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The Incentive to Invest in Environmental-Friendly Technologies: Dynamics Makes a Difference[1]

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Abstract

The established view on oligopolistic competition with environmental externalities has it that, since firms neglect the external effect, their incentive to invest in R&D for pollution abatement is nil unless they are subject to some form of environmental taxation. We take a dynamic approach to this issue, using a simple differential game to show that the conclusion reached by the static literature is not robust, as the introduction of dynamics shows that firms do invest in R&D for environmental-friendly technologies throughout the game, as long as R&D is accompanied by an output restriction exhibiting a distinctively collusive flavour. We also examine the social planning case and the effects of Pigouvian taxation, to show that there exists a feasible tax rate inducing profit-seeking firms to choose a combination of output and R&D such that the resulting social welfare level is the same as in the first best.

Keywords: pollution, environmental externality, R&D, differential games, social planning

JEL Codes: H23, L13, O31, Q55

1 Introduction

The enormous amount of data being assembled by the IPCC (Intergovernmental Panel on Climate Change) on the anthropic responsibility in generating (or at least increasing) global warming, and the debate on how to cope with it along the guidelines of the Kyoto Protocol and its follow-ups, are clearly identifying the control of polluting emissions damaging the environment as one of the hottest scientific issues of our times. As such, it is receiving an increasing amount of attention in the current literature in the field of environmental economics, with particular attention to the general equilibrium implications of environmental aspects on trade and growth.[1]

Most of the existing contributions adopting a partial equilibrium approach investigate the design of optimal Pigouvian taxation aimed at inducing firms to reduce damaging emissions, both in monopoly and oligopoly settings.[2] A related stream of literature examines the incentive for firms to carry out R&D activities in order to introduce environmental-friendly technologies. In static games, this requires the introduction of some form of taxation/subsidy by the policy maker, in order to induce firms to take into account the presence of the externality, that they would clearly neglect otherwise.[3] A third line of research investigates the optimal design of minimum quality standards and/or profit taxation in vertically differentiated industries affected by environmental externalities.[4]

[1]On the optimality of free trade with environmental externalities, see Copeland and Taylor (1994, 2004) and Antweiler *et al.* (2001). As to the role of environmental issues in growth theory, see Grossman and Krueger (1995), Bovenberg and de Mooij (1997), Bartz and Kelly (2008) and Itaya (2008), *inter alia*.

[2]See Karp and Livernois (1994) and Benchekroun and Long (1998, 2002), *inter alia*.

[3]To this regard, see Downing and White (1986), Milliman and Prince (1989), Damania (1996), Scott (1996), Chiou and Hu (2001), Mohr (2002), Hart (2004), Greaker (2006) and Poyago-Theotoky (2007), *inter alia*.

[4]See Lutz *et al.* (2000), Amacher *et al.* (2004), Lombardini-Riipinen (2005), André *et*

In the present paper, we take a differential game approach to the investigation of environmentally-oriented R&D efforts in a dynamic Cournot oligopoly model where (at least in the first version of the game) there may not be any tax or subsidy linked to the external effect, in order to show that the main message emerging from the corresponding static version of the same game falls short of telling the whole story of the issue at hand. In particular, we describe a scenario where the stock of pollution increases in proportion to industry output, and each firm may invest in R&D in order to diminish its individual contribution to the emission of pollutants.

Our first result consists in showing that unregulated firms may indeed fully neglect the environmental effects of their productive activity and replicate the static Cournot-Nash equilibrium forever, without putting any effort whatsoever in R&D activities for cleaner technologies at any point in time. However, we also show that the alternative may in fact be more attractive, if R&D efforts go along with an output contraction closely resembling cartel behaviour, although the setup remains fully non cooperative. That is, we identify a path along which, by taking explicitly into account the externality, firms performs environmental R&D investments not because of some altruistic or environmental concern but for pure profit-seeking reasons.

The game among unregulated firms yields multiple steady state equilibria, all of them (except of course the quasi-static solution replicating the Cournot outcome forever) being characterised by positive R&D efforts at all times, except possibly doomsday. In summary, the appraisal of our analysis of private incentives can be outlined as follows. First, the static game captures the main feature of one of the steady states we identify, but cannot grasp the essence of what happens along the optimal path to this long run equilibrium. Secondly, the remaining two equilibria, both emerging whenever the stock of polluting emissions vanishes, are linked by saddle point trajectories which

al. (2009) and Bottega and De Freitas (2009), *inter alia*.

exit the least preferable point to enter the most desirable one, as far as profit, consumer surplus and social welfare are concerned. This is a desirable property, entirely driven by profit incentives, which in the present case are not in conflict with social preferences.

Then, we examine two modified versions of our setup: in the first one, a social planner concentrates the production of the good in a unique plant, whereby the activity of R&D takes place in N different structures (due to the decreasing returns to scale characterising the R&D technology). In this case, five steady state points exist, one of which replicates the perfectly competitive allocation that would emerge under social planning in the corresponding static version of the model. Yet, a relevant feature of this equilibrium is that the planner would be able to reach it only in the very specific (and totally unrealistic) case where the production of the final good were not polluting the environment at all.

The second extension takes into account the possibility of regulating profit-seeking firms via the introduction of a Pigouvian tax associated to the environmental externality. In this case, we show that the tax can be designed so as to induce the industry to yield the first best level of social welfare that is unattainable under planning, although of course the associated surplus distribution is not the same as it would be at the first best.

The remainder of the paper is structured as follows. Section 2 briefly outlines the static version of the game. The setup of the dynamic problem and the related trajectory analysis are laid out in section 3, where we also compare the profit and welfare performance of the industry in correspondence of the multiple steady state equilibria. In section 4 we examine the behaviour of the model under social planning. In section 5 we illustrate the effects of Pigouvian taxation on the equilibrium behaviour of profit-seeking firms as well as the related welfare levels. Section 6 contains concluding remarks.

2 A summary of the static problem

As a preliminary step, we revisit the static Cournot game in order to highlight the lack of R&D incentives to decrease the amount of polluting emissions characterising firms. The market is supplied by N single-product homogeneous-good firms. The market demand function is $p = a - Q$, with $Q = \sum_{i=1}^{N} q_i$, q_i being firm i's output. Technology is the same for all firms alike, and it is summarised by the cost function $C = cq_i$. Supplying the final good entails a negative environmental externality $S = \sum_{i=1}^{N} b_i q_i$, where $b_i = \overline{b} - k_i \geq 0$; $\overline{b}$ measures the marginal contribution of each firm to the stock of pollutants; k_i is the R&D effort of firm i to decrease its individual amount of pollution,[5] and it involves a convex cost $\Gamma_i = rk_i^2$, $r > 0$. Consequently, firm i's instantaneous profits are $\pi_i = (p - c)\, q_i - \Gamma_i$. This game has a two-stage structure: in the first stage, firms non-cooperatively and simultaneously set their respective R&D efforts; in the second, they compete *à la* Cournot-Nash. The solution concept is subgame perfection by backward induction.

