WP 26_09

Ramazan Gençay
Simon Fraser University, Canada
and
The Rimini Centre for Economic Analysis, Italy

Nikola Gradojevic
Lakehead University, Canada

OVERNIGHT INTEREST RATES AND AGGREGATE MARKET EXPECTATIONS

Overnight Interest Rates and Aggregate Market Expectations

Nikola Gradojevic* Ramazan Gençay†

April 2007

Abstract

This paper introduces an entropy approach to measuring market expectations with respect to overnight interest rates in an inter-bank money market. The findings for the Turkish 2000-2001 borrowing crisis suggest that a dynamic, non-extensive entropy framework provides a valuable insight into the degree of aggregate market concerns during the crisis.

Keywords: Non-additive Entropy, Tsallis Entropy, q-Gaussian Distribution.

JEL No: G0; G1; C40

*Faculty of Business Administration, Lakehead University, 955 Oliver Road, Thunder Bay, ON P7B 5E1, Canada. Email: ngradoje@lakeheadu.ca

†Department of Economics, Simon Fraser University, 8888 University Drive, Burnaby, British Columbia, V5A 1S6, Canada. Email: gencay@sfu.ca

1. Introduction

The Turkish financial crisis that peaked in February 2001 is an example of extreme risks in financial markets. The events that led to the crisis began in 1999 with a government reform program that was supposed to curb inflation by fixing the value of the Turkish Lira against a basket of foreign currencies. The structure of the program was such that it left the interest rate to be market determined. In other words, the reform implied that the volatility of interest rates would increase over the course of the program implementation (Gençay and Selçuk, 2006). However, the increase in the (simple annual) overnight interest rate that followed was unprecedented and reached 873 per cent on December 1, 2000, 2,058 per cent on February 20, 2001, and 4,019 per cent on February 21, 2001. The government immediately abandoned the *tablita* program, the Turkish Lira depreciated against the U.S. dollar by 40 per cent in one week, and the interest rate declined to stable levels.

We study the dynamics of market expectations of the overnight interest rate during the crisis using a non-extensive (or Tsallis) entropic measure.[1] We argue that the sentiment of a financial market can be summarized through the aggregation of the subjective expectations of its participants. If the expectations of market participants are highly dispersed and independent, extreme interest rate movements are less likely to occur. If however, market participants have highly dependent and less dispersed expectations, the aggregate market sentiment could drive the interest rate to extraordinary levels. Our approach extracts aggregate market expectations from a past sequence of interest rates via time-dependent Tsallis entropy. By utilizing this particular measure we concentrate on long-range, time-dependent, interactive instability in the market (Gell-Mann and Tsallis, 2004; Martin *et al.*, 2000). Moreover, the Tsallis (or a q-Gaussian) distribution that is estimated by maximizing the Tsallis entropy can capture the frequency of extreme events together with ordinary frequencies satisfactorily (Borland, 2002; Gell-Mann and Tsallis, 2004). The findings show that from an entropic perspective market concerns between December 2000 and February 2001 were particularly strong and that subsequent high interest rate levels were to some extent predictable.

[1] Gell-Mann and Tsallis (2004) provides an excellent survey of this methodology and its interdisciplinary applications. In some other applications, Stutzer (2000) and Stutzer and Kitamura (2002) study option and asset pricing through an entropic methodology.

2. q-Gaussianity and Tsallis entropy

We will motivate our approach of modeling aggregate market expectations through a time-dependent Tsallis entropy by relating how the number of states (or regimes) in a market translate themselves into a probability distribution of the aggregate market sentiment. One well-known entropy is the Shannon information measure (S_S):

$$\begin{aligned} S_S(f(x)) &= \int f(x) \ln \left[\frac{1}{f(x)} \right] dx \\ &= -\int f(x) \ln \left[f(x) \right] dx, \end{aligned} \tag{1}$$

or, in discrete setting S_S is,

$$S_S = -\sum_{i=1}^{n} p_i \ln p_i, \qquad \sum_{i=1}^{n} p_i = 1 \tag{2}$$

where the number of states $i = 1, \ldots, n$, p_i is the probability of outcome i, and n is the number of states. Namely, the entropy is the sum over the product of the probability of outcome (p_i) times the logarithm of the inverse of p_i. This is also called i's surprisal and the entropy of x is the expected value of its outcome's surprisal. It is worthwhile to note that if two states A and B are independent from one another, $p(A \cup B) = p(A)p(B)$, then S_S is additive $S_S(A \cup B) = S_S(A) + S_S(B)$.

