THE RIMINI CENTRE FOR ECONOMIC ANALYSIS

WP 15-41

Andrea Mantovi
Università di Parma, Italy
The Rimini Centre for Economic Analysis, Italy

Augusto Schianchi
Università di Parma, Italy

A NEO-AUSTRIAN PERSPECTIVE ON THE VALUE OF GROWTH PROSPECTS

The Rimini Centre for Economic Analysis
Legal address: Via Angherà, 22 – Head office: Via Patara, 3 - 47921 Rimini (RN) – Italy
www.rcfea.org - secretary@rcfea.org

DIPARTIMENTO DI ECONOMIA
UNIVERSITÀ DEGLI STUDI DI PARMA
Via J. F. Kennedy, 6 – 43125 Parma – Italia

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ANDREA MANTOVI
andrea.mantovi@unipr.it

AUGUSTO SCHIANCHI
augusto.schianchi@unipr.it

WORKING PAPER EP01/2015

Parole chiave: Neo-Austrian Capital Theory; Convolution Theorem; Investment; Tobin's q.

This draft: June 10, 2015

A NEO-AUSTRIAN PERSPECTIVE ON THE VALUE OF GROWTH PROSPECTS

Andrea Mantovi, Augusto Schianchi

Dipartimento di Economia, Università di Parma

Abstract. The valuation framework inherent to the neo-Austrian theory of capital set forth by Hicks' (1973) is discussed in terms of a fundamental formula which disentangles the profitability associated with the scale of operation from the internal rate of return of the production process. The formula is employed to tailor a perspective on the value of investment prospects, meant to complement the insights embodied by Tobin's q metric. Balance sheet recessions are briefly discussed as a cogent line of application of our formula. Potential lines of progress are envisioned.

Keywords: Neo-Austrian Capital Theory; Convolution Theorem; Investment; Tobin's q.

JEL classification: D92, O12, O40.

1. INTRODUCTION

It is the aim of the present contribution to deepen the valuation framework inherent to the theory of capital set forth by Hicks (1973). By the properties of the Laplace transform (LT), we shall establish a "scale and scope" formula which separates the present value of a growth prospect from the internal rate of return of the production technique. We shall apply such a formula to envision insightful complements to Tobin's q, and an essential framework for the discussion of balance sheet recessions.

As pointed out by Dorfman (1969), "Capital theory is the economics of time", to the extent that lasting instruments of production are expected to produce value during their lifetime, enable accumulation of further instruments, and endow time with value. In such respects, as pointed out by Zamagni (1984), the sequential causal relations inherent to the 'working' of capital represent a major challenge for capital theory. In fact, the history of the theory features manifold interpretations (and controversies as well) of the "economics of time", with the benchmark stationary state assumption fixing the framework for classical equilibrium analysis.[1] The 'Austrian' (problematic) notions of average time of production and roundaboutness have been introduced precisely with the aim of deepening the economic role of time in the analysis of production (see for instance Gehrke and Kurz, in Hagemann and Scazzieri, 2009).[2] In such respects, Hicks (1973) succeeded in building a solid "neo-Austrian" (NA) theoretical framework, upon which further advances may be, and in fact have been, built (see for instance Nardini, 1994, and references therein). Our point is to uncover basic properties of the valuation framework inherent to Hick's (1973) theory of capital.

The cornerstone of such a theoretical construction is the *convolution* structure of production activity (see formula 1 below), which fixes the economic role of the time in which production takes place, as distinct from the role of historical ("calendar") time, with respect to which economic dynamics should, in principle, be symmetric. Hicks' (1973) theory is widely held as the natural framework for the representation of the complementarities in production connected with the diachronic heterogeneity of capital

[1] Hayek (1941, Chapter II) pointed out that "most of the shortcomings of the theory of capital in its present form are due to the fact that it has in effect only been studied under the assumptions of a stationary state." (ivi, p. 14).

[2] More generally, a number of scholars have contributed sophisticated approaches to the dynamic analysis of production and capital; we refer to the contributions in Landesmann and Scazzieri (1996) for a thorough account of such challenging theoretical perspectives.

(Zamagni, 1984); true, in the authors' vision, the potentialities of such a theory may not have been fully exploited. For instance, the LT has long been employed in the analysis of the Hicksian traverse (see for instance Gozzi and Zamagni, 1982; Violi, 1984); still, the properties of the LT do not seem to play a crucial role in the valuation of investment, and things do not seem to differ substantially from the situation pictured by Buser (1986), according to whom the properties of the LT seemed to be underexploited in financial analysis.

It is a well established principle that the value of a going concern at date T is gauged in first instance by the present value at T of its expected stream of earnings. Such *discounted cash flow* (DCF) method lies at the foundations of economic, financial and accounting analysis; the modern appraisal of the relevance of such an approach can be traced back to Hicks' (1939) celebrated treatise *Value and Capital.* Definitely, via the convolution theorem, we shall we be in a position to deepen the valuation framework for a generalization of Hicks' (1973) theory in which an exogenous hurdle rate sets the benchmark of present value comparisons. We shall thereby fix a unified framework for a theory of capital, financial valuation an accounting, based on a fundamental unity of analysis (the NA process), a framework capable of supporting both positive and normative approaches to the problem of growth. Admittedly, our approach is not meant to exploit the full potential of Hicks' (1973) framework, and we shall confine to the convolution structure of production activity, for which our reference to the LT is meant to be *conceptual* more than analytical, on account of the following considerations.

