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David Andolfatto
Federal Reserve Bank of St. Louis,
Simon Fraser University,
and the Rimini
Centre for Economic Analysis.

On the Societal Benefits of Illiquid Bonds

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David Andolfatto[†]

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Abstract

Kocherlakota (2003) presents an example of a monetary economy where efficiency is enhanced with the introduction of a nominally risk-free bond that is specifically designed to be illiquid. In his environment, an asset market involving swaps of money for bonds effects a socially desirable redistribution of purchasing power that might otherwise be replicated by a policy of type-contingent money transfers.

In this paper, I recast Kocherlakota's model in a fully dynamic quasi-linear model and characterize optimal interventions when type-contingent transfers are feasible and when they are not. When they are not, an illiquid bond is essential. However, I also find that an illiquid bond may remain essential even when type-contingent transfers are feasible.

1 Introduction

Nominally risk-free interest-bearing bonds frequently coexist with non-interest-bearing money. This observation can be explained by appealing to the fact that bonds typically have physical properties that render them less liquid than money. But if this is so, then why should risk-free claims to money be designed in a such a manner? Are bonds rendered illiquid by accident or by design?

In reply to this question, Kocherlakota (2003) presents an argument for why the existence illiquid bonds may be necessary to enhance social welfare. He notes that in the equilibria of monetary models, individuals may have different intertemporal marginal rates of substitution; so that the equilibria are inefficient relative to the first-best. When this is so, the introduction of a nominal bond may allow agents to realize additional intertemporal gains to trade. A liquid bond cannot serve this purpose; in equilibrium, it becomes a perfect substitute for money and is therefore redundant. But if bonds can somehow be rendered illiquid, they will trade at a discount relative to money. The implied positive

interest rate induces those who value consumption less to save in the form of bond purchases. This allows those who value consumption more to acquire additional money to finance their consumption expenditures via bond sales.

Kocherlakota (2003) demonstrates this result in the context of a model where the welfare-enhancing properties of an illiquid bond last for one period only. Moreover, his analysis limits attention to a welfare-improving policy; he does not characterize the optimal intervention. His simple set up also abstracts from the inflationary consequences that might be associated with a policy of discounting bonds.

Recent contributions by Boel and Camera (2006), Sun (2007), and Shi (2008) overcome some or all of these limitations. One goal of my own paper is to do so as well; albeit, in what I consider to be a much simpler manner that is also much closer to Kocherlakota's original specification. The way I do so is to embed Kocherlakota's model in a quasilinear environment with competitive markets; see, for example, Rocheteau and Wright (2005).[1] My results here do not differ in substance from these earlier contributions; but they are, I think, delivered in a more transparent manner.

The second goal of my paper is to investigate the claim made by Kocherlakota (2003) that an illiquid bond policy is not necessary when type-contingent money transfers are feasible. The basic idea here is that type-contingent transfers of money might, in principle, be used to redirect purchasing power away from those with a low desire to consume toward those who have a high desire to consume (this is precisely the role that an illiquid bond plays). In Kocherlakota (2003), this is always true. But I find that this result need not hold when the inflation costs of a type-contingent transfer policy are taken into account. I demonstrate that illiquid bonds may remain essential even if type-contingent transfers are feasible.

In Section 2, I describe the economic environment and characterize the first-best allocation. Heterogeneity is generated by way of idiosyncratic shocks that affect the marginal utility of consumption over time. Agents lack commitment and are anonymous so that a fiat money instrument is essential. Trade is restricted to be voluntary, so that lump-sum taxation is infeasible.[2] Lump-sum transfers of money, however, are allowed. If preference shocks are observable, then type-contingent lump-sum transfers are feasible. If preference shocks are not observable, then these type-contingent transfers are no longer feasible.

In Section 3, I formulate the choice problems that households face under the assumption that preference shocks are observable. In Section 4, I characterize

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[1] Boel and Camera (2006) also adopt a quasilinear model. But they do so within the context of a model that features permanent differences in discount rates and trading frequencies; which renders their environment somewhat more cumbersome that it need be to demonstrate the essential result.