Tsallis (1988) entropy (S_q) is a generalization to a non-additive measure

$$S_q(f(x)) = \frac{1 - \int f(x)^q dx}{q - 1} \tag{3}$$

where q is a measure of non-additivity such that $S_q(A \cup B) = S_q(A) + S_q(B) - (1 - q)S_q(A)S_q(B)$. Larger values for q emphasize long-range interactions between regimes (states) and can be interpreted as a long memory parameter. Tsallis entropy recovers the Shannon entropy when $q \to 1$ such that $\lim_{q \to 1} S_q = S_S$.

S_q in its discrete version can be written as

$$S_q = \frac{1 - \sum_{i=1}^{n} {p_i}^q}{q - 1} \tag{4}$$

The maximum entropy principle for Tsallis entropy under the constraints

$$\int f(x)dx = 1, \quad \frac{\int x^2 f(x)^q dx}{\int f(y)^q dy} = \sigma^2 \tag{5}$$

yields[2] the q-Gaussian probability density function

$$f(x) = \frac{\exp_q(-\beta_q x^2)}{\int \exp_q(-\beta_q x^2)dx} \propto [1 + (1-q)(-\beta_q x^2)]^{\frac{1}{1-q}} \tag{6}$$

where β_q is a function of q and $\exp_q(x)$ is the q-exponential function defined by

$$\exp_q(x) = \begin{cases} [1 + (1-q)x]^{\frac{1}{1-q}} & \text{if } 1 + (1-q)x > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

For $q \to 1$, q-Gaussian distribution[3] Equation (6) recovers the usual Gaussian distribution.

As Gaussian distribution is unable to approximate fat tails (or extreme events) that are observed in many high-frequency empirical distributions in finance, we turn our attention to q-Gaussian probability distribution. Borland (2002) shows that q-Gaussian distribution provides a much better fit to the empirical distribution of high-frequency S&P 500 and Nasdaq returns than the log-normal.

Through a moving window approach, the evolution of S_q for the overnight interest rate is calculated over time.[4] The calculation of a time-dependent entropy is influenced by the following considerations (Thakor and Tong, 2004):

1. *Number of states.* With too few states, one may not be able to characterize the underlying market sentiment reliably, and with too many states, tracking fine changes becomes difficult. Without loss of generality, we set n=10.

2. *Partitioning method.* There are two different methods for partitioning the range of a time series: (a) fixed partitioning (equipartition is performed on all available data) and (b) adaptive partitioning (equipartition is performed on each moving-window of data, i.e., it changes over time). The adaptive partitioning approach can track transient changes better than the fixed partitioning and is more suitable for our application.

[2] See, for instance, Suyari (2006).
[3] For a zero mean process with unitary variance $\beta_q = 1/(5-3q)$ with $q < 5/3$.
[4] See Gamero *et al.* (1997) or Tong *et al.* (2002) for more information.

3. *Estimation of q.* The entropic index q is the degree of long-memory in the data. Gell-Mann and Tsallis (2004) estimate $q \approx 1.4$ for high-frequency financial data (returns and volumes) and stress that as the data frequency decreases, q approaches unity. Larger q values ($1 < q \leq 2$) emphasize highly volatile activities in the signal when a time-dependent entropy is plotted against time, i.e., the entropy is more sensitive to possible disturbances in the probability distribution function. In this paper, we find the optimal q by minimizing the sum of the squared errors of the logarithms of the q-Gaussian probability density and the data-implied empirical density.

4. *Sliding step (Δ) and moving window size (K).* The sliding step (the number of observations by which the moving window is shifted forward across time) and moving window size (the number of observations used in calculating the entropy) determine the time resolution of S_q. If the focus is on tracking the local changes, the sliding step is set to be very small (e.g., one observation: Δ=1). Non-overlapping windows ($\Delta \geq K$) are useful only when one is interested in monitoring the general trend of a time series. To get a reliable probability distribution function, K should not be too small. We set Δ=1 and K=200.

3. Results

The data are the daily overnight interest rates (simple annual percent) from January 2, 1990 to November 30, 2001 (2,999 observations). They are calculated by the Central Bank of Turkey as a weighted average of intraday transactions in the interbank money market. Excess skewness and kurtosis are observed for both the sample period before the crisis and the full sample. Noteworthy, fat-tailness and skewness substantially increased during the crisis.[5]

First, we estimate the long memory parameter ($\hat{q}$) for the sample period before the crisis and find that $\hat{q}$=2.66 (0.02).[6] Using the optimal q, next, we investigate the evolution of the time-dependent Tsallis entropy based on overnight interest rates.

Table 1 presents how the probabilities of the states are distributed before and during the crisis. Initially, in February, 2000, the probabilities of the states were roughly evenly distributed and that continued until early November, 2000. Consequently, the entropy level was stable which implies heterogeneity of the expectations of market participants. A sudden change in the distribution of probabilities (concentration increases towards states $s_1 - s_4$) is

[5] For more information about the data see Gençay and Selçuk (2006).

