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"Last Resort Gambles, Risky Debt and Liquidation Policy"

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LAST RESORT GAMBLES, RISKY DEBT AND LIQUIDATION POLICY*

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Abstract. This paper develops a real option model in which the interaction between debt, liquidation policy and risky investments is studied. We consider a manager who owns the firm and faces the opportunity to invest in risky projects which may boost current profits at the cost of bankruptcy if they turn out to be unsuccessful. These investments are "last resort gambles" in the sense that, if successful, they save the company from insolvency, while, if unsuccessful, they make liquidation unavoidable. We show that last resort gamble strategies delay liquidation. We study how the liquidation and the last resort gamble strategies are affected by the firm's capital structure.

Keywords: Last resort gambles; risky investments; liquidation policy; real options.

JEL: G3, G32, G33

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1 Introduction

In a situation of financial distress managers, acting as rational economic agents trying to resurrect their company, may end up engaging in “last resort gambles”, that is, they employ an investment strategy that, if successful, would save the company from insolvency, if unsuccessful, would make liquidation unavoidable. On the verge of bankruptcy managers employ this strategy with the aim to "weather out the storm", that is, they invest in risky projects trying to bridge bad and good times.

In this context it is often difficult to ascertain whether real investment are incurred with the objective to manipulate earnings or just for strategic considerations. Failures at Enron, WorldCom and Tyco in the US together with some other prominent companies in Europe (Vivendi, Ahold, Adecco, Parmalat, etc.) are iconic examples of corporate scandals combined with excessive risk taking which is influenced by moral hazard in the hopes of extraordinary returns that could rescue a company from bankruptcy (Lev, 2003; Jensen, 2004).

In this paper we consider a manager who owns the firm and chooses the debt policy and the liquidation policy. If the market is in a downturn the manager may exercise the option of liquidating the company, leaving the claimants with a positive liquidation value. At each point in time the manager faces also the opportunity to change the firm’s strategy by investing in a risky project. Such investment project may be part of a diversification strategy where the company invests in activities not strictly related with its core business. Specifically, we assume that at each point in time the project may either fail or not. In case of failure the company is inevitably forced into bankruptcy. On the other hand, given that the project is active, the project may either generate a positive cash flow or nothing, where the positive cash flow is assumed to be independent of the demand levels since it is not related with the firm core business. We model the failure probability and the positive cash flow probability of the project (conditional on survival of the project) as Poisson processes, where we assume that the lower (larger) is the mean waiting time of project failure, the lower (larger) is the mean waiting time of a first payoff. We show that the opportunity cost of the project, represented by the loss if the project fails, depends positively on the demand level. Since the expected gains from this risky investment strategy are independent of the demand levels of the company core business while its opportunity costs are increasing in the demand level, it turns out to be optimal to invest in this type of project in a market downturn. Thus, we interpret this particular form of risk-taking as a "last resort gamble", in that the firm is thrown into a new venture that, if successful, would save the company from insolvency and let it remain operative; if unsuccessful it would force the company into bankruptcy.

We characterize the firm's optimal liquidation policy and optimal gambling strategy showing reluctance of the manager to shut the company down. We use real option analysis to address the problem of optimal liquidation and gambling decisions. Such methodology is most appropriate since it allows us to model and quantify this risk-taking problem in a continuous time framework. Real options represent the formal modeling technique which is more suitable to serve the purposes of decision making in a dynamic context under uncertainty. Traditional static approaches are not proper in the study of liquidation policies, because of the high uncertainty and costs of irreversible decisions; moreover, they underestimate the upside potentials of the operational strategies. Real options methods allow to incorporate all these elements of corporate policy.[1]

In this paper we abstract from agency problems arising from a conflict between managers and equity-holders and analyze a model where the management owns the firm and a principal/agent problem eventually arises between debt-holders and the firm. Conflicts between debt-holders and equity-holders may arise because of the equity-holders incentive to invest in risky but poor projects, affecting the value of the debt. In this framework we study the impact of the company indebtedness on the optimal liquidation policy and how capital structure and bankruptcy decisions

[1] The literature on real options is vast. See, for example, Dixit and Pindyck (1994), Grenadier (2000), Schwartz and Trigeorgis (2001) and Smit and Trigeorgis (2004) for the methodology of real options. We recall here only a few recent contributions about investment, timing, agency and real options, that is, Grenadier and Wang (2005, 2007), Lambrecht and Perraudin (2003), Wong (2007). Our paper contributes to the debate about risk taking employing a real option methodology which has not been used to study both liquidation policy and optimal gambling strategies.

are affected by the investment strategy.

The problem we tackle in this paper is closely related to the literature on moral hazard and excessive risk-taking, for which many colorful descriptions have been used, namely "gambling for resurrection", "heads I win, tails I break even", "fourth-quarter football", etc. (Hart, 2000; Akerlof and Romer, 1993).

Our paper is closely aligned with the literature on excessive continuation induced by equity-holders' limited liability when a moral hazard problem arises between equity- and debt-holders. Knot and Vychodil (2006) examine debt contracting in the case of gambling for resurrection under different bankruptcy regimes in a 3-period model. They show that under the absolute priority rule (such that nothing can be paid to a class of claimholders unless the claims of all superior classes are satisfied) equity-holders tend toward excessive risk-taking and delaying bankruptcy filing; in contrast, a softer law or the possibility of creditors' verification of the firm's situation mitigate the problem of avoiding bankruptcy and represent an alternative solution to the gambling for resurrection problem. Decamps and Faure-Grimaud (2002), using a compound exchange option model, study a setting where excessive continuation always occur and such excessive continuation is even exacerbated as debt repayment increases. In our paper excessive continuation results from the last resort gamble strategy employed by the manager. We find excessive continuation both in the all-equity firm and in the case where an agency problem between equity- and debt-holders may arise.

The issue we address is linked to the asset substitution problem first discussed by Jensen and Meckling (1976), where the equity-holders wish to switch to a riskier portfolio after debt is issued, to transfer value from debt to equity[2] . This action leads to a delay in liquidation and excessive continuation. In Leland (1998) a firm can choose between two exogenous levels of the volatility of its value. In the leveraged case, it is shown that the choice which is optimal before issuing debt is

[2] The asset substitution problem has been much developed in the agency literature. Galai and Masulis (1976) first pointed out that a shareholder aligned manager faced with a choice between two different projects would invest in the project of higher variance. Jensen and Meckling (1976) even raise the possibility of a manager investing in a negative NPV investment for the sake of increasing the volatility of the firm's assets.

not the same after debt has been issued. Our problem is similar to asset substitution in the sense that the manager switches to a riskier project with his last resort gamble strategy. In the case of asset substitution equity-holders benefit from an increased upside volatility by engaging in riskier project. In our case, because of the last resort gamble strategy, equity-holders may benefit from increased current cash flows if the project is successful at the cost of an increased downside risk. Our model is more concerned with "real" asset substitution rather than with "financial" asset substitution since we consider a firm's real option of changing its strategy into a new branch of activity through its last resort gamble strategy. Real asset substitution instead of financial asset substitution has been studied also in Decamps and Djembissi (2007) where the authors tackle the problem of a company facing the option to switch irreversibly to a "poor" project, that is an activity whose cash flow dynamics are characterized by a geometric Brownian motion with a larger volatility and a lower drift.

Excessive risk-taking and bankruptcy postponing tendency of managers are often inextricably linked with a tendency of the management to misreporting. In some cases gambling for resurrection involves unlawful risk-taking, which means that it is a complementary strategy to earnings manipulation and corporate fraud[3] . When faced with the threat of firing, liquidation or in order to increase the value of stock options, managers are encouraged to take substantial risk and to boost short term profit through legal and sometimes fraudulent means. In some cases (see Johnson, Ryan and Tian, 2006) executives commit fraud to avoid under-performance resulting from significant slowdowns in their earnings growth, so that frauds are committed more likely during industry downturns.

Research on the determinants of fraud has indicated external financial needs (Povel, Singh and Winton, 2007), proximity to debt covenant violations and executive compensation as the main causes for violations of accounting principles and earnings manipulation. A few recent papers

[3] Earnings manipulation has been discussed in several papers, among them we recall Stein (1989), Narayanan (1985) and Von Thadden (1995).

have examined the relation between executive equity-based compensation and corporate fraud (Goldman and Slezak, 2006; Burns and Kedia, 2006; Bergstresser and Philippon, 2006; Gao and Shrieves, 2002; Bebchuk and Fried, 2003, 2004; Johnson, Ryan and Tian, 2006; Erickson,Hanlon and Maydew, 2006), and have emphasized that executives at fraud firms have significantly large equity-based compensation and greater financial incentives to commit fraud than executives at non-fraud firms. Anecdotal evidence suggests that analysts, investors and financial markets commentators often focus on firms' abilities to consistently increase earnings per share. A few papers have highlighted how earnings manipulation is not directly linked to an agency problem between managers and equity-holders (Bolton, Scheinkman and Xiong, 2005, 2006; Friebel and Guriev, 2005), and have found that top-management and initial shareholders have often aligned interests in over-reporting short-term earnings, because they can sell stocks at higher prices to uninformed outside investors who base their evaluations on the accounting reports.

We study how last resort gambles influence liquidation policy and the interaction between the firm's financial structure, gambling and closure decisions. In Section 2 we study the value of the firm if the firm faces the option of investing in a risky project, trying to boost current profits at the cost of liquidation, if the project fails. We find that engaging in this gamble is optimal in a market downturn: it increases the firm value and affects the firm's closure policy, delaying liquidation. Thus, in engaging in a last resort gamble, the firm bets on a market upturn, trying to bridge good and bad times.

In Section 3 we extend the basic setting to the case where the firm is financed by issuing debt and equity and where it must pay interest to its creditors continuously. We derive the equity-holders and debt-holders claims and study how the firm's financial structure influences optimal liquidation and gambling policies. Last resort gambling boosts the equity value, inducing a delay in liquidation. Furthermore, we find that increasing the firm's indebtedness speeds up liquidation. Debt financing mitigates the conflict between share- and debt-holders because debt service reduces the amount of free cash flows available to equity-holders (Myers, 1977; Jensen, 1986). A larger indebtedness reduces the equity-holders' gains from a last resort gamble, reducing the appeal of such a strategy. We find that increasing the company's indebtness reduces the range of demand levels for which it is optimal to engage in a last resort gamble. Moreover, we find a threshold level for the debt above which engaging in last resort gambling is never optimal. The intuition behind this result is the following. A last resort gamble, increasing the downside risk, makes the company discount future profits at a higher discount rate. Consequently, an increase in the coupon value increases the burden of debt service with a last resort gamble more than without a last resort gamble, reducing the incentives to invest in such a project. Moreover, for a sufficiently large coupon value the benefits of a last resort gamble are more than offset by its cost in terms of debt burden and thus it is never optimal to invest in such a project. We compare the debt value in the case of a last resort gamble and in the case where a last resort gamble is not available. We show that a last resort gamble has an ambiguous effect on the debt value and the debt capacity of the company. The delay in liquidation due to the risky investment strategy increases the probability that the company is going to service the debt and increases the debt value (delay-in-liquidation effect). On the other hand, the increased downside risk associated with the gamble reduces the debt value (increased downside risk effect) and thus the overall effect is ambiguous. We discuss which of the two effects prevail according to the parameter values, and, in particular, we show that the difference between the two debt values depends, among others, on the coupon value. A larger coupon value reduces the distortions induced by a last resort gamble and consequently it is more likely that last resort gambles increase the debt value.

In Section 4 we address the question of how an endogenous capital structure interacts with liquidation and gambling strategies. We model the benefits of debt issuance by introducing taxation so that the firm can take advantage of the tax shield on the coupon payment. Through numerical simulations we find that the coupon value that maximizes the firm value at time zero is increasing in the marginal tax rate and in the initial demand level. As a consequence, the larger is the marginal tax rate and/ or the initial demand level, the lower is the range of demand levels for which it is optimal to engage in a last resort gamble and the lower are the distortions in the liquidation policy induced by such a strategy.

Section 5 generalizes the model to the scenario where different degrees of gambling intensities are possible. While in Section 3 the firm can choose between engaging in a last resort gamble or not, in Section 5 we introduce a choice between different gambling intensities. We find that as the firm's indebtedness increases, gambling intensity decreases.

Section 6 contains the conclusion and final remarks. All proofs are in the Appendix.

2 The model

A firm generates total operating profits of $Kx_t - f$, where f is a fixed cost, K is a constant parameter, x_t a geometric Brownian motion representing exogenous demand shocks

$$dx_t = \mu x_t dt + \sigma x_t dB_t$$

μ is a drift term and σ measures volatility. As x falls, the firm faces the opportunity to close the activity irreversibly and is left with a constant liquidation value. Let H be the constant liquidation value of the firm net of bankruptcy costs. It is assumed that at each time period the firm can engage in a risky project which boosts current profits if successful at the cost of liquidation in case of failure. The firm acts to maximize the present value of the expected cash flows. In this section we specify the value of the firm in the absence of debt. Then, in Section 3, both the debt policy and the closure policy are considered and the value of the firm and the debt-holders' claims are specified.

