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ENTREPRENEURIAL UNDER-DIVERSIFICATION: OVER OPTIMISM AND OVERCONFIDENCE

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Entrepreneurial Under-Diversification: Over Optimism and Overconfidence

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Abstract

Our model wants to explain how overconfidence and over optimism lead entrepreneurs to overinvest in their companies, underestimating risks and overestimating expected returns. The entrepreneur has to choose which part of her wealth to invest in her private company and which one in the stock market. Overconfidence and over optimism are parameters in our model, and they bias the entrepreneur's portfolio allocation. With a simulation analysis, we calculate overconfidence and over optimism levels implicit in the entrepreneurs' observable portfolio, instead of using proxies or indirect measures. Our explicit measure of entrepreneurial under-diversification could be used in empirical analyses.

Keywords: Entrepreneurship, Overconfidence, Over Optimism, Portfolio Optimization, Simulation Analysis, Under-Diversification

JEL Classification: G02, G11, L26

1. Introduction

Several empirical findings show that entrepreneurs often invest a large share of their personal wealth in their own company, over exposing themselves to idiosyncratic risk because their stake in the company is higher than the one that a rational risk-return analysis would suggest (Heaton and Lucas, 2000; Moskowitz and Vissing-Jørgensen, 2002). This exposure to idiosyncratic risk can be very costly (Kerins *et al*., 2004; Pattitoni *et al*., 2013). Since some studies point out that entrepreneurs demand compensation for their exposure to idiosyncratic risk (Müller, 2011), one possible explanation for this puzzling evidence – i.e., that entrepreneurs 'do not understand' idiosyncratic risk – can be ruled out.[1] Other justifications mostly rely on non-pecuniary benefits such as achieving greater control over their work environment: entrepreneurs obtain substantial rewards from being their own boss and, thus, they are willing to accept a suboptimal risk-return trade-off (Puri and Robinson, 2008; Ødegaard, 2009; Puri and Robinson, 2009; Shefrin, 2011). An alternative explanation is that entrepreneurs may invest in what they think they know better (Vardas and Xepapadeas 2012). Despite these justifications, it is still debated why entrepreneurs overinvest in their private company given the suboptimal risk-return trade-off.

While the behavioral approach has been extensively used in the field of corporate finance (e.g., managerial overconfidence), or with regard to individual or professional investors (Broihanne *et al*., 2014; Glaser and Weber, 2007) there are still few theoretical studies in the entrepreneurship field (notable exceptions are the studies collected in (Yazdipour, 2011). We intend to fill this gap, proposing a theoretical framework that proves how behavioral biases may explain the decision to become entrepreneur or to continue overinvesting in their already existing companies. We focus on overconfidence and over optimism as two of the most prominent examples of the entrepreneur's behavioral biases.

[1] For a study which analyses the role of bounded rationality in investment decisions, see Magni (2009).

Overconfidence generally leads the entrepreneur to underestimate the risk on the investment in her private company, while over optimism typically causes the entrepreneur to overvalue the return on the investment in her company. Thus, it is likely that both behavioral biases would lead the entrepreneur to overinvest in her company.

With regard to risk underestimation caused by overconfidence, we refer to Slovic's and Olsen's notion of perceived risk (Slovic, 1987; Olsen, 2011; Slovic, 2000) and to the 'two-component' total perceived risk formula proposed by Yazdipour (2011) that distinguishes between the objective ('resident risk' in Yazdipour's terminology) and subjective ('behavioral') components of risk.

We include their framework within a theoretical model that distinguishes between the objective component of the entrepreneur's private company expected future returns and the over optimistic component that leads to overestimate these returns. With over optimism, we refer to unrealistic optimism as a domain-specific bias in expectations (Weinstein 1980), rather than to dispositional optimism, i.e., the positive personality trait by which a person holds positive generalized expectations regarding the future (Scheier and Carver, 1985).

We consider both overconfidence and over optimism as parameters in our model, and show how they may affect a risk-return analyses *à la* Markowitz (Markowitz, 1952, 1959). Through this parameterization, we measure the potential bias in the entrepreneur's portfolio allocation induced by over optimism and/or overconfidence, thus explaining entrepreneurial under-diversification.[2] Using a simulation analysis, we calculate how distinct parameters of overconfidence and over optimism affect the entrepreneur's portfolio choices. Our simulations also allow calculating overconfidence and over optimism levels that, given a set of assumptions on the model parameters, are implicit in the entrepreneurs' observable portfolio choices.

[2] Under-diversification is an important and well-known issue for entrepreneurs and investors, which has both micro-level and macro-level implications (James *et al*., 2012).

2. Theoretical setup

We consider an entrepreneur that has to choose her portfolio allocation, i.e., which part of her wealth to invest in her private company and which to invest in the stock market. The portfolio optimization problem is dual: either the entrepreneur minimizes the risk for a given portfolio expected return, or she maximizes the return for a given portfolio risk. In the following sections, we discuss the impact of overconfidence on risk minimization, and the impact of over optimism on return maximization.

