THE RIMINI CENTRE FOR ECONOMIC ANALYSIS

WP 30_08

Chiara Binelli
Oxford University, UK - Institute for Fiscal Studies, UK
and
The Rimini Centre for Econonic Analisys, Italy

"Returns to Education and Increasing Wage Inequality in Latin America"

Copyright belongs to the author. Small sections of the text, not exceeding three paragraphs, can be used provided proper acknowledgement is given.

The Rimini Centre for Economic Analysis (RCEA) was established in March 2007. RCEA is a private, non-profit organization dedicated to independent research in Applied and Theoretical Economics and related fields. RCEA organizes seminars and workshops, sponsors a general interest journal *The Review of Economic Analysis*, and organizes a biennial conference: *Small Open Economies in the Globalized World* (SOEGW). Scientific work contributed by the RCEA Scholars is published in the RCEA Working Papers series.

The views expressed in this paper are those of the authors. No responsibility for them should be attributed to the Rimini Centre for Economic Analysis.

The Rimini Centre for Economic Analysis
Legal address: Via Angherà, 22 – Head office: Via Patara, 3 - 47900 Rimini (RN) – Italy
www.rcfea.org - secretary@rcfea.org

Returns to Education and Increasing Wage Inequality in Latin America*

Chiara Binelli†
Oxford University and Institute for Fiscal Studies

First draft: June, 2005. This draft: September, 2008.

Abstract

This paper studies a central feature that characterized the changes in wage inequality in Latin America in the 1990s: log wages became a convex function of the level of education. The wage gap between Higher and Intermediate Education increased and the one between Intermediate and Basic Education declined. The double change in the wage differentials was driven by a significant drop in the mean wage at Intermediate. I develop and simulate a dynamic general equilibrium model of savings and educational choices under credit constraints and uninsurable earnings' risk in which ability is an important component of individual wages. I estimate the parameters of the model using micro data from Mexico. The results show that the convexification was the result of changes in the prices of education due to changes in its supply. Absent the general equilibrium price effects, the changes in ability composition by education needed to produce the convexification would have been unrealistically high.

Key Words: Latin America, Wage Inequality, Education Choices, General Equilibrium.
JEL Codes: J23, J24, J31, C68.

*I thank Orazio Attanasio for many helpful comments and discussions. I am also grateful to Richard Blundell, Jeremy Lise, Nicola Pavoni, Victor Rios-Rull and Claudia Trentini for useful discussions and to seminar participants at the 2008 European Economic Association Conference in Milan, the 2008 Labor Markets Workshop in Rimini, the 2008 Small Open Economies in a Globalized World Conference in Canada, the 2008 European Workshop in Macroeconomics in Glasgow, the 2007 Annual Conference of the Northeast Universities Development Consortium in Boston, the First Riccardo Faini Conference in Italy and the 2007 Annual Conference of the Society for the Study of Economic Inequality in Berlin for useful comments. A previous version of this paper circulated under the title "Wage Inequality and Education Choices in Latin America: a General Equilibrium Approach". All errors are mine.

†Address: Institute for Fiscal Studies, 7 Ridgmount Street, London WC1E 7AE. Email: chiara_b@ifs.org.uk

1 Introduction

In the decade of the 1990s Latin American Countries (LACs) have been characterized by a process of educational expansion and significant changes in the wage differentials between education levels. Compulsory education became almost universal and the share of individuals that completed secondary and high school increased from around 32 to 48 per cent.[1] Graduation rates increased also at higher education: the proportion of college graduates went from an average of 13 per cent in 1987 to an average of 18 per cent in 2002.[2] At the same time, the wage differential between higher and intermediate education increased and the one between intermediate and basic education declined. Between 1987 and 2002 relative wages rose on average by 45 per cent at higher education and decreased by 26 per cent at intermediate education.

The result of the double change in the relative wages at higher and at intermediate education is shown in Figure 1 in section 2. From a linear relationship between mean wages and the level of education at the end of the 1980s, the wage profile became convex with proportional increases in education translating into higher than proportional increases in income for the more educated. The changes in relative wages were driven by a decline in the mean wage at intermediate that fell below the value at the end of the 1980s. Between 1987 and 2002 the mean wage at intermediate decreased in real terms by three and two per cent in Brazil and Colombia and by as much as five per cent in Mexico.

The convexification of the wage profile has received almost no attention. The empirical literature on LACs has focused primarily on the increase in the premium to higher education. To my knowledge, Bouillon, Legovini and Lustig (2005) is the only exception. They focus on Mexico and estimate Mincer-type wage equations for the year 1984 and 1994 with the number of years of education entering as a second order polynomial. They find that the quadratic term is highly significant in 1994 in almost all regressions while barely significant or non-significant in 1984. Their analysis is descriptive in nature while the aim in this paper is to investigate the determinants of the convexification.

The mean wages in Figure 1 are a combination of three main components: the return to observable (such as cohort, sector of employment and work experience) and to unobservable

[1] In the year 1987 the proportion of the adult population with completed secondary education was 32 per cent in Brazil, 34 per cent in Colombia and 30 per cent in Mexico. In the year 2002 these proportions increased, respectively, to 55, 43, 45 per cent in each of the three countries.

[2] In the year 1987 the proportion of college graduates among the adult population was around 12 per cent in Brazil, 17 per cent in Colombia and 10 per cent in Mexico. In the year 2002 these proportions increased to, respectively, 14, 21 and 17 per cent.

(unobserved ability or quality of the workers) characteristics of those completing each level of education, and the price of education, which is the market value of completing a given education level independently of any (observable and unobservable) individual characteristic. Therefore, the change of the wage profile from linear to convex could be produced for three main reasons: a change in the composition on observables and their returns, a change in the ability composition by level of education of the cohorts of graduates in 1987 and in 2002 or a change in the prices of education.

In the first case, the convexification would be the result of changes in the composition by and the returns to observable characteristics that are relevant for wage determination, such as cohort, gender, sector of employment and work experience, by level of education. This first explanation is easy to check by estimating wage equations that control for observables and their interaction with the level of education. The results are qualitatively very similar to the ones reported in Figure 1. Controlling for age and its interaction with the level of education is particularly important: the convexification is not the result of changes in the age structure of the population or in the returns to a specific age group.

In the second case, the change in the shape of the wage function would be produced by changes in the ability composition of workers with intermediate and higher education. The educational expansion at intermediate could have resulted into a decline in the mean ability level and therefore a decrease in the observed wage at this level. At the same time, if the expansion at intermediate enabled more high-ability people than before to progress to higher education, mean ability and thus observed wages would have increased for college graduates.

In the third case, the production technology and the interactions between the demand and the supply of skill would be of crucial importance: any change in the demand for skills would affect the incentives to invest in human capital and return a value for the skill prices that reflect the degree of complementarity and substitutability between the different production inputs. There are no empirical studies of wage inequality in LACs that allow for endogenous education prices, so nothing is known on the importance of the general equilibrium effects of changes in the prices due to changes in the supply of skills.

Differently from the first one, the second and third explanations can not be investigated by a simple look at the data. Instead, we need to develop a structural framework with two key characteristics: being of general equilibrium with the inclusion of the demand and the supply of skills and able to account for self-selection on ability into education. This paper provides such a framework.

I develop an incomplete market, dynamic, general equilibrium model of savings and educational choices where the interest rate is taken as given and the education prices are the equilibrium outcomes of changes in the supply and in the demand for skills. The model has two important features. First, educational (and savings) choices are taken by altruistic parents that face constraints on resources. The economy is made of overlapping generations of parents and children with three distinct market failures: parents cannot buy insurance against their own earnings' risk, cannot borrow against future earnings of their children and face credit constraints. In each period parental resources are given by a stochastic labor income and the stocks of a risk-free asset. Credit constraints are modelled as a limit to the amount that individuals can borrow ranging from zero to the maximum amount that an individual can repay with certainty over the life cycle, which is a function of the level of education, an insurable i.i.d. shock and an endowment of ability that is received at birth and is perfectly transmitted across generations. By having ability as an important component of wages, the model endogenously generates a process of self-selection of higher ability individuals into higher education.

Second, the production function allows for complementarities between intermediate and higher education. I assume that three education levels - basic, intermediate and higher education - build up the economy human capital endowment and I allow the elasticity of substitution (ES) between intermediate and basic education to differ from the one between intermediate and higher education. I estimate the production function using micro data from Mexico between 1987 and 2002. I find that the substitution elasticities are consistent with the complementarity between intermediate and higher education. A key implication is that the growth of the supply of intermediate education increases the relative demand for higher education and therefore its marginal product while it decreases the relative return of intermediate with respect to basic education.

Starting from an initial distribution of ability, wealth and education in 1987 estimated with micro-data from Mexico, I solve for the steady state of the model under different scenarios characterized by a skill-biased technological change (SBTC - hereafter) and different levels of credit constraints and uninsurable earnings' risk. Following Heckman, Lochner and Taber (1998) (HLT - hereafter), I estimate the value of the share of skilled labour in production in 1987 and its average annual increase in the 1990s and I model a SBTC as a permanent increase in this share.

The results show that the convexification came about through changes in the prices of schooling due to changes in its supply. If the supply by education had been constant at the level observed at the end of the 1980s, a SBTC would have produced an increase in both the relative return to higher and to intermediate education. The simulations identify two main determinants of the convexification. First, the complementarities in the production function are responsible for the double change in the wage differentials. In addition to the standard supply effect, an increase in the supply of intermediate further decreases the relative return to intermediate with respect to basic education and increases the relative return to higher with respect to basic education. Second, the level of the credit constraints determines the extent of the supply increase at intermediate education and therefore the change in the wage at this level. Finally, an increase in the level of uninsurable earnings' risk produces an equilibrium prices' schedule that is "not enough" convex to match what is observed in the data: a high level of risk would have prevented the sizable supply increase at intermediate and the drop of the price at this level that characterizes the convexification.

1.1 Literature review

Several contributions are related to the analysis developed in this paper. They can be classified in two main categories: the empirical literature on education and wages in Latin America and the literature that uses dynamic general equilibrium (GE) models to study the changes in wage inequality and their determinants.

With respect to the literature on wages and education in LACs, both the object of interest and the approach taken in this paper are rather novel. Up to now all the contributions have been focusing on the increase in the premium to higher education rather than on the convexification and have adopted a partial equilibrium approach explaining the rise in the premium with changes in the supply or in the demand of skills.

The "supply-side" literature focuses on financial constraints on educational choices and self-selection on ability into higher education as two alternative determinants of the increase in the relative wage of higher versus intermediate education. If ability is an important component of wages, the premium to higher education might simply reflect a composition effect of higher ability into higher education. Alternatively, a decrease in the tightness of the financial constraints at the intermediate level with the constraints remaining binding at higher education could also have produced the increase in the skill premium by allowing the supply to increase more at intermediate than at higher education.

The main contribution for Mexico is Jacoby and Skoufias (2002). They consider a sample of young aged 19 to 21 between 1995 and 1999 and estimate the probability of attending college conditional on high school completion controlling for unobserved ability, labor market conditions, family background, and proxies for current and permanent parental income. They find current parental income being an important determinant of college attendance and weak evidence of selection bias.

A related contribution is Binelli, Meghir and Menezes-Filho (2008) (BMM - hereafter) for Brazil. They distinguish between four levels of education and estimate the changes in the ability distribution by level of education and birth cohort during the educational expansion in Brazil in the 1990s. They find evidence of sizable changes in the mean ability level at intermediate and higher education. However, the changes in ability composition can not fully account for the steep increase in the College premium that remains largerly unexplained.

The "demand-side" literature focuses on the impact of trade liberalization and a series of labor market reforms promoted in Latin America in the 1990s that increased the demand for skilled labor in production. This second branch of literature is vast and counts on many different contributions for each Latin American country.[3] Most empirical studies have found evidence of a technological change that increased the demand for skilled labour, while a smaller set of studies have estimated a positive impact of trade opening on the skill premium.[4] However, when the relative importance of the trade liberalization and the technology explanation has been compared, a SBTC appears as the driving force behind the increase in the skill premium.[5]

Behrman, Birdsall and Szekely (2007) have performed an overall evaluation of the major market reforms that characterized LACs in the 1980s and 1990s. They construct six indexes for six main policy reforms and use a panel data set on eighteen LACs for the period between 1980 and 1998 to estimate the impact of the reforms on the relative wages to higher versus primary and intermediate education.[6] They find that trade liberalization did not have any significant

[3] Goldberg and Pavcnik (2004) and Winters, McCullach and McKay (2004) provide two complete surveys of the literature.

[4] On the impact of a skill-biased change in production, see, among the others, Attanasio, Goldberg and Pavcnik (2002) for Colombia and Corseuil and Muendler (2003), Pavcnik, Blom and Goldberg (2002) and Holm-Nielsen and Verner (2001) for Brazil, Bustos (2005) for Argentina and Verhoogen (2008) for Mexico. On the impact of opening to trade, see, among the others, Hanson and Harrison (1999) for Mexico and Lisboa, Menezes-Filho and Schor (2004), Gonzaga, Menezes-Filho and Terra (2006) and Giovannetti and Menezes-Filho (2006) for Brazil.

[5] In practice trade opening and skilled-biased technological change are very related since a change in production towards skilled labour could be the effect of increased exposure to international markets.

[6] The six reforms they consider are: privatization of former state enterprises, trade liberalization, capital account liberalization, domestic financial market liberalization, tax reforms and labor market reforms.

impact on the changes in the wage differentials while the return to higher education significantly increased when the economies became more technologically advanced and the exports of high-technology products increased. Avalos and Savvides (2003) also find evidence suggesting a positive association between increasing skill premium and technological change. Using a cross-section of Latin American countries they show that exposure to foreign direct investment, greater imports of machinery and equipment and R&D transfer from developed economies are associated with increasing wage differentials between high and low-educated.

The analysis developed in this paper is also related to the literature that uses dynamic GE models to study wage inequality. Up to now all papers have been focusing on the US economy and, most importantly, have been trying to explain changes in relative returns. No paper has analyzed the relevance of using a dynamic GE model to explain the convexification, which amounts at explaining changes in relative returns and in the level of wages. The closest contributions are HLT and Lee and Wolpin (2006) (LW - hereafter).

HLT is the first paper to have quantified the importance of the GE effects to explain changes in wage inequality. They develop and simulate a GE model of training on the job, investment in education and savings that they use to explain the increase in the skill premium observed in the US between the 1970s and the 1990s. They model SBTC as a permanent technological shift towards skilled labor and find that the increase in wage inequality can be explained as a result of skill-specific supply responses to a SBTC.

As in HLT, I jointly model savings and educational choices and I characterize SBTC as a permanent shift towards skilled labour in production. However, my model differs in at least three important aspects. First and most importantly, I do allow for credit constraints that are assumed away in HLT framework. Second, I develop an OLG model of parents and children where parents can use financial bequests and education as two alternative strategies to increase child lifetime utility. This creates important interactions between education and saving investments, while HLT do solve for an individual problem and savings are exogenously redistributed to the new born at the end of life with no direct links between saving choices of successive generations. Finally, I do not model on-the-job investment.