At each time period the firm may invest in a risky project trying to inflate current operating profits at the cost of bankruptcy if the project fails. In particular, we assume that there are $i = 0, 1, 2, ...$ projects available, corresponding to different gambling strategies. In investing in a risky project the company incurs certain costs of size κ_i. If the project fails then bankruptcy is forced; if the project survives then it may either generate a positive cash flow or nothing.

We model the project failure and the positive cash flow event as Poisson processes with mean arrival rate p_i and $\frac{\gamma_i}{1+p_i}$, respectively, with $p_i, \gamma_i > 0$. Thus, over a given time period of length τ, the probability that the project does not fail is $e^{-p_i \tau}$ and, given that the project survived, the probability that the project pays a positive amount off is $1 - e^{-\frac{\gamma_i}{1+p_i}\tau}$. Hence, over an infinitesimal time interval dt the probability of project failure is $p_i dt$ and the probability of success, given that the project survived, is $\frac{\gamma_i}{1+p_i}dt$. The mean waiting time of a project failure, which in turn induces the company bankruptcy, is $\frac{1}{p_i}$ while the mean waiting time of a first success is $\frac{1+p_i}{\gamma_i}$ and thus the larger is p_i, the lower is the mean waiting time of bankruptcy and the larger is the mean waiting time of success. Thus, the larger is p_i, the lower is also the payoff frequency of the project, given its survival. In other words, the lower the survival probability of the project, the lower is also its positive payoff probability[4] . We assume that if the project is successful, then its positive payoff is K and the expected value of the project is positive, i.e. $\frac{\gamma_i}{1+p_i}K - \kappa_i > 0$.[5] Notice that the expected gains from the risky investment is independent of x, since it is an industrial project not related with the core business activity of the firm. We set $\gamma_0 = 0$, $p_0 = 0$ and $\kappa_0 = 0$. Thus, project 0 may be interpreted as the "business as usual" case. Projects $i > 0$ are risky, where we assume that $\frac{\gamma_{i+1}}{1+p_{i+1}} > \frac{\gamma_i}{1+p_i}$ and $p_{i+1} > p_i$, for each $i \geq 0$. Without loss of generality, we normalize $\kappa_i = 0$ for each $i > 0$ and thus interpret $\frac{\gamma_i}{1+p_i}Kdt$ as the net expected payoff of project i over an infinitesimal time interval dt. Note that as γ_i increases, that is the capability of boosting current profits by investing in the project increases, the probability of failure over an infinitesimal time period dt (and thus the riskiness of the project) has to increase in order to maintain the different gambling strategies relevant. We refer to γ_i as a measure of the intensity of the gambling strategy. By investing in a risky project a company tries to increase its current profits at the cost of increasing the downside risk.

[4] As will be shown further on, this assumption implies that for each $\gamma_i > 0$ there exists a finite threshold for p_i such that for p_i larger than this threshold it is never optimal to invest in the risky project since the increased downside risk and the associated expected loss is larger than the project expected gain.

[5] For simplicity, the positive payoff is normalized to K, the size parameter which appears in the expression of the total operative profits. As will be made clearer further on, it is not a restriction and what matters for the optimal gambling and liquidation strategies is that the expected value of the project is positive.

In this section we consider only two projects 0 and 1, that is, the company may either employ a gambling strategy (invest in project 1) or not (invest in project 0). In Section 5 we extend the framework to the case where the company may choose between different gambling intensities.

Let $V^i(x)$, $i = 0, 1$, be the company value in the case the firm invests in a risky project ($i = 1$) and in the case the firm does not ($i = 0$). We assume that all investors are risk-neutral. The firm value $V^i(x)$ satisfies the following equilibrium condition[6]

$$rV^i = Kx + \frac{\gamma_i}{1+p_i}K - f + \mu V_x x + \frac{1}{2}\sigma^2 V_{xx}x^2 + p_i\left(H - V^i\right) \tag{1}$$

for $i = 0, 1$ and where r is the risk-free interest rate with $r > \mu$.

Note that while the expected gains from the gamble, $\frac{\gamma_1}{1+p_1}K$, are constant and independent of x, the loss in case of failure, $\left(V^1 - H\right)$, depends on x. It will be shown that V^1 is increasing in x and hence the opportunity cost of engaging in last resort gambles ($p_1\left(V^1 - H\right)$) is increasing in x. Thus, the lower (larger) is x, the larger (lower) are the relative expected gains from the risky investment. It will be shown that engaging in the gamble is optimal for sufficiently low values of x.

In what follows we shall use the notation:

$$\widehat{x}_i = -\frac{\lambda_i}{1-\lambda_i}\frac{rH - \frac{\gamma_i}{1+p_i}K + f}{K}\frac{r + p_i - \mu}{r + p_i} \tag{2}$$

for $i = 0, 1$, where λ_i is the negative root of the fundamental quadratic equation

$$\mu\lambda_i + \frac{1}{2}\sigma^2\lambda_i\left(\lambda_i - 1\right) = r + p_i \tag{3}$$

for $i = 0, 1$.

Over an infinitesimal time period dt, if the expected gains from investing in the risky project ($K\frac{\gamma_1}{1+p_1}dt$) are sufficiently low while the probability of the project failure ($p_1 dt$) is sufficiently large (i.e. the mean waiting time of the project failure is sufficiently low) then investing in this risky project is never optimal. On the other hand, if the expected gains from investing in the risky

[6] See, for example, Dixit and Pindyck (1994) Section 5.B.

project are sufficiently large while the probability of project failure is sufficiently low then it is never optimal to exercise the option of liquidating the company. Thus, in order to rule out the trivial cases throughout the paper we make the following assumption.

Assumption 1. *p_1 and γ_1 are such that*

$$0 < -\frac{\lambda_1}{1-\lambda_1}\left(H + \frac{f}{r} - \frac{\gamma_1}{1+p_1}\frac{K}{r}\right)\frac{r-\mu+p_1}{r+p_1} < -\frac{\lambda_0}{1-\lambda_0}\left(H + \frac{f}{r}\right)\frac{r-\mu}{r}$$

Assumption 1 poses restrictions on the parameter values p_1 and γ_1. In particular, for each value p_1, Assumption 1 defines an upper and a lower bound for the parameter value γ_1 such that both the closure problem and the gambling option remain relevant, respectively. To see this, we observe that a too large value of γ_1 leads to a violation of the first inequality, posing an upper bound on the value of γ_1. If γ_1 is very large, then expected "inflated" profits are such that the optimal closure problem becomes irrelevant. On the other hand, a too low value of γ_1 leads to an infringement of the second inequality [7] , setting a lower bound on the value of γ_1. A too low gambling intensity makes the option to engage in a last resort gambling strategy unattractive since the increased downside risk and the opportunity costs associated with the risky investment are larger then the expected gains. On the other hand, for each γ_1 Assumption 1 poses an upper bound for the parameter value p_1, say $\overline{p}_1$: a too large value of p_1 leads to an infringement of the second inequality. A too large failure probability of the risky project and a too low payoff probability ($\frac{\gamma_1}{1+p_1}$) imply that the expected costs of the project due to the increased downside risk, are larger than the expected gains[8] .

Proposition 1 characterizes the optimal firm value:

[7] For $\gamma_1 = 0$ the second inequality is always violated since inequality $-\lambda_0 < -\lambda_1 \frac{1-\lambda_0}{1-\lambda_1}\frac{r}{r+p_1}\frac{r-\mu+p_1}{r-\mu}$ holds for each value of $p_1 > 0$.

[8] Note that for $p_1 \to \infty$, $\lambda_1 \to \infty$ and thus since $1 > -\frac{\lambda_0}{1-\lambda_0}\frac{r-\mu}{r}$, the second inequality of Assumption 1 is violated. Note that for $p_1 \to 0$ the second inequality of Assumption 1 always holds and since $-\frac{\lambda_1}{1-\lambda_1}\left(H + \frac{f}{r} - \frac{\gamma_1}{1+p_1}\frac{K}{r}\right)\frac{r-\mu+p_1}{r+p_1}$ is strictly increasing in p_1 there exists a finite threshold $\overline{p}_1$ such that the second inequality of Assumption 1 is satisfied for values of p_1 lower than this threshold.

Proposition 1 *The firm value is*

$$V(x) = \begin{cases} V^0(x) & \text{for } x \geq x_0 \\ V^1(x) & \text{for } \widehat{x}_1 \leq x < x_0 \\ H & \text{for } x < \widehat{x}_1 \end{cases}$$

where

$$\begin{aligned} V^0(x) = {} & \tfrac{Kx}{r-\mu} - \tfrac{f}{r} + \left(\tfrac{f}{r} - \tfrac{Kx_0}{r-\mu} + \tfrac{Kx_0}{r+p_1-\mu} + \tfrac{p_1 H + \frac{\gamma_1}{1+p_1}K - f}{r+p_1} \right) \left(\tfrac{x}{x_0} \right)^{\lambda_0} + \\ & \left(H - \tfrac{K\widehat{x}_1}{r+p_1-\mu} - \tfrac{p_1 H + \frac{\gamma_1}{1+p_1}K - f}{r+p_1} \right) \left(\tfrac{x_0}{\widehat{x}_1} \right)^{\lambda_1 - \lambda_0} \left(\tfrac{x}{\widehat{x}_1} \right)^{\lambda_0} \end{aligned} \tag{4}$$

$$\begin{aligned} V^1(x) = {} & \tfrac{Kx}{r+p_1-\mu} + \tfrac{p_1 H + \frac{\gamma_1}{1+p_1}K - f}{r+p_1} + \\ & \left(H - \tfrac{K\widehat{x}_1}{r+p_1-\mu} - \tfrac{p_1 H + \frac{\gamma_1}{1+p_1}K - f}{r+p_1} \right) \left(\tfrac{x}{\widehat{x}_1} \right)^{\lambda_1} \end{aligned} \tag{5}$$

where $\widehat{x}_1$ *is defined in (2) and* x_0 *is the solution of* $F(x) = 0$, *where*

$$\begin{aligned} F(x) = {} & Kx(1-\lambda_0) \left(\tfrac{1}{r-\mu} - \tfrac{1}{r-\mu+p_1} \right) + \tfrac{\lambda_0 - \lambda_1}{1-\lambda_1} \left(\tfrac{rH - \frac{\gamma_1}{1+p_1}K + f}{r+p_1} \right) \left(\tfrac{x}{\widehat{x}_1} \right)^{\lambda_1} + \\ & \lambda_0 \left(\tfrac{f}{r} + \tfrac{p_1 H + \frac{\gamma_1}{1+p_1}K - f}{r+p_1} \right) \end{aligned} \tag{6}$$

and where Assumption 1 guarantees that $x_0 > \widehat{x}_1$. *Moreover,* $\frac{\partial x_0}{\partial p_1} < 0$ *and* $\frac{\partial x_0}{\partial \gamma_1} > 0$, *while* $\frac{\partial \widehat{x}_1}{\partial p_1} > 0$ *and* $\frac{\partial \widehat{x}_1}{\partial \gamma_1} < 0$.

Proof. In the Appendix. ∎

Proposition 1 identifies two thresholds, the first ($\widehat{x}_1$) being the closure cut-off level and the second (x_0) being a gamble cut-off level. If demand is sufficiently large (i.e. $x > x_0$), the company chooses not to engage in the risky investment. If demand decreases to intermediate values (i.e. for $\widehat{x}_1 < x \leq x_0$), then gambling becomes optimal. The company tries to boost current profits, betting on a recovery of demand and thus trying to bridge good and bad times. If demand decreases further (i.e., $x \leq \widehat{x}_1$), closure becomes optimal.

The firm's values as described in (4) - (5) have a straightforward interpretation. The first and the second term in (4) represent the present value of the firm's cash flow, the third expression in round brackets represents the option value of engaging in a last resort gamble, and the forth expression in round brackets represents the option value of shutting the firm down. The first and the second term in (5) represent the expected present value of the inflated cash flow. Note that in this case the values are discounted at a larger rate since the risky project fails with probability

$p_1 dt$ over the infinitesimal time interval dt, in which case the company goes bankrupt. The third expression in round brackets in (5) represents the option value of closure.

Observe that $V(x)$ is increasing in x and therefore the loss if the risky project fails, is increasing in x as well.