2.1 Overconfidence

2.1.1 Risk minimization

Consider an entrepreneur who holds a portfolio composed of two risky assets with weights $\boldsymbol{\omega} = [\omega_I, \omega_M]'$ and a risk free asset with weight ω_F. Asset I is the entrepreneur investment in her private company, while asset *M* is the entrepreneur investment in a well-diversified stock market portfolio.

The excess return of the entrepreneur's portfolio can be expressed by $\mu_P = \boldsymbol{\omega}'\boldsymbol{\mu}$, where $\boldsymbol{\mu} = [\mu_I, \mu_M]'$ is the vector of the excess returns over the risk free rate r_F. The portfolio variance is given by $\sigma_P^2 = \boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega}$, where $\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_I^2 & \sigma_{IM} \\ \sigma_{IM} & \sigma_M^2 \end{bmatrix}$ represents the positive-definite variance-covariance matrix of the returns of risky assets with $\det \boldsymbol{\Sigma} = \sigma_I^2 \sigma_M^2 - \sigma_{IM}^2 > 0$.

For a given value of portfolio expected excess return, $\mu_P = k$, the entrepreneur prefers the portfolio with the lowest variance. She faces the problem

$$\begin{cases} \min \boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega} \\ \boldsymbol{\omega}'\boldsymbol{\mu} = k \end{cases} \tag{1}$$

Note that the constraint $\omega_I + \omega_M + \omega_F = 1$ is implicit in $\boldsymbol{\omega}'\boldsymbol{\mu} = k$.

Setting up the Lagrangian and solving the problem (Pattitoni and Savioli, 2011), the optimal portfolio weights are

$$\boldsymbol{\omega}(k)=\frac{k\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}} \tag{2}$$

The first element of $\boldsymbol{\omega}(k)$ represents the weight in the private company, namely

$$\begin{aligned}\omega_I(k) &= \frac{k\left(\sigma_M^2\mu_I-\rho_{IM}\sigma_I\sigma_M\mu_M\right)}{\sigma_M^2\mu_I^2-2\rho_{IM}\sigma_I\sigma_M\mu_I\mu_M+\sigma_I^2\mu_M^2}\\ &= \frac{k\sigma_M^2\alpha}{\sigma_M^2\mu_I^2-2\rho_{IM}\sigma_I\sigma_M\mu_I\mu_M+\sigma_I^2\mu_M^2}\end{aligned} \tag{3}$$

where $\rho_{IM}=\sigma_{IM}/(\sigma_I\sigma_M)$ and α is the Jensen's alpha, i.e. $\alpha=\mu_I-\rho_{IM}(\sigma_I/\sigma_M)\mu_M$.

2.1.2 Overconfidence-driven under-diversification

Overconfidence causes the entrepreneur to undervalue the actual risk of the investment in her private company. In this case, the biased standard deviation of the company returns, indicated by $\tilde{\sigma}_I$,[3] is lower than the actual standard deviation, i.e., $\tilde{\sigma}_I<\sigma_I$.[4] We model $\tilde{\sigma}_I$ as

$$\tilde{\sigma}_I=\sigma_I(1-\delta_C),\delta_C\in[0,1) \tag{4}$$

where δ_C is the overconfidence parameter, ranging from 0 (no overconfidence) to 1 (maximum overconfidence).[5] When δ_C tends to 1, then $\tilde{\sigma}_I$ tends to zero. To analyze the impact on the portfolio weight in the private company caused by overconfidence, we define $\tilde{\omega}_I(k)$ as $\omega_I(k)$ in Equation (3) with $\tilde{\sigma}_I$ in place of σ_I, i.e., with biased standard deviation instead of the rational one, to take into account that overconfident entrepreneurs tend to underestimate the risk of their company. We then consider two cases: first, the private

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[3] From now on, the tilde over a symbol (e.g., $\tilde{\sigma}_I$) indicates a biased parameter or variable.

[4] Since the covariance between the private company returns and the market returns is given by $\sigma_{IM}=\rho_{IM}\sigma_I\sigma_M$, if the perceived standard deviation of the company returns, $\tilde{\sigma}_I$, differs from the actual one, σ_I, then overconfidence leads to a biased perception of the covariance, $\tilde{\sigma}_{IM}$.

[5] Choosing $\delta_C\in(-\infty,1)$, we would allow for underconfidence.

company returns and the market portfolio returns are uncorrelated; second, they are correlated either negatively or positively .