[2] Both Boel and Camera (2006) and Shi (2008) demonstrate that illiquid bonds can remain essential if lump-sum taxation is feasible. In the former, this is accomplished by assuming differences in discount factors; and in the latter, by assuming a trading externality.

the monetary equilibrium under a given type-contingent transfer policy and then describe an optimal policy. I demonstrate here that an optimal policy may or may not be consistent with first-best implementation.

In Section 5, I assume that preference shocks are not observable; so that type-contingent money transfers are not feasible. Following Kocherlakota (2003), I introduce a nominal risk-free bond that cannot be used to purchase output (it is this sense in which a bond is illiquid). Instead, bonds can only be traded in an asset market; this allows agents to rearrange their money-bond holdings in response to their preference shocks. I find that an optimal policy can implement the first-best allocation. Hence, an illiquid bond is essential when type-contingent transfers are not possible. Indeed, as even an optimal type-contingent transfer policy may not be consistent with first-best implementation, an illiquid bond may remain essential even when preference shocks are observable. Section 6 concludes.

2   Environment

There is a unit measure of infinitely-lived 2-member households (worker-shopper pairs). Households belong to one of two permanent types, labelled *group 1* and *group 2*. There is an equal measure of each group. Let A and B denote the sets of group 1 and group 2 households, respectively.

Each period $t = 0, 1, 2, ..., \infty$ is divided into two subperiods, labeled *day* and *night*. A household $i \in A \cup B$ has quasi-linear preferences of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[x_t(i) + \omega_t(i)u(c_t(i)) \right] \tag{1}$$

where $x_t(i) \in \mathbb{R}$ denotes consumption (production, if negative) in the day and $c_t(i) \in \mathbb{R}_+$ denotes consumption at night. Assume that $u'' < 0 < u'$, $\lim_{c \to 0} u'(c) = \infty$, $\lim_{c \to \infty} u'(c) = 0$ and $0 < \beta < 1$.

The parameter $\omega_t(i)$ represents an idiosyncratic preference shock, realized at the beginning of the night. These shocks are *i.i.d.* across households and across time. Following Kocherlakota (2003), assume that $\omega_t(i) \in \{\omega_l, \omega_h\}$, $0 < \omega_l < \omega_h < 1$; and that each type occurs with equal probability. Define $\eta \equiv \omega_h/\omega_l > 1$.

All households reside at a central location in the day. Output (utility) generated in the day is transferable and nonstorable; an aggregate resource constraint implies

$$\int_{A \cup B} x_t(i)di \leq 0; \tag{2}$$

for all $t \geq 0$.

Each household is endowed with a nonstorable output $0 < y < \infty$ at night. There are two spatially separated locations at night, labelled *location 1* and *location 2*. Subsequent to households realizing their types, workers in group 1(2) travel to location 1(2); while shoppers in group 2(1) travel to location 1(2). Hence, the two locations are symmetric in terms of the composition of types at night. The implication of this spatial structure is that households cannot consume their own output at night. The resource constraints at night are given by

$$\int_A c_t(i)di \leq \int_B ydi \text{ and } \int_B c_t(i)di \leq \int_A ydi \qquad (3)$$

Weighting all agents equally, a planner maximizes (1) subject to the resource constraints (2) and (3). As utility is linear in $x_t(i)$, agents are indifferent across any lottery over $\{x_t(i) : t \geq 0\}$ that delivers a given expected value. Without loss of generality, a planner may set $x_t(i) = 0$ for all i and all $t \geq 0$.