An increase in the failure frequency of the risky project (an increase in p_1) or a reduction in the gambling intensity (γ_1) speeds up liquidation and reduces the range of values for x where engaging in last resort gambling is optimal, that is it increases $\widehat{x}_1$ and reduces x_0. The intuition for this result is quite straightforward. An increase in p_1 increases the opportunity cost of investing in the risky project and the expected gains from the project, while a reduction in γ_1 reduces the expected gains from the project. Hence, as p_1 (γ_1) increases (decreases), the effectiveness of the project in terms of boosting the firm value is reduced and consequently the incentives to invest in this project are reduced. Note that a violation of the second inequality in Assumption 1 implies that $\widehat{x}_1 \geq \widehat{x}_0$. In this case, it follows from Proposition 1 that investing in the risky project is never optimal. The company value is larger if it does not engage in a last resort gamble even if it is available and, consequently, liquidation is optimal once exogenous demand decreases below the liquidation threshold $\widehat{x}_0$. In other words, the risky investment is ineffective in delaying the company's liquidation. In particular, as the project failure probability p_1 reaches the upper bound set by Assumption 1 (i.e. $p_1 \to \overline{p}_1$), the range of values for x where it is optimal to invest in the risky project becomes infinitesimal small and finally vanishes for $p_1 = \overline{p}_1$.

We compare the result with the case where a last resort gamble is not available. We denote by $V^{NG}(x)$ the firm value satisfying (1) in the case where $\gamma_0 = p_0 = 0$, then [9]

$$V^{NG}(x) = \frac{Kx}{r-\mu} - \frac{f}{r} + \left(H - \frac{K\widehat{x}_0}{r-\mu} + \frac{f}{r}\right)\left(\frac{x}{\widehat{x}_0}\right)^{\lambda_0}$$

where $\widehat{x}_0$ is defined in (2).

Proposition 2 . *If a last resort gamble is not available, then closure occurs at $\widehat{x}_0$, where $\widehat{x}_0 > \widehat{x}_1$.*

[9] See also Proposition 1 in Lambrecht and Myers (2005, 2007).

Proof. It follows from Assumption 1. ■

Thus, the firm closes later, if it can engage in a last resort gamble. The company invests in a risky project, trying to inflate current profits and thus delays the firm's liquidation.

A final remark concerns the effect of volatility on closure.

Remark 1 *An increase in the volatility parameter σ^2 decreases the closure thresholds $\widehat{x}_1$ and $\widehat{x}_0$.*

The intuition is that as volatility increases, so does the value of the firm for a given closure threshold. With the terminology of real option theory, the premium to keep the closure option alive is weaker. This lowers the thresholds $\widehat{x}_1$ and $\widehat{x}_0$.

3 Debt and Equity

In this section we suppose that the firm is financed by issuing debt and equity and examine the effect of debt on the closure and gambling decisions. We assume that debt guarantees the payment of a constant perpetual coupon C unless liquidation occurs. The liquidation value, net of bankruptcy costs, is denoted by H. Two cases can be distinguished: (i) risk-free debt, where the company's liquidation value covers the value of the debt ($H \geq \frac{C}{r}$), so that debt is fully collateralized, and (ii) risky debt, where the company's liquidation value is insufficient ($H < \frac{C}{r}$). Let $E(x)$ denote the equity-holders' claim and $D(x)$ the debt-holders' claim.

In this section we use the following notation, which takes debt into account:

$$x_i^* = -\frac{\lambda_i}{1-\lambda_i} \frac{r\Delta - \frac{\gamma_i}{1+p_i}K + C + f}{K} \frac{r+p_i-\mu}{r+p_i} \tag{7}$$

where $\Delta = \max\left\{H - \frac{C}{r}, 0\right\}$ and $i \in \{0,1\}$. Observe that as long as debt is risk-free $x_i^* = \widehat{x}_i$, while if debt is risky then $x_i^* > \widehat{x}_i$.

Consider first the case of risk-free debt. We denote by $e(x)$ the payout policy to equity-holders, being $e(x) = Kx - f - C$ as long as the company remains operative and $rH - C$ in case of liquidation. We denote by $d(x)$ the payout policy to debt-holders, where $d(x) = C$. It is easy to see that

$$E(x) = V(x) - D(x)$$

$$D(x) = \frac{C}{r}$$

where $V(x)$ is defined in Proposition 1. Thus, as long as debt is risk-free, the company's closure and gambling strategies are not affected by its capital structure.

Consider next the case of risky debt. Now, the payout policy to equity-holders is $e(x) = Kx - f - C$, as long as the company remains operative, and 0 in case of liquidation, while the payout policy to debt-holders is $d(x) = C$ as long as the company remains operative, and rH in the case of liquidation. The following Proposition can be proved:

Proposition 3 *(i) For coupon values lower than $\widehat{C}$, equity and debt values are*

$$E(x) = \begin{cases} E^0(x) \text{ , for } x > x_{0R} \\ E^1(x) \text{ , for } x_1^* < x \leq x_{0R} \\ 0, \text{ for } x \leq x_1^* \end{cases}$$

$$D(x) = \begin{cases} D^0(x) \text{ , for } x > x_{0R} \\ D^1(x) \text{ , for } x_1^* < x \leq x_{0R} \\ H, \text{ for } x \leq x_1^* \end{cases}$$

where

$$E^0(x) = \tfrac{Kx}{r-\mu} - \tfrac{f}{r} - \tfrac{C}{r} + \left(\tfrac{f}{r} + \tfrac{C}{r} - \tfrac{Kx_{0R}}{r-\mu} + \tfrac{Kx_{0R}}{r+p_1-\mu} - \tfrac{C - \frac{\gamma_1}{1+p_1}K + f}{r+p_1} \right) \left(\tfrac{x}{x_{0R}} \right)^{\lambda_0} + \left(\tfrac{C - \frac{\gamma_1}{1+p_1}K + f}{r+p_1} - \tfrac{Kx_1^*}{r+p_1-\mu} \right) \left(\tfrac{x_{0R}}{x_1^*} \right)^{\lambda_1 - \lambda_0} \left(\tfrac{x}{x_1^*} \right)^{\lambda_0}$$

$$E^1(x) = \tfrac{Kx}{r+p_1-\mu} - \tfrac{C - \frac{\gamma_1}{1+p_1}K + f}{r+p_1} + \left(\tfrac{C - \frac{\gamma_1}{1+p_1}K + f}{r+p_1} - \tfrac{Kx_1^*}{r+p_1-\mu} \right) \left(\tfrac{x}{x_1^*} \right)^{\lambda_1}$$

where x_1^* is defined in (7) and x_{0R} is the solution of $F^R(x) = 0$, where

$$F^R(x) = Kx(1-\lambda_0) \left(\tfrac{1}{r-\mu} - \tfrac{1}{r-\mu+p_1} \right) + \tfrac{\lambda_0 - \lambda_1}{1-\lambda_1} \tfrac{C - \frac{\gamma_1}{1+p_1}K + f}{r+p_1} \left(\tfrac{x}{x_1^*} \right)^{\lambda_1} + \lambda_0 \left(\tfrac{f}{r} + \tfrac{C}{r} - \tfrac{C - \frac{\gamma_1}{1+p_1}K + f}{r+p_1} \right)$$

and where

$$D^0(x) = \tfrac{C}{r} \left[1 - \left(\tfrac{x}{x_{0R}} \right)^{\lambda_0} \right] + \tfrac{C + p_1 H}{r+p_1} \left(\tfrac{x}{x_{0R}} \right)^{\lambda_0} + \left(H - \tfrac{C + p_1 H}{r+p_1} \right) \left(\tfrac{x_{0R}}{x_1^*} \right)^{\lambda_1 - \lambda_0} \left(\tfrac{x}{x_1^*} \right)^{\lambda_0}$$

$$D^1(x) = \frac{C + p_1 H}{r + p_1}\left[1 - \left(\frac{x}{x_1^*}\right)^{\lambda_1}\right] + H\left(\frac{x}{x_1^*}\right)^{\lambda_1}$$

Moreover, $\frac{\partial x_{0R}}{\partial p_1} < 0$ *and* $\frac{\partial x_{0R}}{\partial \gamma_1} > 0$*, while* $\frac{\partial x_1^*}{\partial p_1} > 0$ *and* $\frac{\partial x_1^*}{\partial \gamma_1} < 0$*.*
(ii) For coupon values $C \geq \widehat{C}$ *engaging in a last resort gamble is never optimal and closure occurs for* $x \leq x_0^*$*, where* x_0^* *is defined in (7).*
(iii) The critical coupon value $\widehat{C}$ *depends negatively on* p_1*, i.e.* $\frac{\partial \widehat{C}}{\partial p_1} < 0$*.*

Proof. In the Appendix. ■

The coupon value is critical for the company's decision to engage or not to engage in a last resort gamble. For sufficiently low coupon values, engaging in a last resort gamble is optimal in the case of a market downturn (i.e. for low values of x). Thus, the company tries to inflate current profits betting on a market upturn. For sufficiently large coupon values engaging in a last resort gamble is never optimal. In this case, equity-holders are not able to gain from the gambling strategy which may benefit debt-holders, and thus it becomes an unattractive option. Moreover, akin to the results shown in the previous section, last resort gambles introduce a distortion in liquidation decision. If a last resort gamble is not available, then closure occurs for values of x lower than x_0^*, where for each $C < \widehat{C}$, $x_1^* < x_0^*$. Thus, last resort gambles delay liquidation.

The intuition behind the role of the threshold $\widehat{C}$ and the importance of the coupon value for the gambling and liquidation strategy is the following. For $p_1 > 0$ and $C < \widehat{C}$, the effect of C on the closure threshold is stronger if last resort gambles are available than if they are not, and this effect is stronger the larger is p_1. Indeed, since the risky investment increases the downside risk, it makes the company discount future profits at a higher discount rate (i.e. $r + p_1$ instead of r). Consequently, the burden of an increase in the coupon value is larger, the larger is p_1. Thus, there exists a $C = \widehat{C}$ such that $x_1^* = x_0^*$, that is, the threshold for the demand level below which the company decides to liquidate its activity is the same with and without the last resort gamble. In this case the last resort gamble is ineffective in delaying liquidation since the increased burden of debt service exactly compensates the expected gains from the risky investment and hence nothing is gained from investing in the risky project. For $C > \widehat{C}$ we have that the increased debt burden more than compensates the expected gains from the gamble and hence it is not optimal to invest in the project.

The effects of p_1 and γ_1 on the gambling and closure decision are similar to those described in the previous section. An increase in the downside risk due to a last resort gamble (an increase in p_1) or a reduction in the gambling intensity (γ_1) speeds up liquidation and reduces the range of values for x where engaging in last resort gambling is optimal, that is it increases x_1^* and reduces x_{0R}.

The expressions of equity and debt values have a straightforward interpretation. The first two terms of $E^1(x)$ represent the present value of cash flow, given that the company engages in a last resort gamble; the other terms of $E^1(x)$ represent the closure option. Analogously, the first three terms of $E^0(x)$ represent the present value of profits, given that the firm does not invest in the risky project; the second expression in brackets represents the last resort gambling option value while the third part represents the closure option. The debt value can be interpreted in a similar way. Note that, while $\left(\frac{x}{x_1^*}\right)^{\lambda_0}$ can be interpreted as the probability that the manager shuts the company down because demand is too low[10] , $(1-\left(\frac{x}{x_1^*}\right)^{\lambda_0})$ can be interpreted as the probability that this event does not occur. Consider first $D^1(x)$ where the company employs the last resort gamble strategy. $\frac{C}{r+p_1}$ represents the present value of the constant perpetual coupon C, given that the company is not liquidated, where the discount factor takes into account the fact that the project fails over the infinitesimal time interval dt with probability $p_1 dt$; $\frac{p_1 H}{r+p_1}$ represents the present value of the liquidation value if the risky project fails, given that the company is not liquidated. Thus, the first part of $D^1(x)$ represents the debt value if the company is not liquidated, while the second part consists of the expected debt value in the case of liquidation. The debt value $D^0(x)$, corresponding to the case where the company does not employ the last resort gamble strategy, consists of three parts. The first represents the present value of debt, given that the company does not engage in a last resort gamble, the second expression represents the

[10] This probability is different from the case where bankruptcy is induced by the failure of the risky project.

debt value due to the company's gambling option and the final term represents the debt value due the company's liquidation option. Note that in the case of risky debt ($\frac{C}{r} > H$) the third expression is always negative.

A further remark concerns the optimality of the liquidation threshold x_1^* from a "social" point of view. Notice that $\frac{\partial E^1(x)}{\partial x_1^*} = 0$ and $\frac{\partial^2 E^1(x)}{\partial (x_1^*)^2} < 0$, that is, the choice of the closure threshold x_1^* is optimal for equity-holders. On the contrary, $\frac{\partial V(x)}{\partial x_1^*} < 0$, where $V(x)$ is the overall value of the firm, that is, $V(x) = E(x) + D(x)$. Since in this framework the socially optimal bankruptcy trigger is the one that maximizes the overall value of the firm $V(x)$, we get that, when equity-holders choose the timing of bankruptcy and the firm has issued debt, then the socially optimal bankruptcy strategy cannot be achieved, that is, x_1^* is not socially optimal. The cause is equity-holders' limited liability. In particular, since by the envelope theorem $\frac{\partial V(x_1^*)}{\partial x_1^*} = \frac{\partial D(x_1^*)}{\partial x_1^*} < 0$ the "socially optimal" liquidation threshold is lower than x_1^*.[11]

The following Remark can be proved straightforwardly.