Case 1: Uncorrelated returns ($\rho_{IM} = 0$)

In this case, $\partial\tilde{\omega}_I(k)/\partial\delta_C > 0$. Thus, when the private company returns and the market returns are uncorrelated, then the overconfident entrepreneur tends to overinvest in her private company and to be under-diversified. The overconfidence bias is thus positive and equal to

$$b_c = \tilde{\omega}_I(k) - \omega_I(k) > 0 \tag{5}$$

Case 2: Correlated returns ($\rho_{IM} \neq 0$)

Using the definition of $\tilde{\omega}_I(k)$, we get the partial derivative

$$\frac{\partial\tilde{\omega}_I(k)}{\partial\delta_C} = \frac{k\sigma_I\sigma_M\mu_M\left[2\sigma_I(1-\delta_C)\sigma_M\mu_I\mu_M - \rho_{IM}\sigma_M^2\mu_I^2 - \rho_{IM}\sigma_I^2(1-\delta_C)^2\mu_M^2\right]}{\left[\sigma_M^2\mu_I^2 - 2\rho_{IM}\sigma_I(1-\delta_C)\sigma_M\mu_I\mu_M + \sigma_I^2(1-\delta_C)^2\mu_M^2\right]^2} \tag{6}$$

Looking at Equation (6), we can conveniently divide our analysis in two subcases.

Case 2.1 Negatively correlated returns ($\rho_{IM} < 0$)

When the private company returns and the market portfolio returns are negatively correlated, then it is easy to show that $\partial\tilde{\omega}_I(k)/\partial\delta_C > 0$. Therefore, the result in Equation (5) continues to hold, i.e., the entrepreneur underestimates the risk of her private company and overinvest in it.

Case 2.2 Positively correlated returns ($\rho_{IM} > 0$)

When the private company returns and the market portfolio returns are positively correlated, the sign of $\partial\tilde{\omega}_I(k)/\partial\delta_C$ is not straightforward. Imposing the condition $\partial\tilde{\omega}_I(k)/\partial\delta_C = 0$, we find two stationary points. In the space $(\delta_C, \tilde{\omega}_I)$, the coordinates of these two points are

$$\begin{aligned}\left(\delta_C^-,\tilde{\omega}_I^-\right) &= \left(1-\frac{\mu_I}{\mu_I-\alpha}\left(1+\sqrt{1-\rho_{IM}^2}\right),\frac{k}{2\mu_I}\frac{\rho_{IM}^2}{-\sqrt{1-\rho_{IM}^2}-\left(1-\rho_{IM}^2\right)}\right)\\ \left(\delta_C^+,\tilde{\omega}_I^+\right) &= \left(1-\frac{\mu_I}{\mu_I-\alpha}\left(1-\sqrt{1-\rho_{IM}^2}\right),\frac{k}{2\mu_I}\frac{\rho_{IM}^2}{\sqrt{1-\rho_{IM}^2}-\left(1-\rho_{IM}^2\right)}\right)\end{aligned} \tag{7}$$

Since $\left(\delta_C^-,\tilde{\omega}_I^-\right)$ is a minimum and $\left(\delta_C^+,\tilde{\omega}_I^+\right)$ is a maximum, $\partial\tilde{\omega}_I(k)/\partial\delta_C > 0$ in the interval $\left(\delta_C^-,\delta_C^+\right)$, and $\partial\tilde{\omega}_I(k)/\partial\delta_C \leq 0$ elsewhere. We notice that $\alpha > 0$ is a sufficient condition for $\delta_C^- < 0$ to hold.[6] However, we assume $\delta_C \in [0,1)$. Thus, $\tilde{\omega}_I$ reaches its minimum when $\delta_C = 0$ and $\tilde{\omega}_I = \omega_I$. When overconfidence approaches its limiting value, we find a particular weight

$$\lim_{\delta_C \to 1} \tilde{\omega}_I(k) = \frac{k}{\mu_I} \tag{8}$$

When $\delta_C \in \left(\delta_C^+,1\right)$, $\partial\tilde{\omega}_I(k)/\partial\delta_C < 0$. In this case, the entrepreneur invests so much in her private company that, to meet the constraints of the portfolio selection problem, the weight in the well-diversified market portfolio needs to be negative,[7] i.e., she should sell the market portfolio short. However, we can consider short selling of the market portfolio as not significant from an economic point of view.

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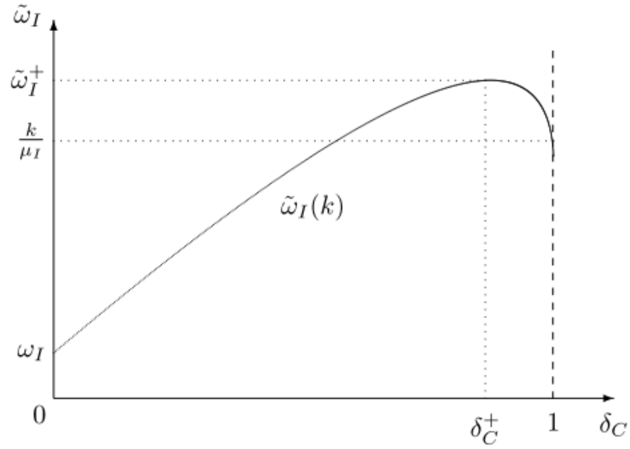
[6] $\alpha > 0$ is a prerequisite to justify investments in private companies because it can be interpreted as a positive Net Present Value (NPV > 0). If financial markets are efficient, then positive NPV investments are feasible only for real investment projects (e.g., investing in the entrepreneur's private company), and not for financial investment projects, for which the NPV should be zero.