Assume that all agents of a given type are treated symmetrically and restrict attention to stationary allocations of the form (c_l, c_h). In this case, *ex ante* welfare is represented by

$$U(c_l, c_h) = \left(\frac{0.5}{1 - \beta}\right)[u(c_l) + \eta u(c_h)]. \qquad (4)$$

Maximizing this welfare function subject to the resource constraint $2y \geq c_l + c_h$ implies that the first-best allocation is characterized by:

$$u'(c_l^*) = \eta u'(c_h^*) \text{ and } c_l^* + c_h^* = 2y. \qquad (5)$$

Clearly, we have $c_h^* > y > c_l^* > 0$.

Following Kocherlakota (2003), I impose the following additional restrictions on this environment. First, households lack commitment and it is impossible to monitor individual trading histories, so that a fiat money instrument is essential. Second, assume that society cannot impose any penalties on individuals. Among other things, this rules out lump-sum taxation (but obviously does not preclude lump-sum transfers of money). Third, assume that trade among agents occurs on a sequence of competitive spot markets (money for goods).

Kocherlakota (2003) makes one other restriction explicit: agent types at night (ω_l, ω_h) are private information. The reason he does so is to rule out the possibility of type-contingent money transfers at night (given the lack of record-keeping, all types would strictly prefer to report a type that results in the highest transfer). In what follows, I consider both public and private information structures in turn.

Whether an illiquid bond (type-contingent money transfers) can improve welfare in this environment is not a foregone conclusion. For example, Berentsen, Camera and Waller (2007) demonstrate that there is no welfare-enhancing role for an illiquid bond in an environment very similar to the one described here.

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As it turns out, their result can be overturned by increasing the number of agent types. I demonstrate below that their result can also be overturned when there are only two types, as long as one embeds the spatial structure described above (also adopted by Kocherlakota, 2003).

3   Decision Making

Households enter the day with nominal money balances $z \geq 0$ and enter the night with nominal money balances $m \geq 0$. Let (v_1, v_2) denote the value of money in the day and night, respectively. Define real money balances $a \equiv v_1 z$ and $q \equiv v_2 m$; and define $\phi \equiv v_1/v_2$.

Let M denote the aggregate supply of money during the day. The supply of money expands at the constant (gross) rate $\mu \geq 1$. Let T_j denote type-contingent money transfers that are delivered to shoppers of each household at night and define $\tau_j \equiv v_2 T_j$. These transfers are financed by money creation; so that

$$\tau_j = (\mu_j - 1)Q$$

where $Q \equiv v_2 M$ and where $\mu_j \geq 1$. Hence, the government's budget constraint is given by $0.5\tau_l + 0.5\tau_h = (\mu - 1)Q$; or

$$0.5\mu_l + 0.5\mu_h = \mu \tag{6}$$

In what follows, I restrict attention to stationary equilibria; in which case $(v_1/v_1^+) = (v_2/v_2^+) = \mu$.

3.1   The Day Market

Let $W(a)$ denote the value of a household entering the day with real money a and let $V(q)$ denote the expected value associated with entering the night with real money q. These value functions must satisfy the following recursive relationship

$$W(a) \equiv \max_{q \geq 0} \{a - \phi q + V(q)\} \tag{7}$$

If V is strictly increasing and concave (a property that can be shown to hold in equilibrium), then the demand for real money in the day is characterized by

$$\phi = V'(\hat{q}) \tag{8}$$

That is, all households enter the night with identical real money balances $0 < \hat{q} < \infty$. Moreover, by the envelope theorem

$$W'(a) = 1 \tag{9}$$

3.2 The Night Market

Households realize their type at the beginning of the night. A shopper in one location travels to the other location with real money balances $\hat{q}$ and receives a real lump-sum transfer of money equal to τ_j. Hence, each shopper faces the cash constraint

$$\hat{q} + \tau_j - c_j \geq 0 \tag{10}$$

Let $a_j^+ \equiv v_1^+ z_j^+$ denote the real money balances taken into the next day by a household of type j. Utilizing the fact that $(v_1/v_1^+) = \mu$, we can write $a_j^+ = (\phi/\mu) [\hat{q} + \tau_j - c_j] \geq 0$. Hence, for a type $j \in \{l, h\}$ household, the choice problem at night may be formulated as

$$V_j(\hat{q}) \equiv \max_{c_j, a_j^+} \left\{ \omega_j u(c_j) + \beta W(a_j^+) : a_j^+ = (\phi/\mu) [\hat{q} + \tau_j - c_j] \geq 0 \right\} \tag{11}$$

