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# Exploration for Nonrenewable Resources in a Dynamic Oligopoly: An Arrovian Result\*

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## Abstract

The model proposed in this paper investigates a differential Cournot oligopoly game with nonrenewable resource exploitation, in which each firm may exploit either its own private pool or a common pool jointly with the rivals. Firms use a deterministic technology to invest in exploration activities. There emerges that (i) the individual exploration effort is higher when each firm has exclusive rights on a pool of its own, and (ii) depending on whether each firm has access to its own pool or all firms exploit a common one, the aggregate exploration effort is either increasing or constant in the number of firms.

**JEL Codes:** C73, L13, Q30

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# 1 Introduction

The point of departure of the literature on natural resource economics is that unregulated profit-seeking firms will not, in general, spontaneously internalise the consequences of their behaviour on the environment, much the same as we know about polluting emissions. This raises the issue of extinction/exhaustion.

Indeed, a large part of the debate on nonrenewables includes exploration into the picture, with and without uncertainty affecting the exploration process (see Peterson, 1978; Deshmuk and Pliska, 1980; Arrow and Chang, 1982; Mohr, 1988; Quyen, 1988, 1991; and Cairns and Quyen, 1998). In the early literature on this matter, exploration is motivated by two reasons. The first is the incentive to obtain information about the uncertain size or features (e.g., quality) of the resource stock; the second is the incentive to abate extraction costs through exploration, in situations where the level of such costs is inversely related with the size of proven reserves. In Mohr (1988), where uncertainty is assumed away and the extraction cost is nil, the investments in expensive exploration activities are driven by an incentive to preempt rivals by appropriating some portion of a still unexplored common pool resource, thereby ensuring exclusive property rights on this portion and preventing its use by other firms.<sup>1</sup>

A recent contribution (Boyce and Vojtassak, 2008), adds costly exploration under uncertainty into the framework known as the *theory of 'oil'igopoly* dating back to Salant (1976), and further developed by Loury (1986) and Polasky (1992, 1996). This approach investigates the relationship between known reserves, production and exploration incentives. It predicts that firms holding larger proved reserves will tend to produce outputs which are larger in absolute size but smaller as a proportion of their reserves as compared

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<sup>1</sup>For an overview of the literature, see Lambertini (2013).

to rivals with smaller proved reserves. Adding exploration activities to the model, Boyce and Vojtassak (2008) conclude that firms whose proved reserves gets exhausted before their owners are able to convert their unproved reserves into proved ones have a strict incentive to overinvest in exploration activities, a feature which should be easily observed, and this empirical prediction is consistent with available data concerning the last decades.

Another approach deals with the search for the so-called *backstop technologies*, i.e., substitute technologies using renewable (and possibly, but not necessarily, green) resources replacing the traditional ones. The incentives of firms or countries to devote R&D efforts in search substitutes of an exhaustible resources have also been investigated by a number of authors (Davidson, 1978; Hoel, 1978; Dasgupta, Gilbert and Stiglitz, 1983; Olsen, 1988; Harris and Vickers, 1995). Under perfect certainty (i.e., if the nature of the innovation is known *a priori*) and the date of its discovery is deterministic), importers may strategically affect the extraction path of the exporters by manipulating the timing of innovation.<sup>2</sup>

My aim here is to set up a model describing a differential game taking place in a Cournot industry, in which firms exploit a nonrenewable resource and may invest in costly exploration activities to enlarge the stock of the resource. This is modelled under two alternative assumptions, either allowing each firm exclusive rights on the exploitation of a private pool, or compelling all firms to extract the resource from a common pool. Under both assumptions alike, there emerges a multiplicity of steady state equilibria, among which one is the ‘myopic’ outcome leading to depletion, while another shows the existence of spontaneous incentives to invest in exploration activities even in absence of a dedicated regulatory policy. In this respect, the present analysis offers results largely differing from those typically emerging in dynamic

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<sup>2</sup>On this, see, in particular, the debate between Dasgupta, Gilbert and Stiglitz (1983), Gallini, Lewis and Ware (1983) and Olsen (1988).

market models dealing with polluting emissions (see, e.g., Benchekroun and Long, 1998, 2002).