[7] Using the constraint in the problem (1), we get $\tilde{\omega}_M = \left(k-\tilde{\omega}_I\mu_I\right)/\mu_M$. Therefore, when overconfidence reaches its limiting value and $\tilde{\omega}_I(k) = k/\mu_I$, then $\tilde{\omega}_M = 0$. When $\tilde{\omega}_I(k) > k/\mu_I$, as happens for $\delta_C \in \left(\delta_C^+,1\right)$, then $\tilde{\omega}_M < 0$. Imposing $\tilde{\omega}_M \geq 0$ means that only corner solutions are possible, where the maximum weight that the overconfident entrepreneur may invest in her company is given by $\tilde{\omega}_I(k) = k/\mu_I$. Thus, the overconfident entrepreneur may even be frustrated by not being able, without short selling the market portfolio, to invest the desired amount of wealth in her private company.

Thus, excluding this extreme case,[8] there is no ambiguity on the sign of the derivative, and we may conclude that, in general, overconfidence leads to overinvestment in the entrepreneur's private company and to portfolio under-diversification. Figure 1 offers a graphical representation of all the aforementioned results.

Figure 1. Private company weight in case of overconfidence



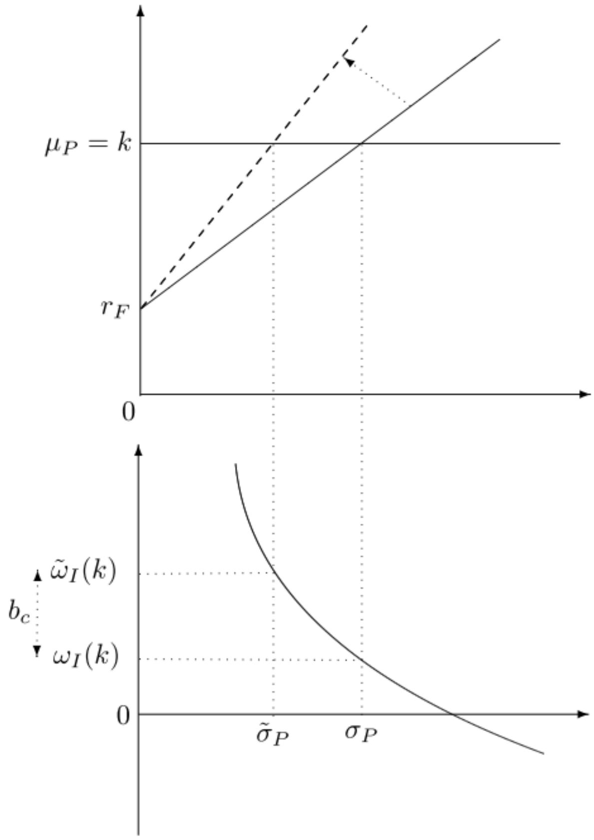
Overconfidence implies suboptimal portfolio weights and a biased perception of portfolio risk. Since the perceived private company risk decreases with the level of overconfidence (i.e., $\partial\tilde{\sigma}_I/\partial\delta_C = -\sigma_I < 0$), whenever the perceived portfolio risk increases with the perceived private company risk (i.e., $\partial\tilde{\sigma}_P/\partial\tilde{\sigma}_I > 0$) and overconfidence leads to overinvest in the

[8] In this extreme case, $\tilde{\omega}_I(k) > 0$, $\tilde{\omega}_M < 0$, and $\rho_{IM} > 0$. Since the portfolio variance is given by the formula $\tilde{\omega}_I^2\sigma_I{}^2(1-\delta_C)^2 + 2\tilde{\omega}_I\tilde{\omega}_M\rho_{IM}\sigma_I(1-\delta_C)\sigma_M + \tilde{\omega}_M^2\sigma_M^2$, following an increase in δ_C, the change in the contribution of the covariance term (the second term in the expression) to the portfolio variance is positive and dominates the change in the contribution of the variance term (the first term in the expression), which is negative. Thus, the higher the overconfidence, the higher the perceived portfolio risk for a given $\tilde{\omega}_I$ and, thus, the lower the $\tilde{\omega}_I$ chosen by the entrepreneur. Of course, this case is at the opposite of what previous studies have shown, i.e., that overconfidence leads to underestimation of risk. Thus, we can consider this extreme case as not significant from a theoretical point of view.

company (i.e., $\partial\tilde{\omega}_I(k)/\partial\delta_C > 0$),[9] then it follows that $\partial\tilde{\omega}_I(k)/\partial\tilde{\sigma}_P = (\partial\tilde{\omega}_I(k)/\partial\delta_C)(\partial\delta_C/\partial\tilde{\sigma}_I)(\partial\tilde{\sigma}_I/\partial\tilde{\sigma}_P) < 0$.