The optimal individual output in the second stage is $q^* = (a - c) / (N + 1)$, whereby the profit function at the first stage reads as $\pi_i = (q^*)^2 - rk_i^2$. This clearly entails that $\partial \pi_i / \partial k_i < 0$, and therefore the optimal R&D investment is nil, yielding the static Cournot-Nash profits $\pi^{CN} = (q^*)^2$. On this basis, one has to introduce some form of environmental taxation, no matter whether it is firm-specific or not, to induce firms to take into account the presence of the externality and indeed carry out some R&D efforts to reduce it. As we shall see in the following sections, this is not necessarily the case if one adopts a properly dynamic approach to this issue.

[5] Here we assume firm-specific externalities and R&D activities, as it appears to be reasonable in examining investments in environmental-friendly technologies. Hence, we rule out the possibility of spillovers in R&D.

3 The dynamic setup

As in the static model, consider a Cournot oligopoly with N single-product homogeneous-good firms interacting over continuous time $t \in [0, \infty)$. At any time t, the demand function is $p(t) = a - Q(t)$, with $Q(t) = \sum_{i=1}^{N} q_i(t)$, $q_i(t)$ being the instantaneous individual output of firm i. All firms use the same productive technology, described by the cost function $C(t) = cq_i(t)$. The production of the final output involves a negative environmental externality $S(t)$, evolving according to the following dynamics:

$$\dot{S}(t) = \sum_{i=1}^{N} b_i(t) q_i(t) - \delta S(t), \tag{1}$$

where $\delta > 0$ is a constant decay rate and $S(0) = S_0 > 0$ is the initial condition. The coefficient $b_i(t) \geq 0$, with $b_i(0) = b_{i0} \geq 0$, measures the marginal contribution to the stock of pollution that the production of firm i entails. Depending on the R&D effort $k_i(t)$ of i,, it evolves over time according to the following equation:

$$\dot{b}_i(t) = b_i(t)[\eta - k_i(t)], \ \eta > 0. \tag{2}$$

That is, until k_i is smaller than the threshold value η, b_i is increasing. As in the static game, the instantaneous cost associated with the R&D activity is $\Gamma_i(t) = rk_i^2(t)$, with $r > 0$. Hence, firm i's instantaneous profits are $\pi_i(t) = [p(t) - c] q_i(t) - \Gamma_i(t)$, and each firm i has to set $q_i(t)$ and $k_i(t)$ so as to maximise

$$\Pi_i = \int_0^{\infty} \{[p(t) - c] q_i(t) - \Gamma_i(t)\} e^{-\rho t} dt, \tag{3}$$

under the state equations (1) and (2) and the initial conditions. Parameter $\rho > 0$ is a constant discount rate common to all firms.

3.1 Equilibrium analysis of the game

The solution concept is the open-loop Nash equilibrium. The current-value Hamiltonian of firm i is:

$$\mathcal{H}_i(\cdot) = \left\{ [p(t) - c]\, q_i(t) - \Gamma_i(t) + \lambda_i \dot{S}(t) + \mu_{ii} \dot{b}_i(t) + \sum_{j \neq i} \mu_{ij} \dot{b}_j(t) \right\}, \quad (4)$$

with the following necessary conditions (FOCs):

$$\frac{\partial \mathcal{H}_i}{\partial q_i} = \sigma - 2q_i(t) - Q_{-i}(t) + \lambda_i(t)\, b_i(t) = 0 \quad (5)$$

where $Q_{-i}(t) \equiv \sum_{j \neq i} q_j(t)$ and $\sigma \equiv a - c$;

$$\frac{\partial \mathcal{H}_i}{\partial k_i} = -2rk_i(t) - \mu_{ii}(t)\, b_i(t) = 0, \quad (6)$$

Notice that $\mu_{ij}(t)$ does not appear in the FOCs. The adjoint equations read as follows:

$$\dot{\lambda}_i(t) = (\rho + \delta)\, \lambda_i(t) \quad (7)$$

$$\dot{\mu}_{ii}(t) = [\rho - \eta + k_i(t)]\, \mu_{ii}(t) - \lambda_i(t)\, q_i(t) \quad (8)$$

$$\dot{\mu}_{ij}(t) = [\rho - \eta + k_j(t)]\, \mu_{ij}(t) - \lambda_i(t)\, q_j(t). \quad (9)$$

From (5) and (6) one obtains, respectively:

$$\lambda_i(t) = -\frac{\sigma - 2q_i(t) - Q_{-i}(t)}{b_i(t)} \quad (10)$$

$$\mu_{ii}(t) = -\frac{2rk_i(t)}{b_i(t)}. \quad (11)$$

The associated transversality conditions are:

$$\begin{aligned} \lim_{t\to\infty} e^{-\rho t} \lambda_i(t)\, S(t) &= 0; \\ \lim_{t\to\infty} e^{-\rho t} \mu_{ii}(t)\, b_i(t) &= 0; \\ \lim_{t\to\infty} e^{-\rho t} \mu_{ij}(t)\, b_j(t) &= 0. \end{aligned} \quad (12)$$

Before carrying out the equilibrium analysis, it is worth dwelling upon the interpretation of the above necessary conditions. First of all, note that (7) admits the solution $\lambda_i(t) = 0$ at all times, which in turn allows $\mu_{ii}(t) = 0$ to be a solution to (8). In such a case, the dynamic model would immediately reproduce the very same outcome of the static game, with no investments at all at any time and the static Cournot-Nash equilibrium replicated at all t:

Proposition 1 *Adjoint equations admit the solution* $\lambda_i(t) = \mu_{ii}(t) = 0$ *at all* $t \in [0, \infty)$. *This entails* $q_i(t) = \sigma/(N+1)$ *and* $k_i(t) = 0$ *for all* $i = 1, 2, 3, ...N$ *at all* $t \in [0, \infty)$.

However, if the R&D control is always nil and the output control is always equal to the static Cournot-Nash solution, the level of pollution would explode to plus infinity unless $b_i(0) = b_{i0} = 0$, i.e., unless the polluting features of productive technology are not an issue because technology itself is already clean at the very outset (which of course makes the entire story a trivial one). From a technical standpoint, this is equivalent to saying that transversality condition would be violated.

Additionally, adjoint equations (7-8) also admits non-nil solutions which, by definition, do not appear in the static version of the game. This has some interesting implications as to the firms' incentive to invest in environmental-friendly technologies. To shed light on this aspect, we may propose the following observations.