The problem of investment lies at the core of the theory of capital, to the extent that investment is meant to *add value* to a stock of capital already in place. In fact, the normative problem of the intensity of investment represents a classical theme in the history of economics, which, according to Tobin and Brainard (1977) can be addressed sharply: "Clearly, it is the q ratio *on the margin* that matters for investment" (ivi, p. 243). It is this marginal criterion which our NA perspective is meant to complement by means of a sharp perspective on the value of growth prospects.

A long established line of analysis fixes the investment margin in terms of adjustment costs, i.e. of the "increasing costs associated with integrating new equipment into a going concern." (Eisner and Strotz, 1963). Noticeably, Dixit and Pindyck (1994) have shown that such a line of inquiry enables one to fix a marginal q in which the value of growth opportunities is measured in terms of real options. On the one hand, our approach is orthogonal to the problem of adjustment costs, which do not enter Hick's (1973) approach; on the other hand, our approach somehow compares to the one set forth by Dixit and Pindyck, at least to the extent that the value of growth opportunities is at stake.

Definitely, among the landmark contributions to the theory of optimal investment, Dorfman (1969) stands out for having devised a transparent economic interpretation of the theory of optimal control. The Author addresses the very long run problem of the firm (possibly, a stylized economy), in which, following the classical approach set forth by Ramsey (1928), production and accumulation are instantaneous. The explicit role of time is embodied by the rate of depreciation of productive capital and by the rate of intertemporal preference gauging the optimal control problem. The time lags in production and accounting activities are disregarded. In such a model, the "value" of the long run investment prospect is inextricably connected with the path of "utility". Such a framework for intertemporal optimization shapes the standard theories of growth (Acemoglu, 2008), and provides useful insights about the growth path of firms (as Dorfman, 1969, repeatedly points out). Still, the extant literature does not seem to fix a pregnant connection of such model with DCF valuation and Tobin's q.

Definitely, Hicks' (1973) approach to investment and growth is inherently consistent with DCF valuation (ivi, chapter II). We shall capitalize on such a property in order to show that the NA framework does fit nicely into the perspective on investment tailored by Tobin and Brainard (1977), which focuses the "sensitivity of capital formation to interest rates in financial markets."[3]

The plan of the rest of the paper is as follows. In section 2 we motivate our approach and recall basic elements of the NA approach. In section 3 we introduce our fundamental formula. In section 4 we discuss our NA complements to Tobin's q. Section 5 sketches an approach to balance sheet recessions. A final section tailors potential lines of progress.

[3] The sustainability of the policies of all-time lows short run interest rates implemented by central banks in the last years represent a dramatic empirical setting with respect to which to frame such issues.

2. ELEMENTS OF THE NEO-AUSTRIAN FRAMEWORK

Economic phenomena, we all know, occur at different time scales. At one extreme, macroeconomic models of growth typically represent phenomena emerging in the "very long run", in which the time scale is much larger than that at which an economy 'adjusts' to shocks. Such models thus represent histories built out of equilibrium states.[4] At the other extreme, the quite short time intervals in which the fluctuations of stock prices occur set the scale for the activity of a large number of traders (some human, some automated). In between such opposite extremes, the various time scales at which economic activities take place may vary from days to weeks, or from months to years. However, whatever the scale, the very representation of the intermediate steps between an initial situation and a final (expected) state is mandatory once relevant economic (for instance, the intertemporal allocation of scarce resources), financial (like matching maturities) or accounting (perhaps a depreciation scheme) issues are at stake. It is this kind of phenomena "less 'temporary' than those characterizing the classical adjustment process of market prices to natural prices, but less 'permanent' than those through which the accumulation process takes way" (Zamagni, 1984, p. 137) which our valuation framework is meant to focus.

The NA approach to production and growth developed by Hicks (1973) was meant to address such phenomena, which lie beyond the reach of a production function approach in which inputs produce outputs "instantaneously", in the sense that we are supposed not to have economic interest[5] in the *in between* events. Hicks' approach was meant to focus vertically integrated economies; in fact, it has been soon interpreted as a 'special case' of the activities approach named after von Neumann, Leontief and Sraffa (Burmeister, 1974), and such a connection has been the subject of substantial research effort (see for instance Hagemann and Scazzieri, 2009). Let us recall the basic element of the NA framework upon which our valuation framework builds.

Hicks' (1973) unit of analysis is the *process*, an economic construct built out of the time profiles of a primary production input a (meant to represent labor[6]) and of the (single) output b. In connection with prices for such variables, one can define a time profile of value $q(t)=(b-wa)(t)$, assuming unit price for output, and writing w for the (relative) price of input.

[4] In close resemblance with equilibrium thermodynamics. Recall, the thermodynamic analogy for economic equilibrium dates back at least to the works of Irving Fisher.

[5] Park (2015) addresses the theoretical consistency of modeling production as instantaneous.

[6] Keynes (1936, 16.II) admitted his sympathy for the "pre-classical doctrine that everything is *produced by labor*".

The process is supposed to have finite time length (possibly subject to optimization), and is to be sequentially restarted in order for production to carry on. In essence, the NA framework addresses the *reinvestment* in the same project. Hicks' (1973) assumption of constant returns to scale (CRS) enables one to scale the activation of processes at will.

The economic intuition of investment expenditure preceding the generation of receipts leads one to conceive of processes as starting with no output ("construction phase") and therefore negative profile values; then, output emerges (in the "utilization phase") and the profile $q(t)$ switches to positive values once the value of output exceeds that of input. Thus, a flow-input-flow-output approach is at stake in the NA approach. A qualitative representation of a NA process in continuous time is given in Figure 1, in which the net output q starts at negative values, and then turns positive.

