WP 07_14

Stuart McDonald
The Universty of Queensland, Australia

Joanna Poyago-Theotoky
La Trobe University, Australia
The Rimini Centre for Economic Analysis (RCEA), Italy

GREEN TECHNOLOGY AND OPTIMAL EMISSIONS TAXATION

Green Technology and Optimal Emissions Taxation[*]

Stuart McDonald[†]

School of Economics

The Universty of Queensland

s.mcdonald@uq.edu.au

Joanna Poyago-Theotoky[‡]

School of Economics, La Trobe University

Rimini Centre for Economic Analysis (RCEA)

j.poyago-theotoky@latrobe.edu.au

March 2014

Abstract

We examine the impact of an optimal emissions tax on research and development of emission reducing green technology (E-R&D) in the presence of R&D spillovers. We show that the size and effectiveness of the optimal emissions tax depends on the type of the R&D spillover: input or output spillover. In the case of R&D input spillovers (where only knowledge spillovers are accounted for), the optimal emissions tax required to stimulate R&D is always higher than when there is an R&D output spillover (where abatement and knowledge spillovers exist si-

multaneously). We also find that optimal emissions taxation and cooperative R&D complement each other when R&D spillovers are small, leading to lower emissions.

Key Words: Environmental R&D, Green Technology, R&D Spillover, Emissions Tax.

JEL Classification: H23, L11, Q55.

1   Introduction

Public policies targeting R&D on emissions reducing technology offer the greatest scope for achieving prolonged and sustained reduction in emissions (Kneese and Schultze, 1975; Jung et al., 1996; Jaffe et al., 2003; OECD, 2010). Without the presence of some form of policy to internalize the cost of pollution, the relevant incentives are not sufficient to promote investment in green technology (environmental R&D). Benefits generated by R&D do not always accrue to the investing firm (Griliches, 1984, 1992; Jaffe, 1986, 1998). This is accentuated with emissions reducing technologies, as they provide two types of spillovers: the first is associated with the public good aspect of knowledge generated from research (standard R&D spillovers), while the second is a consequence of implementing an emissions reducing technology targeted at rectifying a public bad.

In this paper we examine the effectiveness of an emission tax policy in the presence of these two types of R&D spillover: we explore the interplay between process-focused environmental R&D (E-R&D), abatement effort and spillovers in a non-tournament R&D model. In particular, we provide a comparison of two different approaches to modelling research and development of emissions reducing (green) technology. The first approach is based on d'Aspremont and Jacquemin (1988) (AJ henceforth), where spillovers occur in abatement and firms are able to free-ride off the abatement efforts of competitors. Here the spillover operates on the output side of the E-R&D process (E-R&D output spillover).[1] The second modelling approach is based on Kamien et al. (1992) (KMZ henceforth), where the spillover operates on the input side of the the R&D process (E-

---

[1]This approach is widely used in the literature on environmental R&D (e.g., Scott (1996), Chiou and Hu (2001), Petrakis and Poyago-Theotoky (2002) and Poyago-Theotoky (2007, 2010)).

R&D input spillover). Here spillovers occur as a consequence of firms applying technology developed by other firms. This second approach, to the best of our knowledge, has not appeared in the environmental R&D literature. Hence this paper provides the first comparison of the impact of these two types of R&D spillover on abatement and emissions in the specific context of an optimal (second-best) emission tax.[2]

We find that there are substantial differences between the KMZ and AJ models. When firms do not cooperate in R&D, the AJ model predicts that when R&D spillovers are small, there is a negative relationship between the emission tax and spillovers. This implies that R&D spillovers and emission taxes offset each other, and that as R&D spillovers become large, there is an incentive for firms to engage in cooperative R&D either by sharing information or sharing costs. This trade-off does not exist in the KMZ model under non-cooperative R&D regimes and emissions taxes rise as the R&D spillover increases. The R&D input spillover always creates a disincentive when firms invest independently. By contrast, when firms cooperate in E-R&D, there is always a trade-off between the emissions tax and R&D spillovers for both models. In the AJ model, firms free-ride off the abatement effort of others, whereas in the KMZ model this is not possible. As a consequence, in the KMZ model there always exist decreasing returns to scale in abatement investment.