Equation (5) produces firm i's instantaneous best reply:

$$q^*(Q_{-i}(t)) = \frac{\sigma - Q_{-i}(t) + \lambda_i(t)\, b_i(t)}{2} \tag{13}$$

which shifts inwards (resp. outwards) w.r.t. its static counterpart for all $\lambda_i(t) < 0$ (resp., $\lambda_i(t) > 0$). Equivalently, (10) takes a negative value for all $Q(t) < N\sigma/(N+1)$, i.e., whenever the industry output is lower than its static Cournot-Nash level (and conversely). Now, if $\lambda_i(t) < 0$, the inward shift of best reply functions entails a quasi-collusive behaviour on the part of firms, via an output contraction that, nonetheless, is driven by a fully non cooperative behaviour. Also, note that (8) yields $\mu_{ii}(t) < 0$ for all $k_i(t) > 0$. The fact that adjoint variables are negative indicates that firm i attaches a negative shadow value to its marginal contribution to the increase in the pollution stock. Yet, the output contraction opens the possibility that the firm increases its profits instant by instant, even if a costly R&D project for a greener technology is undertaken.[6] That is, the incentive to adopt the investment strategy associated with $\lambda_i(t) < 0$ is highlighted by the flow of instantaneous gains exemplified by:

$$\pi^{CN}(k=0) \equiv \frac{\sigma^2}{(N+1)^2} < (\widehat{p}-c)\,\widehat{q} - rk^2 \equiv \widehat{\pi}(k>0) \tag{14}$$

for non-empty sets of values of $k > 0$ and $\widehat{q} \in \left(0, \dfrac{\sigma}{N+1}\right)$. During the game, firm i may smooth the R&D investment not because she has developed any environmentally-oriented conscience of her own, but rather in order to be able to keep the output at a quasi-collusive level forever. In other words, from the firms' viewpoint, the R&D cost $\Gamma_i(t)$ is the fee to be paid to build up a path replicating that of a cartel in quantities, without actually taking any implicitly collusive attitude that would constitute a target for the antitrust authority. Conversely, from consumers' viewpoint, a higher market price is what they have to pay in return for a cleaner environment.

Having said that, we may proceed to the characterisation of the equilibrium behaviour. One can impose symmetry across quantities, costate variables and states:

$$q_i(t) = q_j(t) = q(t), \quad \lambda_i(t) = \lambda_j(t) = \lambda(t), \tag{15}$$

[6]Using a repeated game with infinite Nash reversion, Damania (1996) finds that firms may not be willing to buy pollution-abating technologies if the associated exogenous cost is too high.

$$\mu_{ii}(t) = \mu_{jj}(t) = \mu(t), \quad b_i(t) = b_j(t) = b(t) \tag{16}$$

and drop the time argument for brevity. From the FOCs (5) and (6) one also obtains the control equations:

$$\dot{q} = \frac{\lambda\dot{b} + \dot{\lambda}b}{N+1}, \qquad \dot{k} = -\frac{\mu\dot{b} + \dot{\mu}b}{2r} \tag{17}$$

which can be rewritten, using (7-8) and (10-11), leading to the following state-control dynamical system:

$$\dot{S}(t) = Nb(t)q(t) - \delta S(t) \tag{18}$$

$$\dot{b}(t) = b(t)(\eta - k(t)) \tag{19}$$

$$\dot{q}(t) = \frac{[(N+1)q(t) - \sigma][\rho + \delta + \eta - k(t)]}{N+1} \tag{20}$$

$$\dot{k}(t) = \rho k(t) - \frac{q(t)[\sigma - (N+1)q(t)]}{2r} \tag{21}$$

Although the equations (18-19) and (20-21) are not decoupled, we can stress that, given any solution curve $(\overline{q}(t), \overline{k}(t))$ of equations (20-21), we can obtain the state trajectories by applying the methods of separation of variables and Lagrange's variation of constants to (18-19):

$$b(t) = b_0 e^{\eta t - \int_0^t \overline{k}(s)ds}, \tag{22}$$

$$S(t) = \left(S_0 + b_0 \int_0^t \left(e^{(\eta+\delta)s - \int_0^s \overline{k}(\tau)d\tau}\right)\overline{q}(s)ds\right)e^{-\delta t}. \tag{23}$$

Which implies that both $b_i(t)$ and $S(t) > 0$ are non negative at all times (except, possibly, doomsday in which they are nil) Before inspecting the stationary points of the above dynamic system, it is worth observing that, using the time elimination method, we can write the derivative

$$\frac{\dot{q}}{\dot{k}} = \frac{2r[(N+1)q(t) - \sigma][\rho + \delta + \eta - k(t)]}{(N+1)[2r\rho k(t) - q(t)(\sigma - (N+1)q(t))]} = \frac{dq}{dk} \tag{24}$$

indicating the slope of the open-loop Nash trajectory in the control plane.

The sign of (24) is evaluated in

Remark 2 *Take* $q(t) \in \left(0, \frac{\sigma}{N+1}\right)$. *Then,* $dq/dk < 0$ *for all*

$$\begin{aligned} k(t) &\in \left(\min\left\{ \rho+\delta+\eta, \frac{q(t)\left[\sigma-(N+1)\,q(t)\right]}{2r\rho} \right\}, \right. \\ &\left. \max\left\{ \rho+\delta+\eta, \frac{q(t)\left[\sigma-(N+1)\,q(t)\right]}{2r\rho} \right\} \right). \end{aligned}$$

That is to say, for any individual output level lower than the Cournot-Nash output, there exists an admissible range of values for $k(t)$ wherein the two controls are substitutes at a generic point in time, during the game. In such a case, any output contraction with respect to the Cournot-Nash static equilibrium drives some R&D effort for cleaner technologies.

Steady state equilibria are described by the following:

Proposition 3 *The stationary points of the system are:*

$$\begin{aligned} P_A &= (S_A,\ b_A,\ q_A, k_A) = \left(0, 0, \frac{\sigma}{N+1}\ 0\right), \\ P_B &= (S_B,\ b_B,\ q_B, k_B) = (0, 0,\ q_B,\ \delta+\rho+\eta), \\ P_C &= (S_C,\ b_C,\ q_C, k_C) = (0, 0,\ q_C,\ \delta+\rho+\eta), \end{aligned}$$

where

$$\begin{aligned} q_B &= \frac{\sigma - \sqrt{\sigma^2 - 8r(N+1)(\rho+\delta+\eta)\rho}}{2(N+1)}, \\ q_C &= \frac{\sigma + \sqrt{\sigma^2 - 8r(N+1)(\rho+\delta+\eta)\rho}}{2(N+1)}. \end{aligned}$$

Proof. Imposing the stationarity condition $\dot{k} = 0$ yields

$$k(q) = \frac{q\left[\sigma-(N+1)\,q\right]}{2r\rho} \tag{25}$$

which can be plugged into $\dot{q} = 0$ to obtain the following solutions:

$$q_A = \frac{\sigma}{N+1}; q_{B,C} = \frac{\sigma \pm \sqrt{\sigma^2 - 8r(N+1)(\rho+\delta+\eta)\rho}}{2(N+1)} \tag{26}$$

with $q_{B,C} \in \mathbb{R}_+$ for $\sigma > \sqrt{8r(N+1)(\rho+\delta+\eta)\rho}$. By substituting in (25) we have that $k_{B,C} = \delta + \rho + \eta$.