If the cash constraint does not bind, then desired consumption is characterized by

$$\omega_j u'(\hat{c}_j) = (\phi/\mu) \beta \tag{12}$$

If the cash constraint binds, then

$$\hat{c}_j = \hat{q} + \tau_j \tag{13}$$

In either case,

$$V_j'(\hat{q}) = \omega_j u'(\hat{c}_j) \tag{14}$$

3.3 Market-Clearing

The competitive spot markets in the day and night must clear at every date; so that in a stationary equilibrium

$$\hat{q} = Q \tag{15}$$
$$\hat{c}_l + \hat{c}_h = 2y \tag{16}$$

4 Equilibrium

It is easy to verify that for any $\mu > \beta$, the cash constraint for the h type will bind tightly. Hence, there are two cases to consider: in one, both constraints bind tightly and in the other, one constraint remains slack.

Assume, for the moment, that only one constraint binds. Then from (12) and (13), the following must be true,

$$\omega_l u'(\hat{c}_l) = (\phi/\mu) \beta \text{ and } \hat{c}_h = \hat{q} + \tau_h$$

Combining (8) and (14), we have

$$\phi = 0.5\omega_l u'(\hat{c}_l) + 0.5\omega_h u'(\hat{c}_h)$$

These latter two restrictions, together with the market-clearing condition (16), imply

$$\left[2 - \left(\frac{\beta}{\mu} \right) \right] u'(\hat{c}_l) = \left(\frac{\beta}{\mu} \right) \eta u' \left(2y - \hat{c}_l \right) \tag{17}$$

Condition (17) characterizes the equilibrium allocation $\hat{c}_l$ when l type households are not cash constrained. Note that the allocation in this case depends only on μ and not on how new money is allocated across different household types; that is, the lump-sum transfers (μ_l, μ_h) matter only to the extent that they influence the overall money growth rate μ.

Proposition 1 *Assume that $\mu > 1$ and that the equilibrium allocation is characterized by (17). Then the equilibrium allocation is invariant to any type-contingent transfer policy $(\Delta\mu_l, \Delta\mu_h, \mu_j \geq 1)$ such that $\Delta\mu = 0.5\Delta\mu_l + 0.5\Delta\mu_h = 0$.*

This 'neutrality' proposition was first derived by Berentsen, Camera and Waller (2007, Proposition 2). The result is counterintuitive as the type h households are cash constrained; in particular, $\hat{c}_h = \mu_h \hat{q}$. One might expect, *a priori*, that a transfer policy $(\Delta\mu_h > 0, \Delta\mu_l < 0)$ would have the effect of alleviating their cash constraint. In fact, what happens is that any increase in μ_h is offset by a decline in the demand for money $\hat{q}$. Type l households, who are not cash constrained, simply end up saving less money; leaving them free to continue consuming $\hat{c}_l$.

Proposition 1 relies on the assumption that type l households are not cash constrained. The validity of this assumption turns out to depend on parameters.[3] To see this, assume for the moment that $\mu_j = 1$ (so that $\mu = 1$). If the cash constraint for l type households remains slack, then by appealing to (17), the equilibrium allocation is characterized by

$$(2 - \beta)u'(\hat{c}_l) = \beta\eta u' \left(2y - \hat{c}_l \right)$$

with $\hat{c}_h = \hat{q} = 2y - \hat{c}_l$.