Hicks (1973) introduces the process profile as a natural representation of value creation by a firm, and envisages the finite lifetime of a plant as a consistent interpretation for the finite time length of the profile. In fact, for financial firms like banks, any financial contract which specifies *exactly* a cash flow profile of the form represented in Figure 1 represents a cogent example of the NA process. More generally, the *cycles* inherent to working of a business (think for instance of investment and inventory) represent a rich realm of phenomena for which the cogency of NA representation may have been underestimated by the literature. Thus, a number of elements seem to establish the relevance of applying the NA structure of value creation to the problem of the firm, despite Hicks' (1973) main concern with the macroeconomic consequences of technological transitions on wages and demand for labor.

A fundamental property of the process is its *internal rate of return* (IRR), i.e. the discount rate which makes the present value of the process vanish. The process is viable (contingent on the prices of input and output) if its IRR is not negative. *The IRR is unique for the time profiles addressed in the NA framework,* as fixed by a "fundamental theorem", for which a sufficient condition is that the profile switches from negative to positive value only once.

The NA approach is developed by Hicks (1973) in discrete time. The value generated by the ruling production process is a sum of contributions resulting from all the processes already started and still alive, i.e. the convolution in discrete time

$$\sum_{j=0}^{\Omega} q_j x_{T-j} \equiv Q_T \tag{1}$$

in which x represents the profile of intensity (scale) of (re-)investment, the "rate of starts", in historical time. Thus, the term $x_T\, q_0$ represents the contribution to net output at date T coming from the newly started processes; the term $x_{T-1}\, q_1$ represents the contribution from the processes started at the previous date (T–1), and so on, until all the contributions coming from processes still alive are taken into account. Formula (1) establishes Hicks' (1973) "Take-Out" (ivi, p. 30), which gauges the value generated by the economy after the remuneration of the primary input.

Formula (1) embodies the essence of the NA approach, and immediately sets the departure from the "neoclassical" growth models built out of production functions. Such an approach is well suited to address the very long run, and has indeed proved useful in pinning down fundamental tenets of economic growth, like the Kaldor facts (see for instance Acemoglu, 2008, and references therein), but is inherently blind to the properties of transitional phases in which an old and a new production techniques coexist, and inputs must be *properly* allocated between them. This is the difficult problem which Hicks named the *traverse*.

True, we shall not be concerned with the restructuring of an economy. Our sharp goal is to envision NA vertical integration as a significant perspective on the investment valuation problem, without committing to either a positive or normative stance on the problem of growth. The methodological implications of the convolution structure (1) of production represent our sharp focus: value is "produced" much like output, and the time structure of such a production process is 'sequentially consistent', as discussed in the following section. Noticeably, a sequential re-investment process in terms of convolutions has been established as a consistent accounting framework in continuous time (Stauffer, 1971), which strengthens the cogency of our NA approach.

3. THE FUNDAMENTAL FORMULA

The Laplace transform (LT) $\mathcal{L}C(r) \equiv \int_0^\infty dt\, e^{-rt} C(t)$ of the expected cash flow profile $C(t)$ defines its present value at $T = 0$ as a function of the constant discount rate r. The effectiveness of the LT in deepening the analytical characterization of present values has been long established (Grubbström, 1967; Buser, 1986). What the literature may not have adequately addressed are the potentialities of the *convolution theorem*, according to which the LT of a convolution is the product of the LTs of the convolved functions, a results which enables us to deepen Hicks' (1973) theory of capital.

To begin with, translate expression (1) in continuous time, with $f:[0,\Omega]\to\mathbf{R}$ and $x:[T-\Omega,T]\to\mathbf{R}$, and obtain

$$Q(T)=\int_0^{\Omega} ds\, f(s)x(T-s)\equiv(f*x)(T) \qquad (2)$$

in which the net output $Q(T)$ is supposed to be continuously generated, and accounted for, at any date T. It is natural to assume the function $f(s)$ to be piecewise continuous, with possibly a finite number of discontinuities, as a natural correspondence with the case of discrete time; an example is given in the figure below.

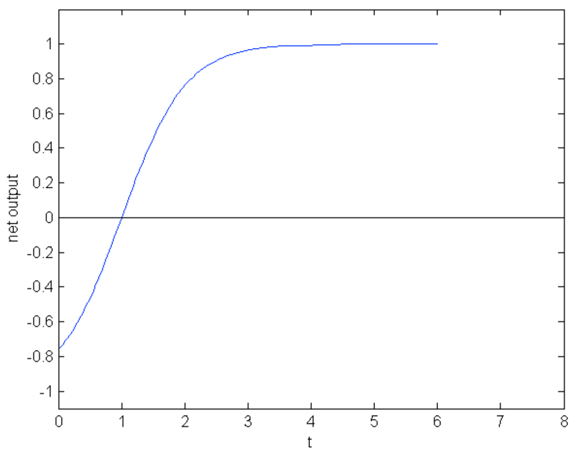

Figure 1. Plot of a sample continuous profile (hyperbolic tangent) with time length 6.