In correspondence of the Cournot-Nash optimal quantity q_A, we have $k_A = 0.$, $S_A = 0$, $b_A = 0$. ■

The following results show the dynamic behaviour of the optimal solutions:

Proposition 4 *P_A, P_B and P_C are saddle points of the system.*

Proof. The Jacobian matrix of the state-control system reads as:

$$J = \begin{pmatrix} -\delta & Nq & Nb & 0 \\ 0 & \eta - k & 0 & -b \\ 0 & 0 & \rho+\delta+\eta-k & -q + \dfrac{\sigma}{N+1} \\ 0 & 0 & \dfrac{1}{2r}[2(N+1)q - \sigma] & \rho \end{pmatrix}. \tag{27}$$

$J(P_A)$ has the eigenvalues $\lambda_1 = -\delta < 0$, $\lambda_2 = \eta > 0$, $\lambda_3 = \rho+\delta+\eta > 0$ and $\lambda_4 = \rho > 0$, subsequently P_A is a saddle point.

The analysis of the remaining two equilibria is slightly more difficult: both $J(P_B)$ and $J(P_C)$ admit the negative eigenvalues $\lambda_1 = -\delta < 0$ and $\lambda_2 = -\rho - \delta < 0$, so the stability properties of those two points depend on the roots of the characteristic polynomials of the submatrices, for $j = B, C$:

$$\begin{pmatrix} \rho+\delta+\eta-k_j & -q_j + \dfrac{\sigma}{N+1} \\ \dfrac{1}{2r}[2(N+1)q_j - \sigma] & \rho \end{pmatrix}, \tag{28}$$

i.e.

$$p_j(\lambda) = \lambda^2 - \rho\lambda - \frac{1}{2r}\left(-q_j + \frac{\sigma}{N+1}\right)[2(N+1)q_j - \sigma]. \tag{29}$$

If $j = B$, the two remaining eigenvalues are complex with real part $\rho/2 > 0$, whereas if $j = C$, they are real and at least one of them is positive, hence P_B and P_C are saddle points too. ■

As is well known, a saddle point can be reached starting from initial states that can be subject to more or less stringent conditions. In particular:

Remark 5 *The steady state P_A is degenerate, as it can be reached only along an equilibrium trajectory which solves (18-19) for $b_0 = 0$ and for any $S_0 > 0$, i.e., it is completely contained in the half-line determined by the intersection of the hyperspaces $b = 0$, $q = \dfrac{\sigma}{N+1}$, $k = 0$, with the stock of pollution asymptotically decreasing to 0.*

That is, the equilibrium reproducing the Cournot-Nash outcome can be attained iff the technology is already fully environmental-friendly from the outset, which makes this case quite peculiar and somewhat uninteresting. Or, put it in other terms, the requirement on b_0 indicates that the prediction of the static game is far from convincing. Completely different considerations apply to the remaining two steady states, that are attainable for $b_0 > 0$.

Proposition 6 *In the half-space $k > \eta$, along each equilibrium trajectory of the system close to P_B and P_C the state variables S and b are monotonically decreasing to 0.*

Proof. The stationary points P_B and P_C belong to the half-space $k > \eta$. The eigenvectors of $J(P_B)$ and $J(P_C)$ imply that the stable subspaces $E_s(P_B)$ and $E_s(P_C)$ are spanned by the vectors of the canonical basis of $\mathbb{R}^4$: $(1, 0, 0, 0)$ and $(0, 1, 0, 0)$, that is the trajectories on the respective stable manifolds are heading towards the equilibrium coordinates $S = 0$, $b = 0$. ■

The economic meaning of the previous results is clear: in correspondence of the two points P_B and P_C the stock of pollution tends to diminish and finally disappears.

From the standpoint of the dynamical behaviour of the system, in the above-mentioned half-space the Nash trajectories approach P_B in the control plane, spiral around it and then head towards P_C, which is a saddle point in the sense that there exists a phase curve contained in the control plane which enters P_C. As we will see in next subsection, this is good news because in that point higher levels of profit and social welfare can be reached with respect to P_B.

The figures we are going to show in the following are sketched with the help of Mathematic@ 5.0, after fixing suitably the relevant parameters:

$$N = 20, \quad \sigma = 1, \quad \rho = 3 \cdot 10^{-2}, \quad \eta = 10^{-2}, \quad r = 10^{-2}, \quad \delta = 10^{-2}.$$

The first parametric plot represents a phase curve in a hyperspace $S = S_0$, being the horizontal plane the control space and the state b the variable on the vertical axis. Choosing $q(0) = 10^{-2}$, $k(0) = 2 \cdot 10^{-1}$, $b(0) = 2$ as initial conditions for this numerical simulation, we obtain the following sketch of a trajectory:

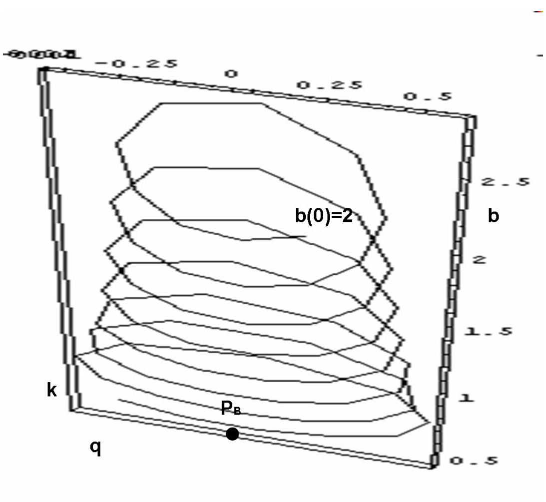

Figure 1. The path spirals down towards the steady state P_B in the control plane as the polluting emissions decrease over time

In the following plot we can visualize the sketches of some equilibrium trajectories on the (k, q) control plane, with the same parameter values as in Figure 1. On such a plane, coherently with the eigenvalues of (28), $P_B = (0.16, 0.08)$ is clearly an unstable focus, whereas $P_C = (0.16, 0.38)$ is a saddle point.

