WP 29_14

Periklis Gogas
Department of Economics, Democritus University of Thrace
The Rimini Centre for Economic Analysis, Italy

Theophilos Papadimitriou
Department of Economics, Democritus University of Thrace

Maria-Artemis Matthaiou
Department of Economics, Democritus University of Thrace

A NOVEL BANKING SUPERVISION METHOD USING THE MINIMUM DOMINATING SET

The Rimini Centre for Economic Analysis
Legal address: Via Angherà, 22 – Head office: Via Patara, 3 - 47921 Rimini (RN) – Italy
www.rcfea.org - secretary@rcfea.org

A Novel Banking Supervision Method using the Minimum Dominating Set

Periklis Gogas, Theophilos Papadimitriou and Maria-Artemis Matthaiou

Abstract The magnitude of the recent financial crisis, which started from the U.S. and expanded in Europe, change the perspective on banking supervision. The recent consensus is that to preserve a healthy and stable banking network, the monitoring of all financial institutions should be under a single regulator, the Central Bank. In this paper we study the interrelations of banking institutions under the framework of Complex Networks. Specifically, our goal is to provide an auxiliary early warning system for the banking system's supervisor that would be used in addition to the existing schemes of control. We employ the Minimum Dominating Set (MDS) methodology to reveal the most strategically important banks of the banking network and use them as alarm triggers. By monitoring the MDS subset the regulators can have an overview of the whole network. Our dataset is formed from the 200 largest American banks and we examine their interconnection through their total deposits. The MDS concept is applied for the first time in this setting and the results show that it may be an essential supplementary tool to the arsenal of a Central Bank.

1 Introduction

The financial crisis of 2008 demonstrated that effective and continuous supervision of economic systems is necessary. More specifically, close monitoring of banking institutions and their interrelations is of outmost importance in the effort to reduce and control systemic risk in the specific industry and the economy as a whole. The banking interconnectedness resulting from interbank lending forms a network of links. Interbank lending between financial institutions creates a network of obligations and claims that's prone to systemic risk. The connections of this network become more complex as more sophisticated financial products (swaps, collateralized

Periklis Gogas, Theophilos Papadimitriou and Maria-Artemis Matthaiou
Department of Economics, Democritus University of Thrace,
e-mail: {pgkogkas, papadimi, mmatthai}@ierd.duth.gr

depth obligations, etc.) are considered [9], necessitating the need for effective monitoring. Moreover, the abolishment of the Glass-Steagall Act resulted in the lack of separation between commercial and investment banking, making banking systems more vulnerable to systemic risk. By systemic risk we refer to the possibility that an institution's default will cause the collapse of the entire system, through consecutive cascading failures.

We can allege that a banking network is exposed to systemic risk when default is spread through contagion to a significant portion of the financial system. According to the above, the minimization of systemic risk is crucial if we want to avoid episodes of banking crises. So an effective and continuous monitoring of financial institutions is necessary in order to preserve a healthy and stable banking system. If the regulators are equipped with proper tools that forecast timely an upcoming expansion of contagion risk, they will be able to take the necessary measures to either restrict or sidestep the domino effect of failures. It is proposed that the supervision of the European banking network should be assigned to a single authority, the ECB. The proponents of this supervising scheme believe that if the supervision and regulation of the network is in the hands of ECB, a prompt and correct intervention can be achieved. Blinder [5] enumerates the reasons why systemic risk regulation (SRS) should be operated by a single authority, in his case, the Fed. The reasons are: a) creation of economies of scale, b) pursuance of financial stability and c) credibility. Vives [23] points out that a single regulatory authority can create and take advantage of the economies of scale between Lender of Last Resort (LOLR) facility, monetary policy and supervision. It can also overcome the obstacles of asymmetric information and is capable to cope with global risk that emerges from the convergence of financial institutions and markets. Boyer and Ponse [7] also support that the optimal supervision of the banking system is preserved when the responsibility of regulation is in the hands of a single supervisor.