A simple difference in the modelling of the E-R&D process, in particular, the way

---

[2]In the context of (non-environmental) cost-reducing R&D, Amir (2000) shows that these two approaches are equivalent only if R&D spillovers are negligible. When non-negligible spillovers exist, the AJ model exaggerates the impact of cost-reducing R&D, hence Amir (2000) argues that the KMZ model is more appropriate. Hauenschild (2003), Hinloopen & Vendekerchhove (2009), Stepanova and Tesoriere (2011) and Burr et al. (2013) further explore the differences between input and output technological spillovers within the non-tournament R&D literature.

spillovers operate, leads to significant differences in policy implications and implementation. In terms of the relative effectiveness of R&D organization in combination with an emissions tax, both models show a similar outcome: R&D cooperation is more effective in reducing emissions when R&D spillovers are small (less than 0.5). When R&D spillovers are relatively large (greater than 0.5), R&D competition is more effective in reducing emissions. This indicates that there is a trade-off between the relative effectiveness of emissions taxes and cooperative R&D. Moreover, the scale of abatement and emissions reduction is quite different between the KMZ and AJ models, with the AJ model generating higher abatement, lower levels of emissions and taxes than the KMZ model under non-cooperative and cooperative R&D regimes.

The paper is set out as follows: Section 2 presents the model, explaining the differences between the AJ and KMZ approaches. Sections 3 and 4 contain the analysis using the two different approaches. Section 5 presents results on the optimal(second-best) emissions tax for the case where the government pre-commits as well as abatement and emissions. Section 6 concludes. All proofs of propositions are in the appendix.

2   The Model

We formulate a generalization of the analysis presented in Chiou and Hu (2001). Consider a Cournot duopoly producing a homogeneous good.[3] The inverse demand function for

---

[3]We restrict attention to a duopoly in the interest of a simplified presentation. The analysis presented here carries over to a model with n identical firms.

this good is given by

$$P(q_1, q_2) := a - Q, \quad Q = q_1 + q_2,$$

where a is the demand intercept and Q is the aggregate amount supplied by both firms. As is standard practice in the R&D spillovers literature, there are no fixed costs of production and the marginal cost of production is normalized to zero without loss of generality. When each firm produces q_i, it also emits pollution at the rate of $\bar{e}$ per unit of production. The cost of pollution is imposed on the firm by a linear emissions tax t. To guarantee non-negative production, $a > t\bar{e}$.

The profit function for firm i is given by

$$\pi_i = (a - (q_i + q_j) - t(\bar{e} - s(r_i, r_j; \beta))) q_i - c(r_i), \quad i, j = 1, 2, i \neq j, \tag{1}$$

where $s(r_i, r_j; \beta)$ is firm i's effective abatement expressed as a function of its own E-R&D effort r_i, firm j's effort r_j, and $c(r_i)$ denotes firm i's R&D costs of investing in abatement reducing technology. The parameter β denotes the degree with which each firm can benefit from its rival's research (spillover).

The effective abatement function $s(r_i, r_j; \beta)$ models the way in which a firm's E-R&D can reduce its marginal emission rate $\bar{e}$ and in turn, its marginal emissions tax payments. The effective abatement function $s(r_i, r_j; \beta)$ is assumed to be continuously twice differentiable, strictly increasing and concave with $s(0, 0; \beta) = 0$ and $s(r_i, r_j; \beta) \leq \bar{e}$. In addition, $c(\cdot)$ is continuously twice differentiable for $r_i \geq 0$, $c(0) = 0, c'(r_i) > 0$ and $c''(r_i) \geq 0$ for $r_i > 0, i = 1, 2$. In the sequel we use the following specification, $c(r_i) = \frac{\gamma r_i^2}{2}$, where γ

4

measures the relative efficiency of R&D investment (higher values of γ lead to steeper marginal costs of R&D, or, in other words E-R&D becomes more difficult).