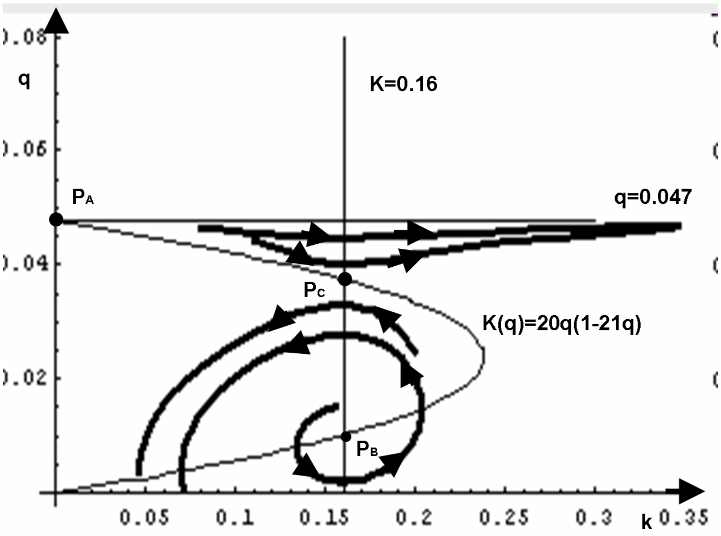

Figure 2. On the control plane, the saddle point trajectories either leave the Cournot-Nash equilibrium P_A or spiral around P_B. The feasibility of P_C is ensured

Moreover, the optimal R&D effort of the representative firm is positive at any time t during the game. Or, put it the other way around, any non-zero value of the co-state variable attached to the dynamics of the individual firm's contribution to the increase of the pollution stock ensures that the firm itself has indeed an incentive to invest in R&D activities for pollution abatement all along the game.

3.2 Profit and welfare assessment

In this section we compare the optimal quantities, the level of profits and of social welfare associated to the three steady states.

Proposition 7 *For every admissible* σ, N, r, ρ, δ, η, *we have* $q_A > q_C > q_B$.

In steady state, the profit levels are the following:

$$\pi(P_A) = \frac{\sigma^2}{(N+1)^2}, \tag{30}$$

$$\pi(P_B) = \sigma q_B - N q_B^2 - r(\rho + \delta + \eta)^2, \tag{31}$$

$$\pi(P_C) = \sigma q_C - N q_C^2 - r(\rho + \delta + \eta)^2. \tag{32}$$

On the basis of (30-32), we can state:

Proposition 8 *The profits* $\pi(P_B)$ *and* $\pi(P_C)$ *are positive if either of the following holds:*

1. $\rho \geq \delta + \eta$;
2. $\rho < \delta + \eta$ *and*
$$2\sqrt{2(N+1)(\delta+\eta-\rho)} < \sigma < [(N+1)(\delta+\eta)+(1-N)\rho]\sqrt{\frac{r(\delta+\eta+\rho)}{\delta+\eta-\rho}}.$$

Assuming that the parameters are such that profits are indeed non negative, we can make a comparison to assess the relative desirability of the three outcomes:

Proposition 9 *The following inequalities hold:*

1. $\pi(P_C) > \pi(P_B)$ *irrespective of parameter values;*
2. $\pi(P_A) > \pi(P_C)$ *if* $\rho \in [0, \delta + \eta)$.

The intuition behind the above result is that P_A is characterised by a larger output level (which, per se, would be detrimental for profits) but the corresponding R&D effort is nil (which in turn is good news for profits), while the remaining two steady states are characterised by lower output levels in combination with positive R&D efforts. In particular, it is noteworthy observing that the Cournot-Nash solution may be worse than the steady state P_C where the firm indeed invests in R&D, despite the fact that pollution does not affect its profits.

Now we turn to consumer surplus $CS(P_i)$, $i = A, B, C$, in the three equilibria. Note that, in principle, the definition of consumer surplus would be $CS(P_i) = Q_i^2/2 - S$; however, $S = 0$ always in steady state. The resulting ranking is summarised in

Proposition 10 *Over the entire admissible range of parameters, we have* $CS(P_A) > CS(P_C) > CS(P_B)$.

Finally, we evaluate social welfare $SW(P_i) = N\pi(P_i) + CS(P_i)$, $i = A, B, C$, to obtain:

Proposition 11 *Over the entire admissible range of parameters, we have* $SW(P_A) > SW(P_C) > SW(P_B)$.

Propositions 7-11 also entail:

Corollary 12 *Any* $\rho \in [0, \delta + \eta)$ *suffices to ensure that private and social preferences over the spectrum of steady state equilibria are reciprocally aligned.*

This essentially relies upon the fact that the industry R&D effort in P_A is nil. Note however that, as we have outlined above, P_A is indeed degenerate.

4 Social planning

We assume that the benevolent planner uses a single plant for the production of the consumption good (in view of the constant returns to scale characterising the related technology), while keeping N R&D labs, as this activity features decreasing returns. Hence, the list of variables reduces to $N+1$ controls and two states, namely, S and b. The Hamiltonian of the planner is:[7]

$$\mathcal{H}_{SP}(\cdot)=\left\{(\sigma-q)\,q+\frac{q^2}{2}-S-Nrk^2+\lambda\,(bq-\delta S)+\mu b\,(\eta-k)\right\} \tag{33}$$

where subscript SP stands for *social planning*. The necessary conditions are:

$$\frac{\partial\mathcal{H}_{SP}}{\partial q}=\sigma-q+\lambda b=0; \tag{34}$$

$$\frac{\partial\mathcal{H}_{SP}}{\partial k}=-N\,(\mu b+2rk)=0; \tag{35}$$

$$-\frac{\partial\mathcal{H}_{SP}}{\partial S}=\dot{\lambda}-\rho\lambda\Leftrightarrow\dot{\lambda}=(\rho+\delta)\,\lambda+1; \tag{36}$$

$$-\frac{\partial\mathcal{H}_{SP}}{\partial b}=\dot{\mu}-\rho\mu\Leftrightarrow\dot{\mu}=(\rho-\eta+Nk)\,\mu-\lambda q. \tag{37}$$

With respect to the case of competition, observe that, under social planning, in steady state it cannot be that $\lambda=\mu=0$. By manipulating the above conditions, we obtain the following state-control system:

$$\dot{S} = bq-\delta S \tag{38}$$

$$\dot{b} = b(\eta-Nk) \tag{39}$$

$$\dot{q} = b+(q-\sigma)\,(\rho+\delta+\eta-Nk) \tag{40}$$

$$\dot{k} = \rho k-\frac{q\,(\sigma-q)}{2r} \tag{41}$$

Unlike the oligopoly game we have investigated above, the planner's problem yields five steady state points:

[7]We attribute to the planner the same time discounting that we have used to measure firms's time preferences in the previous section. One might, however, suppose that the planner's discount rate be significantly lower than firms (possibly even nil), in order to give an appropriate weight to the welfare of future generations. For a thorough appraisal of this issue, see the *Stern Review* (Stern, 2007) as well as Dasgupta (2007), Norhaus (2007) and Weitzman (2007).

