the minimum value of V for the monopoly is V_m and the minimum value for competition is V_c (when $x_B < 1$) and V_o (when $x_B \geq 1$). The comparison is thus meaningful for $V \geq \max\{V_c, V_m\}$ and for $V \geq \max\{V_o, V_m\}$.

Notice that we always have $V_{eq} > V_m$ (and $V_{eqo} > V_m$). This happens as V_m is the V for which $\pi_{Bm} = 0$, whereas, V_{eq} (or V_{eqo}) is the V for which $\pi_{Bm} - \pi_B = 0$ (or $\pi_{Bm} - \pi_{Bo} = 0$). We have three possible cases:

When $\max\{V_c, V_m\} = V_c$, and $V_c < V_{eq}$ we have that $\pi_{Bm} < \pi_B$ for $V_c < V < V_{eq}$ and $\pi_{Bm} > \pi_B$ for $V > V_{eq}$.

When $\max\{V_c, V_m\} = V_c$, and $V_c > V_{eq}$ we only have that $\pi_{Bm} > \pi_B$ as the market is covered in competition only when $V > V_c$.

When $\max\{V_c, V_m\} = V_m$, we have that $\pi_{Bm} < \pi_B$ for $V_m < V < V_{eq}$ and $\pi_{Bm} > \pi_B$ for $V > V_{eq}$.[8]

We first assume that $x_B < 1$. There are two cases: For high values of m (which suggests low density in the main market) we have $V_c > V_m$. As the density $\frac{1}{m}$ in the main market in such case is low, to have competition consumers should have high value V for the product. This would guarrantee that the marginal consumer will have positive profits and the whole market will be covered. The opposite happens for low values of m as in such case α will be relatively closer to the location of A. Recall that $x_B < 1$ when $m < \frac{3}{4}$. We have:

Proposition 7 *$V_c > V_m$ if and only if,*

$$m \in \left(\frac{9(1-b)}{2\sqrt{145}}, \frac{3}{4} \right].$$

If $V_c > V_m$ and in addition we have $V_{eq} > V_c$, then $\pi_{Bm} < \pi_B$ for $V_c < V < V_{eq}$, and $\pi_{Bm} > \pi_B$ for $V > V_{eq}$. If on the other hand $V_{eq} < V_c$ then $\pi_{Bm} > \pi_B$ for $V > V_c$.

We now examine what happens when m is small. In such case the density in the main market is high and we can have the competition outcome for small values of V. From above, when $m < \frac{9(1-b)}{2\sqrt{145}}$ we have $V_c < V_m$. We further show:

[8] The three cases then $x_B = 1$ are:

When $\max\{V_o, V_m\} = V_o$, and $V_o < V_{eqo}$ we have that $\pi_{Bm} < \pi_{B_o}$ for $V_o < V < V_{eqo}$ and $\pi_{Bm} > \pi_{B_o}$ for $V > V_{eqo}$.

When $\max\{V_o, V_m\} = V_o$, and $V_o > V_{eqo}$ we have that $\pi_{Bm} > \pi_{B_o}$ as only for $V > V_o$ the market is covered in competition.

When $\max\{V_o, V_m\} = V_m$, we have that $\pi_{Bm} < \pi_{B_o}$ for $V_m < V < V_{eqo}$ and $\pi_{Bm} > \pi_{B_o}$ for $V > V_{eq}$.

Proposition 8 *We have:*

$$V_{eq} = \frac{(243\alpha(1-b)^2 - m(243 - 486b + 243b^2 + 512m^2))t}{972(\alpha - m)}.$$

Furthermore, $V_{eq} > V_m$.

As $V_{eq} > V_m$ we have $\pi_B > \pi_{Bm}$ for $V \in (V_m, V_{eq})$, and $\pi_B < \pi_{Bm}$ for $V > V_{eq}$.