Figure 2 presents this result, showing the link between the perceived frontier of investments (the dashed line) and the weight in the entrepreneur's private company.

Figure 2. Frontier shift and overconfidence bias



The first plot in Figure 2 shows the shift in the frontier caused by overconfidence, while the second one projects this shift in the private company weight. Note that the slope of the curve in the second plot is determined by $\partial\tilde{\omega}_I(k)/\partial\tilde{\sigma}_P$.

All the aforementioned results describe the effects of under evaluating the actual risk due to overconfidence and are summed up in the following observation.

[9] That is the most common case. Conversely, in case 2.2, when $\delta_C \in (\delta_C^+, 1)$ (i.e., when the entrepreneur should sell the market portfolio short), $\partial\tilde{\sigma}_P/\partial\tilde{\sigma}_I < 0$ but also $\partial\tilde{\omega}_I(k)/\partial\delta_C < 0$. Therefore, the inequality $\partial\tilde{\omega}_I(k)/\partial\tilde{\sigma}_P < 0$ still holds.

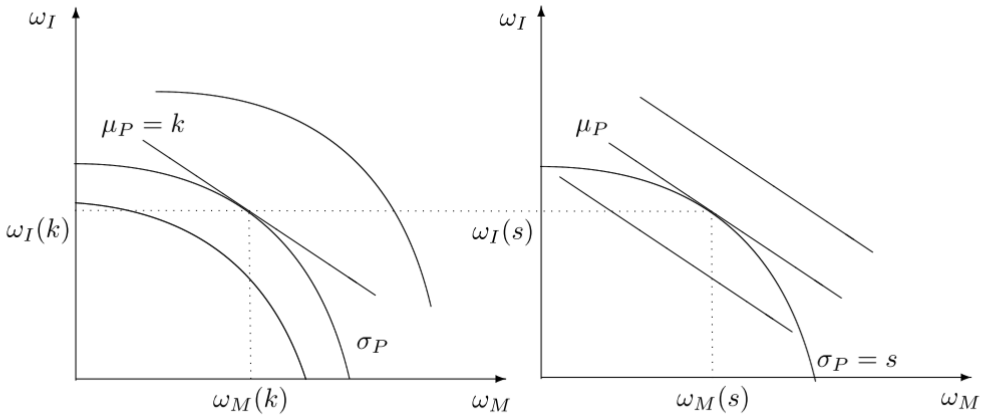
Observation 1. Typically, overconfidence leads the entrepreneur to overinvest in her company, $\partial\tilde{\omega}_I(k)/\partial\delta_C > 0$, and to under-diversify her portfolio. The only case in which $\partial\tilde{\omega}_I(k)/\partial\delta_C < 0$ occurs is when $\rho_{IM} > 0$ and the level of overconfidence is particularly high, $\delta_C \in \left(\delta_C^+, 1\right)$.

2.2 Over optimism

Since overconfidence affects risk perception, we studied its effect on portfolio risk using a risk minimization approach, which keeps the expected return level fixed. In the following, we analyze the effect of over optimism on portfolio expected return using a return maximization approach, which holds the objective risk constant.

Figure 3 shows the duality of the problem showing the tangency conditions that identify lower iso-risk (left plot) and upper iso-return (right plot).

Figure 3. Duality in portfolio optimization



2.2.1 Return maximization

The duality of the problem allows considering return maximization as the solution for the entrepreneur's optimization problem for a given value of portfolio risk, $\sigma_P^2 = s^2$. In such a setting, the entrepreneur faces the problem

$$\begin{cases} \max \boldsymbol{\omega}'\boldsymbol{\mu} \\ \boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega} = s^2 \end{cases} \tag{9}$$

Setting up the Lagrangian and solving the problem, the optimal portfolio weights are[10]

$$\boldsymbol{\omega}(s) = \frac{s\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\left(\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}\right)^{\frac{1}{2}}} \tag{10}$$

The weight in the private company is

$$\begin{aligned} \omega_I(s) &= \frac{s\left(\sigma_M^2\mu_I - \sigma_{IM}\mu_M\right)}{\left[\left(\sigma_I^2\sigma_M^2 - \sigma_{IM}^2\right)\left(\sigma_M^2\mu_I^2 - 2\sigma_{IM}\mu_I\mu_M + \sigma_I^2\mu_M^2\right)\right]^{\frac{1}{2}}} \\ &= \frac{s\sigma_M^2\alpha}{\left[\left(\sigma_I^2\sigma_M^2 - \sigma_{IM}^2\right)\left(\sigma_M^2\mu_I^2 - 2\sigma_{IM}\mu_I\mu_M + \sigma_I^2\mu_M^2\right)\right]^{\frac{1}{2}}} \end{aligned} \tag{11}$$