The present value of net output (2) at $T=0$ is given by the LT

$$\mathcal{L}(f*x)(r)\equiv\int_0^{\infty} dt\, e^{-rt}\int_0^{\Omega} ds\, f(s)x(t-s) \qquad (3)$$

with $x:[-\Omega,\infty)\to \mathbf{R}$. Notice in fact that (2) does not fit exactly the standard notion of convolution addressed by the literature, in which integration ranges from 0 to T and it is assumed that $T > s$ (see for instance Shiff, 1999). In order to fit the standard framework, separate the present value (3) into the contribution from processes already in place at date 0, as defined by $x(T)$ for $T < 0$, and which can be written

$$\int_0^\Omega dt\, e^{-rt} \int_t^\Omega ds\, f(s)x(t-s) \tag{4}$$

and the one resulting from the expected profile $x(T)$ for $T > 0$, i.e.

$$\int_0^\infty dt\, e^{-rt} \int_0^t ds\, f(s)x(t-s) \tag{5}$$

The term (5) does indeed represent the standard case for the convolution theorem (Shiff, 1999), and therefore equals the product of the LTs of the convolved functions, namely, the basic profile and the the *prospect* rate of starts $x(T)$, with $T \geq 0$, so that (5) can be written $\mathcal{L}x(r)\mathcal{L}f(r)$. Correspondingly, one can define $y:[0,\Omega]\to \mathbf{R}$, $y(T) \equiv x(-T)$ for $T > 0$, as a 'mirror' image of the past rate of starts, and obtain the expression $\mathcal{L}y(r)\mathcal{L}f(r)$ for (4). Then, one can state

Proposition 1. The present value at $T = 0$ of the NA firm (stylized economy) admits the representation

$$V(r) = \mathcal{L}y(r)\mathcal{L}f(r) + \mathcal{L}x(r)\mathcal{L}f(r) \tag{6}$$

being $\mathcal{L}y(r)\mathcal{L}f(r)$ the contribution from processes already in place, and $\mathcal{L}x(r)\mathcal{L}f(r)$ the contribution from the prospect rate of starts $x(T)$.

The factorization structure $\mathcal{L}x(r)\mathcal{L}f(r)$ is the key to our approach; such an expression factorizes the functional dependence on *r* in a pair of contributions pertaining to the basic building blocks of the NA framework, thereby disentangling their contribution to value. Let us pin down the main properties of the formula.

First, its dimensional structure: the formula is the product of the dimensionless quantity $\mathcal{L}x(r)$ (the dimension of *x* being time $^{-1}$) and of $\mathcal{L}f(r)$, whose dimension is value. The formula is consistent with CRS, on account of the homogeneity of the LT: for any constant *a*, $\mathcal{L}(af)(r) = a\mathcal{L}f(r)$. For positive profiles $x(t)$, $\mathcal{L}x(r)$ is positive for any positive *r*. Then, if *r* exceeds the IRR of the project, (6) is negative, signalling that the enterprise is burning value. The going concern "breaks even" at *r* = IRR. The firm has positive present value once the IRR of the basic project exceeds *r*; then, (6) enables one to factorize the profitability of the basic project (be it technological or reflecting market power) at the ruling rate *r* (embodied by $\mathcal{L}f(r)$, the *scope* of the going concern) and the effect of the *scale* of the going concern (embodied by $\mathcal{L}x(r)$). It is natural to call $\mathcal{L}x(r)\mathcal{L}f(r)$ *scale and scope* formula.

A benchmark case for the above proposition concerns constant rate of starts, say, x_0, from $-\Omega$ to ∞. That being the case, (6) reduces to

$$x_0 \frac{e^{r\Omega}}{r}\mathcal{L}f(r) = x_0 \frac{e^{r\Omega}-1}{r}\mathcal{L}f(r) + x_0 \frac{1}{r}\mathcal{L}f(r) \tag{7}$$

As expected, the above expression is homogeneous in the level of activity x_0, and in the present value $\mathcal{L}f(r)$ of the profile as well. Expression (7) in fact fixes a transparent setting for the study of the "long run" limit(s) of the NA framework. Consider first the limit $\Omega \to 0$, other things the same. One thereby fixes a "neoclassical" limit in which the time length of the process shrinks to an instant, and value is produced instantaneously. Then, on economic grounds, one expects the contribution from already started process to vanish in this limit (in fact, with $e^{r\Omega}-1 \to 0$, the first terms on the RHS of (7) vanishes), and $\mathcal{L}f(r)$ to converge to $\mathcal{L}f(0)$, which represents the undiscounted amount of output produced by the process.

Evidently, with constant *r*, one has no long run limit. A truly long run limit is obtained once we posit $\Omega \to 0$, $r \to \infty$ *and* a definite limit for the dimensionless product $r\Omega$. For instance, suppose a model exists which endogenizes the parametrization $\Omega \equiv 1/\beta$, $r \equiv 2\beta$, which leads to the finite limit $r\Omega = 2$ as $\beta \to \infty$; then one has a well defined long run limit to work with.

Definitely, the factorization of our fundamental formula sets a sharp line of comparative statics. The empirical relevance of the sensibility of asset values to interest rates has been clearly pointed out by Tobin and Brainard (1977) at the opening of their contribution. In such respects one can envision the relevance of the proposition below.

As is well known, the Hicks-Macaulay *duration*[7] of a profile represents a significant index of the time extension of the process, and gauges the responsiveness of a present value to changes in r. Recall, the duration Θ of the profile $f(t)$ at $t = 0$ reads

$$\Theta[f(t),r] \equiv \frac{\int_0^\Omega dt\, e^{-rt} t\, f(t)}{\int_0^\Omega dt\, e^{-rt} f(t)} = \frac{-\frac{d}{dr}\mathcal{L}f(r)}{\mathcal{L}f(r)} = -\frac{d}{dr}\ln \mathcal{L}f(r) \tag{8}$$

By the Leibniz rule of elementary calculus one obtains immediately

Proposition 2. The value function $\mathcal{L}x(r)\mathcal{L}f(r)$ responds to changes in r according to

$$\frac{d}{dr}\left(\mathcal{L}x(r)\mathcal{L}f(r)\right) = \frac{d\mathcal{L}x(r)}{dr}\mathcal{L}f(r) - \Theta V(r) \tag{9}$$

Proposition 2 fixes the *separation* of the sensitivity problem into the sensitivity associated with the basic profile $f(t)$, which has a clear interpretation, and the sensitivity associated with the re-investment prospect $x(t)$, which does not seem to have a correspondingly well established interpretation (and for which we shall indeed envision an insightful interpretation in the following section).