We now examine the case where $x_B = 1$. Recall that this occurs for $m \geq \frac{3}{4}$. In such case we have:

Proposition 9 *For* $m \geq \frac{3}{4}$ *we have:*

$$V_{eqo} = \frac{(-2 + 9\alpha(-1+b)^2 + (7 + 18b - 9b^2)m - 32m^2)t}{36(\alpha - m)}.$$

Furthermore, $V_o > V_m$.

As a result, when $V_{eqo} > V_o$, for $V \in (V_o, V_{eqo})$ we have $\pi_{Bo} > \pi_{Bm}$, and for $V > V_{eqo}$ we have $\pi_{Bo} < \pi_{Bm}$. When $V_{eqo} < V_o$, for $V > V_o$ we have $\pi_{Bo} < \pi_{Bm}$ as the comparison of profits is meaningful for $V > V_o$.

2 Conclusion

In the paper we modify the standard Hotelling duopoly model to examine a market with a main part and a niche. We initially assume that both firms compete in the main market, and derive the optimal location for both firms. We then examine the incentives of firm B to adopt a niche marketing strategy. To do that it repositions its product, and change its pricing strategy accordingly. We derive the conditions under which a niche marketing strategy is preferable for a firm. In our analysis we are very general regarding the size of the two parts of the market as we only require that the main market is greater than half of the total market. On the other hand, by adopting the duopoly model we end up with two local monopolies when B adopts the niche strategy. Although this is restrictive and it would be more reasonable to assume that there are more firms serving the main market, this analysis provides a bound on the size of profits in the main market that would induce a firm to focus on the niche market only. Furthermore, the analysis can be extended to incoporate markets with more than one niches.

3 Appendix

Proof of Proposition 2:

Proof. From (2) we have that the marginal consumer who is indifferent from buying from either firm is located according to

$$V - t(x - x_A)^2 - P_A = V - t(x - x_B)^2 - P_B,$$

which gives:

$$x = \frac{P_A - P_B + t\left({x_A}^2 - {x_B}^2\right)}{2t(x_A - x_B)}. \tag{6}$$

As described above we assume that

$$\frac{P_A - P_B + t\left({x_A}^2 - {x_B}^2\right)}{2t(x_A - x_B)} < a.$$

As the whole population has mass equal to one, the profits of A can be written as:

$$\frac{P_A \cdot x}{m} = P_A \cdot \left(\frac{P_A - P_B + t\left({x_A}^2 - {x_B}^2\right)}{m2t(x_A - x_B)}\right). \tag{7}$$

The FOC with respect to P_A gives:

$$P_A = \frac{P_B - t\left({x_A}^2 - {x_B}^2\right)}{2}. \tag{8}$$

Similarly, the profits of B are:

$$\frac{P_B \cdot (a - x)}{m} + \frac{P_B \cdot (1 - b)}{n} = \tag{9}$$

$$\frac{P_B}{m} \cdot \left(a - \frac{\left(P_A - P_B + t\left({x_A}^2 - {x_B}^2\right)\right)}{2t(x_A - x_B)}\right) + \frac{P_B}{n} \cdot (1 - b)\,. \tag{10}$$

The FOC with respect to P_B gives:

$$P_B = \frac{nP_A + (x_A - x_B)\left(2(-1 + b)m + (-2a + x_A + x_B)n\right)t}{2n}. \tag{11}$$

If we insert P_B from (11) we get P_A from (8). We have:

$$P_A = -\frac{(x_A - x_B)\left(2m - 2bm + 2an + x_A n + x_B n\right)t}{3n}. \tag{12}$$

If we insert (12) in (11) we have:

$$P_B = \frac{(x_A - x_B)\left(4(-1 + b)m + (-4a + x_A + x_B)n\right)t}{3n}. \tag{13}$$

If we insert P_A from (12) and P_B from (13) in (7) we have the profits of A.

$$\pi_A = \frac{(x_B - x_A)(-2(-1+b)m + (2a + x_A + x_B)n)^2 t}{18mn^2}. \tag{14}$$