Thus, it seems as common view these days that the maintenance of a viable and stable economic system depends on effective bank supervision. Bank supervision is successful when a) we know the structure of the network, b) we can make an accurate estimation of the contagion risk resulting from a potential bank failure, c) we mainly focus on the monitoring of banks, representing the behavior of a networks region. Timely and accurate estimation of economic distress grants benefits to the supervisor, as he can react by taking the necessary precautions to avoid or eliminate contagion risks. Thus, to detect potential banking distress we need reliable monitoring tools.

The Complex Networks theory has been used for modeling banking networks and more precisely for testing the robustness of the network against shocks and simulating contagion patterns. Allen and Galle [3] in their seminal paper assessed the stability of a bank system with a homogenous topology taking under consideration the connectivity of the banking network. They concluded that complete networks (i.e. networks where every possible pair of nodes is connected), are more resilient to economic shocks. On the other hand, the sparser is the network connectivity, the higher is the risk that a bank default will cause a cascading effect of failures to the neighboring institutions. Similar conclusions were drawn by Leitner [15]. He stated that in a complete network, the crisis is effectively dispersed to the overall bank system, and each bank receives a smaller shock from the defaulted bank. Gai and Kapadia [9] claim that while the probability of contagion is diminished when the network is highly interlinked, if the bank that defaults is crucial for the network, contagion can be spread to the economy as a whole. Angelini et al. [4] test an Italian netting system with homogenous topology and estimate the damage from a potential ”domino effect” triggered by an institution’s failure. The results reveal that only a small number of institutions (4%) were crucial enough to cause the collapse of the entire economy.

Taking into account the special structure and dynamics of banking networks, a number of papers examine the topology of them (banking networks). Tabak et al [21], Minoiu and Reyes [16], Iori et al. [13], Inaoka et al. [12] assess banking networks by using several metrics: degree distribution, cluster coefficient, volatility and efficiency. Their primary concern is to detect the degree of network connectivity as well as the network’s distribution, and draw conclusions about the network’s configuration during and after banking crises. They sum up that the network becomes sparser during and after a crisis and that the clustering coefficient and domestic interest rates, are negatively correlated. Using the same metrics Thurner et al. [22] examine the impact of a network’s structure on the wealth of the economy, concluding that a highly connected network, is less unstable because it is not exposed to large wealth changes. Thus the possibility for cascading failures drops. The closest related paper to ours is Papadimitriou, Gogas and Tabak [18] in which they use tools from Complex Network theory, specifically the Minimum Spanning Tree (MST) to find the network’s ”core” banks and potential contagion paths. They support that these banks can serve as an early warning system in case of a bank distress, so if a red flag is raised, regulators should be focused on the directly linked banks and take the necessary measures to minimize systemic risk.

In this study we focus on the monitoring of the banking networks using tools from the Graph Theory. Our primary scope is to provide an auxiliary monitoring mechanism to Central Banks for prompt intervention and regulation of the banking network. We propose a scheme that unveils the banks that best represent the whole banking system. Our goal is to identify the smallest possible subset of banks that allow us to achieve an efficient monitoring of the whole banking system. That subset can be used as a warning system of an upcoming banking crisis. It must be clarified that the proposed system does not nullify the existing monitoring tools and should be used in addition to the existing supervision mechanisms. The rest of the paper is organized as follows. In Section 2 we first give a brief description of complex networks and then we provide a detailed description of the basic MDS model pointing out the problems that arise and their solutions. Section 3 is a presentation of the initial dataset and the transformation of that matrix in order to be used by the model. Section 4 is a report of the empirical results and finally the paper concludes in Section 5.