Remark 2. *Since under risky debt $C > rH$, liquidation occurs earlier than with risk-free debt, or an unleveraged firm, i.e. $x_1^* > \widehat{x}_1$.*

Observe that the closure threshold x_1^* is increasing in the coupon value C. Thus, debt speeds up closure: the leveraged firm closes earlier than the unleveraged one. Remark 2 is in keeping with what is established in the "debt overhang problem" literature (Myers, 1977).

In Figure[12] 1 we depict an example with two different coupon values $C_1 > C_0$. For coupon value C_0 (C_1), investing in a risky project is optimal for values of $x \in (x_1(C_0), x_{0R}(C_0))$ ($x \in (x_1(C_1), x_{0R}(C_1))$), while for values of $x \geq x_{0R}(C_0)$ ($x \geq x_{0R}(C_1)$) it is not. Note that $x_{0R}(C_0) -$

[11] In view of this remark and following Leland (1994), one can study how to design debt contracts where debt is protected by positive net worth covenant and determine the optimal covenant rule. See also Decamps and Djembissi (2007) for a discussion of optimal covenant rules.

[12] Throughout the paper the parameter values used in the simulations, if not stated otherwise, are $\sigma = .25$, $r = .05$, $\mu = .001$, $p_1 = .001$, $\gamma_1 = .005$, $f = 1$, $K = H = 100$.

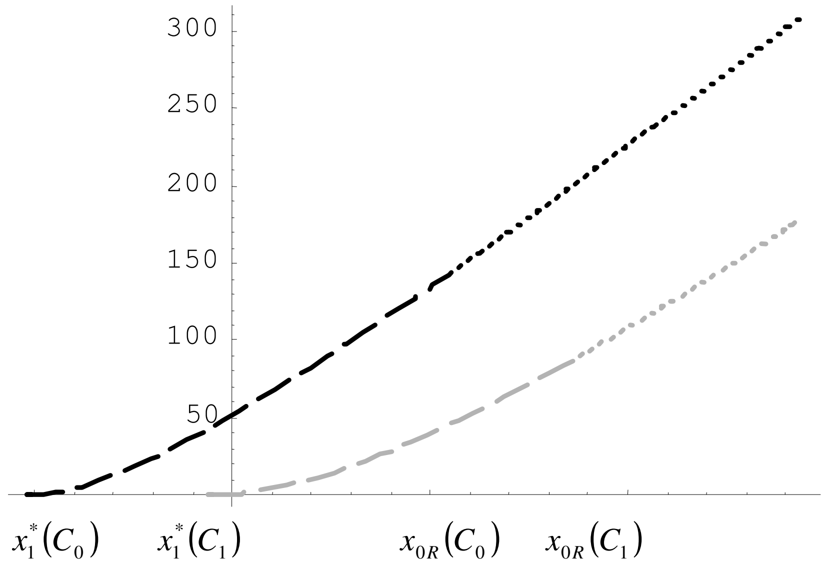

Figure 1: Equity values $E^1(x)$, dashed line, and $E^0(x)$, dotted line, for coupon values C_0 (black line) and C_1 (grey line) with $C_0 = 10$, $C_1 = 20$.

$x_1^*(C_0) > x_{0R}(C_1) - x_1^*(C_1)$ and thus a larger coupon value reduces the range of values for x where engaging in a last resort gamble is optimal. Closure is optimal for values of $x \leq x_1^*(C_0)$ $(x \leq x_1^*(C_1))$. Thus, an increase in the coupon value speeds up liquidation $(x_1^*(C_0) < x_1^*(C_1))$. The following proposition states this result more formally.

Proposition 4 *Increasing the coupon value C reduces the range of values where employing a last resort gamble strategy is optimal (i.e. $x_{0R} - x_1^*$ is decreasing in C) and the distortion in liquidation induced by last resort gambles (i.e. $x_0^* - x_1^*$ is decreasing in C).*

Proof. In the Appendix. ■

As we explained above, the driving mechanism of the result that $x_0^* - x_1^*$ is decreasing in the coupon value C is that the debt burden increases more with a last resort gamble than without, because of an increased downside risk in the former case. The same mechanism drives also the result that $x_{0R} - x_1^*$ is decreasing in the coupon value: the increased downside risk associated with the gamble leads to a larger burden if the coupon value is increased, reducing the incentives to

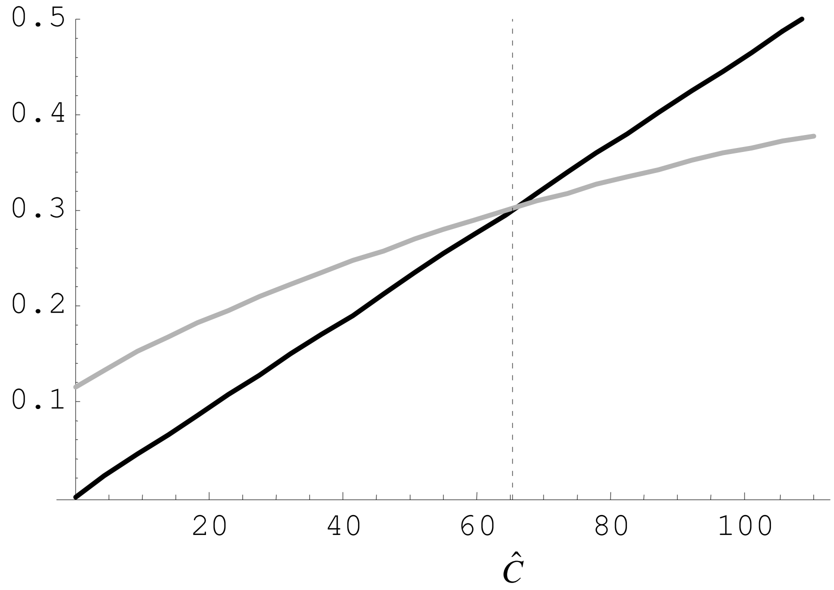

Figure 2: Liquidation (black line) and gambling (grey line) thresholds as a function of the coupon value C.

engage in a last resort gamble. This effect is stronger, the larger is p_1 (i.e. $x_{0R} - x_1^*$ is decreasing in p_1 as shown in Proposition 3).

In Figure 2 we depict the liquidation (black line) and the gambling (grey line) threshold as a function of the coupon value C. For each $C < \widehat{C}$ the gambling threshold is larger than the liquidation threshold (i.e. $x_1^* < x_{0R}$) and thus the last resort gamble strategy is optimal for values of x between these two thresholds. As C increases both the gamble and the liquidation threshold increase while the difference between the two decreases, reducing the values of x where last resort gambling is optimal. For the coupon value $C \geq \widehat{C}$ investing in the risky project is never optimal.

Remark 3. *An increase in the volatility parameter σ^2 decreases the closure thresholds x_1^* and x_0^*.*

As asset risk rises, so does the value of equity for a given closure threshold. Hence, equity-holders' incentive to default on the interest payment, i.e. on the premium to keep the option alive is weaker. Notice that $\frac{\partial E(x)}{\partial \sigma^2} > 0$, as long as bankruptcy has not been declared, that is, equity value is enhanced by greater risk in case of debt. This lowers the triggers x_1^* and x_0^*. Notice that the effect of σ^2 on x_1^* has the same sign of the effect of γ_1.

We investigate how a last resort gamble strategy affects the debt value comparing the debt value in the case of a last resort gamble strategy and the case where these are not available. If last resort gambles are not available then the debt value is

$$D^{NG}(x) = \begin{cases} \frac{C}{r}\left[1 - \left(\frac{x}{x_0^*}\right)^{\lambda_0}\right] + H\left(\frac{x}{x_0^*}\right)^{\lambda_0}, \text{ for } x > x_0^* \\ H, \text{ for } x \leq x_0^* \end{cases}$$

That is, the value of risky debt equals the value of the risk-free debt $\frac{C}{r}$ times the probability that bankruptcy does not occur plus the value of the proceeds from asset liquidation in the event of bankruptcy H times the probability of bankruptcy. To see how risky investments affect the debt value we compute $\Delta D(x, C) \equiv D(x) - D^{NG}(x)$, where $D(x)$ is defined in Proposition 3

$$\Delta D(x, C) = \left(\frac{C}{r} - H\right) \begin{cases} \left(\frac{x}{x_{0R}}\right)^{\lambda_0}\left[\left(\frac{x_{0R}}{x_0^*}\right)^{\lambda_0} - \frac{p_1}{r+p_1} - \frac{r}{r+p_1}\left(\frac{x_{0R}}{x_1^*}\right)^{\lambda_1}\right], \text{ for } x > x_{0R} \\ \left\{\frac{r}{r+p_1}\left[1 - \left(\frac{x}{x_1^*}\right)^{\lambda_1}\right] - \left[1 - \left(\frac{x}{x_0^*}\right)^{\lambda_0}\right]\right\}, \text{ for } x_0^* < x \leq x_{0R} \\ \frac{r}{r+p_1}\left[1 - \left(\frac{x}{x_1^*}\right)^{\lambda_1}\right], \text{ for } x_1^* < x \leq x_0^* \\ 0, \text{ for } x \leq x_1^* \end{cases}$$

Consider first the case $x_1^* < x \leq x_0^*$, where the company is liquidated if a last resort gamble is not available, while the company is gambling for resurrection if a risky investment opportunity is available. In this case $\Delta D(x, C)$ is always positive. A last resort gamble delays liquidation and hence increases the probability that the debt will be repaid, increasing the debt value. Note that the larger is the default probability of the gamble p_1, the larger is the liquidation threshold (i.e. the earlier closure occurs) and consequently the lower is $\Delta D(x, C)$, for $x_1^* < x \leq x_0^*$. For values of x larger than x_0^* it may happen that $\Delta D(x, C)$ is negative. Consider, for example, the case where $x_0^* < x \leq x_{0R}$ where the company remains operative if both a last resort gamble is available and if it is not. The difference in the debt values $\Delta D(x, C)$ depends on the difference between the two probabilities that the company is going to service the debt (term in curled brackets). If the company invests in a last resort gamble then this probability depends on the failure probability of the risky project p_1 and on the probability that the company decides to shut the activity down $\left(\frac{x}{x_1^*}\right)^{\lambda_1}$, while if a last resort gamble is not available then the default probability depends just on the probability that the company decides to declare bankruptcy $\left(\frac{x}{x_0^*}\right)^{\lambda_0}$. While the probability that the company decides to liquidate is lower if the company invests in the risky project (delay-in-liquidation effect), the increased downside risk due to the last resort gamble increases the default probability (increased-downside-risk effect), leading to an ambiguous effect. For $p_1 \to 0$ the delay-in-liquidation effect dominates and hence $\Delta D(x, C) > 0$ for $x_0^* < x \leq x_{0R}$. The larger is the failure probability p_1 the lower is the probability that the company is going to service the debt in the presence of a last resort gamble and thus the more likely it is that $\Delta D(x, C)$ is negative for some values of $x_0^* < x \leq x_{0R}$. For large values of x the probability that the company decides to close the company down (with and without last resort gambles available) is negligible and hence the delay-in-liquidation effect is small. If the difference $x_{0R} - x_0^*$ is sufficiently large (this may happen, for example, for large values of γ_1 and a low coupon value), then the company may still engage in a last resort gamble. In this case the increased-downside-risk effect associated with the risky investment dominates the delay-in-liquidation effect leading to a negative $\Delta D(x, C)$. In this case we have also that $\left(\frac{x_{0R}}{x_0^*}\right)^{\lambda_0} < \frac{p_1}{r+p_1} + \frac{r}{r+p_1}\left(\frac{x_{0R}}{x_1^*}\right)^{\lambda_1}$, and consequently $\Delta D(x, C) < 0$ for $x > x_{0R}$ and also for some values of $x_0^* < x \leq x_{0R}$. This result is in keeping with Leland and Toft (1996), where it is shown that the incentives for increasing risk become positive for equity holders and debt holders as bankruptcy is approached and that the incentives to increase risk become positive for equity holders before they become positive for debt holders.

We summarize this discussion in the following remark.

Remark 4. *A last resort gamble strategy changes the debt value according to the composition of the delay-in-liquidation and the increased-downside-risk effects.*

The coupon value has an important role on the difference $\Delta D(x, C)$. A larger coupon value, reducing the incentives to engage in a last resort gamble and reducing the distortion induced by this strategy (see Proposition 4), increases the probability that $\Delta D(x, C)$ is positive.

A related issue concerns the debt capacity of the firm, that is, the maximal value of total debt[13] . For a given value of x, we study how the debt value $D(x)$ and $\Delta D(x, C)$ change as a function of the coupon value C. Note that the debt capacity is increasing in x: the larger is the exogenous demand level, the larger is the debt capacity. For $C \leq rH$ the debt is risk-free and thus for $C = rH$ the debt value is H. For $C > rH$ the debt is risky and for a sufficiently large coupon value C'' the company defaults on its debt. Thus, since the debt value is a continuous function of C, there exists a $C \in (rH, C'')$ where the debt value is maximized.