Note that $\hat{c}_l$ is strictly increasing in β; so that $\hat{c}_h = \hat{q}$ is strictly decreasing in β. For sufficiently low β, the condition $\hat{c}_l < \hat{q}$ will be violated. Evidently, there exists a critical $\beta_0 \in (0, 1)$ defined by

$$\beta_0 \equiv \left(\frac{2}{1 + \eta} \right) \tag{18}$$

such that only one cash constraint binds for all $\beta > \beta_0$; and such that both cash constraints bind for all $\beta < \beta_0$. When both cash constraints are binding, then the equilibrium allocation is given by $\hat{c}_l = \hat{c}_h = y$.

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[3]This is not true in Berentsen, Camera and Waller (2007). In their environment, one of the two agents types remains unconstrained in any monetary equilibrium.

4.1 An Impatient Economy

By an impatient economy, I mean $\beta < \beta_0$. With zero intervention ($\mu = 1$), the cash constraints for both types of households are binding. The question I wish to address here is whether an activist monetary policy ($\mu > 1$) can be welfare improving (in which case, it is non-neutral). In what follows, I assume without loss that $\mu_l = 1$ so that $\mu_h = 2\mu - 1$.

It should be apparent that any cash transfer to the h type households will not relax the cash constraint that is binding for the the type l households; hence $\hat{c}_l = \hat{q}$. If the cash constraint continues to bind for the h households as well, then $\hat{c}_h = \mu_h \hat{q} = (2\mu - 1)\hat{q}$. When this is the case, the market-clearing condition (16) implies

$$\hat{q} + (2\mu - 1)\hat{q} = 2y$$

or $\hat{q} = y/\mu$. The equilibrium allocation is, in this case, given by

$$\hat{c}_l = \left(\frac{1}{\mu}\right) y \text{ and } \hat{c}_h = \left(2 - \frac{1}{\mu}\right) y \tag{19}$$

Condition (19) suggests the following proposition.[4]

Proposition 2 *When $\beta < \beta_0$, the type-contingent transfer policy $\mu^* = (y/c_l^*) > 1$, $\mu_h^* = (2\mu^* - 1) > 1$ and $\mu_l^* = 1$ implements the first-best allocation.*

Proposition 2 recovers the intuitive result, first expressed in Kocherlakota (2003), that type-contingent money transfers–if they are available–can be used to improve welfare when lump-sum taxation is unavailable. The implied non-neutrality of this policy hinges critically on the fact that both household types are cash constrained (and the restriction $\beta < \beta_0$ is simply a way to guarantee that this will be the case). In particular, when l type households are cash constrained, a lump-sum transfer policy does not lead them to save less as in Proposition 1 (Berentsen, Camera and Waller, 2007, Proposition 2). The effect of inflation here is to redirect purchasing power in the socially desirable direction.[5]

4.2 A Patient Economy

By an impatient economy, I mean $\beta > \beta_0$. In this case, the cash constraint may remain slack for the l types if inflation is sufficiently low. When this is the case, the equilibrium allocation is characterized by (17); so that inflation harms efficiency.

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[4]It can be shown that the cash constraint for the h type household will bind tightly for any $\beta < \mu < \mu^*$; and bind weakly at $\mu = \mu^*$.

[5]This is in contrast to the effect of inflation in Proposition 1 (Berentsen, Camera and Waller, 2007); where welfare is decreasing with higher inflation.

The prospect of Pareto-improving money transfers requires that the cash constraints for both types of households bind tightly. Intuition suggests that the cash constraint for both types of households can be made to bind for a sufficiently high rate of inflation. To explore this possibility, consider the functions

$$\hat{s}_l(\mu) \equiv \hat{q}(\mu) - \hat{c}_l(\mu)$$
$$\hat{c}_h(\mu) = (2\mu - 1)\hat{q}(\mu)$$

For $\beta > \beta_0$, we know that $\hat{s}_l(1) > 0$ and that $\hat{c}_h(1) = \hat{q}(1)$. By condition (16), the latter expression above may be written as $2y - \hat{c}_l(\mu) = (2\mu - 1)\hat{q}(\mu)$; which, when substituted into the former expression above, yields

$$\hat{s}_l(\mu) \equiv \left(\frac{2}{2\mu - 1}\right)(y - \mu\hat{c}_l(\mu))$$

Note that $\hat{s}_l(\mu)$ is monotonically decreasing in μ; this effect is both direct and indirect (as $\hat{c}_l$ is increasing in μ as long as $\hat{s}_l > 0$). Evidently, there exists an inflation rate $1 < \mu_0 < \infty$ such that $\hat{s}_l(\mu_0) = 0$.