[6] The number in parentheses is the bootstrap standard error. We use one leave-out bootstrap with replacement for a window size of K=200 observations.

apparent when the entropy declined from November 29, 2000 to December 1, 2000, while the interest rate increased from 160.78 to 873.13. Figure 1 also shows a major dip in the entropy on December 1, 2000. This indicates the lack of aggregate expectations heterogeneity and concerns that the interest rate is too high.

[Insert Table 1 about here]

[Insert Figure 1 about here]

The most striking result is the behavior of the entropy following December 1, 2000 when there were concerns about the Treasury's ability to borrow from the domestic market at favorable terms. These concerns were not directly reflected in the interest rate that was relatively stable during this period. However, after the entropy had bottomed out, it did not gradually recover, but, on December 26, 2000, hit a *ceiling* at 0.126 with the probabilities still concentrated in states $s_1 - s_4$. At this value, the entropy remained stable until February 20, 2001, when the interest rate reached 2,000 per cent (the entropy then decreased to 0.039).

Therefore, our approach extracts very useful information regarding the extreme expectations and the entropy provides a powerful platform to measure and predict the concentration of these expectations in a particular direction. In the end, it is important to stress that our findings are not sensitive to various reasonable choices for the size of the moving window K (100-1,000 days) and the number of states n (7-10).

4. Conclusions

By monitoring a time-dependent Tsallis entropy, one can gain insight into the evolution of the aggregate market expectations and obtain an early indication of upcoming crises. We extract the entropy from the daily overnight interest rates before, during and after Turkish financial crisis in February, 2001. The results suggest that the entropy performs well in tracking aggregate market expectations as reflected in interest rates and, thus, complement the work by Gençay and Selçuk (2006) who also found that an interest rate crisis of large magnitude was to be expected. This innovative approach could benefit both investors and policy makers that in this case failed to anticipate such extreme events.

Date	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	TE
Feb 2, 2000	0	0.04	0.03	0.01	0.01	0.01	0.05	0.44	0.17	0.24	0.513
Mar 2, 2000	0	0.06	0.03	0.02	0.05	0.02	0.06	0.45	0.17	0.14	0.521
Apr 3, 2000	0	0.07	0.08	0.05	0.06	0.02	0.06	0.45	0.17	0.04	0.521
May 2, 2000	0.01	0.08	0.12	0.09	0.07	0.02	0.05	0.44	0.12	0	0.528
Jun 2, 2000	0.01	0.08	0.16	0.12	0.10	0.02	0.05	0.42	0.04	0	0.535
Jul 3, 2000	0.01	0.09	0.21	0.13	0.11	0.04	0.03	0.38	0	0	0.543
Aug 2, 2000	0.02	0.08	0.20	0.19	0.13	0.05	0.04	0.03	0.26	0	0.566
Sep 1, 2000	0.02	0.16	0.26	0.21	0.12	0.05	0.02	0.16	0	0	0.565
Oct 2, 2000	0.02	0.17	0.29	0.24	0.13	0.06	0.02	0.06	0.01	0	0.559
Nov 2, 2000	0.02	0.16	0.32	0.26	0.13	0.06	0.03	0.02	0.01	0	0.550
Nov 29, 2000	0.20	0.59	0.13	0.06	0.01	0.01	0	0	0	0	0.446
Nov 30, 2000	0.70	0.25	0.03	0.01	0.01	0	0	0	0	0	0.354
Dec 1, 2000	0.97	0.02	0	0.01	0	0	0	0	0	0	0.046
Dec 26, 00 - Feb 19, 01	0.92	0.07	0	0.01	0	0	0	0	0	0	0.126
Feb 20, 2001	0.98	0.01	0	0.01	0	0	0	0	0	0	0.039
Feb 21, 2001	0.98	0.01	0.01	0	0	0	0	0	0	0	0.031

Table 1: DISTRIBUTION OF AGGREGATE MARKET EXPECTATIONS - OVERNIGHT INTEREST RATES

The time-dependent Tsallis entropy (TE) is calculated with a moving window of 200 days for the overnight interest rate. s_1,...,s_{10} denote non-overlapping intervals (states). The lower boundary of s_1 is the minimum of the moving window. Accordingly, the upper boundary of s_{10} is the maximum of the moving window. Aggregate expectation probabilities (p_i) are calculated from the ratio between the number of observations in each interval and the total number of observations in the moving window. The maximum entropy (expectations heterogeneity) corresponds to equal probability of 10% for each state. The minimum entropy (expectations homogeneity) occurs when all observations concentrate in one particular state such that one state receives 100% of the probability. In this particular case above, distribution of expectations is more evenly distributed until November 29, 2000 which becomes more concentrated towards December 1, 2000 in states $s_1, \ldots, s_4$. The increased concentration leads to a reduction in the entropy. The entropy is bounded between [0,1.48] for $q = 2.66$.