2.2.2 Over optimism-driven under-diversification

Over optimism causes the entrepreneur to overestimate the actual return of the investment in her private company. In this case, the biased expected return, indicated by $\tilde{\mu}_I$, is larger than the actual expected return, i.e., $\tilde{\mu}_I > \mu_I$.[11]

We model $\tilde{\mu}_I$ as

$$\tilde{\mu}_I = \frac{\mu_I}{1-\delta_O}, \delta_O \in [0,1) \tag{12}$$

where δ_O is the over optimism parameter, which ranges from 0 (no over optimism) to 1 (maximum over optimism). When δ_O tends to 1, then $\tilde{\mu}_I$ tends to infinity.[12]

In order to see the variation of the portfolio weight in her private company in case of over optimism, we define $\tilde{\omega}_I(s)$ as the $\omega_I(s)$ in Equation (11) with $\tilde{\mu}_I$ in place of μ_I, to

[10] As the problem is quadratic, we also obtain a second solution with weights equal to minus those of Equation (10). We discard them since they are dominated ($\alpha > 0 \Rightarrow \omega_I(s) > 0$).

[11] The justifications of entrepreneur's under-diversification based on non-pecuniary benefits as the desire for control can be modeled by varying μ_I as well. In that case, the 'biased' μ_I would incorporate the value of non-pecuniary benefits.

[12] Choosing $\delta_O \in (-\infty,1)$, we would allow for under optimism.

underline that over optimistic entrepreneurs will overestimate future expected returns from their company. Using this definition, we get the partial derivative

$$\frac{\partial\tilde{\omega}_I(s)}{\partial\delta_O}=\frac{s\,\frac{\mu_I}{(1-\delta_O)^2}\,\mu_M^2\left(\sigma_I^2\sigma_M^2-\sigma_{IM}^2\right)^{\frac{1}{2}}}{\left[\sigma_M^2\,\frac{\mu_I^2}{(1-\delta_O)^2}-2\sigma_{IM}\,\frac{\mu_I}{(1-\delta_O)}\mu_M+\sigma_I^2\mu_M^2\right]^{\frac{3}{2}}} \tag{13}$$

Since $\partial\tilde{\omega}_I(s)/\partial\delta_O>0$, the over optimist entrepreneur tends to overinvest in her private company and to be under-diversified. The over optimism bias is

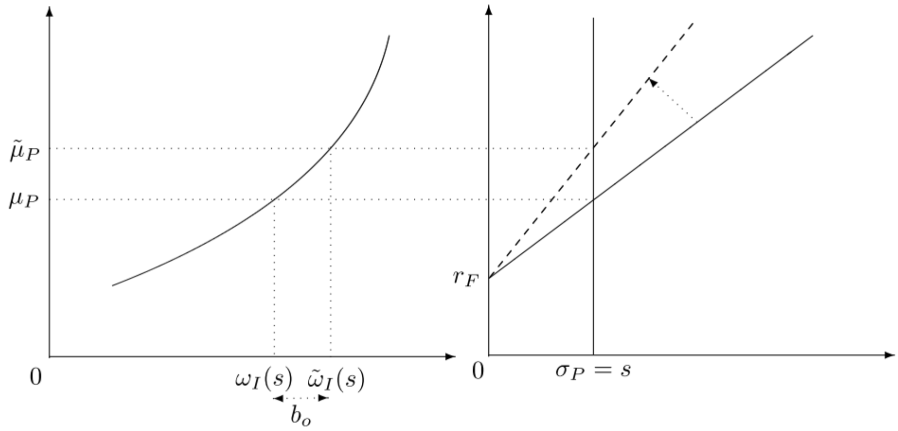
$$b_o=\tilde{\omega}_I(s)-\omega_I(s)>0 \tag{14}$$

The limit case for over optimism identifies a particular weight

$$\lim_{\delta_O\to 1}\tilde{\omega}_I(s)=\frac{s\sigma_M}{\left(\sigma_I^2\sigma_M^2-\sigma_{IM}^2\right)^{\frac{1}{2}}} \tag{15}$$

Over optimism implies suboptimal portfolio weights and a biased perception of portfolio return. Since the perceived portfolio return increases with the perceived private company return (i.e., $\partial\tilde{\mu}_P/\partial\tilde{\mu}_I>0$), the perceived private company return increases with the level of over optimism (i.e., $\partial\tilde{\mu}_I/\partial\delta_O=\mu_I/(1-\delta_O)^2>0$), as well as the weight in the private company (i.e., $\partial\tilde{\omega}_I(s)/\partial\delta_O>0$). Then, it follows that $\partial\tilde{\omega}_I(s)/\partial\tilde{\mu}_p=(\partial\tilde{\omega}_I(s)/\partial\delta_O)(\partial\delta_O/\partial\tilde{\mu}_I)(\partial\tilde{\mu}_I/\partial\tilde{\mu}_P)>0$, i.e., the upwardly biased expectations on her private company returns lead the entrepreneur to over-invest in the company. This result is presented in Figure 4, which shows the link between the perceived frontier of investments (dashed) and the weight in the private company.