2 Methodology

The main objective of this paper is to provide a new monitoring tool that ensures the overview of the entire banking system using a small subset of banks. The proposed monitoring mechanism should be used supplementary to the existing ones, in order to increase the efficiency of the current Central Bank supervision systems. The optimal monitoring of a banking network is achieved when the supervision is authorized to a single supervisor, the central bank. The E.U. leaders decided in October 2012 that all six-thousand European banks should be under the monitoring veil of the European Central Bank, which is a quite demanding task. Respectively, as far as it concerns the American banking network, the Fed should be authorized as the definite SRS for the banking network monitoring. By applying the suggested methodology, we identify which is the most strategically important subset of banks for network's supervision. This subset is the smallest possible that serves as an auxiliary monitoring mechanism for the overview of all financial institutions. In addition to the current supervision system, the banks identified by the proposed scheme can provide an alarm system to the regulator for close and detailed attention not only to the bank that triggered the alarm but to the whole sub-network associated with it. We claim that these banks consist the representative pillars of the network and the Central Bank should be focused on close monitoring them. The proposed methodological mechanism examines the interrelations of banking institutions from a new point of view, regarding the existing literature. To our knowledge, we are the first to apply the MDS metric using Graph Theory for the supervision of a banking network. Graph theory is a field in mathematics that studies the structure of networks and represent them as Graphs. A Graph depicts a collection of nodes which are interconnected. A node can represent anything: humans, entities, computers and so forth. Two nodes are interconnected if there exists a link between them. Steen [20] gives the following definition for graphs:

Definition 1. A graph $\mathbf{G}$ consists of a collection $\mathbf{V}$ of nodes and a collection of links $\mathbf{E}$, for which we write $\mathbf{G} = (\mathbf{V}, \mathbf{E})$. Each link $e \in \mathbf{E}$ is said to join two nodes, which are called its end points. If e joins $u, v \in \mathbf{V}$, we write $e = < u, v >$. Node u and v in this case are said to be adjacent. Link e is said to be incident with nodes u and v, respectively.

The topology (characteristics) of the whole network is examined by calculating specific metrics. Employing graph theory within an economics context, the nodes represent the economic entities (**N**) while the edges (**E**) the links (interrelations) between a pair of nodes, represent the similarity measure of the tested variable. A graph of a network (**G**) is the depiction of nodes and edges. In this study, the nodes of the network represent the largest 200 American banks in terms of their total deposits, and the edges connecting them are drawn by calculating the cross-correlations of banks total deposits (TD). For the calculation of cross-correlations we use the Pearson cross-correlation coefficient $r_{i,j}$ using the following equation:

$$r_{i,j} = \frac{\mathrm{COV}(\mathrm{TD}_i, \mathrm{TD}_j)}{\sqrt{\mathrm{VAR}(\mathrm{TD}_i)\mathrm{VAR}(\mathrm{TD}_j)}} \tag{1}$$

The values of the coefficient r spans from $[-1,1]$: values near 1 indicate a strong positive correlation and values near -1 indicate a strong negative correlation. When the coefficient takes values near 0, the behavior of the bank's total deposit is considered uncorrelated. The formation of a network that depicts the coefficient correlations in terms of the predetermined variable, total deposits in our case, is of outmost importance because based on that network we will implement the proposed methodological tools. Calculating the cross-correlations for every pair of banks, unveils the similarity (or dissimilarity) of behavior for every possible pair of banks leading to the formation of a complete network. The proposed method needs a consistent network in the sense that every edge should imply that the end nodes have similar behavior. Consequently we set an arbitrary threshold for the edges below of which we consider that the edge links two uncorrelated nodes (i.e. they have dissimilar behavior), and these edges should be removed from the network. The thresholding procedure produces a sparser network and may create isolated sub-networks or nodes (i.e. sub-networks or nodes without any link to the rest of the network). The remaining edges, though, are highly correlated, i.e. the nodes consisting the network have similar behavior over time in the examined variable. In our study we will use specific tools from the Graph theory for the evaluation of the network topology, and more specific we will use the Density and the Minimum Dominating Set. We use the metric of density d to quantify the connectedness of a network using the following equation:

$$d = \frac{\sum_{i=1}^{n} k_i}{n(n-1)/2} \tag{2}$$

where k_i represents the degree of node i (i.e. the number of edges that directly link node i with the rest of network's nodes) and n is the total number of nodes. The values of density fluctuate from $[0,1]$ where value 0 indicates a network with no links between any node of the network, respectively the value 1 indicates that the network is complete. Values close to 1 show that the network is highly connected. A highly connected network, is interpreted as a network that comprises more nodes with similar behavior, therefore the subset of nodes that constitute the MDS exhibits an abundance of linkages, a property that sets them reliable gauges of a bank distress. Every node of the MDS will act as a representative of a bigger group of nodes/banks. If a bank distress occurs in an institution of the network, it will be reflected to its respective MDS node, as those two nodes are directly linked. When the MDS node locates instances of bank distress it raises a red flag in a timely manner. By doing so, the banking system supervising authority may have some extra time to react and intervene by implementing the necessary policies for the protection of the whole neighborhood. On the other hand values near zero reflect that the nodes have dissimilarities on their behavior, so a sparser network is formed (network with fewer nodes). The nodes consisting the MDS are more than in the previous case, while the sub-networks that those nodes represent are sparser (i.e. fewer nodes are adjacent/neighbor to every node of the MDS). In a sparser network we can still preserve the overview of the whole system, though by monitoring a larger group of banks. In our attempt to extract the subset of supervised banks, we apply the MDS mechanism:

Definition 2. A dominating set DS of a graph $\mathbf{G}$ is a subset of nodes $\mathbf{S} \subseteq \mathbf{V}$ such that every node not in $\mathbf{S}$ is joined to at least one member of $\mathbf{S}$ by some link. The domination number $\gamma(\mathbf{G})$ is the number of nodes in the smallest dominating set for $\mathbf{G}$ ([19]).

Consequently the MDS is the DS with the smallest cardinality (i.e. the minimum subset of nodes that still can be characterized as DS). The methodology for identifying the MDS is the following: We employ a binary variable x_i, $i = 1, 2, ..., n$ to represent every node of the network.

$$x_i = \begin{cases} 1 & \text{if } i \in DS \\ 0 & \text{if } i \notin DS \end{cases} \tag{3}$$

We present the assumption of the DS in mathematical terms:

$$x_i + \sum_{j \in N(i)} x_j \geq 1 \tag{4}$$

where $N(i)$ indicates the neighbors (adjacent nodes) of node i. The assumption is quite easy to be comprehended. The posed constraint preserves that every node of the network will be represented from the DS subset. The MDS is the DS with the smallest cardinality, therefore to calculate the MDS we need to minimize the DS. In mathematic terms:

$$\min_{\mathbf{x}} = \sum_{i=1}^{n} x_i \tag{5}$$

To identify the MDS we must minimize the nodes of the network (eq. 5) under the constraint of eq. 4. It is important to note that according to the presented framework, every isolated node (i.e. nodes that lost all the links with the rest of the network) is an MDS node. Nonetheless, we need to distinguish the isolated nodes from the rest of the MDS nodes. In our study the isolated nodes represent the banks that are totally uncorrelated with the rest of the network. The cross-correlations are based on bank's total deposits and as a result an isolated bank is interpreted as a bank with entirely different behavior than the rest of the network and they should be monitored separately.