LW develop a structural dynamic GE model of work, schooling and sector-specific occupational choice to explain the main facts that characterized the U.S. labor market in the 1970s, 1980s and 1990s. One important fact is the rise in the college wage premium. They find that a SBTC was the main determinant of the increase in the premium. The model I develop is also an attempt to evaluate alternative explanations of the increase in wage inequality. However, it differs from LW in at least one important dimension: I do model saving choices, while LW assume that individuals can not save. Abstracting from savings allows LW to develop a model with a very rich production structure, which they fully structurally estimate. However, it comes at the cost of not being able to assess the impact of credit constraints and risk aversion on occupation (education) choices, which in my model will be crucial to understand the evolution of wage inequality.

A recent contribution that develops a GE model of savings and educational choices with credit constraints and earnings' risk is Gallipoli, Meghir and Violante (2007) (GMV - hereafter). The model of GMV has a much richer structure than the one developed here, the main difference being that it includes endogenous labor supply. Also, GMV use their model to address a very different question. They study the relationship between schooling decisions, wage inequality and education policy. They are mainly interested in studying the long run effects of policy interventions on educational choices and the distribution of earnings. The changes in wage inequality associated with a particular policy is not a direct object of interest but rather an outcome measure to assess the performance of an education policy.

The remainder of the paper is organized as follows. Section 2 presents some empirical evidence on changes in wage inequality in Latin America in the 1990s. Section 3 presents the theoretical model. Section 4 defines the equilibrium. Section 5 discusses the parametrization of the model. Section 6 presents the main results from the simulations. Section 7 gives some concluding remarks. Appendix A describes the data, the construction of the education groups and the estimation of the wage equations. Appendix B gives details of the solution methods.

2 Wage inequality and returns' convexification

Table 1 reports the difference between the mean log hourly real wage at higher and intermediate and at intermediate and basic education for three main LACs. The values have been computed from the National Household Survey of each of the three countries considering the adult population aged between 25 and 60. The data for Colombia refer to 1988 and 1998.

The wage premium to higher education is substantial: in 2002 it is above one hundred per cent in Brazil and Colombia and at around sixty-five per cent in Mexico.[7] In addition, the

[7] The value of the premium to higher education is much bigger than the corresponding figure for developed countries. For example, Carneiro, Heckman and Vytlacil (2005) find a value for the college premium in the US of around 15-20 per cent.

Relative wages			
Year	*1987*	*2002*	*Growth*
Brazil			
Higher vs. intermediate	0.828	1.100	33%
Intermediate vs. basic	0.927	0.557	− 40%
Colombia			
Higher vs. intermediate	0.832	1.092	31%
Intermediate vs. basic	0.551	0.429	− 22%
Mexico			
Higher vs. intermediate	0.375	0.646	72%
Intermediate vs. basic	0.451	0.384	−15%

Table 1: Difference in Log Hourly Real Wages by Level of Education (Source: Author's calculations based on household Surveys)

premium is rising over time, with an increase of around thirty per cent in Brazil and Colombia and of over seventy per cent in Mexico. At the same time intermediate graduates have lost ground. From a value of around ninety-three per cent in Brazil and fifty-five and forty-five per cent in Colombia and Mexico at the end of the 1980s, by the year 2002 relative wages of intermediate with respect to basic education decreased by around forty per cent in Brazil and twenty-two and fifteen per cent in Colombia and Mexico. As a result of the increase in the relative wages at higher education and the decline at the intermediate level the wage profile has become convex.

The convexification appears as a characterizing feature of the changes in wage inequality in Latin America in the 1990s. Brazil, Colombia and Mexico are highly representative of LACs accounting for over seventy per cent of Latin America GDP in this decade. Moreover, in her analysis of the returns to schooling in Argentina between 1992 and 2003 Lopez Boo (2008) does also find increasing convexity in the earnings-education profile due to increasing returns to higher education and decreasing returns to intermediate education. The convexity of the wage profile with respect to the level of education changes the typical concave shape of the wage function observed in Latin America in the 1970s and 1980s.[8]

[8]See Patrinos and Psacharopoulos (2004).

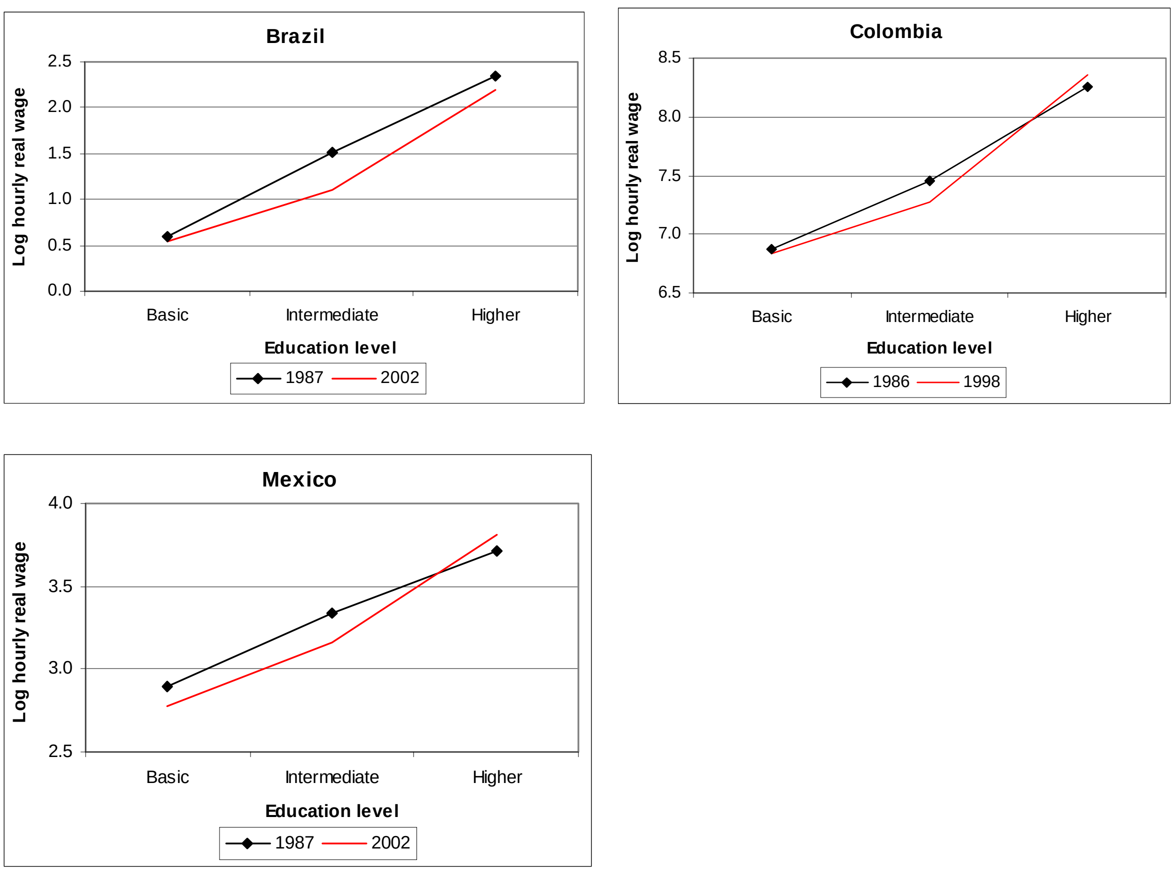

Figure 1: Returns' Convexification in Latin America

Figure 1 presents the mean log hourly real wage in Brazil, Colombia and Mexico at the end of the 1980s and in 2002.[9] In each of the three countries the convexification is apparent: from a linear relationship at the end of the 1980s, wages have become a convex function of the level of education.

In principle, the wage profile could have become convex if both the relative wages at higher and at intermediate education had increased with the relative increase at higher education bigger in magnitude than the one at intermediate. On the contrary, the feature that characterizes the convexification of the wage profile in LACs is the decrease in the relative wages at intermediate with respect to basic education. In Colombia and Mexico the reduction of the wage at intermediate came together with an increase in the mean wage at higher education. On the contrary, in Brazil the mean wage at higher education was lower at the end than at the beginning of the 1990s. However, the relative wage at higher education increased also in this country since the decrease in the wage at higher education was smaller in magnitude than the one at intermediate.[10]

The decrease in the relative wages at intermediate was driven by a drop in the mean wage at this level. There was also a decrease in the wage at basic education but of a much smaller magnitude. The salaries of workers with low education at the bottom of the wage distribution are typically the most affected by changes in labor market institutions such as the minimum wage. It could be that the extent of the fall of wages at basic education was limited by the presence of binding minimum wages. However, with the exception of Colombia, the empirical evidence for Latin America suggests that in the 1990s minimum wages were not binding.[11] The evidence for Mexico shows that the drop in the mean wages at basic was of a much smaller size than the one of the minimum wages: between 1987 and 2002 minimum wages declined in real terms by around 47 per cent while wages at basic education decreased by less than four percentage points.

In the rest of the paper I will focus on Mexico. This country offers an interesting setting to study the convexification. During the 1990s it underwent a series of reforms that changed the structure of production and made the economy more open to foreign investment. The reform

[9] For each of the three countries the difference of the mean real wages by education between the two years is highly statistically significant. A test of the increase of the relative wage at higher education and the decline of the relative wage at intermediate education also returns highly significant results.

[10] Interestingly, a convexified wage profile has also been found to be the driving factor of the steep increase in wage inequality that characterized the US economy in the 1990s (see Deschenes (2002) and Lemieux (2006, 2007)). However, in the US there is no evidence of a decline in the mean real wage at intermediate, which dismisses the possibility of a worldwide decline in the wage for this skill.

[11] See Maloney and Nunez (2003) and Bosch and Manacorda (2007).

effort culminated in 1994 when Mexico became a member of the Organization for Economic Cooperation and Development (OECD) and entered the North American Free Trade Agreement (NAFTA) with the US and Canada. In the same year Mexico was hit by a severe financial crisis, the "Peso crisis", which resulted into a massive devaluation of the national domestic currency. The recovery from the crisis was rather quick and by the end of 1995 Mexico had reentered the international capital markets.

As discussed in section 1.1, several contributions have found that NAFTA and the economic reforms of the mid 1990s increased the demand for skilled labor in production. More recently, Verhoogen (2008) found the Peso crisis more than the opening to trade being associated with an increase in the use of skilled labour and in the production of skill-intensive goods. What is important for the analysis developed here is that, disregarding which explanation has been favoured, all the contributions agree that in the 1990s Mexico underwent a structural change towards the use of skilled labor. I will refer to this change as a skill-biased technological change and I will study its role to explain the convexification.

3 The model

At each time t the economy consists of overlapping generations of parents and children that live together for four periods: a pre-school period and three periods necessary to complete the three education levels - basic, intermediate and higher education. At $t = 1$ each cohort schooling and wealth distribution are taken as exogenous initial conditions. From $t = 2$ these distributions evolve endogenously as a result of parental maximizing behavior. A description of the Mexican schooling system and the construction of the education groups is presented in the first section of Appendix A.

3.1 Supply side: household decision problem

A continuum of individuals is born at each time t. Each individual lives for eight periods, four as a child and four as a parent. As a child the individual lives with the parent that works full time and maximizes utility which is a function of joint household consumption. In the first two periods consumption is the only choice variable. In the first period the child is in pre-school, in the second period is sent to compulsory basic education. In the third and fourth period the child can be sent either to school or to work. If the child is sent to school, the parent has to pay a fixed cost that is education-specific. If sent to work, the child works full time and gives her earnings to the parent. At the end of the fourth period the parent retires and leaves a bequest of financial assets to the child. The child starts the adult life with the level of education completed during childhood and an amount of assets given by parental bequest.

Labor supply is perfectly inelastic and the wages clear the labor markets. The wage of an individual i with education level j and age a in period t is given by:

$$w^i_{j,a,t} = p_{j,t} * \exp(e^i_{j,a,t}) \qquad j = 1, 2, 3 \tag{1}$$

with

$$e^i_{j,a,t} = \eta^i + g_j(age^i_t) + z^i_{j,a,t} \tag{2}$$

where $j = 1, 2, 3$ denotes the education level from basic up to higher education, $p_{j,t}$ is the equilibrium price for education level j in period t and $e^i_{j,a,t}$ denotes labor efficiency of individual i, which is a function of η^i, the individual's ability endowment, $g_j(age^i_t)$, an education-specific polynomial in age which reflects the growth of wages with experience, and $z^i_{j,a,t}$, an education-specific i.i.d. uninsurable shock that is assumed to be normally distributed with mean $\mu_{z_{j,a,t}}$ and variance $\sigma^2_{z_{j,a,t}}$.

The individual's ability endowment, η^i, represents the permanent component of human capital. It is a measure of ability and all unobservable family background factors that have a permanent impact on human capital formation. It is assumed to be perfectly transmitted between successive generations: each individual inherits at birth the ability endowment of her parent and passes it over to her own child.

In order to solve the household maximization problem the adults (parents) need to form expectations on current and future skill prices, $p_{j,t}$, that determine wages. Let us define as $p_t(a)$ the vector of current and future skill prices forecasted from age a onwards. By omitting for simplicity the t time index, parental maximization problem is given by:

$$V_a(X_a) = \max_{\{c_a, I_a\}_{a=\underline{a}}^{a=\overline{a}}} E\left\{\sum_{a=\underline{a}}^{\overline{a}} \beta^{a-\underline{a}} U(c_a) + \beta^3 \lambda V_{\underline{a}}(X_{\underline{a}})\right\} \tag{3}$$

$$A_{a+1} = A_a(1+r) + w_{j^P,a} + [(1-I_a) w_{j^C,a} - I_a F_{j^C}] - c_a \tag{4}$$

$$j^C_{a+1} = \begin{Bmatrix} j^C_a + 1 & if\ I_a = 1 \\ j^C_a & if\ I_a = 0 \end{Bmatrix} \quad \forall\ a = \overline{a} - 1, \overline{a} \tag{5}$$

$$A_a \geq -B_a \qquad \forall\ a = \underline{a}, ..., \overline{a} - 1 \tag{6}$$

$$A_a \geq 0 \qquad a = \overline{a} \tag{7}$$

where X_a denotes the vector of state variables at age a, which includes the level of adult education, j^P, that is fixed throughout adulthood, the level of child education, j_a^C, the amount of assets at age a, A_a, the vector of current and future skill prices forecasted from age a onwards, $p(a)$, the ability endowment, η, and the idiosyncratic shock to wages, z_a. Then $X_a = (j^P, j_a^C, A_a, p(a), \eta, z_a)$, with j_a^C normalized to zero when consumption is the only choice variable. λ is the degree of parental altruism, $\underline{a}$ ($\overline{a}$) denotes the age of the parent at start (end) of the adult life and $V_{\underline{a}}(.)$ is the child's lifetime utility once adult with child education normalized to zero since in the first period of adulthood consumption is the only choice variable.

c_a and A_a denote, respectively, joint household consumption and financial assets at age a. $w_{jP,a}$ is parental wage at age a given parental education level j^P. I_a is an indicator function taking the value of one when the child is sent to school and zero otherwise. If the child is sent to work, the parent receives the child's wage, $w_{jC,a}$. If the child is sent to school, the parent pays the fixed costs, F_{jC}, for the j^C schooling level attended by the child. E denotes expectations that reflect uncertainty due to the presence of the uninsurable idiosyncratic shocks to earnings, z, and β is the discount factor. The utility function is assumed to be strictly increasing and concave in consumption, so that absolute risk aversion is decreasing in individual's wealth, the impact of risk on investment decisions being higher for poorer than for wealthier households.[12]

The optimization problem specified in (3) is solved under four main constraints. The first constraint is a standard period budget constraint with the term in square brackets switching on when child education becomes a choice variable. The second constraint defines the law of motion of child's education. The third constraint is a borrowing restriction imposing a limit B_a on the amount of net indebtedness at age a. The fourth constraint is a terminal condition that prevents parents from dying in debt: they can not leave debts to their children.