Observe that $\Delta D(x, C) = 0$ as long as the debt is risk-free. Once debt becomes risky, the option of engaging in a last resort gamble has an ambiguous effect on the debt capacity, depending, amongst others, on the value of x. For low values of x, as it has been argued above, the delay-in-liquidation effect dominates the increased-downside-risk effect and thus the option of engaging in a last resort gamble increases the company's debt capacity. On the other hand, for large values of x, the increased-downside-risk effect dominates the delay-in-liquidation effect and thus the option of engaging in a last resort gamble reduces the company's debt capacity for some values of C. The larger is C, the lower are the incentives to engage in a last resort gamble and so are the distortions induced by it and, as a consequence, the more likely it is that $\Delta D(x, C)$ is positive.

In Figure 3 we depict two examples based on the numerical example of Figure 2 where we plot $D(x)$ (black line) and $\Delta D(x, C) * 100$ (grey line)[14] as functions of C for two different values of x. For small values of C the debt is risk free (black continuous line). Increasing the value of C the debt becomes risky but the company does not engage in a last resort gamble (black dashed line for $D(x)$ and grey dashed line for $\Delta D(x, C) * 100$); increasing further C engaging in the last resort

[13] See Kim (1978) and Leland (1994) for a definition and a discussion of the concept of debt capacity.

[14] We consider $\Delta D(x, C) * 100$ instead of $\Delta D(x, C)$ in order to be able to plot $D(x)$ and $\Delta D(x, C)$ together in the same graph.

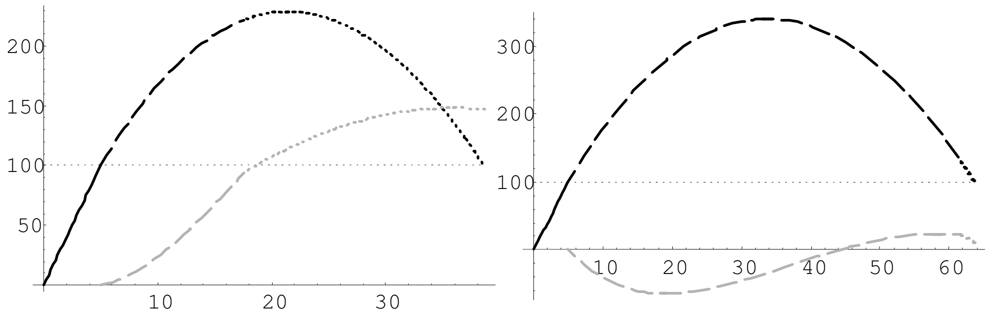

Figure 3: Debt capacity. Debt value $D(x, C)$ (black line) and $\Delta D(x, C) * 100$ (grey line) as a function of C with $x = 0.295$ (RHS) and $x = 0.18$ (LHS).

gamble becomes optimal (black dotted line for $D(x)$ and grey dotted line for $\Delta D(x, C) * 100$).

In Figure 3 we put $x = 0.18$ and $x = 0.295$. We observe that in the first case the debt capacity is lower than in the second case, and furthermore the last resort gamble increases the debt capacity in the first case while in the second case it decreases the debt capacity for low values of C and increases the debt capacity for large values of C.

The debt capacity is relevant to determine the debt contract at an initial time. If x at time 0 is relatively large, then the option of engaging in a last resort gamble reduces the company's debt capacity and consequently has a negative effect on the firm value.

4 Endogenous capital structure: some remarks

In the previous section we considered the capital structure as exogenous and made some comparative statics analyses to show how optimal last resort gambling and liquidation policies are affected by the company's indebtedness. In this section we address the question of how an endogenous capital structure interacts with liquidation and gambling strategies. The simplest way to tackle this question and to model the benefits of debt issuance is to introduce taxation so that the firm can take advantage of the tax shield on the coupon payment.

Let τ be the marginal corporate tax rate. Consider first the case of risk-free debt. The payout policy to equity-holders is $e(x) = (1 - \tau)(Kx - f - C)$ as long as the company remains operative and $e(x) = (1-\tau)(rH - C)$ in case of liquidation while the payout policy to debt-holders is $d(x) = C$. Then the company value is $\widetilde{V}(x) = \widetilde{E}(x) + D(x)$ where $\widetilde{E}(x) = (1-\tau)[V(x) - D(x)]$ with $D(x) = \frac{C}{r}$ and $V(x)$ as defined in Proposition 1.

Consider next the case of risky debt. The payout policy to equity-holders is $e(x) = (1-\tau)(Kx - f - C)$, as long as the company remains operative, and 0 in case of liquidation, while the payout policy to debt-holders is $d(x) = C$ as long as the company remains operative, and rH in the case of liquidation. Then, following the steps outlined in Section 3, the company value is $\widetilde{V}(x) = \widetilde{E}(x) + D(x)$, where $\widetilde{E}(x) = (1-\tau)E(x)$ with $E(x)$ and $D(x)$ given in Proposition 3.

Let $x(t_0)$ denote the demand level at the initial time t_0 when the coupon value is chosen. In Figure 4 we plot the coupon rate $\frac{C}{D(x(t_0))}$ computed at the coupon value C that maximizes the firm value $\widetilde{V}(x)$ at time t_0 as a function of $x(t_0)$ and for values of $\tau = 0.25$ (black line), $\tau = 0.3$ (dotted line) and $\tau = 0.35$ (grey line). Actually, the optimal C is increasing in τ and $x(t_0)$. From Proposition 4, the greater is τ and/or $x(t_0)$ the lower the range of values of x such that engaging in a last resort gamble is optimal and the lower are the distortions induced by these gambles. Moreover, if $x(t_0)$ and/or τ is sufficiently large, then from Proposition 3 we know that it will never be optimal to engage in last resort gambles. Notice that for large values of $x(t_0)$ the optimal coupon rate is decreasing in $x(t_0)$, given that for large values of $x(t_0)$ there is no last resort gamble effect and larger values of $x(t_0)$ correspond to a smaller credit risk. As shown in the previous section, for low values of $x(t_0)$ debt capacity is increased by a last resort gamble and decreases as $x(t_0)$ increases. Hence, for low values of $x(t_0)$ the last resort gamble effect leads to an increase in the riskiness and therefore in the optimal coupon rate, which vanishes as $x(t_0)$ increases.

A final issue concerns the agency costs of debt. Following Leland (1998) we define agency costs as the difference between the optimal firm value before debt is in place (that is ex ante) and the optimal firm value after debt is in place (that is ex-post). In Figure 5 we plot $\frac{V^0(x(t_0))}{\widetilde{V}(x(t_0))} - 1$, where $V^0(x)$ is defined in Proposition 1 and $\widetilde{V}(x(t_0))$ is the value of the firm with the optimal capital

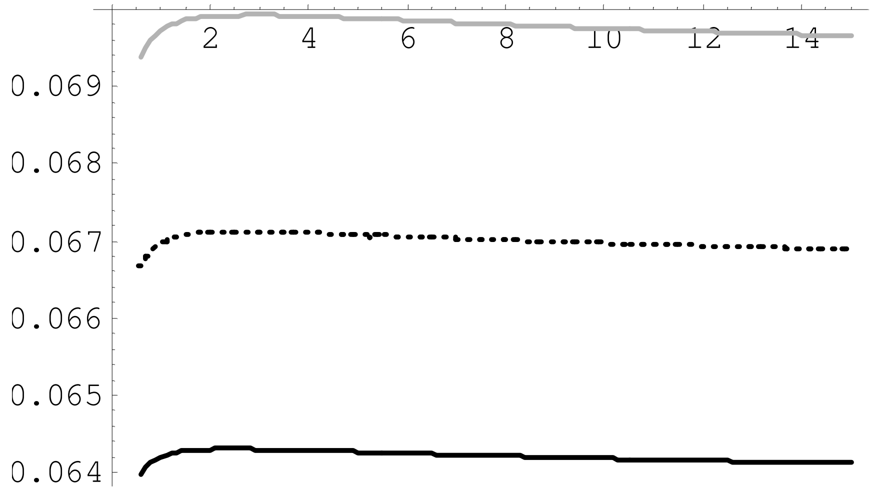

Figure 4: Optimal coupon rate $C/D\,(x\,(t_0))$ as a function of $x\,(t_0)$ for $\tau = 0.25$ (black line), $\tau = 0.3$ (dotted line) and $\tau = 0.35$ (grey line).

structure in place, as a function of $x\,(t_0)$ and for different values of τ.

From Figure 5 we observe that agency costs are increasing in τ. $\widetilde{V}\,(x\,(t_0))$ is decreasing in τ and thus the larger is τ the larger are the agency costs. We also observe that $x\,(t_0)$ has an ambiguous effect on agency costs. For large values of τ tax benefits are most important. Thus increasing $x\,(t_0)$ increases distortions due to tax benefits and thus increases agency costs. For low values of τ tax benefits are less important while distortions due to last resort gambling are more important, which may lead to decreasing agency costs for low values of $x\,(t_0)$ and increasing agency costs for large values of $x\,(t_0)$ once the last resort gamble effect vanishes.

5 Gambling intensity

In this section we generalize the results obtained in Section 3 introducing different gambling intensities. While in the previous sections the choice was either to invest in a risky project or not, here we introduce the choice among different risky projects with different gambling intensities. As an example we restrict our analysis to the case of two risky projects. The available projects are $i = 0, 1, 2$. The model can be extended straightforwardly to the case of n degrees of gambling

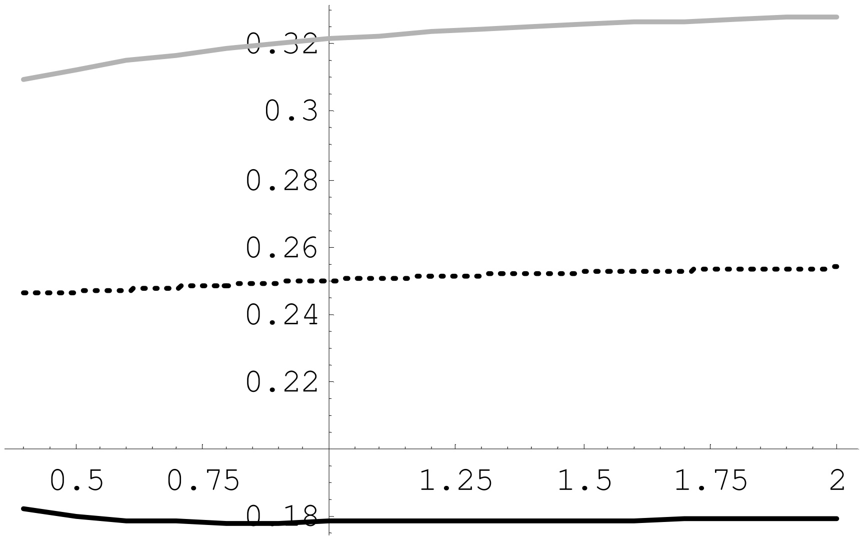

Figure 5: Agency costs as a function of $x(t_0)$ for $\tau = 0.2$ (black line), $\tau = 0.275$ (dotted line) and $\tau = 0.35$ (grey line).

intensities.

For the remaining part of the paper we make use of Assumption 2 which generalizes Assumption 1.

Assumption 2 *p_i and γ_i, for $i = 0, 1, 2$ are such that*

$$0 < -\frac{\lambda_{i+1}}{1-\lambda_{i+1}}\left(H + \frac{f}{r} - \frac{\gamma_{i+1}}{1+p_{i+1}}\frac{K}{r}\right)\frac{r-\mu+p_{i+1}}{r+p_{i+1}} < -\frac{\lambda_i}{1-\lambda_i}\left(H + \frac{f}{r} - \frac{\gamma_i}{1+p_i}\frac{K}{r}\right)\frac{r-\mu+p_i}{r+p_i}$$

for each $i = 0, 1$ and γ_1 is sufficiently large.

For given values of p_1 and p_2, Assumption 2 poses restrictions on gambling intensities γ_1 and γ_2. To maintain the closure problem relevant, the capability to inflate current profits must be limited, posing an upper bound on γ_2 (first part of the inequality in Assumption 2 for $i = 1$). To maintain the gambling problem relevant at different gambling levels, the second part of the inequality in Assumption 2, for $i = 0, 1$, establishes a relationship between γ_1 and γ_2, defining a lower bound on γ_2 as well as an upper and a lower bound on γ_1.

Assumption 3 is required in order to have the choice of different gambling levels meaningful for the relevant parameter configurations.

Assumption 3 *p_i and γ_i, $i = 0, 1, 2$ are such that*

$$-\frac{\lambda_2}{1-\lambda_2}\frac{r-\mu+p_2}{r+p_2}+\frac{\lambda_1}{1-\lambda_1}\frac{r-\mu+p_1}{r+p_1}>-\frac{\lambda_1}{1-\lambda_1}\frac{r-\mu+p_1}{r+p_1}+\frac{\lambda_0}{1-\lambda_0}\frac{r-\mu}{r}$$

The following Proposition characterizes the equity-holders and debt-holders claims in the case of risky debt and shows that a gradual increase in gambling intensity is optimal as demand decreases for low values of the coupon, while for large values of the coupon high gambling intensity is never optimal.