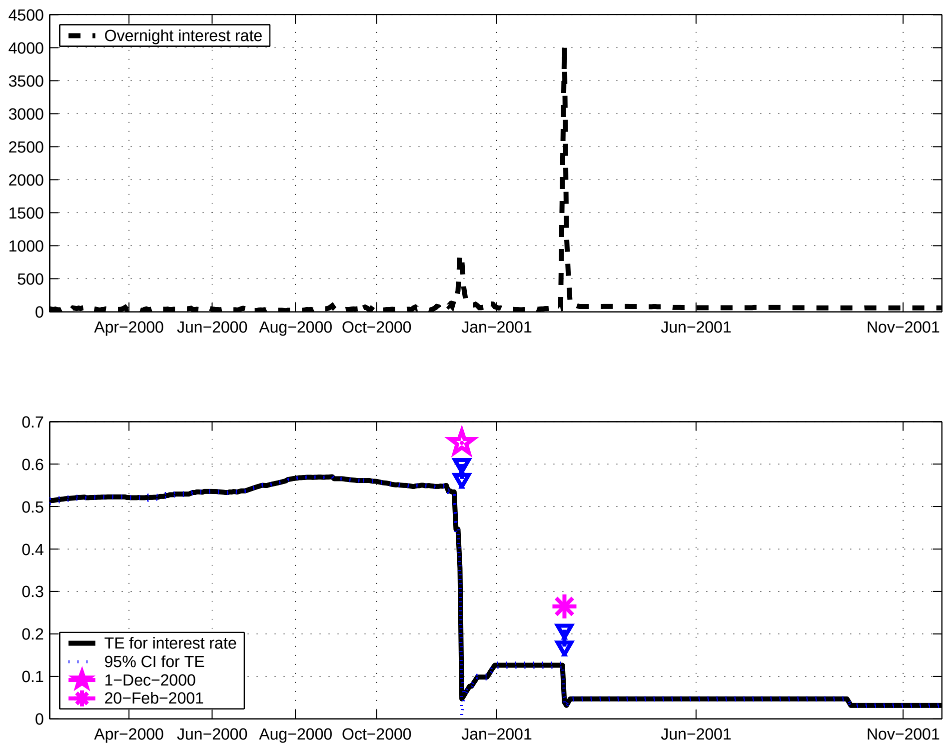

Figure 1: Daily Overnight Interest Rate and Dynamic Entropy

Tsallis entropy (TE) values are plotted on the bottom panel (solid line) and the daily overnight interest rates for February 2000 - November 2001 are given on the top panel (dotted line). The time-dependent, discrete entropy is calculated from the daily overnight interest rate based on the size of the moving window of K=200 days. After the initial drop on December 1, 2000, the entropy stays low and stable until February 20, 2001, indicating strong aggregate market concerns during this period.

References

Borland, L. (2002). A theory of non-gaussian option pricing. *Quantitative Finance*, **2**, 415–431.

Gamero, L. G., Plastino, A., and Torres, M. E. (1997). Wavelet analysis and nonlinear dynamics in a nonextensive setting. *Physica A*, **246**, 487509.

Gell-Mann, M. and Tsallis, C. (2004). *Nonextensive Entropy: Interdisciplinary Applications.* Oxford University Press, Oxford.

Gençay, R. and Selçuk, F. (2006). Overnight borrowing, interest rates and extreme value theory. *European Economic Review*, **50**, 547563.

Martin, M. T., Plastino, A. R., and Plastino, A. (2000). Tsallis-like information measures and the analysis of complex signals. *Physica A*, **275**(1), 262271.

Stutzer, M. (2000). Simple entropic derivation of a generalized black-scholes option pricing model. *Entropy*, **2**, 70–77.

Stutzer, M. and Kitamura, Y. (2002). Connections between entropic and linear projections in asset pricing estimation. *Journal of Econometrics*, **107**, 159–174.

Suyari, H. (2006). The unique self-referential q-canonical distribution and the physical temperature derived from the maximum entropy principle in tsallis statistics. In *Faculty of Engineering, Chiba, University Working Paper.*

Thakor, N. V. and Tong, S. (2004). Advances in quantitative electroencephalogram analysis methods. *Annual Review of Biomedical Engineering*, **6**, 453495.

Tong, S., Bezerianos, A., Paul, J., Zhu, Y., and Thakor, N. V. (2002). Nonextensive entropy measure of eeg following brain injury from cardiac arrest. *Physica A*, **305**, 619628.

Tsallis, C. (1988). Possible generalization of boltzmann-gibbs statistics. *Journal of Statistical Physics*, **52**, 479–487.