The government (or regulator) pre-commits to an emissions tax by maximising society's net social surplus.[4] The simple game we present is an non-tournament R&D model consisting of the following three-stages:

1. **Stage 1:** The regulator decides on the emissions tax t, to maximize welfare

$$W = CS + PS + T - D,$$

   where CS denotes consumer surplus, PS denotes producer surplus, T denotes aggregate revenues from emissions taxes and D denotes the social cost of environmental damages caused by polluting firms.

2. **Stage 2:** Given the optimal tax t, firms choose their E-R&D with the intention to reduce pollution. This choice is based on whether firms cooperate and share costs of R&D or not. This stage is designed according to either the AJ or the KMZ approach to modelling spillovers.

3. **Stage 3:** Firms engage in Cournot competition. Their production decisions, in turn, determine total emissions.

---

[4]Alternatively, the government could make an interim decision regarding the emissions tax after firms have made their E-R&D decisions. This case corresponds to a situation where the government pursues a time-consistent policy. This is not considered in this paper, having been examined in Poyago-Theotoky (2007, 2010), albeit for a slightly different emissions structure. Petrakis and Xepapadeas (2001, 2003) provide a comparison of these two types of emissions tax structure under monopoly and Cournot oligopoly but in the absence of R&D spillovers, showing that time consistent policies generate less emissions.

For both the AJ and KMZ specifications, we explore the following four scenarios:[5]

1. **Case N (non-cooperative R&D):** Firms behave non-cooperatively in setting R&D, choosing neither to coordinate on R&D nor share information.

2. **Case NJ (non-cartelized research joint venture):** Firms behave as in Case N, choosing R&D separately. However, they share information so that the R&D spillover parameter β is set equal to one.

3. **Case C (cartelized R&D):** Firms coordinate their R&D by choosing r_i to maximize the sum of their profits $\pi_1 + \pi_2$.

4. **Case CJ (cartelized research joint venture):** Firms behave similar to Case C, choosing r_i to maximize their joint profits. In addition they also share information, so that the R&D spillover parameter β is set equal to one.

3 Cournot Market Stage Game

In the following, we use $\tilde{\beta}$ and $\bar{\beta}$ to denote spillovers in the AJ and KMZ models respectively. We use this notation because these spillovers are not equivalent in the two models and act in very different ways. In the AJ model, the spillover $\tilde{\beta}$ acts on abatement and for this reason incorporates free-riding on both abatement effort and firm specific E-R&D; while in the KMZ model, the spillover parameter $\bar{\beta}$ acts only on E-R&D expenditure.

---

[5]Note that because, in the final stage of the game, Cournot competition ensues, we use the behaviour of firms in the E-R&D stage game to describe each of these scenarios.

3.1 The AJ Model

Spillovers are regarded as leakages in technological know-how and take place in final outcomes (emission reduction). Each firm's final emission reduction is the sum of its autonomously acquired part and a fraction (equal to the spillover parameter $\tilde{\beta}$) of the other firms' part. Hence effective abatement effort is given by

$$s\left(r_i, r_j; \tilde{\beta}\right) = r_i + \tilde{\beta} r_j, \quad i, j = 1, 2, i \neq j. \tag{2}$$

Hence, each firm can reduce its marginal emission rate $\bar{e}$, and in turn its tax burden, by undertaking r_i of emissions reducing abatement.