Proposition 5 *There exists a coupon value $\overline{C}$ such that for $C \leq \overline{C}$ the equity and debt values are*

$$E(x)=\begin{cases} E^0(x) \ \ \textit{for } x_{0L}<x \\ E^1(x) \ \ \textit{for } x_{1L}<x\leq x_{0L} \\ E^2(x) \ \ \textit{for } x_2^*<x\leq x_{1L} \\ 0 \ \textit{for } x\leq x_2^* \end{cases}$$

$$D(x)=\begin{cases} D^0(x) \ \ \textit{for } x_{0L}<x \\ D^1(x) \ \ \textit{for } x_{1L}<x\leq x_{0L} \\ D^2(x) \ \ \textit{for } x_2^*<x\leq x_{1L} \\ 0 \ \textit{for } x\leq x_2^* \end{cases}$$

where the closure threshold x_2^* is defined in (7), for $i = 2$, x_{1L} is the solution of $F^1(x) = 0$, and x_{0L} is the solution of $F^0(x) = 0$, where $F^1(x)$, $F^0(x)$ and $E^i(x)$, for $i = 0, 1, 2$, are defined in the Appendix, and where Assumptions 2 and 3 guarantee that $x_{0L} > x_{1L} > x_2^*$. For $C > \overline{C}$ Proposition 3 applies, where Assumption 3 guarantees that $\overline{C} < \widehat{C}$.

Proof. In the Appendix. ■

For coupon values lower than $\overline{C}$ the company increases the gambling intensity as x decreases. For large values of x the company does not engage in a last resort gamble strategy ($x > x_{0L}$). As x decreases, the company starts to invest in the risky project 1 which corresponds to a low gambling intensity γ_1 (for $x \in (x_{1L}, x_{0L})$). As x decreases further, the company increases its gambling intensity, investing in the risky project 2 (for $x \in (x_2^*, x_{1L})$), delaying further liquidation. For coupon values larger than $\overline{C}$ investing in the risky project 2 is never optimal, while it remains

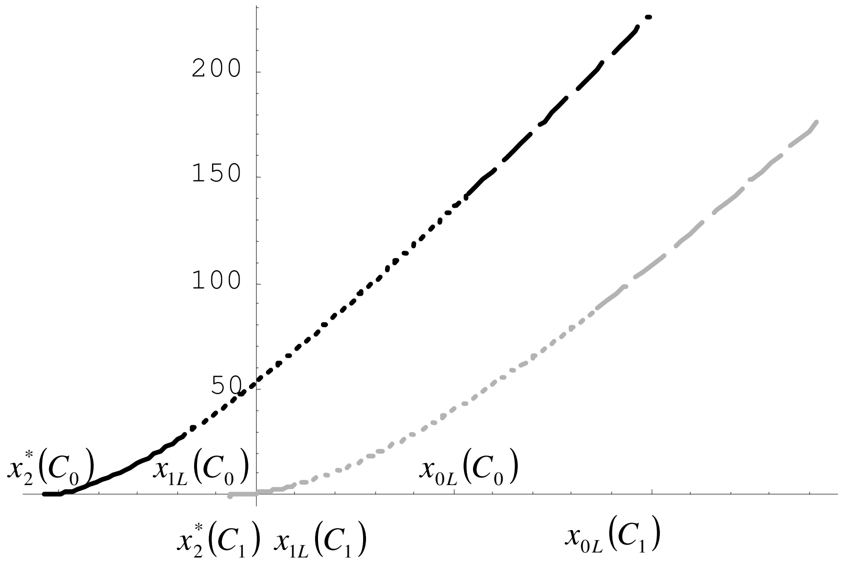

Figure 6: Equity values $E^2(x)$, continuous line, $E^1(x)$, dotted line, and $E^0(x)$, dashed line, for coupon values C_0 (black line) and C_1 (grey line) with $C_1 > C_0$. Parameter values: $\sigma = .25$, $r = .05$, $\mu = .001$, $p_1 = .001$, $p_2 = .005$, $\gamma_1 = .005$, $\gamma_2 = .0125$, $f = 1$, $K = H = 100$, $C_0 = 10$, $C_1 = 20$.

optimal to invest in the risky project 1 for some values of x. Thus, as C increases the intensity of the last resort gambles decreases.

Remark 4 *For values of $C > rH$, increasing the coupon value C reduces the last resort gambling intensity.*

Note that while in the previous section the gambling intensity was given, in this section here the firm chooses between different gambling intensities. A change in the firm's financial structure leads the firm to choose a different last resort gambling intensities. A higher coupon value leads a firms to engage a less intense gambling strategy. In Figure 6 we depict a typical situation with two different coupon values $C_1 > C_0$. Observe that a larger coupon value reduces the range of values of x where investing in project 1 is optimal $(x_{0L}(C_1) - x_{1L}(C_1) > x_{0L}(C_0) - x_{1L}(C_0))$ and where investing in project 2 is optimal $(x_{1L}(C_1) - x_2^*(C_1) > x_{1L}(C_0) - x_2^*(C_0))$. Moreover, observe that the reduction in the latter range is larger than the reduction in the former one. For

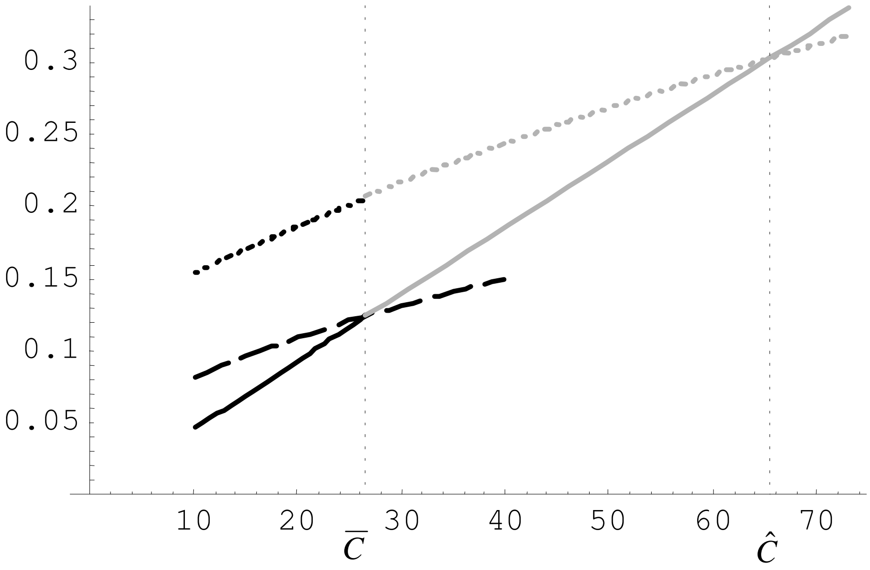

Figure 7: For $C < \overline{C}$ we depict the gambling threshold for project 1 (black dotted line), for project 2 (black dashed line) and the liquidation threshold (black continuous line) as a function of C. For $\overline{C} \leq C < \widehat{C}$ we depict the gambling threshold for project 1 (gray dotted line) and the liquidation threshold (gray continuous line) as a function of C.

a sufficiently large coupon value, investing in project 2 will never be optimal, while investing in project 1 remains optimal for some values of x.

In Figure 7 we depict an example of gamble and liquidation thresholds as a function of C.[15]

6 Conclusion

The problem of the relation between last resort gambles, debt and liquidation policies is set out in this paper within a real option model. It is studied how the firm's capital structure affects corporate liquidation and optimal last resort gamble strategies. Last resort gambles delay liquidation. On the other side, increasing indebtedness reduces the range of values where it is optimal to engage in last resort gambles. The way corporate financial structure and last resort gamble strategies interact is examined in Sections 3 and 4, where we characterize equity holders optimal strategies and debt holders claims explicitly and provide both a formal analysis and a numerical implementation of

[15] Parameter values are the same as in Figure 6.

the model. Our analysis produces results which go beyond those obtained in asset-substitution effects models, both in terms of the impact of a last resort gamble on the debt value and the firm's debt capacity and in terms of the behaviour of the optimal coupon rate and agency costs. When equity holders can play a last resort gamble strategy in order to increase the value of their assets, some unexpected results may occur, that is, the debt capacity may increase, optimal coupon rates may increase, agency costs may decrease, as it is shown in Sections 3 and 4.

Our model abstracts from conflicts of interests between manager and shareholders, arising from the former's propensity to divert firm resources to his own benefit. One way to extend our model and introduce also such conflict of interests is to follow Lambrecht and Myers (2005) framework, where the manager maximizes the present value of the expected cash flows, given the debt policy, but is subject to the threat of collective action by shareholders, who can either close the firm or manage it directly. Thus, the manager has to take into account such constraint and make payouts to shareholders to prevent collective action. In this more complex setting, however, the main results of our paper still hold, since the manager and equity holders basically split the surplus from the firm according to fixed fractions even in the event of liquidation, where both get nothing, and also the effects of a last resort gamble strategies are split between them.

Management could also misreport the state of the world to creditors. By introducing asymmetric information it would be interesting to study the fraudulent aspect of last resort gambles. Presumably, the last resort gamble effects are weakened if the possibility of verifying the firm's situation by creditors is introduced. One could explore how endogenous capital structure may act as an incentive device and under which circumstances the optimal debt policy can be used to affect last resort gamble strategies.

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7 Appendix

Proof of Proposition 1. The solution to the differential equation (1) is

$$V^0(x) = \frac{Kx}{r-\mu} - \frac{f}{r} + A_1 x^{\lambda_0} + A_2 x^{\beta}$$

where λ_0 and β are the negative and positive roots of (3), respectively. A no bubble condition requires $\lim_{x\to\infty}(V^0(x) - \frac{Kx}{r-\mu} + \frac{f}{r}) = 0$ and thus $A_2 = 0$. The solution to the differential equation (1) is

$$V^1(x) = \frac{Kx}{r+p_1-\mu} + \frac{p_1 H + \frac{\gamma_1}{1+p_1}K - f}{p_1 + r} + B_1 x^{\lambda_1} + B_2 x^{\varepsilon}$$

where λ_1 and ε are the negative and positive roots of (3), respectively. Since for large values of x the option value of closure becomes negligible, $B_2 = 0$. The value matching condition $V^1(\widehat{x}_1) = H$ together with the smooth pasting condition $V_x^1(\widehat{x}_1) = 0$ define the closure threshold $\widehat{x}_1$ and B_1, while the value matching condition $V^1(x_0) = V^0(x_0)$ together with the smooth pasting condition $V_x^1(x_0) = V_x^0(x_0)$ define the gamble threshold x_0 and the constant A_1.

Substituting $\widehat{x}_i$ for $i = 0, 1$, as defined in (2), into $F(x)$ we obtain

$$F(x) = (1-\lambda_0)\tfrac{K}{r-\mu}(x - \widehat{x}_0) - (1-\lambda_0)\tfrac{K}{r+p_1-\mu}(x - \widehat{x}_1) +$$
$$-\tfrac{\lambda_0-\lambda_1}{\lambda_1}\tfrac{K\widehat{x}_1}{r+p_1-\mu}\left[\left(\tfrac{x}{\widehat{x}_1}\right)^{\lambda_1} - 1\right]$$

Assumption 1 guarantees that $F(\widehat{x}_1) < 0$ and since $F(x)$ is a convex function where $\lim_{x\to\infty} F(x) = \infty$, a unique solution to $F(x_0) = 0$ exists and moreover $x_0 > \widehat{x}_1$. Moreover, $F(\widehat{x}_0) < 0$ and thus, following the same argument, we obtain $x_0 > \widehat{x}_0 > \widehat{x}_1$.

The result $\frac{\partial \widehat{x}_1}{\partial p_1} > 0$ is straightforward since $-\lambda_1$ and $\frac{r+p_1-\mu}{r+p_1}$ are both increasing in p_1. Also $\frac{\partial \widehat{x}_1}{\partial \gamma_1} < 0$ is straightforwardly obtained from $\widehat{x}_1$. Taking the derivative of $F(x)$ with respect to p_1 we obtain

$$\frac{\partial}{\partial p_1} F(x) = (1-\lambda_0) \frac{K}{(r+p_1-\mu)^2} (x - \widehat{x}_1) + (1-\lambda_0) \frac{K}{r+p_1-\mu} \frac{\partial \widehat{x}_1}{\partial p_1} +$$
$$+ \left[\frac{\partial \lambda_1}{\partial p_1} \frac{\lambda_0 - 1}{(1-\lambda_1)^2} - \frac{\lambda_0 - \lambda_1}{1-\lambda_1} \frac{1}{r+p_1} \right] \frac{rH - \frac{\gamma_1}{1+p_1} K + f}{r+p_1} \left[\left(\frac{x}{\widehat{x}_1} \right)^{\lambda_1} - 1 \right] - \frac{\lambda_0 - \lambda_1}{1-\lambda_1} \frac{rH - \frac{\gamma_1}{1+p_1} K + f}{r+p_1} \lambda_1 \left(\frac{x}{\widehat{x}_1} \right)^{\lambda_1} \frac{1}{\widehat{x}_1} \frac{\partial \widehat{x}_1}{\partial p_1}$$

Since $\frac{\partial \lambda_1}{\partial p_1} > 0$, $\frac{\partial \widehat{x}_1}{\partial p_1} > 0$ and $\left(\frac{x}{\widehat{x}_1} \right)^{\lambda_1} < 1$ for each $x > \widehat{x}_1$, we have $\frac{\partial}{\partial p_1} F(x) > 0$ for each $x > \widehat{x}_1$. As a consequence, x_0 is decreasing in p_1.