3 Data and Results

The source of our dataset was the U.S. Bank Association. We collected quarterly data for the largest 200 American banks, in terms of their total deposits. The period under examination spans from March 2003 to September 2012. The predetermined variable for the formation of our network, is the total bank's deposits. We choose the specific variable because a) it can secure us with a representative perception about a bank's health and b) it is a variable with no extreme fluctuations over time. By calculating the correlations of the banks' total deposits we can observe how they move in time compared to the rest banks deposits and efficiently estimate which of them exhibit similar behavior. Additionally we are able to locate isolated banks i.e. banks with unlike behavior and monitor them individually. After collecting all banks that comprise the dataset, our next step is to calculate the cross-correlations using eq.1 and shape our complete network. A crucial step of the employed methodology is setting the level of correlations threshold: the cardinality of the MDS depends on it. During our trial and error search for the optimal value, we witnessed that as the value of the threshold increased, a simultaneous rise in the size of the MDS and the number of the isolated nodes was observed, while the network's density became more sparse. Respectively, a drop in the correlation threshold value, yielded smaller MDS, fewer isolated nodes and a denser network. Table 1 depicts the above ascertainment for various values of threshold. By implementing higher thresholds, we set a higher hurdle in behavior's resemblance, and as a result the final network includes fewer nodes. This is the outcome of removing more edges from the network. The procedure shrinks the neighborhood of every node, thus more MDS nodes are necessary to efficiently represent the network. As we can clearly see from Table 1 the number of isolated nodes becomes larger, the MDS size also grows and network density drops. In that case the supervision is obtained by monitoring a larger group of banks, which is an aftereffect of the higher threshold. We tested various values for the correlation threshold and we report the structure of the resulting network in Table 1. Considering that we propose the MDS method as an auxiliary monitoring tool, we can claim that our goal is to achieve the maximum supervision, while having the minimum monitoring. Taking under concern that goal, we regard that the values that granted the most notable results, are the following:

- Threshold = 0.75 creates an MDS of 6 nodes, the isolated nodes are 7, while the density of network is 0.61.
- Threshold = 0.80 creates an MDS of 7 nodes, the isolated nodes are 11 and the network's density 0.53.
- Threshold = 0,85 creates an MDS of 8 nodes, the isolated nodes are 16, and network's density 0.42.

We believe that the above values yield noteworthy results because the constructed networks a) require the supervision of fewer network's nodes/banks, b) are denser, so they include more nodes with similar behavior, c) complete network's cross-correlations fluctuate close to those values.

Figures 1, 2 depict the transformation of the network's composition after the implementation of different threshold values. In an attempt to stress the influence of the threshold's value in the structure of the network, we intentionally choose values that differ by far. In Figure 1 and 2 the threshold value is 0.8 and 0.9 respectively. As we can see in the first case, the network is composed of a larger number of nodes,

Table 1 Values for threshold, MDS, isolated nodes and density Threshold

Threshold	MDS	Isolated Nodes	Network's Density
0.75	6	7	0.61
0.8	7	11	0.53
0.85	8	16	0.42
0.87	10	17	0.37
0.9	13	22	0.28
0.95	20	46	0.09
0.97	24	81	0.03