The borrowing limit, B_a, can take any value between zero, which corresponds to the maximum level of credit constraints of no possible borrowing, and an upper bound that is given by the present discounted value of lifetime earnings at age a under the lowest possible realization of individual labor efficiency, that is under the lowest possible realization of the idiosyncratic shock

[12] We assume that the utility function takes a simple CRRA formulation:
$U(c) = \frac{(c)^{1-\gamma}}{1-\gamma}$
where γ is the reciprocal of the intertemporal elasticity of substitution.

z.[13] The upper bound represents the maximum amount that an individual will always be able to repay without violating the no-debt condition specified in equation (7).[14]

The household's problem is solved recursively by backwards induction as described in Appendix B.

3.2 Demand side: aggregate production function

The representative firm operates a constant returns to scale technology over physical and human capital. I assume that there are no adjustment costs for physical and human capital and no shocks to the aggregate production. The production function in year t is given by:

$$Y_t = Z_t K_t^{\alpha} HH_t^{1-\alpha} \tag{8}$$

where Y_t denotes aggregate output, K_t is aggregate physical capital and HH_t is aggregate human capital.[15] α denotes the share of physical capital in production which is assumed to be constant over time and Z_t is the technology factor that is normalized to one in all years.[16] I assume that the economy is small and open to the world financial markets. Capital flows in or out of the country so that the marginal product of physical capital equals the world interest rate, r.[17]

I consider three types of human capital corresponding to the three education levels that the individuals can complete and I specify the aggregate human capital in year t, H_t, as a nested CES function over the three types of human capital, H_1, H_2, H_3, which represent, respectively, the human capital of those completing basic, intermediate and higher education.

The choice of how to combine the three human capital inputs in the HH aggregate is very important since it determines the elasticity of substitution (ES) between the H factors, which drives the direction and the magnitude of the general equilibrium effects. I choose a flexible specification for the aggregate human capital that allows for different elasticities of substitution

[13] The empirical distribution of z_j is defined over a finite support with a minimum value, $\underline{z}_j$, and a maximum value, $\overline{z}_j$. Therefore, wages are assumed to be always positive and different from zero.

[14] The value of the upper bound arises naturally from the assumption that the utility function satisfies the Inada condition $\lim_{c\to 0} U(c) = -\infty$ and that parents have to repay all debts before retiring.

[15] This specification of the production function assumes that there are no complementarities between physical and human capital. This assumption is motivated by the near-constancy of the share of physical capital in production estimated for LACs in the 1990s (see Bosworth (1998), Harrison (1994) and Hoffman (1993)).

[16] Given the assumption of no population growth and the normalization of Z, there is no growth in steady state. Growth in the model will only occur during the transition towards a steady state as a result of the reallocation of efficiency units of labor from less to more productive combinations of the different types of human capital.

[17] In the absence of aggregate shocks, the constancy of the world interest rate implies that the economy's physical to human capital ratio is fixed over time. Also, this assumtion implies that firms face no credit constraints. Differently from individual households, they can freely borrow in the international capital markets at the fixed rate r. There are no financial intermediaries that can borrow money from firms and lend it to households.

(ES) between human capitals' pairs. A convenient way to allow for a different ES between pairs of human capital is to combine them within a CES specification. I use the CES specification because it is simple, has few parameters and restricts the substitution elasticities to be constant. An alternative to the CES is the translog function. However, the translog has many more parameters to estimate, which would significantly reduce the degrees of freedom in an already small sample.

Given three human capital inputs, there are three ways of nesting them within a CES aggregate: $\widetilde{HH}_1 = \Gamma_1(H_3, \Gamma_2(H_2, H_1))$, $\widetilde{HH}_2 = \Gamma_2(H_2, \Gamma_2(H_3, H_1))$ and $\widetilde{HH}_3 = \Gamma_3(H_1, \Gamma_2(H_2, H_3))$, where Γ_1, Γ_2 and Γ_3 are CES aggregators. The CES functional form imposes symmetry restrictions on substitution elasticities. For $\widetilde{HH}_1$, the ES between H_3 and H_1 is restricted to be the same as the one between H_3 and H_2. For $\widetilde{HH}_2$, the ES between H_2 and H_3 is restricted to be the same as the one between H_2 and H_1. These restrictions contrast with factor elasticities estimated for LACs suggesting that the ES between higher and intermediate education differs from the one between either higher or intermediate and basic education.[18]

For $\widetilde{HH}_3$, the CES restricts the ES between H_1 and H_2 to be the same as the one between H_1 and H_3. This specification fits an economy with two main sectors, a first one that employs low-skilled labor and a second one where production is carried out by using semi and high-skilled labor. This structure of production is a good description of the Mexican economy that is characterized by two main sectors: a formal sector of semi and high-skilled workers and an informal sector of low-skilled workers.

Therefore, HH_t is specified as it follows:

$$HH_t = [(1 - \delta_{s,t})H_{u,t}^{\rho} + \delta_{s,t}H_{s,t}^{\rho}]^{\frac{1}{\rho}} \tag{9}$$

where $H_{u,t}$ and $H_{s,t}$ are, respectively, the human capital aggregate for unskilled and skilled labour at time t.

$H_{u,t}$ correspond to $H_{1,t}$ and H_s is given by:

$$H_{s,t} = [(1 - \alpha_{3,t})H_{2,t}^{\theta} + \alpha_{3,t}H_{3,t}^{\theta}]^{\frac{1}{\theta}} \tag{10}$$

The time-varying and skill-specific parameters δ and α in equations (9) and (10) denote the shares of the human capital factors in production. Changes in δ and α reflect variations in the

[18] See Manacorda, Sanchez-Paramo and Schady (2006).

productivity and in the demand of the different inputs. The parameter ρ determines the ES between unskilled and skilled labor, which is given by $ES_{u,s} = ES_{1,2} = ES_{1,3} = \frac{1}{1-\rho}$, while θ determines the ES between intermediate and higher education, which is given by $ES_{2,3} = \frac{1}{1-\theta}$.[19] If either ρ or θ is zero, the corresponding nesting is Cobb-Douglas. If $\rho > \theta$, the elasticity of substitution between higher and intermediate is lower than the one between either higher or intermediate and basic education, which means that there are complementarities in production between intermediate and higher education.

Differently from physical capital, labor is not internationally mobile and its remuneration is set domestically. Under the assumption of perfectly competitive markets and profit maximization by firms, the price for education level j in year t, $p_{j,t}$, is given by the marginal product of the jth aggregate human capital:

$$p_{1,t} = G_t(1-\delta_{s,t})H_{1,t}^{\rho-1} = \frac{\partial Y_t}{\partial H_{1,t}} \tag{11}$$

$$p_{2,t} = G_t(\delta_{s,t})[\alpha_{3,t}H_{3,t}^{\theta} + (1-\alpha_{3,t})H_{2,t}^{\theta}]^{\frac{\rho-\theta}{\theta}}(1-\alpha_{3,t})H_{2,t}^{\theta-1} = \frac{\partial Y_t}{\partial H_{2,t}} \tag{12}$$

$$p_{3,t} = G_t(\delta_{s,t})[\alpha_{3,t}H_{3,t}^{\theta} + (1-\alpha_{3,t})H_{2,t}^{\theta}]^{\frac{\rho-\theta}{\theta}}\alpha_{3,t}H_{3,t}^{\theta-1} = \frac{\partial Y_t}{\partial H_{3,t}} \tag{13}$$

where $G_t \equiv Y_t(1-\alpha)\frac{1}{[\delta_{s,t}H_{s,t}^{\rho}+(1-\delta_{s,t})H_{u,t}^{\rho}]}$.

By taking the ratios of the equations above, I can derive the expressions for the relative returns to schooling:

$$\frac{p_{2,t}}{p_{1,t}} = \frac{\delta_{s,t}}{(1-\delta_{s,t})}(1-\alpha_{3,t})\left(\frac{H_{1,t}}{H_{2,t}}\right)^{1-\rho}\left\{(1-\alpha_{3,t}) + \alpha_{3,t}\left[\frac{H_{3,t}}{H_{2,t}}\right]^{\theta}\right\}^{\frac{\rho-\theta}{\theta}} \tag{14}$$

$$\frac{p_{3,t}}{p_{2,t}} = \frac{\alpha_{3,t}}{(1-\alpha_{3,t})}\left(\frac{H_{3,t}}{H_{2,t}}\right)^{\theta-1} \tag{15}$$

$$\frac{p_{3,t}}{p_{1,t}} = \frac{\delta_{s,t}}{(1-\delta_{s,t})}\alpha_{3,t}\left(\frac{H_{1,t}}{H_{3,t}}\right)^{1-\rho}\left\{\alpha_{3,t} + (1-\alpha_{3,t})\left[\frac{H_{2,t}}{H_{3,t}}\right]^{\theta}\right\}^{\frac{\rho-\theta}{\theta}} \tag{16}$$

where equations (14), (15) and (16) define, respectively, the relative returns to intermediate versus basic, higher versus intermediate and higher versus basic education.

The degree of complementarity between intermediate and higher education is an important

[19] There are different ways of measuring the ES when the aggregate output is produced with more than two inputs. We use the definition of the direct ES. One alternative commonly used definition is the Allen elasticity of substituion. The direct elasticity of substitution between Intermediate and Higher Education is solely a function of the curvature parameter, θ, while the Allen definition involves both the curvature parameter and the factor shares.

determinant of the changes in relative returns. An increase in the amount of human capital at intermediate level has two different effects: a standard supply effect (SE) and a complementarity effect (CE). The standard SE is clear from the human capitals' ratio in round brackets in equation (14) and (15). For a given supply of basic and higher human capital, an increase in H_2 decreases the relative return to intermediate with respect to basic education and increases the relative return to higher with respect to intermediate education. The CE is given by the term in the curly brackets in equations 14 and 16. If $\rho > \theta$, that is if higher and intermediate education are more complementary than higher and basic (or intermediate and basic), an increase in H_2 further decreases the relative return to intermediate with respect to basic education and increases the relative return to higher with respect to basic education.

Activities of the government are not central to the analysis. The government neither collects taxes nor redistributes them.

4 Equilibrium

Given an initial distribution of ability, financial assets and education and the world interest rate, r, a competitive equilibrium is given by a sequence of vectors of skill prices, $p_t = [p_{1,t}, p_{2,t}, p_{3,t}]$, aggregate labor inputs, $H_t = [H_{1,t}, H_{2,t}, H_{3,t}]$, parental decision rules for consumption and education choices, $[c_{a,t}, I_{a,t}]$, individual labor supply of skill j, $j_{a,t}$, individual labor efficiency, $e_{j,a,t}$, age, time and education specific measures, $\varphi_{j,a,t}$, for $t = 0, 1, 2, ...$, and $a = \underline{a}, ..., \overline{a}$ such that:

1. Given the prices $[p_{1,t}, p_{2,t}, p_{3,t}]$, the contingent plans $c_{a,t}$ and $I_{a,t}$ solve the household maximization problem 3.

2. Given the prices $[p_{1,t}, p_{2,t}, p_{3,t}]$, firms choose optimally the production factors and prices are marginal productivities:

$$p_{j,t} = \frac{\partial Y_t}{\partial H_{j,t}} \quad \forall j$$

3. The labor markets clear:

$$H_{j,t} = \sum_{a=\underline{a}}^{a=\overline{a}} \int_S (j_{a,t}(s) * \exp(e_{j,a,t})) d\varphi_{j,a,t}(s) \quad \forall j$$

where S defines the state vector at age a, time t, minus the education states, i.e. $S \equiv (A_{a,t}, p_t(a), \eta, z_{a,t})$.

An equilibrium steady state is a competitive equilibrium with stationary prices and distributions, that is an equilibrium such that $[p_t, H_t] = [p, H]$ for all t.

5 Parametrization of the model

This section discusses how I parametrize the model economy.

5.1 Initial distribution of wealth and education

I set the initial distribution of education using the 1987 wave of the Mexican Employment Survey, the ENEU (*Encuesta Nacional de Empleo Urbano*). A description of the Survey is presented in Appendix A. I divide the workers into two categories: the adult population that is made up by all heads of households aged between 25 and 60 with basic, intermediate and higher education and the population of young living with their parents aged between 15 and 24 with completed basic and intermediate education. I use the mean proportions by education in the adult population to set the initial education distribution of the parents and the mean proportions of the young with basic and intermediate education for the education distribution of the children in the third and fourth periods. In the pre-school period all children have by definition zero education. In the second period they all complete compulsory basic education.

The ENEU does not record information on wealth. I use instead the Mexican Expenditure Survey, the ENIGH (*Encuesta Nacional de Ingresos y Gastos de los Hogares*), which is available in 1984, 1987 and every two years since then and contains detailed information on individuals' consumption and assets. I set the initial wealth distribution to a lognormal with mean and standard deviation from the distribution of financial assets of workers aged between 25 and 60 in 1992.[20]

5.2 Preferences and costs of schooling

The coefficient of relative risk aversion, γ, is set to 0.9, which gives a value of around 1.1 for the elasticity of intertemporal substitution (EIS). The value is taken from Arrau and Wijnbergen (1991) that estimate for Mexico a value for the EIS between a lower bound of 0.8 and an upper bound of 1.4. The limit on net indebtedness, B, is set to zero, which corresponds to the maximum level of credit constraints. Consumption is adjusted to account for the presence of the child. I

[20] The ENIGH reached its final structure only in 1992 with significant changes in the questionnaire and data collection in the years before. For this reason, we parametrize the wealth distribution using the first and second moments of the 1992 instead of the 1987 wave. The results of the simulations are not sensitive to this choice.

use an equivalence scale equal to 0.7 for a child reflecting the average calories intake of a child relative to an adult as reported by the Mexican National Nutritional Institute.[21] Assuming that parent-child dynasties are linked by fully altruistic preferences, the altruism parameter, λ, is set to one.

I set the values of the fixed costs of schooling, F_j, for each jth education level so that the model matches the education distribution of the workers aged between 25 and 60 in the ENEU in 1987. I find $F_1 = 0.035$, $F_2 = 0.26$ and $F_3 = 0.64$, which implies that the costs at intermediate level are around seven times the ones at primary and the costs at higher education are around eighteen times the ones at primary.[22]

5.3 Interest rate and capital share

The value of the real interest rate, r, is set to a US benchmark value. I choose a value of five per cent, which is the average real interest rate on the US 6-months Treasury Bills published by the Federal Reserve Board for the period between the year 1990 and 2000. Given an average working life of the adult Mexican population of approximately thirty years, the model period is set to seven years. Therefore, the interest rate in the model is $r = (1.05^7 - 1) \cong 0.41$. Setting the yearly discount factor equal to the inverse of $(1 + r)$, $\beta = 1.05^{-7} \cong 0.71$.