Hence, for any $\mu \geq \mu_0$, the cash constraints for both types of households bind tightly. When this is so, we can appeal to our earlier analysis for the impatient economy to claim that the equilibrium allocation is given by (19). It follows then that if $\mu_0 \leq \mu^*$, then the first-best allocation is implementable for the patient economy under policy $\mu^* = (y/c_l^*) > 1$, $\mu_h^* = (2\mu^* - 1) > 1$ and $\mu_l^* = 1$.[6]

First-best implementation will, however, fail in the patient economy if $\mu_0 > \mu^*$. The problem here is that to render type-contingent transfers 'non-neutral,' both cash constraints must be made to bind. In turn, this may require a very high rate of inflation with an implied transfer that exceeds what is necessary to implement the first-best allocation. Still, it is possible that a constrained-efficient transfer policy may improve welfare beyond what is achievable under zero intervention. But it is also possible that zero intervention constitutes the constrained-efficient policy. Which of these two outcomes prevails depends on

$$U(\hat{c}_l(1), \hat{c}_h(1)) \gtrless U(\hat{c}_l(\mu_0), \hat{c}_h((\mu_0));$$

and this, in turn, depends on the properties of U.

5   An Illiquid Bond Market

If household types are not observable, then a type-contingent transfer policy is not feasible. But in this case, Kocherlakota (2003) demonstrates how an illiquid

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[6]One can demonstrate that for the log preferences considered by Kocherkota (2003), $\mu_0 < \mu^*$.

bond market might replicate the desired redistribution of purchasing power at night.

The government now creates two distinct tokens M and B, called money and bonds. New bonds are issued during the day at the discount price $\delta_1 \leq 1$. A bond issued during the day turns into money at the beginning of the next day. Hence, a bond represents a risk-free claim to future cash. Assuming a constant bond price δ_1, the money supply evolves according to

$$M^+ = M + B - \delta B^+$$

Assuming a constant bond-money ratio $\theta \equiv B/M$, this latter expression implies

$$\mu = \left[\frac{1+\theta}{1+\delta_1\theta} \right] \tag{20}$$

Hence, a zero discount policy ($\delta_1 = 1$) implies a constant money supply ($\mu = 1$).

5.1   The Day Market

Let b denote real bond purchases made by a household during the day. The choice problem in the day is given by

$$W(a) \equiv \max \left\{ a - \phi(q + \delta_1 b) + V(q,b) \right\} \tag{21}$$

where $V(q,b)$ now denotes the value of entering the night market with a money-bond portfolio (q,b). Desired money and bond holdings are characterized by

$$\phi \quad = \quad \frac{\partial V}{\partial q}(\hat{q}, \hat{b}) \tag{22}$$

$$\phi\delta_1 \quad = \quad \frac{\partial V}{\partial b}(\hat{q}, \hat{b}) \tag{23}$$

Once again, note that $W'(a) = 1$.

5.2   The Night Market

Following Kocherlakota (2003), assume that two competitive spot markets open in sequence at night. First, just subsequent to the realization of each household's type, an asset market is available on which money can be swapped for bonds at the discount price $\delta_2 \leq 1$. Consider a household that enters the night with portfolio $(\hat{q}, \hat{b})$. Let b_j denote the quantity of bonds sold in the asset market at night (where $b_j < 0$ denotes a purchase of bonds). As the quantity of bonds sold cannot exceed the quantity available,

$$\hat{b} - b_j \geq 0 \tag{24}$$

Subsequent to these asset market trades, a household is left with $\hat{q} + \delta_2 b_j$ units of money; this is money that can be used to purchase output from other households in the goods market. The critical assumption made here is that households may only purchase output at night by using money (it is in this sense that bonds are illiquid). This leads to the following cash constraint

$$\hat{q} + \delta_2 b_j - c_j \geq 0 \tag{25}$$

with the evolution of money balances given by

$$a_j^+ = (\phi/\mu) \left[\hat{b} - b_j + (\hat{q} + \delta_2 b_j - c_j) \right] \geq 0$$