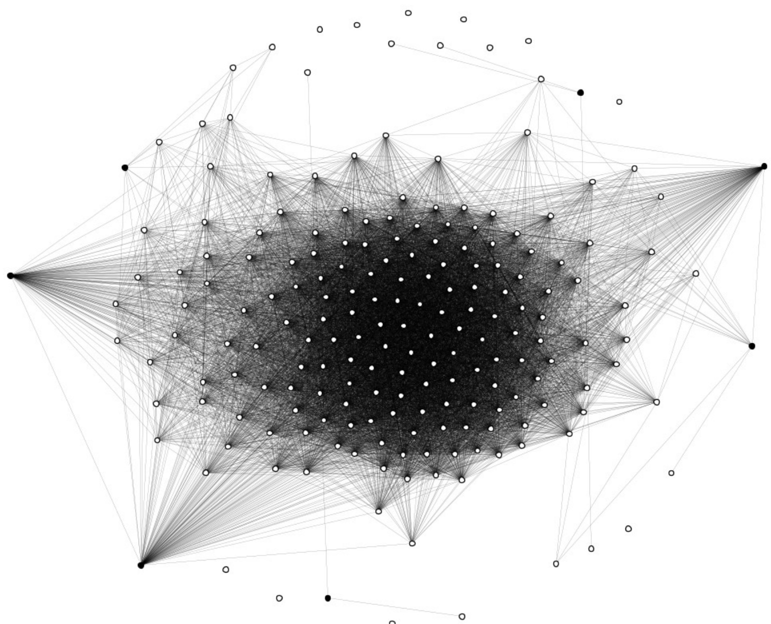

Fig. 1 Network with threshold 0.8

most of the network's nodes are connected, so for the monitoring of the system we only need 7 MDS nodes (the darker nodes). By that, we can claim that each MDS node represents a large number of neighbor (adjacent) nodes, so if an episode of bank distress occurs, the banks belonging to the respective neighbor will be affected. The turmoil of the neighbor will be reflected to its MDS node and that node will immediately raise a red flag. In such a case the central bank should be focused on

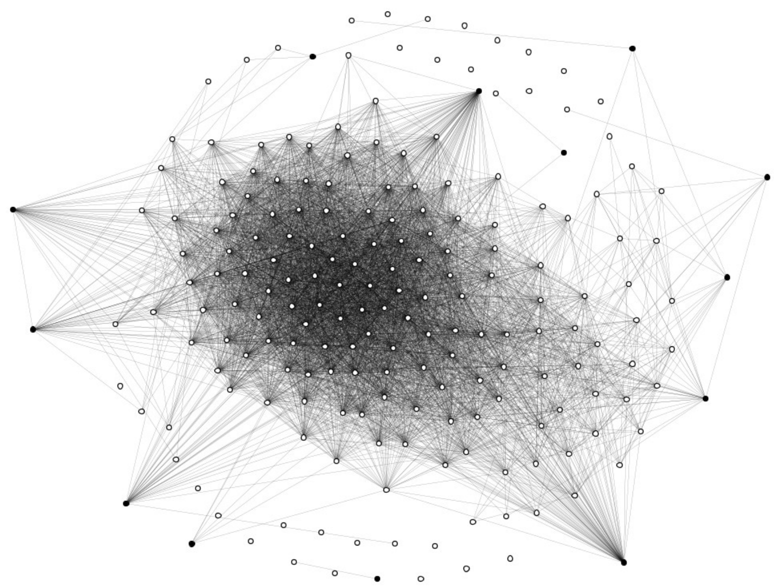

Fig. 2 Network with threshold 0.9

the neighboring (directly linked) MDS nodes of the one that acted as warning light, and implement the necessary policies to prevent the expansion of contagion. The isolated nodes are just 11, so by monitoring 18 banks we have the supervision of the total 200 banking network. In Figure 2, we have applied a higher threshold 0.9 and a sparser network is formed, as fewer nodes belong to the network. The size of the MDS nodes increase, while we have an increasing number of isolated nodes and a decreasing networks density. In this case the overview of the entire network is achieved by monitoring a larger group of banks. The MDS nodes which now are 13 form smaller clusters of adjacent nodes, the isolated nodes are 22, so by monitoring 34 banks we can have the overview of the whole network.

4 Conclusion

Maintaining a stable and viable banking network, ranks first in the to-do list of supervisors. In order to eliminate the expansion of contagion in case of an institution's default, supervisors must be equipped with potent monitoring tools. By using Minimum Dominating Set (MDS) and Density metrics from Graph theory we propose an additional monitoring tool, aimed at improving the existing supervising method. The proposed scheme signals the most strategically important banks (i.e. by monitoring those ones, an overview of the entire system is guaranteed). Furthermore those banks are capable to act as measures of bank distress and timely raise a red flag, for the prevention of an potential imminent banking crisis. When an MDS bank gives a signal of distress, central bank should be focused on its adjacent banks, as those are most prone to collapsing. Considering the foregoing, the proposed methodology can be used as an extra monitoring layer resulting in the improvement of supervision and the minimization of systemic risk. A prompt and targeted intervention to the most exposed institutions is sufficient to deter the expansion of contagion.