The capital share, α, is set equal to 0.35, which is the average value between the lower and the upper bound that has been estimated for LACs.[23]

5.4 Labor efficiency

The human capital aggregates are defined as the sum of the efficiency weighted labor supplies of all individuals with a given education level. In order to simulate the model, I need to obtain an estimate of the mean and variance of the distribution of unobserved heterogeneity, η, and of the idiosyncratic shock, z, and an estimate of the coefficients of the education specific age polynomials, g_j. Ideally, I would need a panel data set with individual information on wages, a

[21] See Hernández, Chávez and Bourges (1987).

[22] The F_j intend to measure the total direct costs of education, which include the costs of school material and maintanance. An empirical counterpart of these costs is provided by the nationally representative Mexican Family House Survey (MxFLS), which collects high quality data on a rich set of variables for a cross section of Mexican households in 2002. The Survey contains a detailed set of questions on education costs and distinguishes between tuition fees, the costs of exams, books, school material, uniforms and the maintenance costs for public and private schools. Summing over the different categories and computing the mean for public and private education, the costs of intermediate education are around eight times the ones at primary while the ones at higher education are around nineteen times the ones at primary.

[23] See Bosworth (1998) for a discussion of the empirical issues involved in the estimation of the capital share in Mexico and Harrison (1994) and Hoffman (1993) for two cross-countries empirical studies that use a capital share that varies between the value of 0.3 and 0.4 for a group of LACs.

measure of permanent heterogeneity with a measurable impact on wages, such as individual test scores, and age, spanning over many years. However, for Mexico there are no available data sets that follow individuals over many years.[24] Also, until 2004 there were no standardized measures of test scores.[25]

The only available data set with individual-level information on wages and a panel dimension is the ENEU, which collects wage information on the Mexican urban population of workers at least twelve years old over five consecutive quarters, the four quarters of a given year and the first of the following year. I consider the four quarters of a given year and I specify the following log wage equation for individual i with education level j in quarter qr:

$$\ln w^i_{j,qr} = \ln w_{j,qr} + g_j(age^i_{qr}) + u^i_{j,qr} \qquad j = 1, 2, 3 \tag{17}$$

$$u^i_{j,qr} = \eta^i + z^i_{j,qr} \tag{18}$$

$$z_{j,qr} \sim N(0, \sigma^2_{z_{j,qr}}) \tag{19}$$

where $\ln w^i_{j,qr}$ is the log hourly real wage of individual i with education level j in quarter qr, $\ln w_{j,qr}$ is the mean log wage among those with education level j in quarter qr, $g_j(.)$ is an education-specific quadratic polynomial in age, η^i is a permanent individual-specific effect and $z^i_{j,qr}$ is an i.i.d. shock received by the individual i with education level j in quarter qr.

I construct panels of individuals by matching workers by the position in an identified household, number of years of education and age. I consider all wage workers between the age of 15 (minimum legal working age) and 60 (average retirement age) and I follow them over the four successive quarters in a given year. For each year of the sample between 1987 and 2002 I run the following fixed effects regression:

$$(\ln w^i_{j,qr} - \ln \overline{w}^i_j) = (\ln w_{j,qr} - \ln \overline{w}_j) + g_j(age^i_{qr}) - g_j\overline{(age^i)} + (u^i_{j,qr} - \overline{u}^i_j) \tag{20}$$

where the upper-bar variables denote time averages over the four quarters in a given year. $\ln \overline{w}^i_j$ is the average log wage over the four quarters for the ith individual with the jth education level, $\ln \overline{w}_j$ is the mean log wage over the four quarters for education level j. The term $(\ln w_{j,qr} - \ln \overline{w}_j)$ is modelled as quarter-education dummies' interactions. $g_j(age^i_{qr})$ is an education-specific

[24] The first survey that collects individual information on wages in Mexico over many years is the Mexican Family Life Survey (MxFLS). The first wave of the MxFLS was collected in 2002 and it will be followed by two additional waves with retrospective information in order to build up a uniquely rich longitudinal database that spans over a period of at least ten years. However, as of today, only the 2002 wave is available for use by the general public.

[25] Non-standardized test scores were collected in Mexico since 1998 (*Estandares Nacionales*). In 2001 the ENLACE (*ENgaging LAtino Communities for Education*) initiative was launched to support Latino students to progress from primary to secondary and college education. Standardized test scores started to be collected as part of ENLACE in 2004.

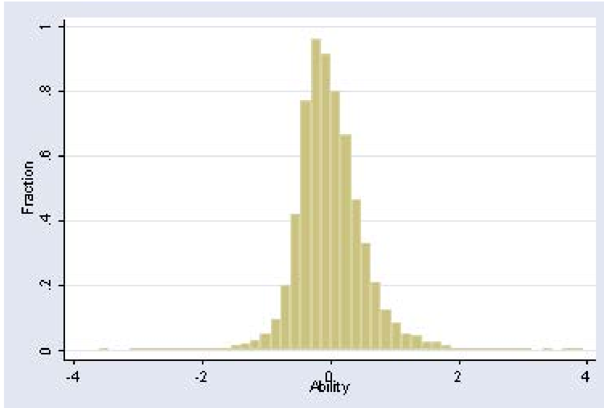

Figure 2: Ability Distribution in 1987 (Source: ENEU)

quadratic polynomial in age.

For the purposes of the simulation we require the unconditional distribution of ability as reflected by the fixed effect, η^i. I use the estimate:

$$\widehat{\eta}_i = \frac{\sum_{t=1}^{T(i)} \ln w_{qr}^i - \widehat{\ln w_{qr}} - g(\widehat{age_{qr}^i})}{T(i)} \tag{21}$$

where $T(i)$ is the total number of observations available on individual i. The estimated fixed-effects give an estimate of the distribution of η over the working population.

Figure 2 presents the empirical frequencies of η obtained for 1987, the first year of the sample. The first and second moment of the distribution of η are used to parametrize the distribution of ability in the model. I find $\widehat{\sigma}_\eta^2 = 0.25$ and $\widehat{\mu}_\eta = 0$.[26]

The short length of the panel with only four observations on wages for each individual biases the estimates of the fixed effects. Despite this limitation, the lack of alternative data sources with successive observations on wages and measurable proxies of permanent heterogeneity restricts to this regression the best that we can do to obtain an estimate of η that is consistent with the estimated human capital aggregates. One alternative that I am exploring with is to use the distribution of the results from the Raven test for adult individuals collected by the MxFLS in 2002.

However, disregarding the specific initial ability distribution estimated from the data, what is relevant for the research question addressed in this paper is to assess the contribution of the

[26] The distribution of ability is taken to be time-invariant. We therefore ignore any possible heterogeneity in terms of ability endowments between successive cohorts active in the labour market in different years.

changes in the ability composition by education with respect to the changes in the skill prices to produce the convexification.

The results in BMM provide a precise benchmark of the size of the changes in mean ability by education in Latin America in the 1990s. At the end of section 6.3 I will use model's simulations to compare the changes in composition estimated by BMM to the changes in mean ability by education that are needed in this model to produce the convexification in the absence of any general equilibrium price effect.

Finally, given the estimation of equation (20), I can treat as observable the following:

$$z^i_{j,qr} = \ln w^i_{j,qr} - g_j(age^i_{qr}) - \ln w_{j,qr} - \eta^i \tag{22}$$

I use the residuals from the wage equation to obtain an estimate of the distribution of the idiosyncratic shock. I assume that $z_{j,qr}$ is a normally distributed i.i.d. shock with mean zero and variance $\sigma^2_{z_j}$. I use the second moment of the distribution of $z_{j,qr}$ for each education group to parametrize the distribution of z in the model. For 1987 I find $\widehat{\sigma}^2_{z_1} = 0.065$, $\widehat{\sigma}^2_{z_2} = 0.07$, $\widehat{\sigma}^2_{z_3} = 0.079$. Table 6 in Appendix A presents the estimated variances for each year of the sample between 1987 and 2002.

Finally, the coefficients of the quadratic polynomials $g_j(age^i_{qr})$ provide the estimates of the education-specific experience effects in quarter qr. Table 7 in Appendix A presents the (yearly rescaled) estimates of the age and age squared term for each education group and year between 1987 and 2002.

5.5 Human capital aggregates

Following Heckman, Lochner and Taber (1998), I identify the human capital aggregates, $H_{j,t}$, by combining the data on the wage bills paid in each year to the different education groups with the time series of the skill prices estimated for each year between 1987 and 2002.

Let us define the wage bill $WB_{j,t}$ as the total earnings' payments received by the individuals of a given education group in a given year. Then:

$$WB_{j,t} = \widehat{p}_{j,t} * H_{j,t} \qquad j = 1, 2, 3 \tag{23}$$

where $\widehat{p}_{j,t}$ is the estimated market price of workers with education level j in year t.

Therefore:

$$H_{j,t} = \frac{WB_{j,t}}{\widehat{p}_{j,t}} \qquad j = 1, 2, 3 \tag{24}$$

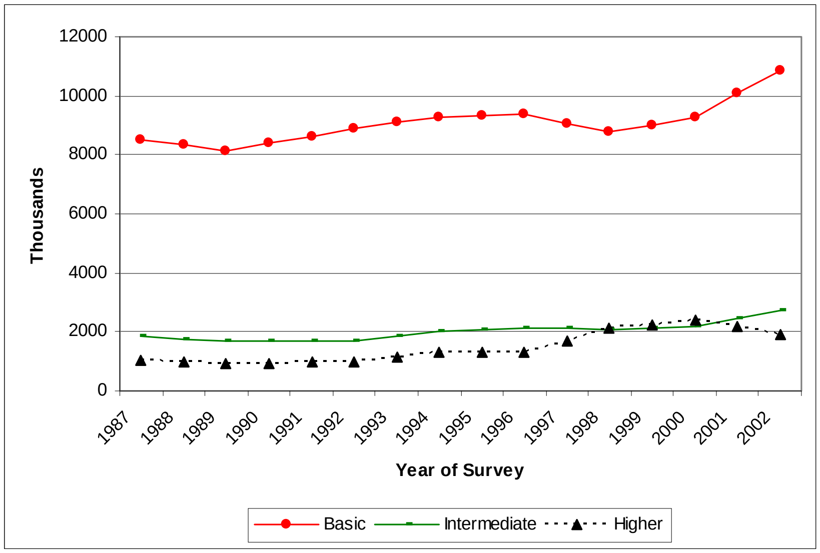

Figure 3: Wage Workers (Source: ENIGH)

Normalizing to one the value of each of the $\widehat{p}_j$ in 1987, it is possible to identify the series of the utilized human capital stocks normalized to 1987.

In order to compute the wage bills we need a data set that is representative of the entire Mexican population. The ENEU collects information on urban areas only so it can not be used to this purpose. I use instead the ENIGH, that is nationally-representative and reports individual earnings together with detailed information on assets and consumption.

For each year and education group I compute the wage bill by summing over the individual earnings of all primary wage earners between the age of 15 and 60. I linearly interpolate the available data for the missing years. The total number of workers and the wage bills by education for each year between 1987 and 2002 are reported in Figure 3 and 4. The drop of the wage bills between 1994 and 1996 is the result of the Peso crisis that hit Mexico in 1994: the total wage bill declined in real terms by around 29 per cent at basic and by around 35 and 48 per cent at intermediate and higher education. Wages started increasing again steadily for all education levels from the end of the 1990s.

In order to obtain an estimate of the $p_{j,t}$ I run the fixed effect regression described in equation (20) for each year of the sample between 1987 and 2002 and I compute the predicted mean log hourly real wage for each education level net of the fixed effects. Given the wage bills and the (log) skill prices, I divide the wage bills by the exponentiated value of the skill prices to

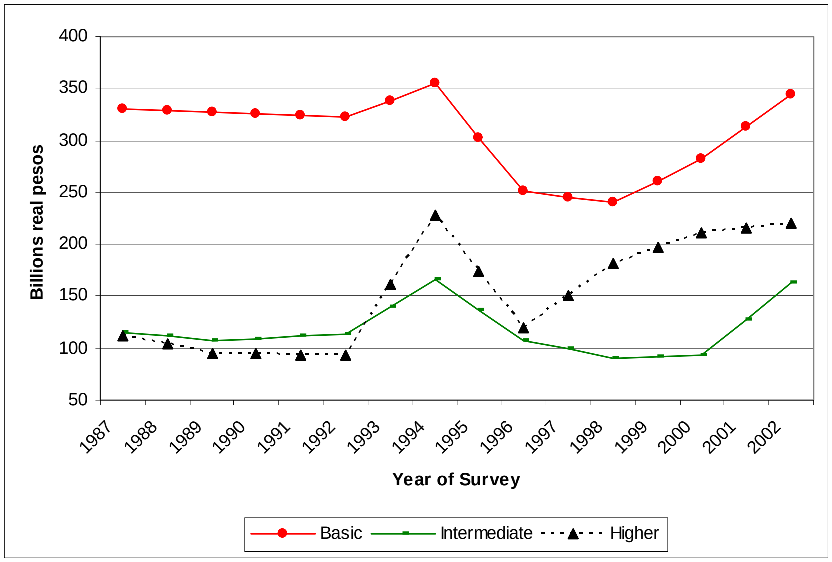

Figure 4: Wage Bills (Source: ENIGH)

obtain the time series of the human capital aggregates for each year and education group. The identification of the H factors is then consistent with the ability distribution estimated from the data and used to simulate the model.[27]

Figure 5 and 6 present the series of the estimated skill prices and human capital stocks normalized to 1987. As in the aggregate wage bills, the impact of the Peso crisis does also shows up in the skill prices: between 1994 and 1996 the log skill price decreased by around 12 per cent at basic and by around 10 per cent at intermediate and higher education.

Each of the three human capital aggregates show a positive growth in the 1990s but with a rather non-monotonic trend. In particular, the growth of H_3 dropped sharply between 1994 and 1996 and the one of H_2 became positive only from the year 2000. The decrease (increase) in the value of the estimated H is due to an increase (decrease) in the level of the estimated prices of the corresponding H factor which grew proportionally more (less) than the total remuneration of the factor.

[27] Heckman, Lochner and Taber (1998) assume that at older ages changes in wages are solely due to changes in skill prices and do not depend any more on ability. Under a normalization assumption that $\widehat{p}_{j,t=0} = 1 \; \forall j$, it is possible to estimate the time series of the skill prices from a cross-section of wages of adult individuals. Then, if the ratios have mean zero measurement error, we can estimate the ratio of the skill prices without bias and, with these relative skill prices at hand, obtain an estimate of the human capitals. Following Heckman, Lochner and Taber's procedure, we can estimate the skill price ratios by using the mean log wages by education for the ENEU workers in each year of the sample between 1987 and 2002. The main results of the estimation of the production funtion remain unchanged.