[7] The notion has been introduced by Hicks (1939), with the expression "average period of the stream", as the elasticity of the value of the stream with respect to the discount factor. The same notion has been introduced independently by Macaulay in 1938.

For the time being, consider the simple example of a 'step' process, whose construction phase extends from $t = 0$ to $t = 1$, with negative unit cash flow, and whose utilization phase extends from $t = 1$ to $t = \Omega$ with positive constant unit cash flow. The LT of such a profile f results in

$$\mathcal{L}f(r) = -\frac{1-e^{-r}}{r} + \frac{e^{-r} - e^{-r\Omega}}{r} \tag{10}$$

which is monotonically decreasing as r increases, and a threshold $\bar{r} = \bar{r}(\Omega)$ exists at which $\mathcal{L}f(r)$ turns negative and the process is no more viable (such a threshold defines the IRR). In Figure 2 the above function is represented for $\Omega = 10$, with the downward sloping characteristic of present value functions.

The analytical tractability of this simple case enables one to set up transparent representations of the properties of the step process. For instance, the algebraic equation $\mathcal{L}f(r) = 0$ for the IRR defines a bijection IRR $\leftrightarrow \Omega$ at fixed $\mathcal{L}f(r)$.

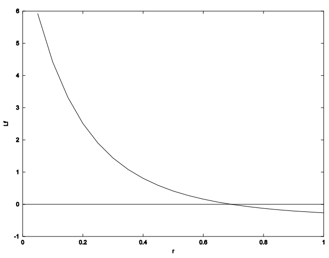

Figure 2. Plot of the function $\mathcal{L}f(r) = -\frac{1-e^{-r}}{r} + \frac{e^{-r} - e^{-r\Omega}}{r}$ for $\Omega = 10$. The function vanishes at $\bar{r} \cong 0.68$.

It is not difficult to convince oneself that the Hicks-Macaulay duration of the process represented by (10) results in

$$\Theta[f(t),r] \equiv \theta(r,\Omega) = -\frac{1}{r} + \frac{2e^{-r} - e^{-r\Omega}}{1 - 2e^{-r} - e^{-r\Omega}} \tag{11}$$

which defines a simple bijection $\theta \leftrightarrow \Omega$ at fixed r. In fact, Hicks' (1973) assumption of the *simple profile* was meant to exploit the analytical simplicity of step profiles, which enhances the tractability of the problem of the *bias* of technical innovation (ivi, ch. 7).

Strictly speaking, technical progress is not ruled out by the assumption of a fixed profile; after all, progresses in the efficiency of production may be offset, in economic terms, by changes in the relative price of input. In fact, a more general framework than the one fixed by Proposition 1 entails an expected profile of evolution for the functional form $f(t)$. Suppose technological progress is expected to ameliorate the profile in historical time T according to the explicit function $\psi(t;T)$, such that $\psi(t;0) = f(t)$, and with increasing IRR: Then, (6) generalizes to $V(r) = V_0 + \mathcal{L}(x\mathcal{L}\psi(T))(r)$. Evidently, a tractable analytical representation of the general case in obtained once progress can be represented in the additive form $\psi(t;T) = f(t) + \varphi(T)$, so that the previous expression can be written

$$V(r) = \mathcal{L}y(r)\mathcal{L}f(r) + \mathcal{L}x(r)\mathcal{L}f(r) + \mathcal{L}(x\varphi)(r) \tag{12}$$

Such a direction of analysis lies beyond our present goals.

To sum up, the NA convolution structure (2) of value creation sets a significant perspective on DCF valuation. Investments are well known to entail delay in the creation of value (think for instance of the concept of payback period); still, the theoretical content of formula (6) entails something more, namely, the consistency of the investment plan, i.e. its sustainability contingent on the discount rate, which in fact amounts to a (stylized) representation of *growth*. True, on applicative grounds, in order to fix the sustainability of a NA going concern, one should add to Proposition 1 the hypothesis that only the stylized present values in formula (6) matter, and no matching of maturities may trigger technical insolvency. In fact, enlarging the valuation setting to the financing side of the enterprise, our scale and scope formula may define a pregnant framework for the combined representation of both assets and liability in the sustainability of economic activity; in section 5 we sketch a preliminary exploration of such a perspective concerning the representation of balance sheet recessions.

4 A NEO-AUSTRAN PERSPECTIVE ON THE VALUE OF INVESTMENT

As pointed out by Tobin and Brainard (1977), “The valuation of the business as a whole as a going concern is generally much more relevant than the separate valuations of the assets on used goods markets.” The relevance of such a difference is connected with the uniqueness of the earning power embodied by a going concern; as is well known, for a number of reasons, the same assets in place should not be expected to yield exactly the same returns (among the reasons, evidently, goodwill and the value of management stand out). The relevance follows of measuring the distribution of such returns across sectors and players. Such comparisons, though, call for proper normalization.