Taking the derivative of $F(x)$ with respect to γ_1 we obtain

$$\frac{\partial}{\partial \gamma_1} F(x) = \frac{K}{r+p_1-\mu} \frac{\partial \widehat{x}_1}{\partial \gamma_1} \left[1 - \lambda_0 + \frac{\lambda_0 - \lambda_1}{\lambda_1} - \frac{\lambda_0 - \lambda_1}{\lambda_1} \left(\frac{x}{\widehat{x}_1} \right)^{\lambda_1} + (\lambda_0 - \lambda_1) \left(\frac{x}{\widehat{x}_1} \right)^{\lambda_1} \right]$$

Note that the second term in the parenthesis is larger than the second term and consequently the whole parenthesis is positive which, since $\frac{\partial \widehat{x}_1}{\partial \gamma_1} < 0$, implies that $\frac{\partial}{\partial \gamma_1} F(x) < 0$ and consequently $\frac{\partial x_0}{\partial \gamma_1} > 0$. ■

Proof of Proposition 3. We divide the proof into two parts. In part (a) we show the content of Proposition 3 (i). In part (b) we show that $\widehat{C}$ exists.

Part (a). To compute $D^i(x)$, let us solve the following differential equation:

$$rD^i = C + \mu D^i_x x + \frac{1}{2} \sigma^2 D^i_{xx} x^2 + p_i \left(H - D^i \right) \tag{8}$$

for $i = 0$ for $x > x_{0R}$ and $i = 1$ for $x_1^* < x \leq x_{0R}$. The general solution of (8) is $\frac{C + p_i H}{r+p_i} + L_i x^{\lambda_i}$ for some L_i, if we take the no-bubble condition into account. We determine L_1 employing the boundary condition $D^1(x_1^*) = H$ and L_0 employing the value matching condition $D^0(x_{0R}) = D^1(x_{0R})$.

The value of the equity-holders' claim $E(x)$ is obtained solving the differential equations:

$$rE^0 = Kx - f - C + \mu E^0_x x + \frac{1}{2} \sigma^2 E^0_{xx} x^2, \text{ for } x > x_{0R}$$

$$rE^1 = Kx + \frac{\gamma_1}{1+p_1} K - f - C + \mu E^1_x x + \frac{1}{2} \sigma^2 E^1_{xx} x^2 + p_1 \left(-E^1 \right), \text{ for } x_1^* < x \leq x_{0R}$$

whose solutions are $E^0(x) = \frac{Kx}{r-\mu} - \frac{f}{r} - \frac{C}{r} + Ax^{\lambda_0}$ and $E^1(x) = \frac{Kx}{r+p_1-\mu} - \frac{C - \frac{\gamma_1}{1+p_1}K + f}{p_1+r} + Bx^{\lambda_1}$, for some A, B, if we take the no-bubble conditions into account. Then, we determine A, B, x_1^*, x_{0R} employing the value-matching and the smooth-pasting conditions $E^1(x_1^*) = 0$, $E_x^1(x_1^*) = 0$, $E^0(x_{0R}) = E^1(x_{0R})$ and $E_x^0(x_{0R}) = E_x^1(x_{0R})$.

Part (b). Using the definition of x_i^* (7) we can rewrite $F^R(x)$

$$F^R(x) = (1-\lambda_0)\frac{K}{r-\mu}(x - x_0^*) - (1-\lambda_0)\frac{K}{r+p_1-\mu}(x - x_1^*) + \\ -\frac{\lambda_0-\lambda_1}{\lambda_1}\frac{Kx_1^*}{r+p_1-\mu}\left[\left(\frac{x}{x_1^*}\right)^{\lambda_1} - 1\right]$$

and hence $F^R(x_1^*) < 0$ if and only if $x_1^* < x_0^*$. Thus, engaging in a last resort gamble strategy is optimal (i.e.$x_{0R} > x_1^*$) if and only if $x_0^* > x_1^*$. Note that $F^R(x_0^*) < 0$ and consequently $x_{0R} > x_0^* > x_1^*$.

Increasing C reduces the gap between x_0^* and x_1^* since

$$\frac{\partial(x_0^* - x_1^*)}{\partial C} = -\frac{\lambda_0}{1-\lambda_0}\frac{1}{K}\frac{r-\mu}{r} + \frac{\lambda_1}{1-\lambda_1}\frac{1}{K}\frac{r+p_1-\mu}{r+p_1} < 0 \tag{9}$$

Since for $C = rH$, Assumption 1 implies that $x_1^* < x_0^*$, and since, for $C \to \infty$, $x_1^* > x_0^*$, by continuity there exists a unique value of C such that $x_0^* = x_1^*$. Moreover, since the larger is p_1, the lower is $\frac{\partial(x_0^*-x_1^*)}{\partial C}$, $\frac{\partial \widehat{C}}{\partial p_1} < 0$. For each $C < \widehat{C}$, $x_1^* < x_0^*$ and thus $F^R(x_1^*) < 0$ and as a consequence $x_{0R} > x_1^*$, while for each $C \geq \widehat{C}$, $x_1^* \geq x_0^*$ and thus $F^R(x_1^*) > 0$ and as a consequence $x_{0R} \geq x_1^*$.

The proof of the sign of the derivatives $\frac{\partial x_{0R}}{\partial p_1} < 0$, $\frac{\partial x_{0R}}{\partial \gamma_1} > 0$, $\frac{\partial x_1^*}{\partial p_1} > 0$ and $\frac{\partial x_1^*}{\partial \gamma_1} < 0$ is the same as the one in the Proof of Proposition 1. ■

Proof of Proposition 4. The sign of the derivative of $\frac{\partial(x_0^*-x_1^*)}{\partial C}$ has already been proved in the proof of Proposition 3.

In this proof we calculate the sign of the derivative $\frac{\partial(x_{0R}-x_1^*)}{\partial C}$. We rewrite $F^R(x_{0R}) = 0$ as

$$(1-\lambda_0)\left(\frac{1}{r-\mu} - \frac{1}{r+p_1-\mu}\right)(x_{0R} - x_1^*) = (1-\lambda_0)\frac{1}{r-\mu}(x_0^* - x_1^*) + g(x_{0R}, x_1^*) \tag{10}$$

where

$$g(x_{0R}, x_1^*) = \frac{x_1^*}{r+p_1-\mu}\frac{\lambda_0-\lambda_1}{\lambda_1}\left[\left(\frac{x_{0R}}{x_1^*}\right)^{\lambda_1} - 1\right]$$

and where, since $x_{0R} > x_1^*$ and $\lambda_1 < 1$, $g(x_{0R}, x_1^*) > 0$.

Let us define $L(x_{0R}, x_1^*) \equiv \frac{\partial g(x_{0R}, x_1^*)}{\partial x_{0R}} + \frac{\partial g(x_{0R}, x_1^*)}{\partial x_1^*}$. The following Lemma summarizes the properties of $L(x_{0R}, x_1^*)$.

Lemma A1.

(i) $L(x_1^*, x_1^*) = 0$;(ii) $L(x_{0R}, x_1^*)$ is strictly increasing in x_{0R};

(iii) $\lim_{x_{0R}\to\infty} L(x_{0R}, x_1^*) = -\frac{1}{r+p_1-\mu}(\lambda_0 - \lambda_1)\frac{1}{\lambda_1}$.

Proof. The derivatives of $L(x_{0R}, x_1^*)$ are

$$\frac{\partial g(x_{0R}, x_1^*)}{\partial x_{0R}} = \frac{1}{r+p_1-\mu}(\lambda_0 - \lambda_1)\left(\frac{x_{0R}}{x_1^*}\right)^{\lambda_1 - 1} \tag{11}$$

$$\frac{\partial g(x_{0R}, x_1^*)}{\partial x_1^*} = \frac{1}{r+p_1-\mu}\frac{\lambda_0 - \lambda_1}{\lambda_1}\left[(1-\lambda_1)\left(\frac{x_{0R}}{x_1^*}\right)^{\lambda_1} - 1\right] \tag{12}$$

and thus

$$L(x_{0R}, x_1^*) = \frac{1}{r+p_1-\mu}(\lambda_0 - \lambda_1)\left[\frac{1-\lambda_1}{\lambda_1}\left(\frac{x_{0R}}{x_1^*}\right)^{\lambda_1} - \frac{1}{\lambda_1} + \left(\frac{x_{0R}}{x_1^*}\right)^{\lambda_1 - 1}\right] \tag{13}$$

(i) From (13) it is easy to see that $L(x_1^*, x_1^*) = 0$.

(ii) Taking the derivative of (13) with respect to x_{0R} we obtain

$$\frac{\partial L(x_{0R}, x_1^*)}{\partial x_{0R}} \approx (1-\lambda_1)\left(\frac{x_{0R}}{x_1^*}\right)^{\lambda_1} + (\lambda_1 - 1)\left(\frac{x_{0R}}{x_1^*}\right)^{\lambda_1 - 1} > 0$$

(iii) Since $\lambda_1 < 0$, it follows that $\lim_{x_{0R}\to\infty} L(x_{0R}, x_1^*) = -\frac{1}{r+p_1-\mu}(\lambda_0 - \lambda_1)\frac{1}{\lambda_1}$.

From (10) we obtain $\frac{\partial(x_{0R} - x_1^*)}{\partial C}$

$$\begin{aligned}(1-\lambda_0)\left(\tfrac{1}{r-\mu} - \tfrac{1}{r+p_1-\mu}\right)\tfrac{\partial(x_{0R}-x_1^*)}{\partial C} &= (1-\lambda_0)\tfrac{1}{r-\mu}\tfrac{\partial(x_0^*-x_1^*)}{\partial C} + \\ &+ \tfrac{\partial g(x_{0R},x_1^*)}{\partial x_{0R}}\tfrac{\partial x_{0R}}{\partial C} + \tfrac{\partial g(x_{0R},x_1^*)}{\partial x_1^*}\tfrac{\partial x_1^*}{\partial C}\end{aligned} \tag{14}$$

Adding and subtracting $\frac{\partial g(x_{0R},x_1^*)}{\partial x_{0R}}\frac{\partial x_1^*}{\partial C}$ on the left-hand-side of (14) and rearranging terms we obtain

$$\frac{\partial\left(x_{0R}-x_1^*\right)}{\partial C}=\frac{\left(1-\lambda_0\right)\frac{1}{r-\mu}\frac{\partial\left(x_0^*-x_1^*\right)}{\partial C}+L\left(x_{0R},x_1^*\right)\frac{\partial x_1^*}{\partial C}}{\left(1-\lambda_0\right)\left(\frac{1}{r-\mu}-\frac{1}{r+p_1-\mu}\right)-\frac{\partial g\left(x_{0R},x_1^*\right)}{\partial x_{0R}}} \tag{15}$$

In the following we show that the numerator as well as the denominator of the right-hand-side of (15) are positive.

We now proceed to prove that the numerator of (15) is negative. The first term of the numerator is negative (see the first part of this proof) while the second term, since $L\left(x_{0R},x_1^*\right)\geq 0$ and $\frac{\partial x_1^*}{\partial C}>0$, is positive. Since $L\left(x_{0R},x_1^*\right)$ is strictly increasing in x_{0R}, a sufficient condition for the numerator to be negative is

$$-\left(1-\lambda_0\right)\frac{1}{r-\mu}\frac{\partial\left(x_0^*-x_1^*\right)}{\partial C}>\frac{1}{r+p_1-\mu}\left(\lambda_0-\lambda_1\right)\frac{1}{\lambda_1}\frac{\partial x_1^*}{\partial C}$$

which, after substituting the derivatives and simplifying terms, is equivalent to

$$\left(1-\lambda_0\right)\frac{1}{r-\mu}\left(\frac{\lambda_0}{1-\lambda_0}\frac{r-\mu}{r}-\frac{\lambda_1}{1-\lambda_1}\frac{r+p_1-\mu}{r+p_1}\right)>\frac{\lambda_0-\lambda_1}{1-\lambda_1}\frac{1}{r+p_1} \tag{16}$$

Rearranging further terms yields

$$\lambda_0\left(\lambda_1-1\right)\mu p_1+\left(\lambda_0-\lambda_1\right)rp_1>0$$

which is always true.