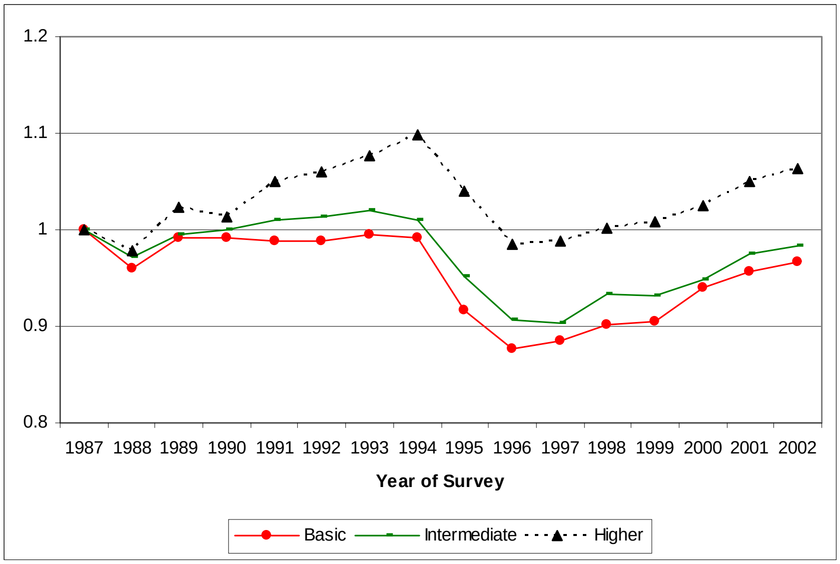

Figure 5: Log Skill Prices (Source: ENEU)

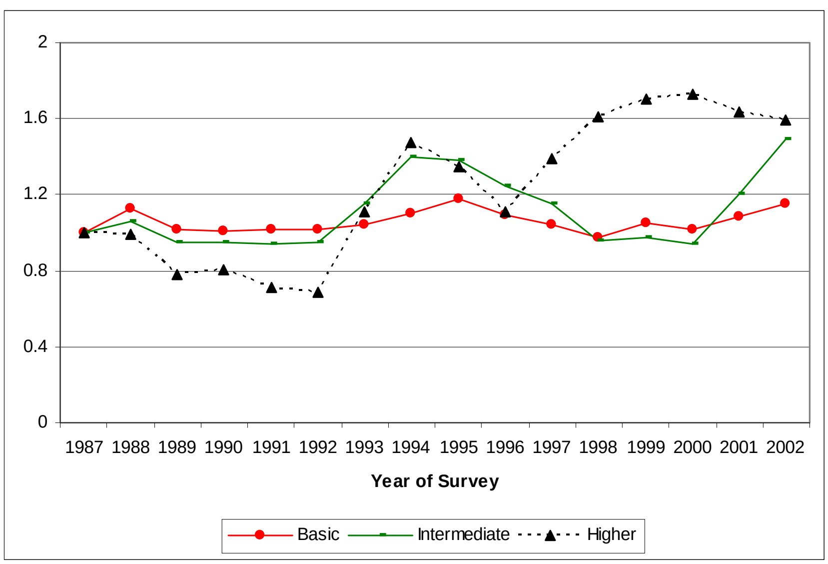

Figure 6: Human Capital Aggregates

5.6 Aggregate technology

The parameters of the production function can be estimated by using the expressions for the returns to schooling in equation (14) to (16).

Log linearizing equation (15), I obtain:

$$(\log p_{3,t} - \log p_{2,t}) = [\log \alpha_{3,t} - \log(1 - \alpha_{3,t})] + (\theta - 1)(\log H_{3,t} - \log H_{2,t}) \tag{25}$$

where $\log \alpha_{j,t}$ denotes the time series of the relative demand shifts for skill level j measured in log quantity units. By rewriting the above expression in terms of wage bills, I obtain:

$$(\log WB_{3,t} - \log WB_{2,t}) = [\log \alpha_{3,t} - \log(1 - \alpha_{3,t})] + \theta(\log H_{3,t} - \log H_{2,t}) \tag{26}$$

Equation 26 can be used to obtain a direct estimate of the elasticity of substitution between higher and intermediate education. The time-varying factor shares $\alpha_{2,t}$ and $\alpha_{3,t}$ reflect changes in the productivity and in the demand of workers with intermediate and higher education. Given the significant shifts in educational attainment observed in Mexico in the 1990s, I express the log of the share parameters as the sum of a constant and a time-varying component:

$$\log \alpha_{j,t} = \phi_{0,j} + \phi_{1,j} * t + e_{j,t} \tag{27}$$

where $\phi_{0,j}$ is a skill-specific constant, t denotes a linear time trend and $e_{j,t}$ is a normally distributed i.i.d. shock at time t for skill level j.[28]

Combining equation (26) and (27), the value of the parameter determining the elasticity of substitution between higher and intermediate education, θ, can be estimated from a regression of the ratio of log wage bills on the ratio of the human capital aggregates, a linear trend and a constant. In order to correct for a possible endogeneity bias I apply an IV estimator using as instrument the first lag of the difference of the logs of the human capital factors.

Then, I use equation (10) above to construct a measure of skilled human capital. To do so, we need an estimate of the log factor shares $\alpha_{3,t}$. Given equations (26) and (27) and the fact that $\alpha_{2,t} = (1 - \alpha_{3,t})$, I have that $\log \left[\frac{\alpha_{3,t}}{(1-\alpha_{3,t})}\right] = (\beta_0 + \beta_1 * t)$, where $\beta_0 = (\phi_{0,3} - \phi_{0,2})$ and $\beta_1 = (\phi_{1,3} - \phi_{1,2})$. Therefore, $\alpha_{3,t} = \frac{\exp(\beta_0 + \beta_1 * t)}{(1+\exp(\beta_0+\beta_1 * t))}$.

Finally, I can estimate a regression of the ratio of log wage bills for skilled and unskilled on the ratio of skilled and unskilled human capital, a linear trend and a constant to obtain an

[28] Given the spike in the wage bills in 1994 (see Figure 3.4), we tried alternative specifications that allow the trend to vary for the pre and post 1994 period. However, the interaction term never turns out to be significant.

estimate of ρ.

Parameter	*Higher versus Intermediate*	*Skilled versus Unskilled*
Difference log H	0.7726	0.8601
	(0.0636)	(0.1362)
Time trend	0.0262	0.0135
	(0.0048)	(0.0065)
Constant	−51.9469	−26.0616
	(9.5317)	(13.0620)
Implied ES	4.4	7.1

Table 2: Estimation of the Production Function, standard errors in parenthesis

Table 2 presents the estimates obtained for higher versus intermediate education and for skilled versus unskilled. The estimates of ρ and θ are consistent with the presence of complementarities between intermediate and higher education: $\rho > \theta$. The value for the ES between higher and intermediate (skilled and unskilled) is of around 4.4 (7.1). These results confirm the presence of important complementarities in production between workers with intermediate and college education that has been found by BMM for Brazil.[29]

A joint estimation of the system of equations to test for the equality of the coefficients of the log relative supplies confirms that ρ and θ are statistically significantly different. The test gives a value of chi-squared of 7.1 with a P-value of 0.0077. I also test for the assumption of equality between $ES_{3,1}$ and $ES_{2,1}$, which is a restriction imposed by the symmetry of the CES operator. The test gives a value of chi-squared of 0.35 with a P-value of 0.5525. Therefore, the test can not reject the null hypothesis of equal coefficients.

Using the wage bill equation for skilled and unskilled, the equivalent of equation 27 for $\delta_{s,t}$ and the definition of the unskilled labor share as one minus the skilled share, I can identify $\delta_{s,t}$ following the same steps used to identify $\alpha_{3,t}$. For the year 1987, I obtain a baseline estimate of 0.55 for α_3 and of 0.692 for δ_s. Given the estimated series of $\widehat{\alpha}_{3,t}$ and $\widehat{\delta}_{s,t}$, $\widehat{\alpha}_{2,t} = (\widehat{\delta}_{s,t} - \widehat{\alpha}_{3,t})$ and $\widehat{\delta}_{u,t} = (1 - \widehat{\delta}_{s,t})$. Figure 7 presents the estimated shares of the three human capital aggregates in each year between 1987 and 2002: an increase in the share of higher education came togheter with a decline in the share of intermediate and basic education.

The coefficient of the time trend gives an estimate of the yearly relative demand-shift of higher

[29] Manacorda, Sanchez-Paramo and Schady (2006) use a cross section of LACs that includes Argentina, Brazil, Chile, Colombia and Mexico in the 1980s and 1990s and find a value of around 4.5 for the ES between higher and intermediate education and a value of around 2.5 for the ES between skilled and unskilled. Therefore, they find a lower ES between skilled and unskilled than between higher and intermediate. They allow for a different ES between age groups and they pool five LACs together. The difference with respect to our results could be due to any of these features.

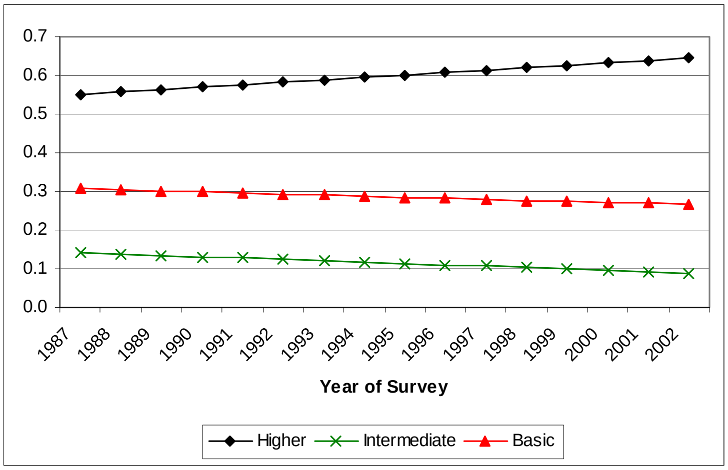

Figure 7: CES Shares, Mexico

with respect to intermediate education, $(\phi_{1,3} - \phi_{1,2})$, and of skilled with respect to unskilled labor, $(\phi_{1,s} - \phi_{1,u})$. I call $(\phi_{1,s} - \phi_{1,u})$ the "skill-bias" parameter. It gives a measure of the annual increase in the relative demand for skilled labour in production. The estimate in Table 2 gives a value of the "skill-bias" of around 1.35 per cent a year. I will use the estimated 1.35 per cent a year to model SBTC as a permanent increase in the share of skilled labor in production starting from the baseline value of δ_s estimated for 1987.

6 Simulations

In this section I simulate the model for a quantitative assessment of the determinants of the convexification. I define the baseline economy as the one with the share of skilled labor fixed at the value estimated in 1987 and the highest level of credit constraints of no possible borrowing. Then, I assess the role of an increased demand for skill by simulating the model in the presence of a SBTC at constant supply and in general equilibrium, with and without the complementarities in production between intermediate and higher education. Finally, I simulate the impact of a SBTC for different levels of credit constraints and earnings' risk. I compare the education prices at steady state in the different scenarios to assess what determines the convex shape.

6.1 Skill-biased technological change

A SBTC is modelled as a permanent increase of δ_s, which measures the contribution of skilled labor to the production of the aggregate output. An increase in its value is to be interpreted as

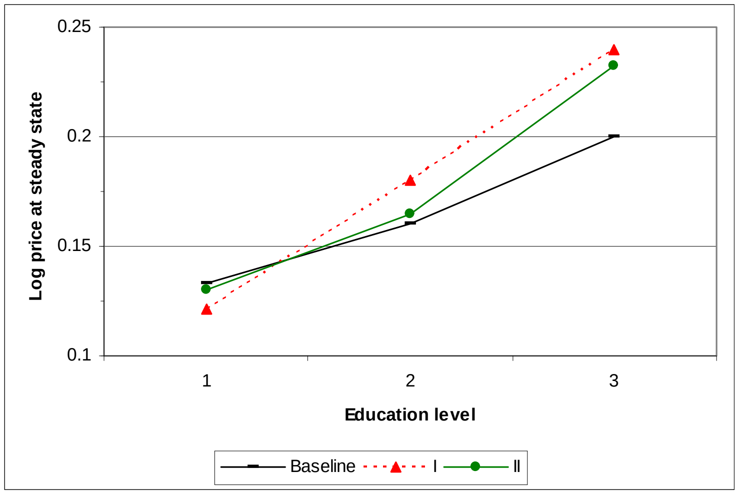

Figure 8: Skill-Biased Technological Change - Partial versus General Equilibrium

an increase in the productivity or in the demand for skilled labor. The size of the increase is given by the "skill-bias" parameter obtained from the estimation of the production function in section 5.6. As reported in Table 2, I estimate a skill-bias of around 0.0135 a year. The value of δ_s estimated for 1987 is used as the baseline case that defines the no-SBTC scenario. Then, starting from this value, I increase δ_s by 1.35 per cent a year for five consecutive years and I keep it constant at the increased value from the sixth year onwards.[30]

I define the baseline economy as the one with no-SBTC and the maximum level of credit constraints of no possible borrowing. I assess the impact of a SBTC in two consecutive steps. First, I compute the equilibrium prices that the model would produce given the increase in δ_s in the absence of any general equilibrium supply effect. I compute the skill prices by using the equilibrium conditions (11) to (13) and I simulate the model assuming that the supply of each skill is kept fixed at the value estimated in the ENEU in 1987. I call this counterfactual Scenario I. Then, I define a second counterfactual where the supply is allowed to change in reaction to the SBTC and thus to the associated changes in the prices of skills. I call this counterfactual Scenario II.

Figure 8 presents the equilibrium log prices at steady state in the baseline case and in Scenario I and II. At baseline the model matches the education shares in 1987 and produces a linear relationship between the mean log prices and the level of education.

Once the share of skilled labor is increased, Scenario I produces a steeper prices' schedule. As

[30]Simulating the model with increases of δ_s of longer and shorter duration produces qualitatively similar results.

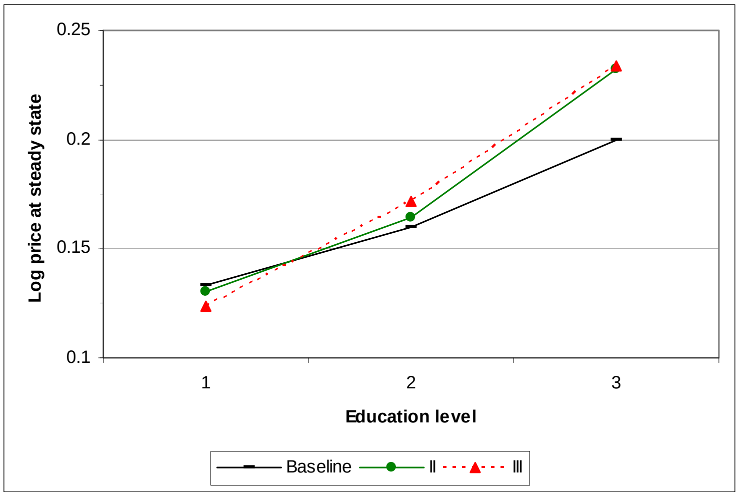

Figure 9: Complementarities Intermediate - Higher Education

clear from equations (11) to (13) and the fact that $\delta_u = (1 - \delta_s)$, an increase in δ_s decreases the equilibrium price at basic and increases the prices at both intermediate and higher education. With respect to the baseline, both the relative returns to higher with respect to intermediate and to intermediate with respect to basic education increase. Moreover, they increase by the same amount; therefore, the price schedule is steeper than at baseline but it is still a linear function of the level of education.

Scenario II allows the supply of skills to react to the changes in the prices. Therefore, the difference with respect to scenario I is due to the supply reaction to a SBTC. The increase in the prices at intermediate and higher education gives incentives to invest after compulsory schooling. However, the supply at the intermediate increases more than at the higher level, so the price at intermediate decreases more and the steady state prices' schedule becomes convex. The complementarity between intermediate and higher education is driving this result. Since I estimated a value of ρ that is higher than the value of θ, in addition to the standard supply effect, an increase in H_2 further decreases the relative return to intermediate with respect to basic education and increases the relative return to higher with respect to basic education.