If we replace (12) and (13) in (9) we have the profits of B:

$$\pi_B = \frac{(x_B - x_A)(4(-1+b)m + (-4a + x_A + x_B)n)^2 t}{18mn^2}. \tag{15}$$

Finally, if we insert P_A from (12) and P_B from (13) in (5) we have the location of the marginal consumer:

$$x = \frac{x_A + x_B}{6} + \frac{an}{3(-1+b+n)} = \frac{x_A + x_B}{6} + \frac{m}{3}.$$

■

Proof of Proposition 3:

Proof. We first assume that $a \geq \frac{1}{2}$. From (14) the derivative of the profits of A with respect to x_A is:

$$\frac{\partial \pi_A}{\partial x_A} = -\frac{(4(-1+b)^2m^2 - 8(a+x_A)(-1+b)nm + (4a^2 + 8ax_A + 3x_A^2 + 2x_Ax_B - x_B^2)n^2)t}{18mn^2} \tag{16}$$

The above is negative for $a \geq \frac{1}{2}$ and $b < 1$. Thus, $x_A = 0$.

From (15) the derivative of the profits of B with respect to x_B is:

$$\frac{\partial \pi_B}{\partial x_B} = \frac{(4(-1+b)m - (4a + x_A - 3x_B)n)(4(-1+b)m + (-4a + x_A + x_B)n)t}{18mn^2}. \tag{17}$$

We solve (17) with respect to x_B. We have

$$x_B = \frac{4(m - bm + an) + nx_A}{3n} \text{ and } x_B = \frac{4(m - bm + an) - nx_A}{n}. \tag{18}$$

From above we have that $x_A = 0$. If we also use (1) we can write the two roots as:

$$x_B = \frac{4an}{3(-1+b+n)} \text{ and } x_B = \frac{4an}{(-1+b+n)}.$$

The second order condition gives:

$$\frac{\partial^2 \pi_B}{\partial x_B^2} = \frac{(8(-1+b)m - 8an + nx_A + 3x_Bn)t}{9mn} = \frac{(-8an + 3x_B(-1+b+n))t}{9an} \tag{19}$$

At $x_B = \frac{4an}{3(-1+b+n)}$ (19) becomes: $\frac{\partial^2 \pi_B}{\partial x_B^2} = -\frac{4t}{9}$ and we have a maximum, whereas at $x_B = \frac{4an}{(-1+b+n)}$ (19) becomes: $\frac{\partial^2 \pi_B}{\partial x_B^2} = \frac{4t}{9}$ and we have a minimum. Thus, the acceptable root is: $x_B = \frac{4an}{3(-1+b+n)}$.

From (1) we have that $m = \frac{an}{-1+b+n}$. Thus, we can write,

$$\frac{4an}{3(-1+b+n)} \geq 1 \Leftrightarrow \frac{4m}{3} \geq 1 \Leftrightarrow \frac{4}{3} \geq \frac{1}{m},$$

the second firm is located at $x_B = 1$. However, for $\frac{4an}{3(-1+b+n)} < 1$, the optimal $x_B = \frac{4an}{3(-1+b+n)} < 1$. We have $x_B < b$ if and only if

$$\frac{4}{3b} < \frac{1}{m}.$$

Finally, we have $b \leq x_B < 1$ if and only if

$$\frac{4}{3} \leq \frac{1}{m} < \frac{4}{3b}.$$

Notice that $x_B > a$. Indeed, $x_B \leq a$ implies that $\frac{4an}{3(-1+b+n)} = \frac{4m}{3} \leq a$ which holds if and only if

$$\frac{4}{3} \leq \frac{a}{m},$$

which is not possible.