Comparing the firms' efforts in the two alternative scenarios where either private pools are exploited or a single pool exists, there obtains - not surprisingly - that the public nature of the common pool curtails firms' incentives to invest in exploration as compared to the situation in which the benefits of private access can be fully appropriated by each firm, a classical free-riding phenomenon consistently characterising the issue of privately financing the supply of a public good.<sup>3</sup>

An additional implication of the ensuing analysis is close in spirit to one of the backbones of the literature on R&D incentives, related with the consequences of market structure (or the intensity of competition) and the innovation incentives at the industry level, dating back to the debate between Schumpeter (1942) and Arrow (1962). In this respects, the model investigated here conveys the message that aggregate exploration efforts are non-decreasing in the number of firms operating in the industry. Although preliminary, as the setting proposed here is admittedly far from general, this conclusion clearly speaks in favour of the Arrovian view. Additionally, the fact that granting exclusive access to a private pool by each firm triggers a volume of aggregate investments increasing in the number of firms has a clearcut policy implication, since it suggests that partitioning a resource pool among a large population of firms is conducive to resource expansion and preservation.

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<sup>3</sup>See Fershtman and Nitzan (1991), Xie (1997), Karp and Lee (2003) and Cellini and Lambertini (2007), among others.

## 2 The model

Examine an industry consisting of  $n$  single-product firms exploiting a non-renewable resource over continuous time  $t \in [0, \infty)$  to produce a final good which is differentiated as in Singh and Vives (1984), so that each firm  $i$  faces the instantaneous demand function

$$p_i(t) = a - q_i(t) - sQ_{-i}(t), \quad (1)$$

where  $Q_{-i}(t) = \sum_{j \neq i} q_j(t)$  is the collective output of rivals,  $a > 0$  is the reservation (or choke) price and  $s \in [0, 1]$  measures the degree of substitutability between any pair of varieties. At any time during the game, the individual cost function is

$$C_i(t) = cq_i^2(t) + \beta k_i^2(t), \quad (2)$$

the first term accounting for extraction and production costs, the second for exploration costs,  $k_i(t) \geq 0$  being the instantaneous exploration effort.<sup>4</sup> Function (2) says that all of the firm's activities take place at decreasing returns to scale.

Accordingly, firm  $i$ 's profit function is

$$\pi_i(t) = [p_i(t) - cq_i(t)]q_i(t) - \beta k_i^2(t). \quad (3)$$

In the remainder, I will investigate the two alternative scenarios in which firms either hold private rights on  $n$  single pools, one for each firm, or jointly (but noncooperatively) extract the resource from a single common pool.

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<sup>4</sup>To this regard, it is worth noting that the exploration and extraction costs in (2) are independent of the resource stock. This modelling choice is admittedly made to simplify the analysis, but observe that using an alternative cost function such as, e.g.,  $C_i(t) = cq_i^2(t) + \beta k_i^2(t)/x_i(t)$  or  $C_i(t) = cq_i^2(t)/x_i(t) + \beta k_i^2(t)$ , would intensify the incentive for the single firm to increase its exploration effort to expand its pool, thus reinforcing the main properties of the equilibrium behaviour which I am about to characterise.