Figure 9 quantifies the importance of the complementarities between intermediate and higher education. I define a third counterfactual, Scenario III, which assumes that the ES between higher and intermediate is the same as the one between higher and basic education. As it is clear from the graph, in the absence of any complementarity between intermediate and higher education, the equilibrium wage profile in steady state is back to being a linear function of the level of education.

All the results up to now have been obtained for the degree of earnings' risk estimated for 1987 and the maximum level of credit constraints of no possible borrowing. Both an increase in the level of risk and in the amount that is possible to borrow would change the stock of available resources and therefore the incentives to invest in education. In the following paragraphs I investigate the role of a change in the level of risk and credit constraints to produce the convexification.

6.2 Uninsurable earnings' risk

The level of uninsurable earnings' risk has a direct impact on the amount of human capital investment. Positive (negative) shocks can change the fortune of dynasties that are born poor (rich) and allow some families to (or not to) have enough resources to finance educational investments up to higher education.

In Section 5.4 I estimated the variance of the earnings' shock by education for each year between 1987 and 2002. The estimates are reported in Table 6 in Appendix A. For each education group, I take the highest estimated value of the variance and I use these estimates to define a fourth counterfactual, Scenario IV, that is given by Scenario II with the high variances of the shocks. From the estimated values of 0.065, 0.07 and 0.079 for the variance of the shock to basic, intermediate and higher education, the values are increased to, respectively, 0.078, 0.094 and 0.125. Figure 10 reports the steady state prices' schedule at baseline and in Scenario II and IV.

The possibility of receiving big positive shocks that could allow financing education up to the highest level increases the option value of education as a profitable investment opportunity and gives incentives to invest in education after compulsory schooling. Therefore, the supply at basic declines and the one at intermediate increases with a corresponding increase in the price of basic and a decrease in the price of intermediate education. However, a high level of risk also means that there is a positive probability of receiving big negative shocks and therefore not being able to invest up to higher education: the increase in the supply at intermediate is not big enough to produce the sizable drop in the price at this level, which is the characterizing feature of the convexification.

The results up to now have been obtained under the assumption of no possible borrowing. I

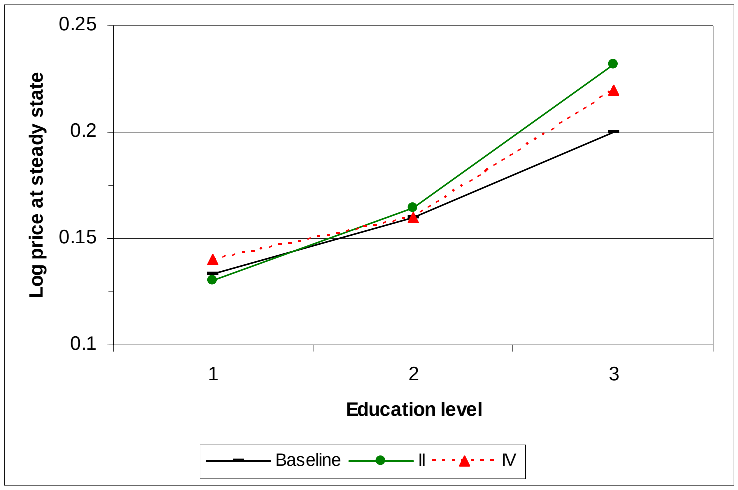

Figure 10: Increase in Earnings' Risk

now investigate how the results change when borrowing is allowed.

6.3 Credit constraints

The assumption of no possible borrowing significantly restricts the set of feasible educational investments and the extent of mobility from basic up to higher education. I am not aware of any estimate of the actual degree of credit constraints in Mexico, so there is no empirical benchmark to fix the value of B_a, the maximum amount that is possible to borrow at each age a.

As I discussed in section 3.1, the internal consistency of the model allows to set an upper bound for the value that B can take at any age, which is given by the present discounted value of the lifetime earnings at age a under the lowest possible realization of the idiosyncratic education-specific shock z_j. Given the distribution of z_j defined over a finite support with a minimum value, $\underline{z}_j$, and a maximum value, $\overline{z}_j$, $\underline{z}_j$ defines the lowest possible value that z_j can take. By substituting $\underline{z}_j$ in $e^i_{j,a}$, I compute $\underline{e}^i_{j,a} = \left(\eta^i + g_j(age^i) + \underline{z}_j\right)$. At each age a, given parental education level j, B_a is defined as $B_a = \sum_{t=0}^{\overline{a}-a} \frac{w^i_{j,a}}{(1+r)^t} = \sum_{t=0}^{\overline{a}-a} \frac{\left(p_j * \exp(\underline{e}^i_{j,a})\right)}{(1+r)^t}$ where r is the world interest rate.

I define a fifth counterfactual, Scenario V, that is given by Scenario II with a relaxation of the credit constraints to the upper bound of the values that B can take. Figures 11 reports the wage profile at steady state in the baseline and in Scenario II and V.

With respect to scenario II, the possibility of borrowing allows more people to complete intermediate and higher education, so investment in human capital at both levels increases

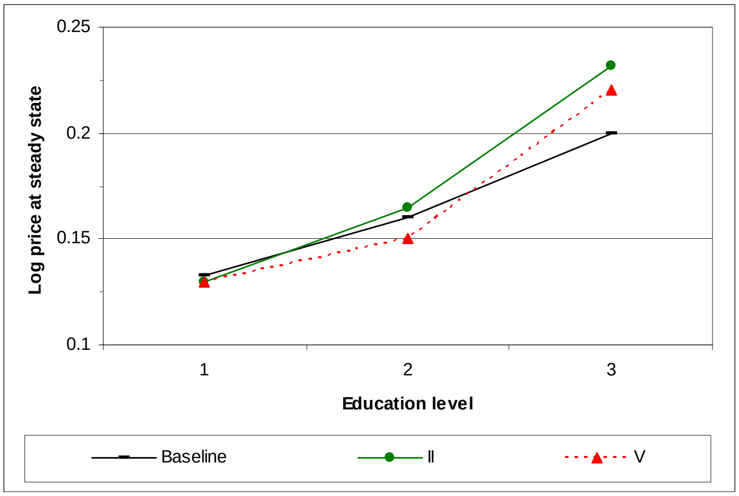

Figure 11: Relaxation of the Credit Constraints

and therefore the prices decrease. However, the supply increase at intermediate level does not translate into a proportional increase at higher education, so the price at intermediate decreases more than at the higher level.

With respect to the baseline, the relative return to higher versus intermediate increases and the relative return to intermediate versus basic education decreases with the price at intermediate falling below the value at baseline.

There is a positive relationship between the borrowing limit and the size of the supply increase at intermediate: the more it is possible to borrow, the higher is the investment in education after compulsory schooling. Unreported results show that there is a borrowing threshold of around forty per cent of individuals' lifetime earnings below which the size of the supply increase at intermediate is not enough to produce a sizeable decrease in the equilibrium price below the value at baseline.

Scenario V presents all the features that characterize the convexification in Mexico. The second column of Table 3 and 4 presents, respectively, the growth of the level and of the log relative prices between 1987 and 2002 estimated from equation 20 using the ENEU; columns 3 to 7 present, respectively, the growth of the level and of the relative log education prices at steady state in each of the five scenarios with respect to the baseline case. Only in the fifth scenario the price at intermediate falls below the value at baseline. Also, the size of the decline is very close to the one estimated in the data. In terms of relative returns, Scenario V predicts an increase of around 75 per cent of the relative returns to higher with respect to intermediate education and a decline of around 26 per cent of the relative returns to intermediate with respect to basic education, which are close in magnitude to the ones estimated from the ENEU.

Log price education	ENEU (1987-200)	I	II	III	IV	V
Basic	−3%	−9%	−2%	−7%	5%	−2%
Intermediate	−5%	13%	3%	7%	0%	−6%
Higher	6%	20%	16%	17%	10%	10%

Table 3: Growth of the Level of Log Education Prices With Respect to Baseline

Log relative prices	ENEU (1987-200)	I	II	III	IV	V
Higher versus Intermediate	72%	50%	69%	50%	200%	75%
Higher versus Basic	25%	78%	52%	19%	139%	34%
Intermediate versus Basic	−15%	119%	28%	−26%	48%	−26%

Table 4: Growth of the Relative Log Education Prices With Respect to Baseline

As expected, the change in the wage profile from linear to convex came together with significant changes in the mean ability level by education. Table 5 presents the mean and standard deviation of the ability level by education at Baseline and in Scenario V. The supply increase at intermediate results into a sharp decline in the mean ability at this level and into an increase in the mean ability at basic education. Due to the entrance of low-ability individuals together with high-ability ones, the mean ability does also decline at higher education. However, the reduction in mean ability at intermediate is of a much bigger magnitude than the one at higher education. Therefore, with respect to the Baseline, in Scenario V the ability gap between higher and intermediate education increases sharply.

Ability by Education	*Baseline*		*Scenario V*	
	mean	sd	mean	sd
Basic	−0.226	0.396	0.311	0.276
Intermediate	0.396	0.314	−0.191	0.436
Higher	0.615	0.283	0.358	0.536

Table 5: Mean and Standard Deviation of Ability by Education in Scenario V and at Baseline

The changes in the mean ability by education were substantial but were not driving the convexification. As shown in Figure 11, the convexification resulted from changes in the prices of education, which is the component of the observed wages that remains once ability has been accounted for.

However, the quantitative importance of the changes in ability composition is sensitive to the specific ability distribution estimated from the data. More importantly, with a big enough drop in the mean ability at intermediate and a big enough increase in the mean ability at higher education, the model would always be able to produce a convexified wage profile, which would be the result of compositional changes with no role left for price effects.

I can test the relevance of this argument by simulating the economy at constant skill prices and compute the changes in mean ability that are needed to produce the convexification. To my knowledge, BMM is the first paper that provides an estimate of the changes in the ability distribution by education in a Latin American country. They distinguish between four levels of schooling and estimated the changes in ability composition by education and birth cohort during the educational expansion in Brazil in the 1990s. These estimates provide a precise benchmark of the magnitude of the shifts in mean ability by education in Latin America in the 1990s.

Unreported results show that the changes in the mean ability by education that were necessary to produce the convexification in the absence of any general equilibrium price effect would be more than five times bigger than the ones estimated by BMM.[31] The convexification was not driven by compositional changes. The fundamental determinants of the shift in the wage profile from linear to convex were changes in the prices of skills.

7 Conclusion

This paper studies a central feature that characterizes the changes in wage inequality in Latin America in the 1990s: the combination of an increase in the wage differential between higher and intermediate education and a decline in the differential between intermediate and basic education driven by a drop in the real wage at intermediate. From being linear at the beginning of the 1990s, by the end of the decade the relationship between mean wages and the level of education became convex. This has important implications for the process of human capital accumulation. From one side, the non-linearity of the wage function changes the opportunity costs of investing in education, from the other, it reflects changes in its supply. Yet, the empirical

[31] A similar argument could be made for the estimates of the fixed costs of education, F_j. By changing the costs of schooling so that enough people move from basic to intermediate and to higher education the model would mechanically produce the convexification. Estimates of average costs by level of education can be computed from the ENIGH. Despite collecting less detailed information on schooling costs than the MxFLS, the ENIGH is available every two years since 1992, thus allowing to estimate a time series of average costs. Unreported results show that in the absence of general equilibrium price effects the changes in F_j that are necessary to produce the convexification differ significantly from the changes estimated from the ENIGH. In particular, while the estimates from the ENIGH show that the average costs increased at all schooling levels, the model would produce a convexification only in the presence of a substantial decrease in the fixed costs of intermediate education.

literature on LACs so far has taken the supply of education as given and has focused primarily on the increase in the wage differential between higher and intermediate education.

Using micro-data from Mexico between 1987 and 2002 I show that the convexification was not the result of changes in the ability composition by level of education but rather of changes in the prices of schooling in a period of economic reforms that many empirical studies have associated with an increased demand for skilled labour in production. The results of the simulations show that there are two main determinants of the convexification. First, the combination of an increase in the relative return to higher and a decline in the relative return to intermediate education is due to the presence of complementarities in production between intermediate and higher education. The increased demand for skill increases the return at intermediate and higher education and gives incentives to invest after compulsory schooling. Since higher education is more complementary with intermediate than is basic education, the increased supply at intermediate increases the relative return to higher with respect to basic education and further decreases the relative return to intermediate versus basic education. Second, the drop in the price at intermediate and therefore the extent of the convexification depends on the extent of binding credit constraints at intermediate education.

The model developed in this paper is rather stylized but it is meant to include all main factors that could have been driving the convexification. An important simplifying assumption is the absence of migration to the US. According to the statistics presented in the 2001 Report published by the US Center for Immigration Studies, in the 1990s Mexico was the largest source of migration to the US accounting for around thirty per cent of all immigrants there. The migration to the US was mainly an outflow of low-skilled workers with two thirds of the adult Mexican immigrants having not completed intermediate education.[32] The low-skilled migration on its own would not be able to explain the double change in the relative wages but it could have contributed to reduce the fall in the wage at basic education and therefore to produce the decline in the relative returns to intermediate education. In this paper I consider cross-sections of adult individuals working in Mexico between 1987 and 2002. I find that the share of workers with basic education decreased from around sixty per cent in 1987 to thirty-eight per cent in 2002. Consistently, from the estimation of the production function I identify an increase in the relative demand for workers with more than basic education. Finally, model's simulations show that the decrease in the demand for basic educated workers more than compensated the price

[32] The Report can be freely downloaded from the website of the US Center for Immigration Studies: www.cis.org

increase induced by the decline in the supply at this level. It could be an interesting topic for future research to disentangle the downward pressure on wages due to the demand decrease for low-skilled workers and the upper pressure due to the migration of this type of workers to the US.

Despite being stylized, the model's findings have strong empirical support. First, as I documented in section 1.1, an increased demand for skilled labour in production has been found by many empirical studies on Mexican economy in the 1990s. Second, a relaxation of the credit constraints does also find empirical support. The decade of the 1990s was characterized by a process of financial liberalization and deregulation of the securities markets, which resulted into an increased availability of credit to the private sector. As documented by Bandiera, Caprio, Honohan and Schiantarelli (2000), two common measures of monetary depth such as the M2 aggregate and the ratio of the total credit to the private sector as a percentage of the gross national GDP steeply increased in Mexico in the first half of the 1990s.

Of course, the question remains as to why the relaxation of the credit constraints was not of a big enough size as to allow the supply increase at intermediate to translate into a proportional increase at higher education. Credit constraints is a general label to capture all financial and non-financial factors that are relevant for grade progression. Among these factors an important role is played by the quality of education received earlier on, which in Latin America differs significantly at all education levels between private and public schools. Estimating a dynamic model of school progression that includes the choice between private and public education would allow estimating the returns to education in the two sectors and help understanding the actual nature of the constraints and the time when they start to be binding. A complementary paper to the analysis developed in this paper is investigating these questions.

The results of this paper confirm the finding by Heckman, Lochner and Taber (1998) and Lee and Wolpin (2006) that the GE effects of changes in the supply of skills on their prices are an important determinant of the evolution of wage inequality. In addition, they provide an empirical test of the steady-state theory of equilibrium wage functions developed by Mookherjee and Ray (2007). In an equilibrium model of savings and occupational choices without any stochastic shock and individual heterogeneity, Mookherjee and Ray derive the theoretical implication of a convex relationship between the skill-intensity of an occupation and its marginal rate of return.