Furthermore, form (6) we have that:

$$x = \frac{5\alpha n}{9(-1+b+n)} = \frac{5m}{9}.$$

As we require $x < a$ we have:

$$\frac{5m}{9} < \alpha \Rightarrow \frac{5}{9} < \frac{\alpha}{m}.$$

We now assume that $a < \frac{1}{2}$. From (16), when $a < \frac{1}{2}$, the derivative of the profits of A with respect to x_A is zero when

$$x_A = \frac{2(-1+b)m - (2a + x_B)n}{n} \text{ and } x_A = \frac{2(-1+b)m + (-2a + x_B)n}{3n}.$$

The second derivative is:

$$\frac{\partial^2 \pi_A}{\partial x_A^2} = \frac{(4(-1+b)m - (4a + 3x_A + x_B)n)t}{9mn}. \tag{20}$$

Notice that $x_A < 0$ at the the first root.

At the second root $x_A = \frac{2(-1+b)m+(-2a+x_B)n}{3n}$ (20) give

$$\frac{\partial^2 \pi_A}{\partial x_A^2} = \frac{2((-1+b)m - (a + x_B)n)t}{9mn} < 0,$$

so we have a maximum.

If we replace in (19), $x_B = \frac{4(m-bm+an)-nx_A}{3n}$ from (18) we get:

$$\frac{\partial^2 \pi_B}{\partial x_B^2} = -\frac{2(2(-1+b)m+(-2a+x_A)n)t}{9mn} > 0,$$

as we have assumed that $x_A < a$. That is, we have a minimum.

If we replace in (19), $x_B = \frac{4(m-bm+an)+nx_A}{n}$ from (18) we get:

$$\frac{\partial^2 \pi_B}{\partial x_B^2} = \frac{2(2(-1+b)m+(-2a+x_A)n)t}{9mn} < 0,$$

that is, for every x_A, when $x_B = \frac{4(m-bm+an)+nx_A}{n}$ we have a maximun.

We therefore have

$$x_A = \frac{2(-1+b)m+(-2a+x_B)n}{3n} \text{ and} \quad (21)$$

$$x_B = \frac{4(m-bm+an)+nx_A}{3n} \quad (22)$$

as the candidates for a maximum.

However, if we solve the above system with respect to x_A and x_B we get:

$$x_A = -\frac{m-bm+an}{4n} < 0 \text{ and } x_B = \frac{5(m-bm+an)}{4n} > 0. \quad (23)$$

The above solution is not acceptable. That is we can not have both $x_A > 0$ and $x_B < 1$.

Notice that when $x_B = \frac{4(m-bm+an)+nx_A}{3n}$, the first derivative of profits of the first firm wrt x_A is negative:

$$\frac{\partial \pi_A}{\partial x_A} = - \frac{2(5(-1+b)^2m^2 - 2(5a+11x_A)(-1+b)nm) + (5a^2+22ax_A+8x_A^2)n^2)t}{81mn^2} < 0$$

Therefore the optimal location for the first firm is $x_A = 0$. Therefore we have a Nash equilibrium. For $x_A = 0$ we have

$$x_B = \frac{4(m-bm+an)+nx_A}{3n} = \frac{4an}{3(-1+b+n)} = \frac{4m}{3}$$

And the analysis is the same as in the $\alpha \geq \frac{1}{2}$ case.

We now show that this is the only equilibrium.

From (21), and (1) we have that x_A may be positive when

$$\frac{2(-1+b)m+(-2a+x_B)n}{3n} \Rightarrow x_B > \frac{2an}{-1+b+n}.$$

Notice that for $x_A = \frac{2(-1+b)m+(-2a+x_B)n}{3n}$, we have:

$$\frac{\partial \pi_B}{\partial x_B} = \tag{24}$$

$$\frac{2(7(-1+b)m-7an+2x_Bn)(5(-1+b)m-5an+4x_Bn)t}{81mn^2}. \tag{25}$$

The above is negative in the interval

$$x_B \in \left(\frac{5an}{4(-1+b+n)}, \frac{7an}{2(-1+b+n)}\right).$$

As a result, when x_B belongs in this interval, the profits of B increase as x_B decreases: Fecall that, for $x_A > 0$, we should have x_B satisfying:

$$x_B > \frac{2(m-bm+an)}{n} = \frac{2an}{-1+b+n}.$$

As

$$\frac{5an}{4(-1+b+n)} < \frac{2an}{-1+b+n} < \frac{7an}{2(-1+b+n)},$$

when x_A is positive, firm B will locate at $\frac{5an}{4(-1+b+n)}$ (which is smaller than $\frac{2an}{-1+b+n}$) to maximize its profits. However, in such case x_A can not be positive.