## 2.1 Private pools

Suppose each firm is allowed to explore its own drilling ground (as, e.g., in Colombo and Labrecciosa, 2013) in order to expand it. The volume of resource existing at any  $t$  is  $x_i(t)$ , and its dynamics is described by the following state equation:

$$\dot{x}_i(t) = vx_i(t)k_i(t) - q_i(t) \quad (4)$$

where  $v > 0$  is a constant common to all firms. It is worth noting that (4) establishes that exploration is effective insofar as the stock has not been altogether exhausted. As far as I know, a resource dynamics like (4) hasn't yet been discussed in the existing literature, and the reason I can put forward to justify its adoption is the following: casual observation suggests that, commonly, exploration (or additional 'drilling') activities are carried out in locations where the resource is either already being extracted or it is known to exist (on the basis of the geological features and genesis of the drilling ground, for instance); hence, I am explicitly disregarding alternative scenarios in which the resource is discovered thanks to serendipity in places where it is not expected to be found.<sup>5</sup>

The dynamic optimization problem of firm  $i$  consists in

$$\max_{q_i(t), k_i(t)} \int_0^{\infty} \pi_i(t) e^{-\rho t} dt \quad (5)$$

s.t. the set of state dynamics (4) and initial conditions  $x_{i0} = x_i(0) > 0$  for all  $i = 1, 2, 3, \dots, n$ . In (5),  $\rho > 0$  is the constant discount rate, common to all firms.

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<sup>5</sup>This possibility - admittedly important in itself - would however require the insertion of stochastic elements into the model, which would in any case go beyond the scope of the present paper, aimed at prospecting the policy of partitioning a pool across a population of firms so as to magnify their aggregate effort to increase its size.

Accordingly, defining as  $\mathbf{x}$ ,  $\mathbf{q}$  and  $\mathbf{k}$  the vectors of states and controls, firm  $i$ 's current-value Hamiltonian is

$$\mathcal{H}_i(\mathbf{x}, \mathbf{q}, \mathbf{k}) = e^{-\rho t} [(p_i - cq_i(t))q_i(t) - \beta k_i^2(t) + \lambda_i(vx_i(t)k_i(t) - q_i(t))] \quad (6)$$

which the firm has to maximise w.r.t. controls  $q_i(t)$  and  $k_i(t)$ . In (6),  $\lambda_i(t) = \mu_i(t)e^{\rho t}$  is the 'capitalised' costate variable. The solution concept is the open-loop Nash equilibrium.<sup>6</sup>

The necessary conditions are (henceforth, the indication of the time argument is omitted for brevity):

$$\frac{\partial \mathcal{H}_i(\cdot)}{\partial q_i} = a - 2(1+c)q_i - sQ_{-i} - \lambda_i = 0 \quad (7)$$

$$\frac{\partial \mathcal{H}_i(\cdot)}{\partial k_i} = \lambda_i vx_i - 2\beta k_i = 0 \quad (8)$$

$$\dot{\lambda}_i = (\rho - vk_i)\lambda_i \quad (9)$$

and the transversality condition is  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_i x_i = 0$  for all  $i = 1, 2, 3, \dots, n$ . At this point, observe that (9) admits the solution  $\lambda_i = 0$  at all times; this, if substituted back into (8), implies  $k_i = 0$  at all times as well. That is, there exists a solution driving the industry to exploit the natural resource without caring about the ultimate consequence, i.e., its exhaustion in finite time. This depicts a perspective in which firms, as in most of the environmental problems we are accustomed with, do not internalise the consequences of their activities if regulation policies are absent or assumed away.

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<sup>6</sup>The technical reason for this choice is that the model does not take a linear-quadratic form, or any other form for which an obvious candidate for the value function to be used under feedback information is available (see Dockner *et al.* 2000, ch. 7). There are, however, sound economic arguments that can be invoked to corroborate the adoption of open-loop rules in dynamic games of resource extraction (for an exhaustive discussion, see Reinganum and Stokey, 1985; and Mohr, 1988).



This, more often than not, is the only possibility when an appropriate policy is not introduced. However, in the present model there exists an alternative and much more productive route that firms can take spontaneously, and it is the following. From (8), we obtain the expression for the optimal value of the costate variable:

$$\lambda_i = \frac{2\beta k_i}{vx_i} \quad (10)$$

and the control dynamics:

$$\dot{k}_i = \frac{v \left( \dot{\lambda}_i x_i + \lambda_i \dot{x}_i \right)}{2\beta}. \quad (11)$$

Imposing symmetry across states and controls, and using (10), the first order condition (7) rewrites as follows:

$$a - [2(1+c) + s(n-1)]q - \frac{2\beta k}{vx} = 0, \quad (12)$$

which delivers the optimal Cournot-Nash output<sup>7</sup>

$$q^N = \frac{avx - 2\beta k}{v[2(1+c) + s(n-1)]x} \quad (13)$$

at every instant, including the steady state, with no need of deriving the kinematic equation of the individual quantity (i.e., in a quasi-static way).