For a household of type $j \in \{l, h\}$ then, the choice problem at night can be formulated as

$$V_j(\hat{q}, \hat{b}) \equiv \max\{\omega_j u(c_j) + \beta W(a_j^+) + [\hat{q} + \delta_2 b_j - c_j] + \xi_j \left[\hat{b} - b_j \right] \tag{26}$$

where (λ_j, ξ_j) denote the Lagrange multipliers associated with constraints (25) and (24), respectively. Desired consumption and asset trades at night are then characterized (in part) by

$$\omega_j u'(\hat{c}_j) - (\phi/\mu)\beta - \hat{\lambda}_j = 0$$
$$(\phi/\mu)\beta(\delta_2 - 1) + \delta_2 \hat{\lambda}_j - \hat{\xi}_j = 0$$

Combining these two expressions yields

$$\hat{\xi}_j = \delta_2 \omega_j u'(\hat{c}_j) - (\phi/\mu)\beta \tag{27}$$

By the envelope theorem

$$\frac{\partial}{\partial q} V_j(\hat{q}, \hat{b}) = \omega_j u'(\hat{c}_j)$$
$$\frac{\partial}{\partial b} V_j(\hat{q}, \hat{b}) = \delta_2 \omega_j u'(\hat{c}_j)$$

so that

$$\frac{\partial}{\partial q} V(\hat{q}, \hat{b}) = 0.5\omega_l u'(\hat{c}_l) + 0.5\omega_h u'(\hat{c}_h) \tag{28}$$
$$\frac{\partial}{\partial b} V(\hat{q}, \hat{b}) = \delta_2 \left[0.5\omega_l u'(\hat{c}_l) + 0.5\omega_h u'(\hat{c}_h) \right] \tag{29}$$

5.3 Market-Clearing

In addition to (15) and (16), the bond market-clearing in the day and night require

$$\hat{b} = \theta \hat{q} \tag{30}$$
$$\hat{b}_l + \hat{b}_h = 0 \tag{31}$$

5.4 Equilibrium

To begin, observe that conditions (22), (23) and (28), (29) imply that $\delta_1 = \delta_2 = \delta$.

Next, combining (22) with (28), we have $\phi = 0.5\omega_l u'(\hat{c}_l) + 0.5\omega_h u'(\hat{c}_h)$; or

$$(\beta/\mu)\,\phi = 0.5\,(\beta/\mu)\,[\omega_l u'(\hat{c}_l) + \omega_h u'(\hat{c}_h)]$$

Assuming that type l households will want to dispose of bonds ($\hat{b}_l < 0$), their bond-sale constraint (24) will remain slack so that $\hat{\xi}_l = 0$. In turn, this implies, together with (27), that

$$\delta\omega_l u'(\hat{c}_l) = (\phi/\mu)\,\beta$$

If this condition holds, then for any $\delta < 1$ it must be the case that $\hat{\lambda}_l > 0$ (so both types of households will be cash constrained). Combining this latter expression with the former, together with condition (16), yields

$$\left[2 - \left(\frac{\beta}{\delta\mu}\right)\right] u'(\hat{c}_l) = \left(\frac{\beta}{\delta\mu}\right)\eta u'(2y - \hat{c}_l) \tag{32}$$

Condition (32) describes an equilibrium relationship that must hold between $\hat{c}_l$ and δ conditional on policy μ. The first-best allocation is implemented if and only if $\delta\mu = \beta$. Is such a policy feasible?