Furthermore, form (5) we have that:

$$x = \frac{1}{6} + \frac{m}{3}.$$

As we require $x < a$ we have:

$$\frac{1}{6} + \frac{m}{3} < \alpha.$$

■

Proof of Proposition 4:

Proof. The profits of A and B can be derived if we replace the appropriate x_A and x_B from the previous Proposition in (3) and (4).

We now derive the minimum value of V for which we have the competition outcome. As the location of B varies, we have two possible Vs. One when $x_B < 1$ and one when $x_B = 1$. From (5), when $x_A = 0$ and $x_B = \frac{4m}{3} < 1$ the marginal consumer has utility:

$$V - t(x - x_A)^2 - P_A = \frac{-145n^2\alpha^2 t + 81(-1+b+n)^2 V}{81(-1+b+n)^2}.$$

Let

$$V_c = \frac{145n^2\alpha^2 t}{81(-1+b+n)^2} = \frac{145m^2 t}{81}$$

be the minimum value for which the marginal consumer has nonegative utility. From above we require that

$$V > V_c \tag{26}$$

for competition.

Similarly, when $x_A = 0$ and $x_B = 1$ the minimum value for which the marginal consumer has nonegative utility is:

$$V_o = \frac{(1+2m)(13+2m)t}{36}.$$

■

Proof of Proposition 5:

Proof. The two local monopolies are located at the center of the markets they serve. A is locaed at $\frac{\alpha}{2}$ and B at $\frac{1+b}{2}$. Firm A sets its price P_{Am} so that the marginal consumer (who is located at α) has zero surplus:

$$V - t\left(\alpha - \frac{\alpha}{2}\right)^2 - P_{Am} = 0 \Rightarrow P_{Am} = \frac{(4V - \alpha^2 t)}{4},$$

and its profits are:

$$\pi_{Am} = P_{Am} \cdot \left(\frac{\alpha}{m}\right) = \frac{4\alpha V - \alpha^3 t}{4m}. \tag{27}$$

The profits are positive for $V \geq \frac{\alpha^2 t}{4}$.

Firm B sets its price P_{Bm} so that the marginal consumer (who is locatd at b) has zero surplus:

$$V - t\left(b - \frac{1+b}{2}\right)^2 - P_{Bm} = 0 \Rightarrow P_{Bm} = \frac{(4V - b^2 t + 2bt - t)}{4},$$

and its profits are:

$$\pi_{Bm} = P_{Bm} \cdot \left(\frac{1-b}{n}\right) = \frac{(1-b)(4V - (1-b)^2 t)}{4n}. \tag{28}$$

The profits are positive for $V \geq \frac{(1-b)^2 t}{4}$. ■

Proof of Proposition 6:

Proof. From (3) we have the profits of A. Notice that we have shown that $x_A = 0$. We first assume that $x_B = \frac{4an}{3(-1+b+n)} < 1$ (which holds for $m < \frac{3}{4}$). We have:

$$\pi_A = \frac{200n^2\alpha^2 t}{243(-1+b+n)^2} = \frac{200m^2 t}{243}.$$

As π_{Am} from (27) are increasing with respect to V, and π_A do not depend on V, to show that $\pi_{Am} > \pi_A$ it suffices to show that this holds for the minimum acceptable V in competition as it is described in (26). At the minimum acceptable V we have:

$$\pi_{Am}\,|_{V_c} = \frac{4\alpha\left(\frac{145m^2t}{81}\right) - \alpha^3 t}{4m} = -\frac{\alpha^3 t}{4m} + \frac{145\alpha m t}{81}$$