Then, (4) and (11) become

$$\dot{x} = \frac{v^2 k [2(1+c) + s(n-1)]x^2 - avx + 2\beta k}{v[2(1+c) + s(n-1)]x}; \quad (14)$$

$$\dot{k} = \frac{k [2\beta k - vx(a - \rho(2(1+c) + s(n-1))x)]}{v[2(1+c) + s(n-1)]x^2}. \quad (15)$$

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<sup>7</sup>Henceforth, superscript  $N$  will be used to indicate the Cournot-Nash output level, while starred values will identify the steady state equilibrium magnitudes.

Imposing stationarity on the system (14-15), we obtain the coordinates of the steady state points in the space  $(x, k)$  :

$$x_{PP}^* = \frac{av \pm \sqrt{a^2v^2 - 8\beta\rho^2 [2(1+c) + s(n-1)]}}{2v [2(1+c) + s(n-1)] \rho}; k_{PP}^* = \frac{\rho}{v}. \quad (16)$$

Note that  $a^2v^2 > 8\beta\rho^2 [2(1+c) + s(n-1)]$  is necessary and sufficient for  $x_{\pm}^* \in \mathbb{R}^+$ . In the remainder, I will assume this condition is satisfied.

Before delving into any further analytical details of the game, we may pause to stress that (16) illustrates a striking but quite intuitive difference between the firms' incentives when pollution and natural resources are, alternatively at stake, since profit-seeking agents will obviously tend to internalise the effects of their productive activities if these may ultimately jeopardise their ability to extract surplus from consumers (which is more likely to be the case with natural resources than polluting emissions, all else equal). As a consequence, firms do invest positive amounts of resources in exploration even if - as here - they are not spurred to do so by any public policy.

Back to the model, the solutions in (16) can be studied by linearising the system around the steady states, and examining the following  $2 \times 2$  Jacobian matrix:

$$J = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial k} \\ \frac{\partial \dot{k}}{\partial x} & \frac{\partial \dot{k}}{\partial k} \end{bmatrix} \quad (17)$$

whereby the stability properties of the state-control dynamics (14-15) depend on the sign and size of the trace  $\mathcal{T}(J)$  and determinant  $\Delta(J)$  of the above Jacobian matrix:

$$\mathcal{T}(J) = \frac{2\beta k - v [a - (2(1+c) + s(n-1)) (vk + \rho) x] x}{v [2(1+c) + s(n-1)] x^2} \quad (18)$$

$$\Delta(J) = \frac{k [2\beta (4vk - \rho) - v^2 (2a - \rho (2(1+c) + s(n-1)) x) x]}{v [2(1+c) + s(n-1)] x^2} \quad (19)$$

In correspondence of the ‘smaller’ solution,  $(x_{PP-}^*, k^*)$ , we have  $\mathcal{T}(J)|_{x_{PP-}^*} = \rho > 0$  and the sign of  $\Delta(J)|_{x_{PP-}^*}$  coincides with the sign of

$$\text{sign} \left\{ \Delta(J)|_{x_{PP-}^*} \right\} = \text{sign} \left\{ a^2 v^2 - 8\beta\rho^2 [2(1+c) + s(n-1)] \right\} \quad (20)$$

which is positive. Moreover, it can be shown that  $\Delta(J)|_{x_{PP-}^*} \leq \left( \mathcal{T}(J)|_{x_{PP-}^*} \right)^2 / 4$ . Consequently,  $(x_{PP-}^*, k_{PP}^*)$  is either an unstable node or an unstable focus.