As is well known, Tobin’s q is meant to fix such ideas by means of the dimensionless ratio between the “market value” of a going concern and the “substitution value” (“replacement cost”) of its assets in place, taken as normalization for the scale of operation. At the aggregate level, typical are the figures for the US stock market q, built out of the Financial Accounts of the United States (Federal Reserve Statistical Release).

A value of q exceeding 1 can be taken as a measure of the market power of the enterprise (Lindenberg and Ross, 1981), whereas a q lower than 1 may signal a (perhaps temporary) misallocation of scarce resources. True, Tobin and Brainard (1977) were not driven by such microeconomic perspective, and were in fact interested in the macroeconomic problem of the inducement to invest driven by a market rate, which may trigger the switch between investment in financial or real assets. Evidently, a temporary $q > 1$ may simply embody an arbitrage opportunity generated by market volatility; as Keynes (1936) put it: “there is an inducement to spend on a new project what may seem an extravagant sum, if it can be floated off on the Stock Exchange at an immediate profit.” The reliability of q as predictor of investment rounds has been the subject of empirical research, to the authors’ knowledge, with nondecisive evidence.

It has been long argued that the identification of replacement costs is far from straightforward. Book measures for fixed assets are typically taken as initial data for algorithms accounting for the effects of capital depreciation, inventory, and inflation. Thus, the *cardinal* status of Tobin's q does depend on such algorithms (and, evidently, on the properness of book values); in turn, such a criticality reverberates on the *ordinal* significance of q as a ranking criterion for business performance. It is not our aim, evidently, to review the subtleties involved in the identification of replacement costs (among others, Lewellen and Badrinath, 1997; Lee and Tompkins, 1999); still, on general grounds, the cycles inherent to the inventory dynamics and fixed asset replacement make the NA framework at least a stimulating idea to look at the sequential pattern of investment and production. In fact, it is our aim to fix a theoretical NA perspective meant to complement Tobin's q with respect to the value of growth prospects. Our proposal builds on the following premises.

On conceptual grounds, the present value of expected earnings in a long run perspective inherently embeds expectations about growth prospects, in which costs of asset replacement are somehow accounted for; the point is clearly discussed by Dixit and Pindyck (1994, 5.2.C) in terms of the growth options embedded in the value of a going concern. On analytical grounds, our scale and scope formula (6) enables us to factorize the profitability connected with the IRR of the production technique, which may be washed out upon considering definite dimensionless ratios build out of the terms in formulas of the type (6).

Evidently, on account of the properties of our NA assumptions, we face neither diminishing returns nor adjustment costs, so that marginal q can be taken to coincide with average q. Given such premises, we need to specify a philosophy underlying our "NAq-measures"; Tobin's q is meant to measure "the value the expected future in units of the present", where the present is measured by replacement cost; our aim is to identify different "units of the present" as normalizations for Tobin's q numerator.

In fact, as for the numerator of Tobin's q, our value function (6) provides a consistent DCF approach to the present value of a NA production unit running the process *f*, once such market value is meant to reflect in first instance the present value of an expected growth prospect. Then, consider the value generated by the processes already started (and still alive) as the normalization. One is thereby led to

Definition 1. The ratio of the prospect value $\mathcal{L}x(r)\mathcal{L}f(r)$ to the current value $\mathcal{L}y(r)\mathcal{L}f(r)$ defines the measure

$$NAq_1 \equiv \frac{\mathcal{L}x(r)}{\mathcal{L}y(r)} \tag{13}$$

of expected performance.

The normalization $\mathcal{L}y(r)\mathcal{L}f(r)$ embodies a ‘faithful’ interpretation of the NA process as a representation of the finite period in which the assets in place (typically, plants) are meant to operate. Such an approach, evidently, makes sense for Ω long enough (at least, a number of years). In such respects, q > 1 signals that a growth prospect based on re-investment on the same type of assets is feasible, and in fact profitable, in a definite range of the opportunity cost of capital. Notice that, despite the fact that the Hicksian process does not appear explicitly in formula (13), it is *implicitly* accounted for, since the profile of rate of starts is defined after a definite NA process.

Definition 1 is consistent with the dimensionless character of Tobin’s q, and paves the way to the duration analysis discussed in the previous section, which enriches considerably the significance of such a definition. True, we are in a position to envision a somewhat more general NA interpretation of expected performance. After all, Hicks (1973, chapter II) himself posits that the NA process is a *perspective* on the representation of productive capital and investment. Thus, a second measure NAq emerges once we normalize in terms of the value generated by the going concern operated forever at the constant rate of starts x_0, for which (7) holds. Such value can be considered an ‘equilibrium’ benchmark for the sequential replacement costs of assets in place, once investment is financed via internal lines. One thereby obtains our second NAq measure:

Definition 2. The ratio of the prospect value $\mathcal{L}x(r)\mathcal{L}f(r)$ to its static correspondent, i.e. the one operated forever at the initial level x_0,

$$\frac{\mathcal{L}x(r)\mathcal{L}f(r)}{\mathcal{L}x_0(r)\mathcal{L}f(r)} = \frac{r}{x_0}\mathcal{L}x(r) \equiv NAq_2 \tag{14}$$

defines our second NAq measure.

Both formulas (13) and (14) exploit the merits of the convolution theorem in decoupling scale and scope effects, and establishing consistent NAq measures. In fact, (14) seems to identify a sharper perspective on the value of a growth prospect, and its sensitivity to the interest rate. Both the numerator and the denominator pertain to a long range prospect; thus, NAq_2 embodies the value of the expected future (potential growth) in units of the present (i.e., a stationary prospect with constant activity level). It is this definition which, in the authors' view, represents the true NA spirit. With that said, both (13) and (14) tailor an insightful microfoundation for performance via a normalized value of growth prospects.