and

$$\pi_A - \pi_{Am}\,|_{V_c} = 800m^3 + 243\alpha^2 - 1740\alpha m^2.$$

If we divide all parts by $\alpha m^2 > 0$ and set $y = \frac{\alpha}{m}$ we can write

$$\pi_A - \pi_{Am}\,|_{V_c} = 800\left(\frac{1}{y}\right) + 243y^2 - 1740.$$

The difference is negative if and only if

$$800\left(\frac{1}{y}\right) + 243y^2 - 1740 < 0. \tag{29}$$

The above has roots: $y_1 = -2.88$, $y_2 = 0.47$ and $y_3 = 2.40$. Notice that we have assume that $\frac{\alpha}{m} \in \left(\frac{1}{2}, 1\right)$, We have that $\frac{1}{2} < y < 1$. The proof for this part is complete as for $y \in \left(\frac{1}{2}, 1\right)$ (29) is negative.

We now assume that $x_B = 1$, which holds for $m \geq \frac{3}{4}$. Notice that as $\frac{1}{2} \leq \frac{a}{m} < 1$, we have $\frac{m}{2} \leq \alpha < m$. In such case, the profits of A are

$$\pi_{Ac} = \frac{(1+2m)^2 t}{18m}$$

and the difference in profits for the minimum acceptable V in competition is:

$$\pi_{Am}\,|_{V_c} - \pi_{Ac} = -\frac{\left(81\alpha^3 - 580\alpha m^2 + 18(1+2m)^2\right) t}{324m}.$$

The derivative with respect to m of the difference is:

$$\frac{\partial(\pi_{Am}\,|_{V_c} - \pi_{Ac})}{\partial m} = \frac{\left(18 + 81\alpha^3 - 72m^2 + 580\alpha m^2\right)t}{324m^2}.$$

The above is positive as $\frac{m}{2} \leq \alpha$. It suffices to show that the difference is positive for $m = \frac{3}{4}$. For that m we have:

$$\left(\pi_{Am}\,|_{V_c} - \pi_{Ac}\right)\Big|_{m=\frac{3}{4}} = -\frac{\left(50 - 145\alpha + 36\alpha^3\right)t}{108}. \tag{30}$$

As we have assumed that $\frac{\alpha}{m} > \frac{1}{2} \Rightarrow \alpha > \frac{m}{2}$ and also we consider the case in which $m \geq \frac{3}{4}$ we have that $\alpha > \frac{3}{8}$. Recall that we also have that $\alpha < 1$. For $\frac{3}{8} \leq \alpha < 1$ the difference in (30) is positive. ■

Proof of Proposition 7:

Proof. We examine the relation between V_c and V_m. If we solve $V_c - V_m$ with respect to m we get:

$$V_c - V_m = 0 \Rightarrow \frac{145m^2t}{81} - \frac{(1-b)^2t}{4} = 0 \Rightarrow$$
$$m_0 = -\frac{9(1-b)}{2\sqrt{145}} \text{ and } m_1 = \frac{9(1-b)}{2\sqrt{145}}.$$

We have that $V_c > V_m$ if and only if $\frac{3}{4} \geq m > m_1$. ■

Proof of Proposition 8:

Proof. We assume that $x_B < 1$. From (4) for $x_A = 0$ and $x_B < 1$ we have:

$$\pi_B = \frac{128m^2t}{243}$$

The profits of B when it covers the niche market only, are:

$$\pi_{Bm} = \frac{(1-b)(4V - (1-b)^2t)}{4n} = \frac{(m-\alpha)(4V - (1-b)^2t)}{4m}.$$

The minimum value for V for which $\pi_B = \pi_{Bm}$ is given by solving:

$$\pi_B - \pi_{Bm} = 0 \Rightarrow$$
$$\frac{128m^2t}{243} - \frac{(m-\alpha)(4V - (1-b)^2t)}{4m} = 0 \Rightarrow$$
$$V_{eq} = \frac{(243\alpha(1-b)^2 - m(243 - 486b + 243b^2 + 512m^2))t}{972(\alpha - m)}.$$