Equilibrium profits can now be written as:

$$\pi_i(r_i, r_j) = \frac{1}{9}\left(a - t\left(\bar{e} - r_i\left(2 - \tilde{\beta}\right) + r_j\left(1 - 2\tilde{\beta}\right)\right)\right)^2 - \frac{\gamma}{2}r_i^2, \quad i, j = 1, 2, i \neq j. \tag{3}$$

The following proposition can now be stated. Proposition 1 shows that firm i's abatement has two effects; a direct effect and a strategic effect. With the direct effect, an increase in firm i's E-R&D, always leads to higher output, regardless of the extent of spillover. This is due to the simple fact that increased effort in emissions reducing technology lowers the tax burden for firm i, making it cheaper for it to produce. The strategic effect ensures that firm i's E-R&D spills over to the opponent, making the opponent's emission tax bill decrease. The strategic effect, if $\tilde{\beta} < 0.5$ and firm i's E-R&D increases, will cause firm j to decrease its output. If $\tilde{\beta} \geq 0.5$ the opposite occurs, so that an increase in firm i's E-R&D effort leads to an increase firm j's output. Hence, the size of the spillover is

important in determining the direction of change in the strategic effect that E-R&D has on output.

Proposition 1 *For a given level of emission tax t:*

1. *An increase in firm i's E-R&D, r_i, leads to an increase in output, q_i, for all values of the spillover $\tilde{\beta}$ (direct effect).*

2. *For $\tilde{\beta} > 0.5$, an increase in firm i's E-R&D output, r_i, has a positive effect on the rival firm j's output, q_j, and negative otherwise (strategic effect).*

3. *An increase in the spillover $\tilde{\beta}$ results in an increase in the output of firm i so long as firm j's E-R&D satisfies $r_j > \frac{r_i}{2}$ (otherwise output decreases).*

3.2 The KMZ Model

In this variant, a firm can reduce its marginal rate of emissions $\bar{e}$, by spending an amount y_i in abatement technology. The cost of generating this technology is given by $c(y_i) = y_i$. The abatement effort of firm i is then given by

$$s\left(y_i, y_j; \bar{\beta}\right) = \sqrt{2(y_i + \bar{\beta} y_j)/\gamma}. \tag{4}$$

There is a spillover effect from applying this technology, which is given by $\bar{\beta} y_j$, $0 \le \bar{\beta} \le 1$. The spillover $\bar{\beta}$ captures the degree to which it is possible for firm i to free ride off the technological investments (E-R&D inputs) of the other firm. This is a distinctly different interpretation to that of the AJ approach. Here each firm's final (or effective) R&D investment in emission reduction is the sum of its own (autonomous) expenditure and a

8

fixed fraction (given by the spillover parameter) of the other firm's expenditure. Hence, all spillovers are purely technological. As first noted in Amir (2000), when $\tilde{\beta} = \bar{\beta} = 0$, the following monotone transformation holds

$$r_i = \sqrt{\frac{2}{\gamma} y_i} \iff y_i = \frac{\gamma}{2} r_i^2, \quad i = 1, 2,$$

implying that the AJ and KMZ models are equivalent.

The equilibrium profit for firm i, is given by:

$$\pi_i = \frac{1}{9} \left(a - t \left(\bar{e} + \sqrt{\frac{2}{\gamma} \left(\bar{\beta} y_i + y_j \right)} - 2 \sqrt{\frac{2}{\gamma} \left(y_i + \bar{\beta} y_j \right)} \right) \right)^2 - y_i, \quad i, j = 1, 2, i \neq j. \quad (5)$$

In the presence of spillovers, it is obvious from equations (3) and (5), that the above transformation cannot work. Hence, the two specifications are not generally equivalent.

Proposition 2 is similar to Proposition 1 above.

Proposition 2 *For a given level of emission tax t:*

1. *When $\bar{\beta} < (>) 1/2 \sqrt{(y_i + \bar{\beta} y_j)/(y_j + \bar{\beta} y_i)}$, if firm j increases (decreases) its R&D expenditure, y_j, then firm i's output q_i, increases (decreases).*

2. *For all values of $\bar{\beta}$, an increase in y_i leads to an increase in q_i.*

3. *An increase in the spillover $\bar{\beta}$ results in an increase (decrease) in q_i whenever the*

following condition holds

$$\frac{y_i}{y_j} < (>) \frac{1}{2}\sqrt{\frac{y_i + \bar{\beta}y_j}{y_j + \bar{\beta}y_i}}, \quad 0 < \bar{\beta} \le 1.$$