References

1. Aguiar, C.L., Soares, J.M.: Business cycle synchronization and the Euro: a wavelet analysis. Journal of Macroeconomics. **33**, 477–489 (2011)
2. Altavilla, C.: Do EMU Members Share the Same Business Cycle?. JCMS: Journal of Common Market Studies. **42**, 869–896 (2004)
3. Allen, F., Gale, D.: Financial contagion. The Journal of Political Economy. **108**, 1-33 (2000)
4. Angelini, P., Maresca, G., Russo, D.: Systemic risk in the netting system. Journal of Banking and Finance. **20**, 853–868 (1996)
5. Blinder, A.S.:How Central Should the Central Bank Be. CEPS. **WP198**, 1–20 (2010)
6. Boss, M., Elsinger, H., Summer, M., Thurner, S.: Network topology of the interbank market. Quantitative Finance. **4**, 677-684 (2004)
7. Boyer, P.C., Ponce, J.: Regulatory capture and Banking supervision reform. Journal of Financial Stability. **8**, 206–217 (2012)
8. Chan-Lau, J.A.: Balance Sheet Network Analysis of Too-Connected-to-Fail Risk in Global and Domestic Banking Systems. IMF. **WP107**, 1–25 (2010)
9. Gai, P., Kapadia, S.: Contagion in financial networks. Proceeding of the Royal Society A: Mathematical, Physical and Engineering Science. **466**, 2401–2423 (2010)
10. Garey, M.R., Johnson G.S.: Computers and Intractability: A Guide to the Theory of NP-Completeness. W.H Freeman and Company. **0716710447**, (1979)
11. Huang, L., Lai, Y.C., Chen G.: Understanding and preventing cascading breakdown in complex clustered networks. Physical Review E. **78**, 36–116 (2008)
12. Inaoka, H., Takayasu, H., Shimizu, T. Ninomiya, T., Taniguchi, K.: Self-similarity of banking network. Physica A. **339**, 621–634 (2004)
13. Iori, G., Masi, G.D., Precup, O.V., Gabbi, G., Caldarelli, G.: A network analysis of the Italian overnight money market. Journal of Economic Dynamics and Control. **32**, 259-278 (2008)
14. Lorenz, J., Battiston, S., Schweitzer, F.: Systemic risk in a unifying framework for cascading processes on networks. European Physical Journal B. **71**, 441-460 (2007)
15. Leitner, Y.: Financial networks: contagion, commitment and private sector bailouts. Journal of Finance. **60**, 925–953 (2005)

16. Minoiu, C., Reyes, J.A.: A networks analysis of global banking: 1978-2009. IMF. **WP74**, 11-41 (2011)
17. Nacher, J.C., Akutsu, T.: Dominating scale-free networks with variable scaling exponent: heterogeneous networks are not difficult to control. New Journal of Physics. **14**, 1–24 (2012)
18. Papadimitriou, T., Gogas, P., Tabak, B.M.: Complex Networks and Banking Systems Supervision. Physica A: Statistical Mechanics and its Applications. **392**, 4429–4434 (2013)
19. Schleich, J., Thi, H., Bouvry, P.: Solving the minimum M-dominating set problem by a continuous optimization approach based on DC programming and DCA. J Comb Optim. **24**, 397–412 (2011)
20. Steen, M.: Graph Theory and Complex Networks. Steen, M., Van. **9081540610**, 1–300 (2010)
21. Tabak, B.M., Takami, M., Rocha, J.M.C., Cajuero, D.O.: Directed clustering coefficient as a measure of systemic risk in complex networks. Working paper of Banco Central do Brazil. **249**, 3–17 (2011)
22. Thurner, S., Hanel, R., Pichler, S.: Risk trading, network topology and banking regulation. Quantitative Finance. **3**, 306–319 (2010)
23. Vives, X.: Central Banks and Supervision. Challenges for Modern Central Banking. Klumer Academic Publishers(USA). 95–113 (2001)