As the elementary example, consider a steady state growth of prospect rate of starts $x(T) = x_0 e^{gT}$, for which (14) reduces to $\frac{r}{r-g}$. One obtains the expected result that the higher g the higher q, with q diverging for $g \to r$. Such a results is hardly surprising; in fact, it is meant to confirm the soundness of our approach.

As a further remark, one should not forget that it is typical to envision a major source of potential for value creation in technical progress. In such respects, notice that, corresponding to (12), one can obtain further NAq expressions accounting for progress. We shall not pursue such a line of inquiry in the present context; still, the analytical efficacy of our NA approach seem promising.

How about estimating the above NAq measures? At first glance, no replacement costs are explicitly at stake, and the properties of the production technique are only implicitly accounted for in formulas (13) and (14). On top of that, one may consider the NA framework as much too distant from ordinary accounting practices for our NAq measures to admit a feasible estimation approach, which, in first instance, would entail translating the problem of value creation of the firm into a NA process, and then translating a growth path for the business into estimates (approximations) for $\mathcal{L}x(r)$ and $\mathcal{L}y(r)$.

Noticeably, an accounting approach which compares to our framework has in fact been developed by Stauffer (1971). The Author assumes constant returns to scale and a point-input-flow-output convolution structure of value creation, which represent the *limit* of a NA convolution approach in which the construction phase shrinks to an instant. It may not be an insurmountable difficulty to extend Stauffer's approach to the general flow-input-flow-output NA framework, and thereby pave the way for estimates of our NAq measures. Recall, our measures are *not* meant to reproduce the figures for Tobin's q, but rather to complement them via an explicit representation of the present value of growth paths, meant to enlighten the profound *unity* of economic, financial and accounting problems (Shubik, 2002), which has come to the fore prominently with the last global financial crisis.

5. BALANCE SHEET RECESSION

As already anticipated, a generalization of our scale and scope formula may account for the present value of both assets and liabilities characterizing a going concern, to the extent that the associated flows can be represented via the convolution structure (1). It is therefore tempting to try and apply our generalized NA approach to the representation of balance sheet recessions, a theme which has gained centerstage in the aftermath of the last great financial crisis.

In such respects, Eggertson and Krugman (2012) discuss the relevance of the *distribution* of debt in an economy, by means of a model in which different agents have different preferences for intertemporal substitution; the Authors find, among other things, that a balance sheet recession can in fact be addressed in terms of debt, a prescription which has long been advocated by Richard Koo (2014 and references therein). Eggertson and Krugman (2012) thereby fix a major weakness of representative agent models. Our aim in this section is more modest: we aim at fixing a NA toy model for the causal link between falling output price and the trigger of a balance sheet recession; a representative agent suffices for such a goal.

Consider an economy (a representative production unit or firm) running a NA process f which entails the (negative) investment profile i in the construction phase, in which no output occurs, and the net output profile g in the utilization phase. Suppose our firm borrows in order to finance the construction phase: the profile defined by such a debt obligation d coincides with $-i$ (a positive flow) in the construction phase, and then entails a profile of negative flows in the utilization phase; write $\mathcal{L}d(r)$ for the (negative) present value of such a debt obligation. Evidently, the cost of debt lowers the present value of the going concern; the rationale for borrowing may be to restructure the time profile of expenditures.

On account of the profitability $\mathcal{L}f(r)+\mathcal{L}d(r)\geq 0$ (a function of the discount rate), the firm (economy) is in a position to sequentially roll up (a scaling of) such a debt contract. For the sake of simplicity, let the time length of the debt contract coincide with the time length of the NA process.

In order to sharpen the insights embodied by the model, let the profit profile g assume the constant value $\lambda(p)$ in the utilization phase [1, Ω], with λ increasing in p, so that the dependence of value on output price p and the discount rate r can be factorized as

$$\mathcal{L}g(r;p)=\frac{e^{-r}-e^{-r\Omega}}{r}\lambda(p) \tag{15}$$

(compare formula 10). Then, the present value of the output that will be generated by the processes already started in connection with the debt obligation, as a function of the discount rate r, is given by

$$\begin{aligned}&\mathcal{L}y(r)\mathcal{L}f(r)+\mathcal{L}y(r)\mathcal{L}d(r)\\ &=\mathcal{L}y(r)\left(\mathcal{L}i(r)+\mathcal{L}d(r)+\lambda(p)\frac{e^{-r}-e^{-r\Omega}}{r}\right)\end{aligned} \tag{16}$$

The pivotal role of the output price p is explicit: the present value of the profit profile is increasing in p, and a fall in p may seriously undermine the sustainability of the economy, which is connected with the sustainability of the debt obligation d.

In order to enhance the readability of the model, suppose both the expenditure profile in the utilization phase, and the flow debt obligation in the utlization phase are constant. Write c and d_u for the level of such flows respectively. The present value of debt can be written

$$\mathcal{L}d(r)=-\mathcal{L}i(r)-d_u\,\frac{e^{-r}-e^{-r\Omega}}{r} \tag{17}$$

and (16) reduces to

$$\mathcal{L}y(r)\mathcal{L}f(r)+\mathcal{L}y(r)\mathcal{L}d(r)=\mathcal{L}y(r)(p-c-d_u)\frac{e^{-r}-e^{-r\Omega}}{r} \tag{18}$$

It is worth emphasizing the transparency of (18) in representing the pro-cyclicality of the incentives implicit in our economy. There is an incentive to inflate a bubble in order to increase $p - c - d_u$, and then justify increasing prospects of rates of starts, which further increase the value of the economy. As long as p exceeds $c + d_u$, the economic activity is sustainable.