From the government budget constraint (20)

$$\delta\mu = \theta^{-1}\,(1 + \theta - \mu)$$

Setting $\delta\mu = \beta$ implies $\beta = \theta^{-1}\,(1 + \theta - \mu)$; or

$$\mu^*(\theta) = 1 + (1 - \beta)\theta > 1 \tag{33}$$

for any $\theta > 0$. Since the bond-money ratio θ is a policy parameter, it can be chosen freely by the government. Condition (33) then describes the optimal money growth rate, conditional on any given $\theta > 0$.

By the government budget constraint (20), a strictly positive inflation implies that bonds will trade at a strictly positive discount. The equilibrium bond price is given by $\delta(\theta) = \beta/\mu^*(\theta) < 1$; which is decreasing in θ. That is, a higher bond-money ratio lowers the discount rate (increases the nominal interest rate). However, note that with policy designed to target $\delta\mu = \beta$, any strictly positive inflation and nominal interest rate will leave the real return on bonds unchanged.

Proposition 3 *In the illiquid bond economy, the first-best allocation is implementable for any bond-money ratio $1 < \theta < \infty$ and money growth rate $\mu^*(\theta) = 1 + (1 - \beta)\theta > 1$.*

Note that Proposition 3 makes no reference to β; i.e., the statement remains valid for both the impatient and patient economies studied earlier. It follows as a corollary that an illiquid bond may remain essential even if type-contingent transfers are feasible (in particular, for parameters that imply $\mu^* > \mu_0$). This latter result follows because type-contingent transfers may require an excessively high inflation rate to render transfers non-neutral. When an illiquid bond is available, the adverse consequences of high inflation are mitigated by the payment of interest on money (via the accumulation of discount bonds).

6   Conclusion

In this paper, I demonstrate that the welfare-enhancing properties of an illiquid bond can persist indefinitely. This result is consistent with Boel and Camera (2006), Sun (2007), and Shi (2008). I also find that this result is robust, in some circumstances, to whether type-contingent transfers are feasible.

In the quasilinear environment that I study, first-best implementation is possible under an appropriately designed policy that necessarily entails some inflation and a strictly positive nominal rate of interest. First-best implementation is likely to fail for more general (nonlinear) preferences; but any constrained-efficient policy is likely to require the use of illiquid bonds.

At one level, the idea that an additional constraint on trading arrangements may be socially beneficial may sound surprising. But this result is in fact consistent with the general theory of the second-best; where it is well-known that relaxing a constraint need not be welfare-improving in a second-best world.[7]

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[7] Jacklin (1987) provides a classic example of where a restriction on *ex post* trading opportunities can enhance the operation of an insurance market.

7   References

1. Berentsen, Aleksander, Camera, Gabriele and Christopher Waller (2007). "Money, Credit and Banking," *Journal of Economic Theory*, 135: 171–195.

2. Boel, Paola and Gabriele Camera (2006). "Efficient Monetary Allocations and the Illiquidity of Bonds," *Journal of Monetary Economics*, 53(7): 1693–1715.

3. Jacklin, Charles (1987). "Demand Deposits, Trading Restrictions and Risk Sharing," in Ed Prescott and Neil Wallace, eds., *Contractual Arrangements for Intertemporal Trade*, Minnesota Studies in Macroeconomics, Volume 1, University of Minnesota Press, Minneapolis.

4. Kocherlakota, Narayana (2003). "Societal Benefits of Illiquid Bonds," *Journal of Economic Theory*, 108: 179–193.

5. Rocheteau, Guillaume and Randall Wright (2005). "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica*, 73(1): 175–202.

6. Shi, Shouyong (2008). "Efficiency Improvement from Restricting the Liquidity of Nominal Bonds," *Journal of Monetary Economics* 55: 1025-1037.

7. Sun, Ling (2007). "Sustained Societal Benefits of Illiquid Bonds," Manuscript: University of Toronto.