Proposition 2 indicates that in the KMZ model, the same qualitative relationships holds as in the AJ model, in that E-R&D expenditure has both a direct and a strategic effect on a firm's production, with the sign of the strategic effect depending on the extent of spillovers. The major difference is that the direction of the strategic effect depends on the relative size of each firm's E-R&D expenditure. An interesting consequence is that $\partial q_i / \partial \bar{\beta} > 0$ will hold only if $\partial q_i / \partial y_j > 0$. This is a stronger result than that implied by the AJ model. It indicates that the presence of an input spillover from firm j's E-R&D expenditure is crucial in determining whether or not firm i is able to expand production.

4   E-R&D Green Technology Stage Game

4.1   The AJ Model

Cases N and NJ: Using equation (3) and solving the first order conditions yields the equilibrium solutions reported in Table I.[6]

The following proposition shows that firm E-R&D decreases monotonically as the spillover increases.

---

[6]The second order necessary condition and stability condition are given by $9\gamma - 2(2-\beta)^2 t^2 > 0$ and $2(2-\beta)(2\beta-1)t^2 < 9\gamma - 2(2-\beta)^2 t^2$. Both conditions are satisfied for $\gamma > 4/3$.

Cases	E-R&D	Output	Emissions
N	$r^N = \frac{(2-\tilde{\beta})(a-\bar{e}t)t}{9\gamma-2(2-\tilde{\beta})(1+\tilde{\beta})t^2}$	$q^N = \frac{3\gamma(a-\bar{e}t)}{9\gamma-2(2-\tilde{\beta})(1+\tilde{\beta})t^2}$	$E^N = \frac{6\gamma(a-t\bar{e})[9\gamma\bar{e}-2(2-\tilde{\beta})(1+\tilde{\beta})at]}{\left(9\gamma-2(2-\tilde{\beta})(1+\tilde{\beta})t^2\right)^2}$
NJ	$r^{NJ} = \frac{t(a-t\bar{e})}{9\gamma-4t^2}$	$q^{NJ} = \frac{3\gamma(a-t\bar{e})}{9\gamma-4t^2}$	$E^{NJ} = \frac{6\gamma(a-t\bar{e})[9\gamma\bar{e}-4at]}{9\gamma-4t^2}$
C	$r^C = \frac{2t(1+\tilde{\beta})(a-t\bar{e})}{9\gamma-2(1+\tilde{\beta})^2t^2}$	$q^C = \frac{3\gamma(a-t\bar{e})}{9\gamma-2(1+\tilde{\beta})^2t^2}$	$E^C = \frac{6\gamma(a-t\bar{e})[9\gamma\bar{e}-2t(1+\tilde{\beta})(a+\tilde{\beta}t\bar{e})]}{9\gamma-2(1+\tilde{\beta})^2t^2}$
CJ	$r^{CJ} = \frac{4t(a-t\bar{e})}{9\gamma-8t^2}$	$q^{CJ} = \frac{3\gamma(a-t\bar{e})}{9\gamma-8t^2}$	$E^{CJ} = \frac{6\gamma(a-t\bar{e})[9\gamma\bar{e}-4t(a+t\bar{e})]}{9\gamma-8t^2}$

Table I: E-R&D Stage solutions - AJ Model

Proposition 3 *For a given emission tax t, increasing the spillover $\tilde{\beta}$ reduces firm E-R&D effort, implying $r^{NJ} \leq r^N$ for all $0 \leq \tilde{\beta} \leq 1$.*

The impact of a change in $\tilde{\beta}$ on firm output is shown by:

$$\frac{\partial q^N}{\partial \tilde{\beta}} = \frac{6\gamma t^2(a-t\bar{e})(1-\tilde{\beta})}{(9\gamma-2(2-\tilde{\beta})(1+\tilde{\beta})t^2)^2} \gtreqqless 0, \quad \tilde{\beta} \lesseqqgtr 1/2.$$

This implies that there are two forces at play: The first is the direct effect making E-R&D more productive. However when $\tilde{\beta} > 0.5$, the strategic effect of free-riding dominates. Furthermore (from Table I) note that $q^{NJ} \leq q^N$ for $0 < \tilde{\beta} \leq 1$ so that emissions must also be smaller within case NJ.