On the contrary, in correspondence of the ‘larger’ solution,  $(x_{PP+}^*, k_{PP}^*)$ , while we have again  $\mathcal{T}(J)|_{x_{PP+}^*} = \rho > 0$ , we see that

$$\text{sign} \left\{ \Delta(J)|_{x_{PP+}^*} \right\} = \text{sign} \left\{ 8\beta\rho^2 [2(1+c) + s(n-1)] - a^2 v^2 \right\} \quad (21)$$

which is negative, qualifying thus  $(x_{PP+}^*, k_{PP}^*)$  as a saddle point. The associated steady state per-firm output (or harvest) is

$$q_{PP}^* = \frac{av + \sqrt{a^2 v^2 - 8\beta\rho^2 [2(1+c) + s(n-1)]}}{2v [2(1+c) + s(n-1)]} = \rho x_{PP+}^*. \quad (22)$$

Finally, for the sake of completeness, it is worth noting that  $\dot{k} = 0$  is also satisfied by  $k = 0$ , in which case, however,  $\dot{x} < 0$  everywhere - which is obvious, as exploitation without exploration leads inevitably to depletion in finite time, as can be verified by substituting  $k = 0$  into (14). By doing so, the derivative of  $x$  w.r.t.  $t$  becomes a constant, whereby the state trajectory is linear in  $t$ :

$$x = x_0 - \frac{at}{2(1+c) + s(n-1)} \quad (23)$$

which is nil at

$$T = \frac{[2(1+c) + s(n-1)] x_0}{a}. \quad (24)$$

As an ancillary remark, it is worth noting that (24) intuitively implies that the terminal (depletion) time is (i) negatively related to the reservation price  $a$  (because industry output increases in  $a$ ) and (ii) positively related to the

number of firms and parameter  $c$ , scaling marginal extraction and production cost (for the opposite reason).

Additionally, in correspondence of  $k = 0$  the determinant of the Jacobian matrix is nil, and therefore we may disregard this special case as it is clearly unstable.

The foregoing discussion can be summarised in the following claim:

**Proposition 1** *The private pool game produces a unique stable steady state equilibrium in  $(x_{PP}^*, k_{PP}^*)$ , which is a saddle point.*

So far, we have dwelt upon the characterization of the equilibrium and its stability properties. However, an interesting qualitative feature of the saddle point is that it has a definite Arrovian flavour. This is self evident, as the optimal steady state investment  $k_{PP}^*$  is independent of industry structure, and therefore, at the aggregate level, the investment in exploration is monotonically increasing in the number of firms:

$$K_{PP}^* = nk_{PP}^* = \frac{n\rho}{v} \Rightarrow \frac{\partial K_{PP}^*}{\partial n} = k_{PP}^* > 0. \quad (25)$$

This very fact proves a key result, stated in the following:

**Corollary 1** *At the steady state, the industry exploration effort is monotonically increasing in the intensity of market competition.*

The above Corollary amounts to saying that fragmentation is conducive to preservation and/or expansion of the resource pool, i.e., a socially desirable outcome which indeed emerges as the endogenous consequence of firms' selfish profit-driven behaviour.

## 2.2 Commons

What if, instead, all firms must exploit a common pool resource, all else equal? The model is unmodified except for a relevant feature, i.e., the pres-

ence of a single state  $x$ , whose dynamic equation is

$$\dot{x} = vx(t) \sum_{i=1}^n k_i(t) - \sum_{i=1}^n q_i(t). \quad (26)$$

Firm  $i$ 's current-value Hamiltonian is

$$\begin{aligned} \mathcal{H}_i(x, \mathbf{q}, \mathbf{k}) = & e^{-\rho t} [(p_i - cq_i(t)) q_i(t) \\ & - \beta k_i^2(t) + \lambda_i (vx(t) (k_i(t) + K_{-i}(t)) - q_i(t) - Q_{-i}(t))], \end{aligned} \quad (27)$$