Next we show that the denominator of (15) is positive. Using (11) the numerator reads

$$M\left(x_{0R},x_1^*\right)\equiv\left(1-\lambda_0\right)\left(\frac{1}{r-\mu}-\frac{1}{r+p_1-\mu}\right)-\frac{1}{r+p_1-\mu}\left(\lambda_0-\lambda_1\right)\left(\frac{x_{0R}}{x_1^*}\right)^{\lambda_1-1}$$

Note that $M\left(x_{0R},x_1^*\right)$ is strictly increasing in x_{0R} and

$$\lim_{x_{0R}\to\infty}\left(1-\lambda_0\right)\left(\frac{1}{r-\mu}-\frac{1}{r+p_1-\mu}\right)>0$$

Thus, since $x_{0R}>x_1^*$ it is sufficient to prove that $M\left(x_1^*,x_1^*\right)>0$, where$M\left(x_1^*,x_1^*\right)>0$ can be rewritten as

$$\frac{r+p_1-\mu}{r-\mu}>\frac{1-\lambda_1}{1-\lambda_0} \tag{17}$$

Subtracting μ from both sides of (3) we have

$$\left(\lambda_i \frac{1}{2}\sigma^2 + \mu\right)(\lambda_i - 1) = r + p_i - \mu$$

Note that the right-hand-side of this expression is positive valued by Assumption 1 (which implies that $\lambda_i \frac{1}{2}\sigma^2 + \mu < 0$). Substituting this last result into (17) we obtain

$$\frac{(1-\lambda_1)\left(\lambda_1 \frac{1}{2}\sigma^2 + \mu\right)}{(1-\lambda_0)\left(\lambda_0 \frac{1}{2}\sigma^2 + \mu\right)} > \frac{1-\lambda_1}{1-\lambda_0}$$

Simplifying and rearranging terms we obtain $-\lambda_1 > -\lambda_0$, which is true. ■

Proof of Proposition 5. Under the risky debt assumption the value of the equity-holders' claim satisfies the following differential equation:

$$rE^i = Kx + \gamma^i K - f - C + \mu E^i_x + \frac{1}{2}\sigma^2 E_{xx} x^2 + p^i\left(-E^i\right) \tag{18}$$

for $i = 0, 1, 2$.

We assume that both gambling intensities are active and calculate the equity value and gamble and closure thresholds, and afterwards we show that this is true for sufficiently low coupon values (i.e. $C < \overline{C}$).

From (18), imposing value matching conditions $E^2(x_{0L}) = E^1(x_{0L})$, $E^1(x_{1L}) = E^0(x_{1L})$ and $E^0(x_2) = 0$ we obtain

$$E^2(x) = \frac{Kx}{r+p_2-\mu} + \frac{\frac{\gamma_2}{1+p_2}K-C-f}{r+p_2} + \\ + \left(-\frac{Kx_2^*}{r+p_2-\mu} - \frac{\frac{\gamma_2}{1+p_2}K-C-f}{r+p_2}\right)\left(\frac{x}{x_2^*}\right)^{\lambda_2}$$

$$E^1(x) = \frac{Kx}{r+p_1-\mu} + \frac{\frac{\gamma_1}{1+p_1}K-C-f}{r+p_1} + \left(\frac{Kx_{1L}}{r+p_2-\mu} + \frac{\frac{\gamma_2}{1+p_2}K-C-f}{r+p_2} - \frac{Kx_{1L}}{r+p_1-\mu} - \frac{\frac{\gamma_1}{1+p_1}K-C-f}{r+p_1}\right)\left(\frac{x}{x_{1L}}\right)^{\lambda_1} + \\ \left(-\frac{Kx_2^*}{r+p_2-\mu} - \frac{\frac{\gamma_2}{1+p_2}K-C-f}{r+p_2}\right)\left(\frac{x_{1L}}{x_2^*}\right)^{\lambda_2-\lambda_1}\left(\frac{x}{x_2^*}\right)^{\lambda_1}$$

$$E^0(x) = \frac{Kx}{r-\mu} - \frac{C+f}{r} + \left(\frac{Kx_{0L}}{r+p_1-\mu} + \frac{\frac{\gamma_1}{1+p_1}K-C-f}{r+p_1} + \frac{C+f}{r} - \frac{Kx_{0L}}{r-\mu}\right)\left(\frac{x}{x_{0L}}\right)^{\lambda_0} + \\ \left(\frac{Kx_{1L}}{r+p_2-\mu} + \frac{\frac{\gamma_2}{1+p_2}K-C-f}{r+p_2} - \frac{Kx_{1L}}{r+p_1-\mu} - \frac{\frac{\gamma_1}{1+p_1}K-C-f}{r+p_1}\right)\left(\frac{x_{0L}}{x_{1L}}\right)^{\lambda_1-\lambda_0}\left(\frac{x}{x_{1L}}\right)^{\lambda_0} + \\ \left(-\frac{Kx_2^*}{r+p_2-\mu} - \frac{\frac{\gamma_2}{1+p_2}K-C-f}{r+p_2}\right)\left(\frac{x_{1L}}{x_2^*}\right)^{\lambda_2-\lambda_1}\left(\frac{x_{0L}}{x_2^*}\right)^{\lambda_1-\lambda_0}\left(\frac{x}{x_2^*}\right)^{\lambda_0}$$

where gambling thresholds x_{1L} and x_{0L} are obtained imposing smooth pasting conditions $E_x^2(x) = E_x^1(x)$ and $E_x^1(x) = E_x^0(x)$, $F^1(x)$ and $F^0(x)$ are defined as follows:

$$F^1(x) = Kx(1-\lambda_1)\left(\frac{1}{r+p_1-\mu} - \frac{1}{r+p_2-\mu}\right) + \frac{\lambda_1-\lambda_2}{1-\lambda_2}\left(-\frac{\frac{\gamma_2}{1+p_2}K-C-f}{r+p_2}\right)\left(\frac{x}{x_2^*}\right)^{\lambda_2} + \\ +\lambda_1\left(\frac{\frac{\gamma_2}{1+p_2}K-C-f}{r+p_2} - \frac{\frac{\gamma_1}{1+p_1}K-C-f}{r+p_1}\right)$$

and

$$F^0(x) = Kx(1-\lambda_0)\left(\frac{1}{r-\mu} - \frac{1}{r+p_1-\mu}\right) + \frac{\lambda_0-\lambda_1}{1-\lambda_1}\left(-\frac{\frac{\gamma_2}{1+p_2}K-C-f}{r+p_2}\right)\left(\frac{x_{1L}}{x_2^*}\right)^{\lambda_2}\left(\frac{x}{x_{1L}}\right)^{\lambda_1} + \\ +\lambda_0\left(\frac{C+f}{r} + \frac{\frac{\gamma_1}{1+p_1}K-C-f}{r+p_1}\right) + \frac{\lambda_0-\lambda_1}{1-\lambda_1}\left(\frac{\frac{\gamma_2}{1+p_2}K-C-f}{r+p_2} - \frac{\frac{\gamma_1}{1+p_1}K-C-f}{r+p_1}\right)$$

and closure threshold x_2^* solve the smooth pasting condition $E_x^2(x) = 0$.

In the following we show that $\overline{C}$ exists. In particular, we first show that Assumption 2 guarantees that as long as debt is risk-free the inequality $x_{0L} > x_{1L} > x_2^*$ holds and then we show that increasing riskiness of the debt value there exists a critical coupon value below which engaging in last resort gambling behavior with intensity 2 is optimal for some values of x, while above this threshold last resort gambling with intensity 2 is never optimal.

Note first that Assumption 2 implies that, as long as debt is risk-free, $x_0^* > x_1^* > x_2^*$. We divide the proof into two parts: (a) $x_{1L} > x_2^*$ and (b) $x_{0L} > x_{1L}$.

Part (a). $F^1(x)$ is a convex function of x. Thus, to prove that $x_{1L} > x_2^*$ we show that $F^1(x_2^*) < 0$. Using (7) we can rewrite $F^1(x)$ as

$$F^1(x) = (1-\lambda_1)\frac{K}{r+p_1-\mu}(x-x_1^*) - (1-\lambda_1)\frac{K}{r+p_2-\mu}(x-x_2^*) - \frac{\lambda_1-\lambda_2}{\lambda_2}\frac{Kx_2^*}{r+p_2-\mu}\left[\left(\frac{x}{x_2^*}\right)^{\lambda_2} - 1\right] \tag{19}$$

and hence $F^1(x_2^*) \leq 0$ if and only if $x_1^* > x_2^*$, which is satisfied by Assumption 2. Moreover, since $x_1^* > x_2^*$ and $\lambda_2 < 0$, from (19) it follows that $F^1(x_1^*) < 0$. Hence, $x_{1L} > x_1^* > x_2^*$. Thus, investing in the risky project 2 is optimal (i.e. $x_{1L} > x_2^*$) if and only if $x_1^* > x_2^*$.

Part (b). To prove that $x_{0L} > x_{1L}$ holds for sufficiently large values of γ_1 we rewrite, using

(7), $F^0(x)$ as

$$F^0(x) = (1-\lambda_0)\frac{K}{r-\mu}(x-x_0^*) - (1-\lambda_0)\frac{K}{r+p_1-\mu}(x-x_1^*) +$$
$$-\frac{\lambda_0-\lambda_1}{1-\lambda_1}\frac{1-\lambda_2}{\lambda_2}\frac{Kx_2^*}{r+p_2-\mu}\left[\left(\frac{x_{1L}}{x_2^*}\right)^{\lambda_2}\left(\frac{x}{x_{1L}}\right)^{\lambda_1} - 1\right]$$

Since by Assumption 2 $x_0^* > x_1^*$ and $\left(\frac{x_{1L}}{x_2^*}\right)^{\lambda_2}\left(\frac{x_0^*}{x_{1L}}\right)^{\lambda_1} < 1$, $F^0(x_0^*) < 0$ and consequently $x_{0L} > x_0^*$. Observe that there always exists a value of γ_1 such that $x_1^* = x_2^*$. In this case condition $F^1(x) = 0$ yields $x_{1L} = x_1^*$, while condition $F^0(x) = 0$ yields $x_{0L} > x_0^*$ and thus we obtain, $x_{0L} > x_0^* > x_1^* = x_{1L}$. By continuity inequality $x_{0L} > x_{1L}$ holds for sufficiently large values of γ_1.

Note that Assumption 3 implies that $\frac{\partial}{\partial C}(x_2^* - x_1^*) > \frac{\partial}{\partial C}(x_1^* - x_0^*)$ and as a consequence there exists a $\overline{C}$ such that $x_2^* = x_1^*$ while $x_0^* > x_1^*$ and thus for $C > \overline{C}$ gambling intensity 2 is no longer optimal, while gambling intensity 1 remains optimal for some values of x as stated in Proposition 3.

To compute $D(x)$, let us solve the following differential equation (8) for $i = 0$ for $x > x_{0L}$, $i = 1$ for $x_{1L} < x \leq x_{0L}$ and $i = 2$ for $x_2^* < x \leq x_{1L}$. The general solution of (8) is $\frac{C+p_iH}{r+p_i} + L_i x^{\lambda_i}$ for some L_i, if we take the no-bubble condition into account. We determine L_2 employing the boundary condition $D^2(x_2^*) = H$ and L_1 and L_0 employing the value matching conditions $D^2(x_{1L}) = D^1(x_{1L})$ and $D^1(x_{0L}) = D^0(x_{0L})$, respectively, yielding

$$D^2(x) = \frac{C+p_2H}{r+p_2}\left(1-\left(\frac{x}{x_2^*}\right)^{\lambda_2}\right) + H\left(\frac{x}{x_2^*}\right)^{\lambda_2}$$

$$D^1(x) = \frac{C+p_1H}{r+p_1} + \left(\frac{C+p_2H}{r+p_2} - \frac{C+p_1H}{r+p_1}\right)\left(\frac{x}{x_{1L}}\right)^{\lambda_1} + \left(H - \frac{C+p_2H}{r+p_2}\right)\left(\frac{x_{1L}}{x_2^*}\right)^{\lambda_2-\lambda_1}\left(\frac{x}{x_2^*}\right)^{\lambda_1}$$

$$D^0(x) = \frac{C}{r} + \left(\frac{C+p_1H}{r+p_1} - \frac{C}{r}\right)\left(\frac{x}{x_{0L}}\right)^{\lambda_0}$$
$$+\left(\frac{C+p_2H}{r+p_2} - \frac{C+p_1H}{r+p_1}\right)\left(\frac{x_{0L}}{x_{1L}}\right)^{\lambda_1-\lambda_0}\left(\frac{x}{x_{1L}}\right)^{\lambda_0} + \left(H - \frac{C+p_2H}{r+p_2}\right)\left(\frac{x_{1L}}{x_2^*}\right)^{\lambda_2-\lambda_1}\left(\frac{x_{0L}}{x_2^*}\right)^{\lambda_1-\lambda_0}\left(\frac{x}{x_2^*}\right)^{\lambda_0}$$

■