The analysis developed in this paper adds to the existing literature by showing that an equilibrium model with endogenous skill prices could be crucial to understand the driving factors of the recent trend in wage inequality. Together with the convexification in Latin America, there is extensive evidence of a convexified wage profile by years of education in the US as well as in some Asian countries.[33] More generally, the growing literature documenting an increase in wage inequality driven by rising remunerations to the top percentiles of the earnings' distribution[34] suggests that an underlying convexification of the returns to skills could be the driving feature of the recent changes in inequality.

Finally, the model developed in this paper could be used to address an interesting and related question concerning the timing of the convexification. In this paper I have focused on the equilibrium steady state in order to explain the equilibrium wage profile at two points in time. Following the recent empirical literature that employs dynamic GE models to explain the changes in wage inequality,[35] the model could be used to study the economy in transition between 1987 and 2002. The main goal would be to explain the path of changes in the skill distribution and in the education prices that resulted into the convexification. A welfare analysis of who has been winning and loosing in the process and by how much would complete the quantitative exercise. This is left for future research.

[33] See Deschenes (2006) and Lemieux (2006, 2007) for the US and Liu (2006) for Vietnam.

[34] See Atkinson (2003) for the UK, Banerjee and Piketty (2005) for India, Piketty (2005) for France, Feenberg and Poterba (2000) and Piketty and Saez (2003) for the US.

[35] See, for example, Lee and Wolpin (2006) and Johnson and Keane (2007).

References

[1] Arrau P., Wijnbergen S., (1991) "Intertemporal Substitution, Risk Aversion and Private Savings in Mexico", The World Bank, WPS 682.

[2] Atkinson A., (2003) "Top Incomes in the United Kingdom over the Twentieth Century", Oxford University Discussion Papers in Economic and Social History.

[3] Attanasio O., Binelli C., (2003) "Redistribution Policy: Theoretical Motivation and Empirical Evidence", Papers and Proceedings AfD/EUDN Conference, Paris, November.

[4] Attanasio O., Goldberg PK., Pavcnik N., (2002) "Trade Reforms and Wage Inequality in Colombia", EdePo discussion paper EWP03/01.

[5] Attanasio O., Szekely M. Editors, (2001) *Portrait of the Poor. An Asset-Based Approach*, IADB, Washington D.C.

[6] Attanasio O., Szekely M. Editors, (2004) "Wage Shocks and Consumption Variability in Mexico During the 1990s", *Journal of Development Economics*, 73, 1-25.

[7] Avalos A., Savvides A., (2003) "On the Determinants of the Wage Differential in Latin America and East Asia: Openness, Technology Transfer and Labor Supply", LABEA Working Paper No. 19.

[8] Bandiera O., Caprio G., Honohan P., Schiantarelli F., (2000) "Does Financial Reform raise or Reduce Saving?", *Review of Economics and Statistics*, 82(2): 239-263

[9] Banerjee A., Piketty T., (2005) "Top Indian Incomes", The World Bank Economic Review, vol. 19, no 1, pp. 1-20.

[10] Behrman J., Birdsall N., Szekely M., (2000) "Economic Reform and Wage Differentials in Latin America", IADB Research Department Working Paper No. 435.

[11] Behrman E., Machin S., (2000) "Skill-Biased Technology Transfer. Evidence of Factor Biased Technological Change in Developing Countries", Mimeo.

[12] Belzil C., Hansen J., (2001) "Heterogeneous Returns to Human Capital and Dynamic Self-Selection", IZA Discussion Paper No. 272.

[13] Binelli C., Meghir C., Menezes-Filho N., (2008) "Educational Attainment and Wages in Brazil", Mimeo.

[14] Bosch M., Manacorda M., (2007) "Minimum Wages in Mexico", Mimeo.

[15] Bosworth B. (1998), "Productivity Growth in Mexico", Mimeo.

[16] Bouillon C., Legovini A., Lustig N., (2005) "Can Education Explain Changes in Income Inequality in Mexico?", Chapter 8 in *The Microeconomics of Income Distribution Dynamics in East Asia and Latin America*, Bourguignon F., Ferreira F. and Lustig N. eds., The World Bank.

[17] Bustos P. (2005), "The Impact of Trade on Technology and Skill Upgrading: Evidence from Argentina", Mimeo.

[18] Calonico S., Nopo H., (2007) "Where did you go to school? Private-public differences in schooling trajectories and their role on earnings", *Well-Being and Social Policy*, Vol. 3, Num. 1, pp. 25-46.

[19] Carneiro P., Heckman J., Vytlacil E., (2005), "Understanding What Instrumental Variables Estimate: Estimating Marginal and Average Returns to Education", Mimeo.

[20] Corseuil C. H., Muendler M., (2003) ”Wage Gaps, Capital and Skills”, Mimeo.

[21] Costa Dias M., (2002) "Essays on the Evaluation of Labor Market Programmes", PhD thesis, UCL.

[22] Cragg M. I., Epelbaum M., (1996) ”Why has wage dispersion grown in Mexico? Is it the evidence of reforms or the growing demand for skills?”, *Journal of Development Economics*, Vol.51, 99-116.

[23] Cunha F., (2006) "A Time to Plant and a Time to Reap", Mimeo.

[24] Davis D. R., (1996) "Trade Liberalization and Income Distribution", NBER Working Paper No. 5693.

[25] Deaton A., (1991) "Saving and Liquidity Constraints", *Econometrica*, Vol. 59, No.5, 1221-1248.

[26] Deschênes O., (2002) "Unobserved Ability, Comparative Advantage and the Rising Return to Education in the United States: A Cohort-Based Approach", UC Santa Barbara, Mimeo.

[27] Feenberg D., Poterba J., (2000) "The Income and Tax Share of Very High-Income Households, 1960-1995", *The American Economic Review*, vol. 90, no 2, pp. 264-270.

[28] Feenstra R. C., Hanson G., (1997) "Foreign Direct Investment and Relative Wages: Evidence from Mexico's Maquiladoras", *The Journal of International Economics*, 42(3-4), 371-393.

[29] Ferreira F., Leite P., Wai-Poi M., (2007) "Trade Liberalization, Employment Flows and Wage Inequality in Brazil", World Bank Policy Research Working Paper 4108.

[30] Fitzsimons E., (2007) "The effect of risk on education in Indonesia", *Economic Development and Cultural Change,* Vol. 56, No. 1, pp. 1-25.

[31] Gallipoli G., Meghir C., Violante G., (2007) "Equilibrium Effects of Education Policies: a Quantitative Evaluation", Mimeo.

[32] Giovannetti B., Menezes-Filho N., (2006) "Trade Liberalization and the Demand for Skilled Labor in Brazil", *Economia*, Vol. 7, No. 1.

[33] Goldberg PK., Pavcnik N., (2004) "Trade, Inequality, and Poverty: What Do I Know? Evidence from recent trade liberalization episodes in developing countries", *Brookings Trade Forum* 2004, Brookings Institute.

[34] Gonzaga G., Menezes-Filho N., Terra C., (2006) "Trade Liberalization and the Evolution of Skill Earnings Differentials in Brazil", *Journal of International Economics*, 68, 2, pp. 345-367.

[35] Hanson G., Harrison A., (1999) "Trade Liberalization and Wage Inequality in Mexico", *Industrial and Labor Relations Review*, 52(2): 271-288.

[36] Harrison A., (1996) "Openness and Growth: A Time Series, Cross-Country Analysis for Developing Countries," *Journal of Developing Economics*, 48: 419-47.

[37] Heathcote J., Storesletten K., Violante G., (2005) "Two Views of Inequality Over the Life-Cycle", *Journal of the European Economic Association*, Papers and Proceedings, Vol. 3, pp. 543-52

[38] Heckman J., Lochner L., Taber C., (1998) "Explaining Raising Wage Inequality: Explorations with a Dynamic General Equilibrium Model of Labour Earnings with Heterogeneous Agents", *Review of Economic Dynamics*, 1, 1-58.

[39] Hernández M, Chávez A, Bourges H., (1987) "Valor nutritivo de los alimentos mexicanos. Tablas de uso práctico." Publication L-12 10^a edition, Mexico. Instituto Nacional de Nutricion INN.

[40] Hoffman A., (1993) "Economic Development in Latin America in the 20th Century: A Comparative Perspective." in Explaining Economic Growth, A. Szirmai, B. van Ark, and D. Pilat eds. New York: North Holland.

[41] Jacoby H. G., Skoufias E., (2002) "Financial Constraints on higher education: Evidence from Mexico", Mimeo.

[42] Jacoby H. G., Skoufias E., (1997) "Risk, Financial Markets, and Human Capital in a Developing Country", *Review of Economic Studies*, 64, 311-335.

[43] Johnson G., (1997) "Changes in Earnings Inequality: The Role of Demand Shifts", *Journal of Economic Perspectives*, Vol. 11, Issue 2, 41-54.

[44] Johnson M., Keane P. M., (2007) "A Dynamic Equilibrium Model of the U.S. Wage Structure, 1968-1996", Mimeo.

[45] Katz L., Murphy K., (1992) "Changes in Relative Wages, 1963-1987: Supply and Demand Factors", *The Quarterly Journal of Economics*, Vol. 107, Issue 1, 35-78.

[46] Krusell P., Ohanian L. E., Rios-Rull V., Violante G., (2000) "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis", *Econometrica*, Vol. 68, No. 5, pp.1029-1053.

[47] Lee D., (2005) "An Estimable Dynamic General Equilibrium Model of Work, Schooling and Occupational Choice", *International Economic Review*, Vol. 46, No. 1, 1-34.

[48] Lemieux T., (2006) "Post-secondary Education and Increasing Wage Inequality", *American Economic Review*, Papers and Proceedings 96(2): 195-199.

[49] Lemieux T., (2007) "The Changing Nature of Wage Inequality", University of British Columbia, Mimeo.

[50] Lee D., Wolpin K., (2006) "Intersectoral Labor Mobility and the Growth of the Service Sector", *Econometrica*, Volume 47, 1-46.

[51] Lee D., Wolpin K., (2006) "Accounting for Wage and Employment Changes in the U.S. from 1968-2000: A Dynamic Model of Labor Market Equilibrium", forthcoming *Journal of Econometrics.*

[52] Lisboa M., Menezes-Filho N., Schor A., (2004) "The Effects of Trade Liberalization on Productivity in Brazil: Competition or Technology?", Mimeo.

[53] Liu A. Y. C., (2006) "Changing Wage Structure and Ecudation in Vietnam, 1992-1998", *Economics of Transitions* Vol. 14(4) 2006, pp. 681–706.

[54] Londono J. L., Szekely M., (2000) "Persistent Poverty and Excess Inequality: Latin America, 1970-1995", *Journal of Applied Economics*, Vol. III, No. 1, 93-134.

[55] Lopez Boo F., (2008) "The Evolution of Returns to Schooling in Argentina", University of Oxford, Mimeo.

[56] Maloney W. F., Nunez J., (2003) "Measuring the Impact of Minimum Wages: Evidence from Latin America", NBER Working Paper No. 9800.

[57] Manacorda M., Sanchez-Paramo C., Schady N., (2006) ”Off and Running? Technology, Trade, and the Rising Demand for Skilled Workers in Latin America”, Mimeo.

[58] Menezes-Filho N., Fernandes R., (2003) ”Demand for Skill in Brazil: Theory and Empirical Evidence”, University of Sao Paulo, Mimeo.

[59] Mookherjee D., Ray D., (2003) "Persistent Inequality", *Review of Economic Studies*, 70, 369-393.

[60] Mookherjee D., Ray D., (2007) "Occupational Diversity and Endogenous Inequality", Mimeo.

[61] Owen A., Weil D., (1998) "Intergenerational Earnings Mobility, Inequality, and Growth", *Journal of Monetary Economics*, Vol. 41(1), 71-104.

[62] Patrinos P., Psacharopoulos G., (2004) "Returns to Investment in Education: A Further Update", *Education Economics*, Volume 12, No. 2, 111-134.

[63] Pavcnik N., Blom A., Goldberg P., (2002) ”Trade Liberalization and Labor Market Adjustment in Brazil”, Mimeo.

[64] Piketty T., (2005) "Top income shares in the long run: an overview", *Journal of the European Economic Association*, vol. 3-4, no 3, pp. 1-11.

[65] Piketty T., Saez E., (2003) "Income Inequality in the United States 1913-1998", *Quarterly Journal of Economics*, vol. 68, Issue 1, pp. 1-39.

[66] Ravallion M., (1997) "Can High-Inequality Developing Countries Escape Absolute Poverty?", *Economics Letters*, Volume 56, 51-57.

[67] Revenga A., (1997) "Employment and Wage Effects of Trade Liberalization: The Case of Mexican Manufacturing", *Journal of Labor Economics*, Vol. 15, 20-43.

[68] Stokey N., Lucas R., Prescott E., (1989) *Recursive Methods in Economic Dynamics*, Cambridge, MA: Harvard University Press.

[69] Székely M., (2003) "The 1990s in Latin America: Another Decade of Pesistent Inequality, but with Somewhat Lower Poverty", *Journal of Applied Economics*, Vol. VI, No. 2, 317-339.

[70] Verhoogen E., (2008) "Trade, Quality Upgrading and Wage Inequality in the Mexican Manufacturing Sector.", *Quarterly Journal of Economics,* Vol. 123, No. 2.

[71] Winters A., McCullach N., McKay A., (2004) "Trade Liberalization and Poverty: the Evidence So Far", *Journal of Economic Literature*, 62, 72-115.

Appendix A - Data

ENEU

The ENEU (*Encuesta Nacional de Empleo Urbano*) is the Mexican national employment survey collected yearly by the Mexican national statistical office, INEGI. The ENEU has a structure similar to the US Consumer Expenditure Survey (CEX). It is a quarterly household survey that collects individual-level data and it has a rotating panel structure: households are interviewed for five consecutive quarters and in each quarter twenty per cent of the households are replaced by new households that are interviewed for the first time. The survey started in 1981 with progressive increase of the geographic coverage. The sample is selected to be geographically and socio economically representative of the Mexican urban population: by the end of the 1990s it covered approximately sixty two per cent of the national urban population and ninety two per cent of the cities with population greater than one hundred thousand people. By the end of the 1990s, approximately seventy four per cent of the Mexican population lived in urban areas. Recent rounds of the ENEU have national coverage, but since the earlier ones survey urban areas only, I restrict the sample to urban areas in all years.

The main questionnaire is divided in three parts. The first part collects socio demographic information on all household members. The second part contains detailed employment information on individuals at least twelve years old. The third part reports information on the characteristics of the house of residence with additional questions on the characteristics of the building, number and type of rooms and ownership status from the 1994 wave onwards. The employment information is very detailed with several questions on individuals' occupation status, type and characteristics of employment, sector of main and secondary job, contract type, number of working hours, monthly earnings, unemployment status and duration and social security taxes paid by the worker's employer in the private and public sector. Earnings are reported net of all labor taxes and social contributions paid in either public or private funds.

Construction of the education groups

The Mexican education system consists of four main cycles: pre-school, primary, secondary (lower and upper) and post-secondary (University/technical institutes or more) education.