where  $K_{-i}(t) = \sum_{j \neq i} k_j(t)$  and  $Q_{-i}(t) = \sum_{j \neq i} q_j(t)$ , accompanied by the single initial condition  $x_0 = x(0) > 0$ . This produces the set of necessary conditions:

$$\frac{\partial \mathcal{H}_i(\cdot)}{\partial q_i} = a - 2(1+c)q_i - sQ_{-i}(t) - \lambda_i = 0 \quad (28)$$

$$\frac{\partial \mathcal{H}_i(\cdot)}{\partial k_i} = \lambda_i vx - 2\beta k_i = 0 \quad (29)$$

$$\dot{\lambda}_i = [\rho - v(k_i + K_{-i})] \lambda_i \quad (30)$$

and the transversality condition is  $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_i x_i = 0$  for each firm. Proceeding much the same way as in the previous version of the model, we can verify that

**Proposition 2** *The common pool game produces a unique stable steady state equilibrium in*

$$x_{CP}^* = \frac{anv + \sqrt{a^2 n^2 v^2 - 8\beta \rho^2 [2(1+c) + s(n-1)]}}{2v [2(1+c) + s(n-1)] \rho}; \quad k_{CP}^* = \frac{\rho}{nv},$$

which is a saddle point.

The proof is omitted for the sake of brevity, as it closely replicates the analysis conducted to prove Proposition 1. The associated steady state individual harvest is

$$q_{CP}^* = \frac{anv + \sqrt{a^2 n^2 v^2 - 8\beta \rho^2 [2(1+c) + s(n-1)]}}{2nv [2(1+c) + s(n-1)]} = \frac{\rho x_{CP}^*}{n}. \quad (31)$$

The relevant implication of this result is that, here, the individual equilibrium effort is monotonically decreasing in  $n$ , while the aggregate effort is altogether independent of industry structure; so, the nature of the aggregate industry behaviour is neither Arrovian nor Schumpeterian:

**Corollary 2** *When exploiting a common pool resource, the individual incentive to invest in exploration decreases in the intensity of competition, while the industry effort is independent of industry structure.*

This, of course, is the consequence of free riding, which also reflects itself into the fact that  $nk_{CP}^* = k_{PP}^*$ , i.e.,

**Corollary 3** *The aggregate exploration effort exerted in expanding a common pool is exactly equal to the optimal effort that each firm would exert if it were allowed to exploit a private pool of its own.*

In a nutshell, this Corollary grasps the essence of the underprovision of a public good, as each firm is aware that any effort on its own part to increase the common pool size creates a positive externality in favour of rivals. That is to say, conversely but equivalently, that every firm plays its role in the tragedy of commons by grasping the advantage made available for free by someone else's efforts.

Additionally, comparing (22) and (31), one obtains that  $q_{CP}^* > q_{PP}^*$  over the whole admissible parameter range, entailing a more intense harvesting activity in the case in which firms are exploiting a common pool resource, the interpretation being analogous to that associated with investment efforts.

### 3 Concluding remarks

I have investigated a simple differential game in which Cournot firms, the lack of a resource policy notwithstanding, spontaneously internalise the con-

sequences of profit-seeking behaviour on the residual stock of natural resources, and consequently activate costly exploration projects.

The foregoing analysis has shown that private access to a single pool creates higher incentives to invest in exploration than the joint exploitation of a common pool does. Additionally, the aggregate exploration effort of the industry is either increasing or constant in the number of firms. The former case has a definite Arrovian flavour.

The elements and conclusions of the present work open a few perspectives to be left for future research, among which (i) the analysis of the consequences of Bertrand behaviour and its comparison with Cournot; (ii) the robustness of the above results in presence of other plausible cost functions or resource dynamics, as well as (iii) the construction of a more comprehensive framework in which the natural resource explicitly appears as a factor of production and, possibly as well as desirably, its use also implies a negative environmental externality (as in Lambertini and Leitmann, 2013).

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