We have $V_{eq} > V_m$ as:

$$V_{eq} - V_m =$$
$$\frac{(243\alpha(1-b)^2 - m(243 - 486b + 243b^2 + 512m^2))t}{972(\alpha - m)} - \frac{(1-b)^2 t}{4} =$$
$$-\frac{128m^3 t}{243(\alpha - m)} > 0.$$

■

Proof of Proposition 9:

Proof. We now assume that $x_B = 1$. From (4) for $x_A = 0$ and $x_B = 1$ we have:

$$\pi_{Bo} = \frac{(4(-1+b)m + n - 4an)^2 t}{18mn^2} = \frac{(1-4m)^2 t}{18m}$$

The profits of B when it covers the niche market only, are:

$$\pi_{Bm} = \frac{(1-b)(4V - (1-b)^2 t)}{4n} = \frac{(m-\alpha)(4V - (1-b)^2 t)}{4m}.$$

The minimum value for V for which $\pi_{Bo} = \pi_{Bm}$ is given by solving:

$$\pi_{Bo} - \pi_{Bm} = 0 \Rightarrow$$
$$\frac{(1-4m)^2 t}{18m} - \frac{(m-\alpha)(4V - (1-b)^2 t)}{4m} = 0 \Rightarrow$$
$$V_{eqo} = \frac{(-2 + 9\alpha(-1+b)^2 + (7 + 18b - 9b^2)m - 32m^2)t}{36(\alpha - m)}.$$

Notice that $V_o > V_m$. This is because their difference is positive even when $1-b$ is maximized (which occurs when $b = 0$). We have

$$(V_o - V_m)\,|_{b=0} = \frac{(1+2m)(13+2m)t}{36} - \frac{(1-b)^2 t}{4} = \frac{(1 + 7m + m^2)t}{9} > 0.$$

■

References

[1] S. P. Anderson, J. K. Goeree and R. Ramer (1997), Location, location location, *Journal of Economic Theory*, 77, 102-127.

[2] A. Ansari, N. Economides, and A. Ghosh (1994), Competitive Positioning in Markets with Nonuniform Preferences, *Marketing Science*, 13, 3, 248-273.

[3] T. Daldic and M. Leeuw (1994), Niche Marketing Revisited: Concept, Applications and Some European Cases, *European Journal of Marketing*, 28,4, 39-55.

[4] J. Dearmon and G. Kosmopoulou (2008), Two Dimensional Spatial Competition with Potential Market Size Effects, *working paper*, University of Oklahoma.

[5] P. Kotler (2003), Marketing Management, 11th ed., Prentice-Hall, Upper Saddle River, NJ.

[6] D. Neven (1986), On Hotelling's competition with non-uniform consumer distributions, *Economics Letters*, 21, 121-126.

[7] E. D. Parrish, N. L. Cassill, and W. Oxenham (2004), Opportunities in the international textile and apparel marketplace for niche markets, *Journal of Fashion Marketing and Management*, 8,1, 41-57

[8] E. D. Parrish, N. L. Cassill, and W. Oxenham (2004), Modeling of niche market behavior of the US textile and apparel firms, *Journal of Textile and Apparel Management*, 4.

[9] E. D. Parrish, N. L. Cassill, and W. Oxenham (2006), Niche market strategy for a mature marketplace, *Marketing Intelligence and Planning* 24, 7, 694-707.

[10] L. Roberts and D. Hall (2004), Consuming the countryside: marketing for 'rural tourism'. *Journal of Vacation Marketing*, 10,3, 253-263.

[11] K. Serfes, and E. Zacharias (2012), Location Decisions of Competing Networks, *Journal of Economics & Management Strategy*, 21,4, 989-1005.

[12] T. Tabuchi and J.-F. Thisse (1995), Asymmetric equilibria in spatial competition, *International Journal of Industrial Organization*, 13, 213-227.

[13] V. Tamagnini and A. Tregear (1998), An assessment of niche marketing opportunities in the delicatessen meat sector, *British food journal*, 100,5, 228-235.