Pre-school education is between age 3 and 6 and is provided free of charge. In December 2001, the Mexican Congress voted to make one year of pre-school education mandatory, a provision that went into effect in 2004. Primary education starts at age 6, it lasts six years and it has always been compulsory. Secondary education comprises two main levels: lower and upper secondary. Lower secondary lasts between three to four years, depending on the program. Upper secondary lasts three years. Both levels includes an "academic" and a "vocational" branch that paves the way, respectively, to University and non-University education. In 1993 lower secondary became compulsory. The policy change mainly affected rural areas with a large increase in the construction of schools and corresponding increasing attainment rates at lower secondary in these areas. Post-secondary education comprises Universities, 4-years technical institutes and 2-years technical institutes. By far the majority of students are enrolled in universities and a very small proportion is enrolled in 2-years technical institutes. University takes four to five years and graduate education lasts between two to four years (two years are necessary to obtain a Master degree and two additional years to obtain a PhD).

In order to construct the three education groups used in the model, the schooling levels have been aggregated as it follows. The "basic education" group includes all individuals that have up to uncompleted upper secondary education, the "intermediate education" group includes all individuals that have up to uncompleted University education and the "higher education" group includes individuals who have completed University or more. As in Manacorda, Sanchez-Paramo and Schady (2006), I aggregated the academic and vocational branch of secondary education by considering in the "intermediate" group all individuals that have completed any of the two branches.

At the beginning of the 1990s attainment rates at primary education were above 90 per cent in all Mexico and at lower secondary they were above 80 per cent in urban areas and below 40 per cent in rural areas. Focusing on urban areas motivates the choice of grouping primary and lower secondary into compulsory basic education. The four model's periods reproduce the four main stages of the education cycle. For simplicity, the length of each period is assumed to be the same and equal to seven years in order to match an average working life of adult Mexican workers of around thirty years.

Estimation of the wage equations

I estimate the wage equations by considering all wage workers between the age of 15 and 60. Hourly wages are computed as the ratio between monthly earnings and hours worked in the main occupation last month. The data are deflated using the Mexican national CPI as of June 2002.

Table 6 and 7 report the estimates of the variance of the residuals and of the yearly rescaled coefficients of the age polynomials by education group for each year between 1987 and 2002.

Year	Basic	Intermediate	Higher
1987	0.065	0.070	0.079
1988	0.065	0.074	0.088
1989	0.070	0.077	0.117
1990	0.068	0.084	0.119
1991	0.069	0.087	0.113
1992	0.069	0.083	0.106
1993	0.065	0.083	0.113
1994	0.070	0.084	0.115
1995	0.073	0.085	0.110
1996	0.068	0.083	0.111
1997	0.064	0.078	0.099
1998	0.066	0.083	0.099
1999	0.078	0.094	0.121
2000	0.078	0.094	0.125
2001	0.063	0.076	0.099
2002	0.071	0.089	0.114

Table 6: Variance of the Residuals - Estimation Wage Equations (Source: ENEU)

Year	Basic		Intermediate		Higher	
	age	age squared	age	age squared	age	age squared
1987	0.098	-0.005	0.122	-0.004	0.274	-0.012
1988	0.079	-0.004	0.188	-0.008	0.302	-0.014
1989	0.092	-0.005	0.194	-0.010	0.320	-0.015
1990	0.116	-0.006	0.167	-0.007	0.303	-0.013
1991	0.118	-0.006	0.102	-0.004	0.264	-0.011
1992	0.096	-0.005	0.279	-0.013	0.224	-0.010
1993	0.121	-0.006	0.170	-0.006	0.204	-0.009
1994	0.168	-0.009	0.257	-0.010	0.184	-0.007
1995	0.171	-0.008	0.253	-0.011	0.216	-0.008
1996	0.161	-0.008	0.166	-0.006	0.243	-0.009
1997	0.108	-0.005	0.236	-0.010	0.281	-0.012
1998	0.104	-0.005	0.106	-0.003	0.182	-0.006
1999	0.131	-0.007	0.131	-0.005	0.228	-0.010
2000	0.118	-0.006	0.162	-0.006	0.253	-0.010
2001	0.108	-0.006	0.104	-0.003	0.186	-0.007
2002	0.124	-0.006	0.129	-0.005	0.196	-0.007

Table 7: Age Polynomials - Estimation Wage Equations (Source: ENEU)

Appendix B - Solution method

This Appendix describes the method used to compute the equilibrium steady state of the model. The model is solved recursively by backwards induction from the last to the first period of adult life.

Step 1. Set an initial guess of the vector of the skill prices $[p_1, p_2, p_3]$ and assume that future prices equal current prices. $[p_1, p_2, p_3]$ gives an initial guess of $p(a)$, the vector of the skill prices forecasted from age a onwards.

Step 2. Solve the optimization problem in the last period of work life before retirement ($a = \overline{a}$).

Define with $V_{\overline{a}}^{Sch}(j^P, j_{\overline{a}}^C, A_{\overline{a}}, p(\overline{a}), \eta, z_{\overline{a}})$ and with $V_{\overline{a}}^{Work}(j^P, j_{\overline{a}}^C, A_{\overline{a}}, p(\overline{a}), \eta, z_{\overline{a}})$, respectively, the conditional value function of sending the child to school and to work and denote with $W_{\underline{a}}^{Sch}(j_{\underline{a}}^P, A_{\underline{a}}, p(\underline{a}), \eta, z_{\underline{a}})$ and $W_{\underline{a}}^{Work}(j_{\underline{a}}^P, A_{\underline{a}}, p(\underline{a}), \eta, z_{\underline{a}})$ the initial guess for child lifetime utility as an adult conditional on having sent the child, respectively, to school and to work in the last period of coresidence. $\underline{a}$ denotes the age of the parent in the first period of adult life.

Given $W_{\underline{a}}^{Sch}(.)$ and $W_{\underline{a}}^{Work}(.)$, $V_{\overline{a}}^{Sch}(.)$ and $V_{\overline{a}}^{Work}(.)$ take the following expressions:

$$\begin{aligned} V_{\overline{a}}^{Sch}(j^P, j_{\overline{a}}^C, A_{\overline{a}}, p(\overline{a}), \eta, z_{\overline{a}}) &= \max_{c_{\overline{a}}} \left\{ U(c_{\overline{a}}) + \lambda\beta E_{z_{\underline{a}}} W_{\underline{a}}^{Sch}(j_{\underline{a}}^P, A_{\underline{a}}, p(\underline{a}), \eta, z_{\underline{a}}) \right\} \\ s.t. \quad c_{\overline{a}} &= A_{\overline{a}}(1+r) + w_{jP,\overline{a}} - F_{jC} - A_{\overline{a}+1} \\ j_{\underline{a}}^C &= (j_{\overline{a}}^C + 1) = j_{\underline{a}}^P \end{aligned} \tag{28}$$

$$\begin{aligned} V_{\overline{a}}^{Work}(j^P, j_{\overline{a}}^C, A_{\overline{a}}, p(\overline{a}), \eta, z_{\overline{a}}) &= \max_{c_{\overline{a}}} \left\{ U(c_{\overline{a}}) + \lambda\beta E_{z_{\underline{a}}} W_{\underline{a}}^{Work}(j_{\underline{a}}^P, A_{\underline{a}}, p(\underline{a}), \eta, z_{\underline{a}}) \right\} \\ s.t. \quad c_{\overline{a}} &= A_{\overline{a}}(1+r) + w_{jP,\overline{a}} + w_{jC,\overline{a}} - A_{\overline{a}+1} \\ j_{\underline{a}}^C &= j_{\overline{a}}^C = j_{\underline{a}}^P \end{aligned} \tag{29}$$

where r is the fixed real interest rate on financial assets, F_{jC} denotes the fixed costs of schooling for child education level j^C and $w_{jP,\overline{a}}$ and $w_{jC,\overline{a}}$ are, respectively, parental and child wage at age $\overline{a}$ given parental (child) education level $jP(jC)$. λ denotes the degree of parental altruism and expectations are taken over next period shock to earnings, z. Equations (28) and (29) describe the evolution of child education that increases by one unit if the child is sent to school. The level of child education at the end of the last period of coresidence defines the (fixed) education level throughout adulthood ($j^C_{\underline{a}} = j^P_{\underline{a}} = j^P$). For simplicity I do not report the credit constraint (6) and the terminal condition (7).

Step 3. Solve the conditional maximization problems in the third, second and first period of adult life.

In the third period child education is a choice variable. The conditional maximization problems read:

$$V^{Sch}_a(j^P, j^C_a, A_a, p(a), \eta, z_a) = \max_{c_a} \left\{U(c_a) + \beta V_{a+1}(j^P, j^C_a + 1, A_{a+1}, p(a+1), \eta, z_{a+1})\right\}$$
$$s.t. \quad c_a = A_a(1+r) + w_{jP,a} - F_{jC} - A_{a+1}$$
$$j^C_{a+1} = (j^C_a + 1)$$

$$\begin{aligned} V^{Work}_a(j^P, j^C_a, A_a, p(a), \eta, z_a) &= \max_{c_a} \left\{U(c_a) + \beta V_{a+1}(j^P, j^C_a, A_{a+1}, p(a+1), \eta, z_{a+1})\right\} \\ s.t. \quad c_a &= A_a(1+r) + w_{jP,a} + w_{jC,a} - A_{a+1} \\ j^C_{a+1} &= j^C_a \end{aligned}$$

where F_{jC} is the fixed cost of child schooling level jC and $V_{a+1}(j^C_a + 1, .)$ and $V_{a+1}(j^C_a, .)$ define, respectively, the expected value over the maximum between the conditional value functions of the schooling and work alternative in the next period given the decision of sending the child, respectively, to school or to work in the current period.

They take the following expressions:

$$\begin{aligned} & V_{a+1}(j^P, j^C_a + 1, A_{a+1}, p(a+1), \eta, z_{a+1}) \\ \equiv\ & E_{z_{a+1}} \max[V^{Sch}_{a+1}(j^P, j^C_a + 1, A_{a+1}, p(a+1), \eta, z_{a+1}), V^{Work}_{a+1}(j^P, j^C_a + 1, A_{a+1}, p(a+1), \eta, z_{a+1})] \end{aligned}$$

$$\begin{aligned} & V_{a+1}(j^P, j^C_a, A_{a+1}, p(a+1), \eta, z_{a+1}) \\ \equiv\ & E_{z_{a+1}} \max[V^{Sch}_{a+1}(j^P, j^C_a, A_{a+1}, p(a+1), \eta, z_{a+1}), V^{Work}_{a+1}(j^P, j^C_a, A_{a+1}, p(a+1), \eta, z_{a+1})] \end{aligned}$$

In the second period the child is sent to compulsory basic education. The maximization problem is given by:

$$
\begin{aligned}
V_a(j^P, A_a, p(a), \eta, z_a) &= \max_{c_a} \left\{ U(c_a) + \beta V_{a+1}(j^P, 1, A_{a+1}, p(a+1), \eta, z_{a+1}) \right\} \\
s.t. \quad c_a &= A_a(1+r) + w_{jP,a} - F_1 - A_{a+1}
\end{aligned}
$$

where F_1 denotes the fixed costs of basic education and $V_{a+1}(1,.)$ defines the expected value over the maximum between the conditional value functions of the schooling and work alternative in the next period given that the child has completed compulsory basic education in the current period:

$$
\begin{aligned}
& V_{a+1}(j^P, 1, A_{a+1}, p(a+1), \eta, z_{a+1}) \\
\equiv \; & E_{z_{a+1}} \max[V_{a+1}^{Sch}(j^P, j_a^C = 1, A_{a+1}, p(a+1), \eta, z_{a+1}), V_{a+1}^{Work}(j^P, j_a^C = 1, A_{a+1}, p(a+1), \eta, z_{a+1})]
\end{aligned}
$$

where $j_a^C = 1$ denotes completed basic education.

In the first period of adult life the child is in pre-school. Child education, j_a^C, is normalized to zero. The parent solves the following maximization problem:

$$
\begin{aligned}
V_{\underline{a}}(j^P, A_{\underline{a}}, p(\underline{a}), \eta, z_{\underline{a}}) &= \max_{c_{\underline{a}}} \left\{ U(c_{\underline{a}}) + \beta E_{z_{\underline{a}+1}} V_{\underline{a}+1}(j^P, A_{\underline{a}+1}, p(\underline{a}+1), \eta, z_{\underline{a}+1}) \right\} \\
s.t. \qquad c_{\underline{a}} &= A_{\underline{a}}(1+r) + w_{jP,\underline{a}} - A_{\underline{a}+1}
\end{aligned}
$$

Step 4. Compute the new initial guesses for $W_{\underline{a}}^{Sch}(.)$ and $W_{\underline{a}}^{Work}(.)$.

The solution of the model in steps two and three provides the complete set of value functions and optimal saving rules for any combination of the state space variables. I can use the optimal value function in the first period of adulthood, $V_{\underline{a}}$, as a new initial guess for the child lifetime utility. Denoting with $j_{\overline{a}}^C$ the level of education of the child at the end of the last period of coresidence, $V_{\underline{a}}(j^P = (j_{\overline{a}}^C + 1), A_{\underline{a}}, p(\underline{a}))$ provides the new initial guess for $W_{\underline{a}}^{Sch}(.)$ and $V_{\underline{a}}(j^P = j_{\overline{a}}^C, A_{\underline{a}}, p(\underline{a}))$ provides the new initial guess for $W_{\underline{a}}^{Work}(.)$. Given the new initial guesses for $W_{\underline{a}}^{Sch}(.)$ and $W_{\underline{a}}^{Work}(.)$, I repeat steps two and three.

Given the conditional value functions for the work and schooling alternative, the child is sent to school when the expected value of investing in schooling is at least as high as the expected value of sending the child to work, that is when the following condition holds:

$$
V_a^{Sch}(j^P, j_a^C, A_a, p(a), \eta, z_a) \geqslant V_a^{Work}(j^P, j_a^C, A_a, p(a), \eta, z_a) \quad \forall \quad a = a\_ed, ...\overline{a}
$$

where a_ed denotes parental age when child education becomes a choice variable.

Step 5. Repeat steps two to four until the following two conditions are satisfied:

$$\begin{aligned}||V_{\underline{a}}^{Sch\_Iter}(j^P, A_{\underline{a}}, p(\underline{a}), \eta, z_{\underline{a}}) - V_{\underline{a}}^{Sch\_(Iter-1)}(j^P, A_{\underline{a}}, p(\underline{a}), \eta, z_{\underline{a}})|| &\leq \varepsilon \\ ||V_{\underline{a}}^{Work\_Iter}(j^P, A_{\underline{a}}, p(\underline{a}), \eta, z_{\underline{a}}) - V_{\underline{a}}^{Work\_(Iter-1)}(j^P, A_{\underline{a}}, p(\underline{a}), \eta, z_{\underline{a}})|| &\leq \varepsilon\end{aligned}$$

where ε is an arbitrarily small number and $||.||$ denotes the distance between the conditional value functions in the first period of adulthood in two consecutive iterations.

Step 6. Compute the equilibrium skill prices as marginal productivities of the human capital factors using the equilibrium conditions (11) to (13).

Compute a new guess of the vector of the skill prices as a linear combination of the guess used to solve the model and the equilibrium prices computed in this iteration.

Step 7. Repeat steps two to six with the new guess for the vector of the skill prices.

Stop when the difference between each element of the vector of the equilibrium skill prices and the initial guess for this price is arbitrarily small.