Proposition 13 *The stationary points of the system are:*

$$\begin{aligned}
P_{SP1} &= (S_{SP1},\ b_{SP1},\ q_{SP1},\ k_{SP1}) = (0, 0, \sigma, 0), \\
P_{SP2} &= (S_{SP2},\ b_{SP2},\ q_{SP2},\ k_{SP2}) = \left(0, 0,\ q_{SP2},\ \frac{\delta+\rho+\eta}{N}\right), \\
P_{SP3} &= (S_{SP3},\ b_{SP3},\ q_{SP3},\ k_{SP3}) = \left(0, 0,\ q_{SP3},\ \frac{\delta+\rho+\eta}{N}\right), \\
P_{SP4} &= (S_{SP4},\ b_{SP4},\ q_{SP4},\ k_{SP4}) = \left(\frac{2r\eta\rho(\rho+\delta)}{\delta N},\ b_{SP4}, \frac{b_{SP4}}{\rho+\delta},\ \frac{\eta}{N}\right), \\
P_{SP5} &= (S_{SP5},\ b_{SP5},\ q_{SP5},\ k_{SP5}) = \left(\frac{2r\eta\rho(\rho+\delta)}{\delta N},\ b_{SP5}, \frac{b_{SP5}}{\rho+\delta},\ \frac{\eta}{N}\right),
\end{aligned}$$

where

$$\begin{aligned}
q_{SP2} &= \frac{\sqrt{N}\sigma - \sqrt{N\sigma^2 - 8r(\rho+\delta+\eta)\rho}}{2\sqrt{N}} \\
q_{SP3} &= \frac{\sqrt{N}\sigma + \sqrt{N\sigma^2 - 8r(\rho+\delta+\eta)\rho}}{2\sqrt{N}} \\
b_{SP4} &= \frac{(\rho+\delta)\left(\sqrt{N}\sigma - \sqrt{N\sigma^2 - 8r\eta\rho}\right)}{2\sqrt{N}} \\
b_{SP5} &= \frac{(\rho+\delta)\left(\sqrt{N}\sigma + \sqrt{N\sigma^2 - 8r\eta\rho}\right)}{2\sqrt{N}}
\end{aligned}$$

Proof. Imposing stationarity on the R&D effort yields

$$k = \frac{q(\sigma - q)}{2r\rho} \tag{42}$$

which can be plugged into $\dot{q} = 0$ to obtain the following solutions:

$$q_{SP1} = \sigma; q_{SP2,3} = \frac{\sqrt{N}\sigma \mp \sqrt{N\sigma^2 - 8r(\rho+\delta+\eta)\rho}}{2\sqrt{N}} \tag{43}$$

with $q_{SP2,3} \in \mathbb{R}_+$ for $\sigma > \sqrt{8r(\rho+\delta+\eta)\rho/N}$. This in turns implies $k_{SP2,3} = (\delta+\rho+\eta)/N$. The corresponding state coordinates are $S = 0$, $b = 0$. On the other hand, if $b \neq 0$, $\dot{b} = 0$ in $k = \eta/N$ and plugging this expression into

$\dot{k} = 0$, we obtain $q_{SP4,5}$. Consequently, $\dot{q} = 0$ yields $b_{SP4,5}$ and finally $\dot{S} = 0$ produces $S_{SP4,5}$. ■

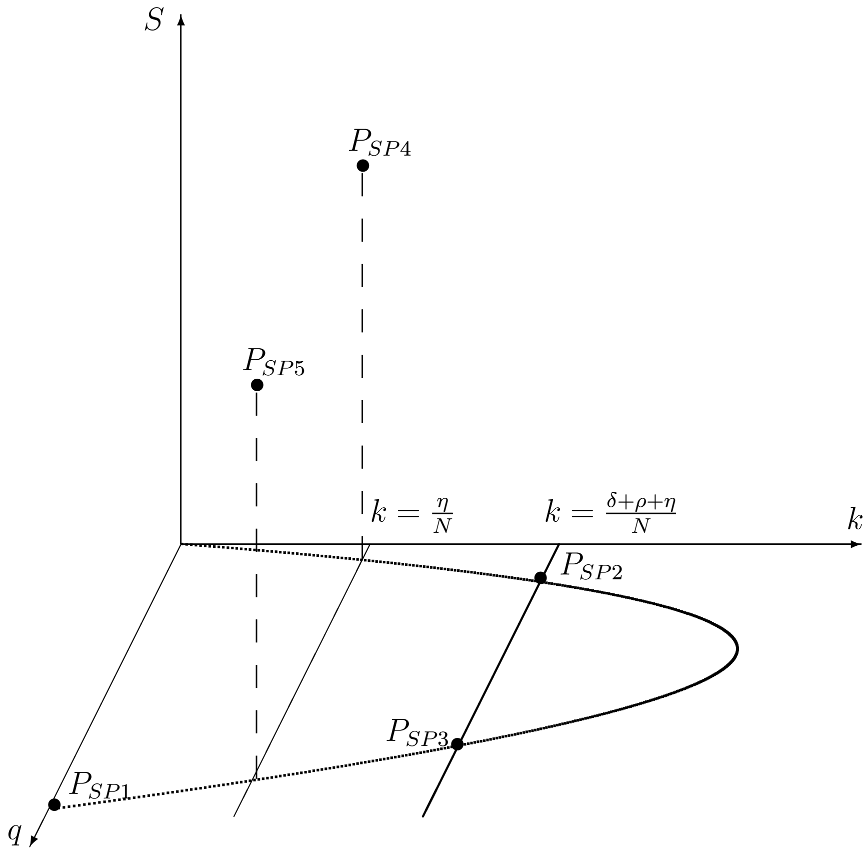

Figure 3. The five steady state points in the space (k, q, S)

Figure 3 locates the five steady state points emerging under social planning in three dimensions, in the space (k, q, S). Note that the equilibrium points P_{SP4} and P_{SP5} entail a positive amount of pollution and therefore do not belong to the control plane. The existence of the fourth and fifth solutions depends on the fact that the dynamics of the output level (40) depends on b, denoting that the planner indeed takes into account the environmental impact of the production technology when choosing the output level..

The Jacobian matrix is:

$$J = \begin{pmatrix} -\delta & q & b & 0 \\ 0 & \eta - Nk & 0 & -Nb \\ 0 & 1 & \rho + \delta + \eta - Nk & -N(q-\sigma) \\ 0 & 0 & \frac{1}{2r}(2q-\sigma) & \rho \end{pmatrix}. \tag{44}$$

By repeating a procedure analogous to the one carried out to produce Proposition 4, we can prove that:

Proposition 14 *P_{SP1}, P_{SP2}, P_{SP3}, P_{SP4} and P_{SP5} are saddle points.*

Next we are going to evaluate the profits and the social welfare levels at each equilibrium point.

$$\begin{aligned} \pi(P_{SP1}) &= 0, \\ \pi(P_{SP2}) &= \pi(P_{SP3}) = \frac{r}{N}(\rho^2 - (\delta+\eta)^2), \\ \pi(P_{SP4}) &= \pi(P_{SP5}) = \frac{\eta r}{N}(2\rho - \eta). \end{aligned} \tag{45}$$

Proposition 15 *1. If $\rho \in (\delta+\eta, \infty)$, then the profits $\pi(P_{SP2})$, $\pi(P_{SP3})$, $\pi(P_{SP4})$ and $\pi(P_{SP5})$ are positive;*

2. if $\rho \in (\delta+\eta, \delta+2\eta)$, then $\pi(P_{SP2}) = \pi(P_{SP3}) < \pi(P_{SP4}) = \pi(P_{SP5})$.