Then, suppose p falls below $c + d_u$, pushing he present value (18) into the negative region. Such a condition triggers a balance sheet recession. The pro-cyclicality embodied by (18) calls for a reduction of d_u in order for (18) to resurface; the policy question therefore is how to accomplish such a reduction in connection with the foreseeable consequences, given the levers that our economy can manoeuvre. In order to lower d_u a restructuring of debt may be a possibility; such a restructuring, though, breaks the perfect correlation between the level of debt and economic activity embodied by our economy (16), typically causing complex intertemporal reallocation problems. In addition, suppose a central bank is in a position to lower the discount rate r, in order to counter the fall in the profitability (IRR) of the production technique; still, according to (18), the role of the discount rate is to gauge the effect of $p - c - d_u$, which is the pivotal variable in our model; lowering the discount rate r does not help recovery.

Thus, the previous considerations enable us to sketch a story which is well known; though, our point is the transparency with which a simple NA toy model (an oversimplification of reality) seems to yield quite convincing a setup for the representation of the incentives underlying a typical economy exposed to the possibility of balance sheet recession. Noticeably, the casual link between the effective change in production technique (caused by the reduction in p) and the following path of economic activity make our economy enter a restructuring path which somehow compares to the Hicksian traverse. As already pointed out, it is this grip on sequential causality which, according to Zamagni (1984), sets the relevance of the NA approach.

According to Koo (2014), the theoretical macroeconomic relevance of balance sheet recession seems to be underestimated in the extant literature. Similar considerations are set forth by Eggertson and Krugman (2012). In the authors' view, the transparency with which our scale a scope formula represents the trigger of the recession makes our toy model worth consideration, on account of the simple link between the level of debt and the level operation of the economy represented in formula (16), which is much more than a DCF formula. All in all, the convolution structure at the core of Hicks' (1973) theory may provide useful elements for modelling progresses in macroeconomic thinking.

6. CONCLUSIONS

It was the aim of the present contribution to deepen the valuation framework inherent to the NA approach. By means of the convolution theorem we have succeeded in establishing a NA version of DCF leading to a "scale and scope" formula, which disentangles the profitabilities associated with the fundamental building blocks of Hicks' (1973) approach, namely, the process profile and the intensity of (re-)investment. A NA perspective on economic performance emerges, which parallels the celebrated metric q introduced by Tobin and Brainard. Our stylized discussion (section 5) of a balance sheet recession seems to deepen the cogency of our approach.

According to Shubik (2002), economic theory and accounting face the need of more solid grounds upon which to build a consistent framework embracing both of them. In fact, on empirical grounds, the balance sheet sustainability of organizations seems to represent a crucial element for both positive and normative analyses on the consequences of the global financial crisis sparked by the Lehman crack (September 2008). Such a tragic event has deepened concerns about the properness of current business valuation practices; for instance the opacity of collateralized mortgage obligations has been (unfortunately, too late) thoroughly discussed. In turn, the crisis has brought renewed interest in Tobin's q as a consistent measure of deviation of prices from their "fair" values, on account of its mean reverting behavior (see for instance Smithers, 2009). On top of that, the increasing complexity of the global financial environment seems to challenge standard visions on the measurement or risk and return, as witnessed for instance by the ongoing debate about accounting standards (the inherent pro-cyclicality of mark-to-market accounting being a major concern). Hopefully, the present paper may contribute pregnant elements of analysis in such respects, meant to align with Hicks' concern on the *unity* of method[8] of economic analysis.

In the authors' view, a natural line of progress for the present contribution may entail relaxing the assumption of a constant discount rate, and consider arbitrary (possibly stochastic) term structures. On purely analytical grounds, such an assumption may be translated in a redefinition of the profile of rate of starts, thereby enhancing the reach of our formulas; in fact, on conceptual grounds, such an occurrence may trigger insightful advances for Hicks' (1973) theory of capital. In addition, a stochastic generalization of the time profile of discounting may be addressed in terms of Feynman-Kac sums over all possible histories.

In fact, it is overdue to envision the possibility of employing our approach in order to generalize Hick's traverse problem. The LT approach to present values has been in fact employed in the analysis of Hicks' traverse problem (see for instance Nardini, 1994, and references therein); hopefully, our foundational approach to a NA valuation framework may help envision lines of progress for existing approaches. Hicks' *economic* approach to the problem of the traverse employs both constraints on available resources (for instance, an exogenously given time path for the primary input) and positive principles for the determination of the traverse path. Such principles in fact amount to fixing *value benchmarks*; for instance, in the fixwage case, a steady state reference path of value creation ("*Q*-assumption"; Hicks, 1973, chapter VIII) gauges the value produced by the economy along the traverse path, in that the value of Take-Out is supposed to compare exactly at any date along reference and actual paths. Enlarging the perspective to our valuation framework, which admits comparison of values in a 'longer run', may set the stage for a consistent generalization of Hicks' traverse analysis, which may in fact complement already existing progresses for the NA approach, like the activity index introduced by Nardini (1994, p. 24)

ACKNOWLEDGEMENTS

The authors gratefully acknowledge a number of stimulating discussions with Stefano Zamagni. Needless to say, the authors themselves are the sole responsible for omissions or errors.

[8] Hicks (1939, Introduction).

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