Figure 4. Frontier shift and over optimism bias.



The plot on the right of Figure 4 shows the shift in the frontier caused by over optimism; the plot on the left projects this shift on the private company weight. Note that the slope of the curve in the plot on the left is determined, as explained above, by $\partial\tilde{\omega}_I(s)/\partial\tilde{\mu}_p$. These results describe the effects of the overestimation of the actual return due to over optimism and can be summarized in the following observation.

Observation 2. Over optimism always leads to overinvestment in the entrepreneur's private company, $\partial\tilde{\omega}_I(s)/\partial\delta_O > 0$, and to portfolio under-diversification.

2.3 Implicit overconfidence and over optimism levels

To understand the effects of overconfidence and over optimism on the weight invested by the entrepreneur in her private company, we consider a couple of numerical examples. Using the estimates in tables 2 and 4 of Kerins *et al*. (2004) who analyzed a sample of IPOs in technologically-oriented industries, we choose the following set of parameters: $\sigma_I = 1.204$, $\sigma_M = 0.162$, $\rho_{IM} = 0.195$, $\mu_I = 0.535$, and $\mu_M = 0.06$. We set $k = 0.300$ and $s = 0.566$. Choosing *k*, we implicitly determine *s*. Based on this set of parameters, in Table 1, we show $\tilde{\omega}_I$, $\tilde{\omega}_M$, and $\tilde{\omega}_F$ by varying the level of overconfidence (Panel A) or the level of over optimism (Panel B).

Table 1. Implicit overconfidence and over optimism levels – parameters from Kerins *et al.* (2004)

Panel A – Overconfidence effects on risk minimization						
δ_C	0.0	0.2	0.4	0.6	0.8	$\cong 1.0$
$\tilde{\omega}_I$	0.343	0.412	0.480	0.534	0.563	0.561
$\tilde{\omega}_M$	1.943	1.328	0.723	0.235	-0.024	0.000
$\tilde{\omega}_F$	-1.286	-0.740	-0.203	0.230	0.461	0.439
$\tilde{\mu}_P$	0.300	0.300	0.300	0.300	0.300	0.300
$\tilde{\sigma}_P$	0.566	0.487	0.387	0.267	0.135	0.000
Panel B – Over optimism effects on return maximization						
δ_O	0.0	0.2	0.4	0.6	0.8	$\cong 1.0$
$\tilde{\omega}_I$	0.343	0.383	0.421	0.452	0.472	0.479
$\tilde{\omega}_M$	1.943	1.544	1.058	0.498	-0.103	-0.694
$\tilde{\omega}_F$	-1.286	-0.927	-0.479	0.050	0.630	1.215
$\tilde{\mu}_P$	0.300	0.298	0.289	0.272	0.247	0.215
$\tilde{\sigma}_P$	0.566	0.566	0.566	0.566	0.566	0.566
Parameters						
k	s	μ_I	μ_M	σ_I	σ_M	ρ_{IM}
0.300	0.566	0.535	0.06	1.204	0.162	0.195

For moderate levels of overconfidence, an increase in δ_C leads to an increase in $\tilde{\omega}_I$. Note that even low levels of overconfidence cause a severe overinvestment in the private company (e.g., when $\delta_C = 0.2$, the weight in the private company is about 20% larger than it should be if the entrepreneur was not overconfident).[13] When the level of overconfidence tends to its limiting value (i.e., $\delta_C \to 1$), $\tilde{\omega}_I = 0.561$ and $\tilde{\omega}_M = 0$. When $\tilde{\omega}_I > 0.561$ (i.e., when δ_C is about 0.8), $\tilde{\omega}_M < 0$, implying that the entrepreneur is short selling the market portfolio. When selling short is not allowed, $\tilde{\omega}_I$ is thus equal to 0.561. Considering the over optimism

[13] This result comes from the comparison between the values assumed by $\tilde{\omega}_I$ when δ_C varies. When $\delta_C = 0$, $\tilde{\omega}_I = 0.343$. When $\delta_C = 0.2$, $\tilde{\omega}_I = 0.412$. Then, the percentage change is (0.412 – 0.343)/0.343=20.12%.

bias, we note that, as with the case of overconfidence, for particularly high levels of over optimism (e.g., $\delta_O \cong 0.8$ or higher), $\tilde{\omega}_M < 0$.