The social welfare associated to the steady states is computed as follows:

$$SW(P_{SPi}) = \pi(P_{SPi}) + \frac{q_{SPi}^2}{2} - S_{SPi}, \qquad i = 1, \ldots, 5, \tag{46}$$

and yields, respectively:

$$\begin{aligned} SW_1 &= SW(P_{SP1}) = \frac{\sigma^2}{2}, \\ SW_2 &= SW(P_{SP2}) = \frac{\sigma^2}{4} - \frac{\sigma\sqrt{\sigma^2 N - 8r(\rho+\delta+\eta)\rho}}{4\sqrt{N}} - \frac{r(\delta+\eta)(\delta+\eta+\rho)}{N}, \\ SW_3 &= SW(P_{SP3}) = \frac{\sigma^2}{4} + \frac{\sigma\sqrt{\sigma^2 N - 8r(\rho+\delta+\eta)\rho}}{4\sqrt{N}} - \frac{r(\delta+\eta)(\delta+\eta+\rho)}{N}, \\ SW_4 &= SW(P_{SP4}) = \frac{\sigma^2}{4} - \frac{\sigma\sqrt{\sigma^2 N - 8r\eta\rho}}{4\sqrt{N}} - \frac{r\eta[2\rho^2+\delta(\eta+\rho)]}{\delta N}, \\ SW_5 &= SW(P_{SP5}) = \frac{\sigma^2}{4} + \frac{\sigma\sqrt{\sigma^2 N - 8r\eta\rho}}{4\sqrt{N}} - \frac{r\eta[2\rho^2+\delta(\eta+\rho)]}{\delta N}. \end{aligned} \tag{47}$$

Proposition 16 *1. $SW_1 > SW_5 > SW_4$ and $SW_3 > SW_2$ over the whole admissible range of parameters;*

2. if $\rho \in (\delta+\eta, \delta+2\eta)$ and $\delta > 2\eta$, then $SW_5 > SW_3$.

The steady state replicating the perfectly competitive outcome of the static model would look like the most desirable one, since the related level of social welfare exceeds all the remaining ones. However, it remains out of reach for all $b_0 > 0$.[8]

Additionally, there exists a subset of the admissible range of parameters in which the steady state P_{SP5} is both privately and socially preferable to all the steady state allocations arising from the open-loop Nash game among unregulated firms. With this in mind, we turn now our attention to the design of a Pigouvian tax/subsidy that may adjust firms' incentives so as to drive them to reproduce P_{SP1}.

[8] That is, the equivalent of Remark 5 holds here. The proof of this fact follows the same lines as for the Cournot equilibrium of the open-loop game among firms. The details have been omitted for brevity.

5 Effects of a Pigouvian taxation

In this section, a Pigouvian tax rate $\theta > 0$ is introduced, with taxation taking the form of a linear function of the environmental externality produced by the industry. Such a taxation affects each current-value Hamiltonian function, which now writes as:

$$\mathcal{H}_i(\cdot) = \left(\sigma - \sum_{j=1}^{n} q_j\right) q_i - rk_i^2 - \theta S + \lambda_i \dot{S} + \mu_{ii}\dot{b}_i + \sum_{j \neq i} \mu_{ij}\dot{b}_j. \tag{48}$$

As in Benchekroun and Long (1998, 2002), our objective here is to investigate whether this Pigouvian tax rate can be designed so as to reproduce the same social welfare level characterising the first best (that the planner himself would be, in general, unable to attain). Clearly, this assumption leaves the FOCs (5) and (6) unchanged, whereby the adjoint equations (7) become as follows:

$$\dot{\lambda}_i(t) = (\rho + \delta)\lambda_i(t) + \theta. \tag{49}$$

The above dynamics implies that the presence of a Pigouvian taxation induces firms to shrink output levels as compared to the unregulated setting, as can be ascertained from (13), whenever $\lambda_i(0) < 0$. That is, the policy maker, being aware of the tradeoff between the price effect and the external effect implied by any change in output, is willing to accept an increase in price (as a result of the related higher degree of quasi-collusion) for the sake of reducing the environmental externality.

Unlike (7), (49) does not admit the nil solution, so Proposition 1 cannot hold, as in the social planning case. The state-control system is as follows (again, omitting the time argument):

$$\dot{S} = Nbq - \delta S \tag{50}$$

$$\dot{b} = b(\eta - k) \tag{51}$$

$$\dot{q} = \frac{[(N+1)\,q - \sigma]\,[\rho + \delta + \eta - k] + \theta b}{N+1} \tag{52}$$

$$\dot{k} = \rho k - \frac{q\,[\sigma - (N+1)\,q]}{2r} \tag{53}$$

As a consequence of taxation, firms' cost structure is modified to account for pollution, and therefore $\dot{q}$ depends on θb. As a consequence, also $\dot{k}$ depends on θb. Therefore,

$$\frac{dq}{dk} = \frac{2r\,[((N+1)\,q - \sigma)\,(\rho + \delta + \eta - k) + \theta b]}{(N+1)\,[2r\rho k - q\,(\sigma - (N+1)\,q)]} \tag{54}$$

and the slope of the Nash trajectory in the control plane becomes sensitive to pollution thanks to the Pigouvian tax rate.

Also in this case, multiple equilibrium points appear. Provided that the market is large enough, $\sigma > 2\sqrt{2(1+N)r\rho(\rho+\delta+\eta)}$, we obtain three steady states corresponding to no pollution:

$$(S_1, b_1,\ q_1,\ k_1) = \left(0, 0, \frac{\sigma}{N+1}, 0\right); \tag{55}$$

$$(S_2, b_2, q_2, k_2) = \left(0, 0, \frac{\sigma - \sqrt{\sigma^2 - 8(1+N)r\rho(\rho+\delta+\eta)}}{2(1+N)}, \rho+\delta+\eta\right);$$

$$(S_3, b_3, q_3, k_3) = \left(0, 0, \frac{\sigma + \sqrt{\sigma^2 - 8(1+N)r\rho(\rho+\delta+\eta)}}{2(1+N)}, \rho+\delta+\eta\right).$$

Moreover, as in the social planning case, two further equilibria with positive stocks of pollution exist:

$$(S_4, b_4, q_4, k_4) = \left(\frac{2\eta N r\rho(\delta+\rho)}{\delta\theta}, \frac{(\delta+\rho)(\sigma - \sqrt{\sigma^2 - 8\eta(1+N)r\rho})}{2\theta}, \frac{4\eta r\rho}{\sigma - \sqrt{\sigma^2 - 8\eta(1+N)r\rho}}, \eta \right); \quad (56)$$

$$(S_5, b_5, q_5, k_5) = \left(\frac{2\eta N r\rho(\delta+\rho)}{\delta\theta}, \frac{(\delta+\rho)(\sigma + \sqrt{\sigma^2 - 8\eta(1+N)r\rho})}{2\theta}, \frac{4\eta r\rho}{\sigma + \sqrt{\sigma^2 - 8\eta(1+N)r\rho}}, \eta \right). \quad (57)$$

The steady states (56) and (57) depend on the Pigouvian tax rate: in particular, notice that $q_4^\theta > q_5^\theta$ and that the associated steady state levels of pollution are decreasing in θ.