Differently from what happens in case of overconfidence, an increase in δ_O always causes an increase in $\tilde{\omega}_I$. It is worth underlining that, in this numerical example, overconfidence leads to higher overinvestment in the company, compared to over optimism.[14] Since the value of σ_I in the previous numerical example is particularly high,[15] in Table 2 we present another numerical example using a new set of parameters that we may consider as an 'average case', in contrast with the 'extreme case' presented above.

Table 2. Implicit overconfidence and over optimism levels – our assumptions

Panel A – Overconfidence effects on risk minimization						
δ_C	0.0	0.2	0.4	0.6	0.8	$\cong 1.0$
$\tilde{\omega}_I$	0.509	0.605	0.696	0.768	0.805	0.800
$\tilde{\omega}_M$	0.727	0.488	0.259	0.079	-0.013	0.000
$\tilde{\omega}_F$	-0.236	-0.093	0.045	0.153	0.208	0.200
$\tilde{\mu}_P$	0.120	0.120	0.120	0.120	0.120	0.120
$\tilde{\sigma}_P$	0.273	0.234	0.185	0.127	0.064	0.000
Panel B – Over optimism effects on return maximization						
δ_O	0.0	0.2	0.4	0.6	0.8	$\cong 1.0$
$\tilde{\omega}_I$	0.509	0.565	0.618	0.660	0.687	0.696
$\tilde{\omega}_M$	0.727	0.571	0.383	0.169	-0.057	-0.278
$\tilde{\omega}_F$	-0.236	-0.136	0.000	0.170	0.369	0.582
$\tilde{\mu}_P$	0.120	0.119	0.116	0.109	0.100	0.088
$\tilde{\sigma}_P$	0.273	0.273	0.273	0.273	0.273	0.273
Parameters						
k	s	μ_I	μ_M	σ_I	σ_M	ρ_{IM}
0.120	0.273	0.150	0.060	0.400	0.200	0.200

[14] We can prove this claim calculating the percentage change in $\tilde{\omega}_I$ when δ_O varies, and showing that is lower than the ones calculated above for δ_C.

[15] Kerins *et al*. (2004) referred to companies going public before and during the Internet bubble.

In particular, we choose $\sigma_I = 0.40$, $\sigma_M = 0.20$, $\rho_{IM} = 0.20$, $\mu_I = 0.15$, $\mu_M = 0.06$, $k = 0.12$, and $s = 0.273$. Even if the latter results quantitatively differ from the ones in Table 1, we notice that their qualitative pattern does not change using this new set of parameters.

Note that, if we assume that our model is sufficiently adequate to describe entrepreneurs' portfolio choices under overconfidence and/or over optimism, we can also analyze Tables 1 and 2 with a "bottom-up" perspective to calculate the 'implicit' overconfidence and over optimism levels given a set of parameters and an observed $\tilde{\omega}_I$. For example, based on Table 1 – Panel A (i.e., using a risk minimization perspective with $k = 0.300$), after observing $\tilde{\omega}_I = 0.480$, we can conclude that $\delta_C = 0.4$. Calculating the implicit overconfidence and over optimism levels is useful to determine how these behavioral biases may affect entrepreneurial decisions. In other words, not only the presence of these biases, but also their magnitude impacts the entrepreneurs' portfolio weight in their private companies and, consequently, the level of idiosyncratic risk they bear due to sub-optimal decisions in portfolio formation.

3. Conclusions

Previous findings in the literature show that entrepreneurs tend to overinvest in their private company, bearing higher levels of idiosyncratic risk, with respect to what would be optimal. We propose a possible explanation for this sub-optimal behavior, based on behavioral biases, that complements other explanations, which rely on non-pecuniary benefits, such as the entrepreneurs' desire of control.

We present a theoretical model that allows not only to show, but also to measure how behavioral biases – overconfidence and over optimism representing the most notable examples – affect the fundamental variables of the risk-return analysis *à la* Markowitz and lead entrepreneurs to overinvest in their private company and hold under-diversified portfolios. Overconfidence leads to underestimation of the risk associated with the entrepreneur's private company, while over optimism to the overestimation of its expected return.

Our model contributes to the literature on risk perception, but it could be modified to consider other motivations for the entrepreneurial behavior, such as the desire for control, since the latter, like over optimism, leads to overestimation of the expected return on the private company. Since we insert both overconfidence and over optimism as parameters in our model, we perform a simulation analysis to ascertain how much the entrepreneur's decision for investing in her private company is affected by variations in the two behavioral biases. As our theoretical model predicts, the entrepreneur invests more in her company as overconfidence and over optimism increase. The simulation can also be used, given other parameters and the weight in the private company, to calculate the implicit levels of entrepreneurial overconfidence and over optimism.

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