At this stage, it is worth carrying out a comparative analysis of the social welfare equilibrium levels again. Let q_i^θ, S_i^θ, π_i^θ and SW_i^θ be, respectively, the i-th steady state values in the present case, the levels of social welfare SW_i^θ is computed by the following formula:

$$SW_i^\theta = \frac{(Nq_i^\theta)^2}{2} + N\pi_i^\theta - S_i^\theta. \quad (58)$$

The only two steady states affected by the tax rate are the fourth and the fifth one and

$$SW_4^\theta = \eta N r \left[\eta \left(-\frac{8Nr\rho^2}{(\sigma - \sqrt{\sigma^2 - 8(1+N)\eta r\rho})^2} - 1 \right) + 2\rho \left(\frac{2\sigma}{\sigma - \sqrt{\sigma^2 - 8(1+N)\eta r\rho}} - \frac{\delta+\rho}{\delta\theta} \right) \right]; \quad (59)$$

$$SW_5^\theta = \eta N r \left[\eta \left(-\frac{8Nr\rho^2}{(\sigma + \sqrt{\sigma^2 - 8(1+N)\eta r\rho})^2} - 1 \right) + 2\rho \left(\frac{2\sigma}{\sigma + \sqrt{\sigma^2 - 8(1+N)\eta r\rho}} - \frac{\delta+\rho}{\delta\theta} \right) \right],$$

so $SW_4^\theta > SW_5^\theta$ irrespective of all the parameter values.

Now we compare SW_4^θ with the maximum social welfare level that would be obtained under social planning case, i.e. $SW_1 = \frac{\sigma^2}{2}$, in order to derive the threshold values of the tax rate that allows to reach SW_1 under oligopolistic competition .

If we consider SW_4^θ as a function of $\theta \in (0, \infty)$, we can stress that it takes negative values when θ is close to zero, :

$$\lim_{\theta \to \infty} SW_4^\theta = \frac{2\eta N r[4\eta r\rho(\eta + \eta N - \rho N) - (2\rho - \eta)\sigma(\sigma - \sqrt{\sigma^2 - 8(1+N)\eta r\rho})]}{(\sigma - \sqrt{\sigma^2 - 8(1+N)\eta r\rho})^2}. \tag{60}$$

Moreover, SW_4^θ is strictly increasing, consequently admitting a horizontal asymptote, whose level is positive if $\rho \in \left(\frac{\eta}{2}, \frac{\eta(1+N)}{N}\right)$. Call $K := K(\eta, r, \rho, N, \sigma)$ such a positive level.

If $K > \frac{\sigma^2}{2}$, then the optimal tax rate θ^* entailing the identity $SW_1 = SW_4^{\theta^*}$ is given by:

$$\theta^* = \frac{4\eta N r\rho(\delta + \rho)}{\delta(2K - \sigma^2)}. \tag{61}$$

An analogous procedure can be carried out with SW_5^θ, where it can be easily ascertained that the tax rate θ^{**} such that $SW_1 = SW_5^{\theta^{**}}$ exceeds θ^* and the inequality with respect to the related externality levels is inverted, i.e. $S_4^{\theta^*} > S_5^{\theta^{**}}$. In other words, the tax rate that allows to reproduce the social welfare SW_1 is higher and corresponds to a higher level of output and pollution.

This seemingly counterintuitive fact relies on the identity leading to the value (61):

$$SW_{4,5}^\theta = \pi_{4,5}^\theta + CS_{4,5}^\theta - S_{4,5}^\theta = \frac{\sigma^2}{2}, \tag{62}$$

implying

$$\theta = \frac{4\eta N r\rho(\delta + \rho)}{\delta[2(\pi_{4,5}^\theta + CS_{4,5}^\theta) - \sigma^2]}. \tag{63}$$

Thus, inequality $q_4^\theta > q_5^\theta$ affects the denominator of the previous relation, because $\pi_4 + CS_4 > \pi_5 + CS_5$. Hence, the first best social welfare can be obtained by moving along two different paths: either with a larger quantity and a lower price but a higher externality level, or conversely with a smaller quantity and a higher price but a lower externality.[9]

The remarkable feature of the latter result is that, starting form a situation where the command optimum (point P_{SP1}) reproducing the perfectly competitive outcome is not, in general, attainable under planning except in the uninteresting case where the productive technology is completely green at the outset, it is nonetheless true that there exist an optimal stationary industrial policy whereby the regulator can drive profit-maximising firms to yield the same steady state welfare level associated with the first best, although of course at the price of a different surplus allocation and environmental externality. If the regulator is interested in the size of the total pie but not in the relative size of its slices, this is a price that he might well be willing to pay.

6 Concluding remarks

We have revisited the issue of the incentive for firms to carry out R&D efforts aimed at introducing environmental-friendly technologies. Contrary to the acquired view establishing that such an incentive is lacking due to the fact that firms fail to internalise the environmental externality, the dynamic approach we have adopted in the foregoing analysis shows that firms do have an

[9]Note that the corresponding steady state profits are independent of θ:

$$\pi_{4,5}^\theta = \frac{\delta\sigma\left(\sigma \pm \sqrt{\Omega}\right) - 2\eta\left(N+1\right)r\Upsilon}{2\delta\left(N+1\right)^2}$$

where $\Omega = \sigma^2 - 8\eta\rho\left(N+1\right)r$ and $\Upsilon = 2\rho^2\left(N+1\right)N + \delta\left[\eta\left(N+1\right) + 2\rho N^2\right]$. There exist admissible parameter regions where the above profits are strictly positive.

R&D incentive in this direction throughout the game, although it may indeed vanish in one specific steady state, which portrays the equilibrium outcome of the corresponding static game. Such an incentive has no altruistic nature, being associated with a quasi collusive decision on output levels whereby any environmentally-oriented R&D is accompanied by a price increase.

Moreover, we have investigated the behaviour of the model under the assumption that a benevolent planner controls production and R&D, showing that the perfectly competitive outcome with marginal cost pricing and a totally clean technology is one of the possible steady states of the system, but is feasible only if initial conditions are such that the environmental externality is not an issue from the very outset.

Yet, as a (partial) remedy, we have found that there exist a feasible stationary Pigouvian tax rate able to induce profit-maximising firms to follow a path leading to the very same aggregate steady state welfare as in the first best.

The foregoing analysis can be extended in several directions, to examine feedback solutions, the implications of international trade with transboundary pollution and uncertainty affecting both the accumulation of pollution and the R&D outcome, all of these issues to be nested into a general equilibrium approach. These extensions are left for future research.

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