Regarding the impact of a change in $\tilde{\beta}$ on emissions, the trade-off is more complicated and reveals a non-monotonic relationship between t and $\tilde{\beta}$. There are two regions identified for which a non-cartelized RJV (NJ) delivers a higher reduction in emissions than non-cooperative R&D (N):

- **Region 1:**

$$0 < \tilde{\beta} < 1 \text{ and } 0 \le \bar{e} \le \frac{a(9\gamma + 4t^2)}{18\gamma t};$$

- **Region 2:**

$$|\tilde{\beta}| < 1/2 \pm 3/2\sqrt{1 + \frac{18\gamma^2(a - 2\bar{e}t)}{t^3(9\bar{e}\gamma - 4at)}} \text{ and } \frac{a(9\gamma + 4t^2)}{18\gamma t} < \bar{e} < \frac{2a(9\gamma^2 + 2t^2)}{\gamma t(36\gamma - 17t^2)}.$$

For all other regions (not listed), non-cooperative R&D delivers lower emissions.

Cases C and CJ: Under cooperative R&D both firms choose their abatement r_i to maximize joint profits

$$V = \sum_{i=1}^{2} \pi_i \left(q_1(r_1, r_2), q_2(r_1, r_2), r_i; \beta \right).$$

In the case of a cartelized RJV $\tilde{\beta} = 1$ (CJ) and in the case of a research cartel $0 < \tilde{\beta} < 1$ (C). Solving this maximization problem, we obtain the equilibrium values reported in Table I.[7]

We now state the following proposition, showing the effect the cartel has on internalising the E-R&D externality:

Proposition 4 *For a given the emission tax t, increasing the spillover $\tilde{\beta}$ increases R&D effort, implying that $r^{CJ} \ge r^C$ for all $0 < \tilde{\beta} < 1$ (C).*

---

[7]The necessary second order condition under a research cartel is $9\gamma > 2(5 - \beta(8 - 5\beta))t^2$, implying that $\gamma > 4/9$ is sufficient for all values of β.

12

The impact of a change in $\tilde{\beta}$ on firm output can be seen by differentiating equilibrium output with respect to $\tilde{\beta}$ for a given emissions tax t. This is given as follows:

$$\frac{12(1+\tilde{\beta})\gamma t^2(a-\bar{e}t))}{(9\gamma-2(1+\tilde{\beta})^2t^2)^2} > 0, \quad 0 < \tilde{\beta} \leq 1.$$

Hence, the equilibrium output is strictly increasing in $\tilde{\beta}$, which implies, $q^C \leq q^{CJ}$ for $0 < \tilde{\beta} \leq 1$.

4.2 KMZ Model

Cases	E-R&D	Output	Emissions
N	$y^N = \frac{2t^2(a-t\bar{e})^2(2-\bar{\beta})^2\gamma}{(1+\bar{\beta})(9\gamma-2(2-\bar{\beta})t^2)^2}$	$q^N = \frac{(a-\bar{e}t)(9\gamma-4(2-\bar{\beta})t^2)}{3(9\gamma-2(2-\bar{\beta})t^2)}$	$E^N = \frac{2(a-\bar{e}t)(9\gamma-4(2-\bar{\beta})t^2)(9\gamma\bar{e}+2t(2-\bar{\beta})(a-2\bar{e}t)}{3(9\gamma-2(2-\bar{\beta})t^2)}$
NJ	$y^{NJ} = \frac{2t^2(a-t\bar{e})^2\gamma}{2(9\gamma-2t^2)^2}$	$q^{NJ} = \frac{(a-\bar{e}t)(9\gamma-4t^2)}{3(9\gamma-2t^2)}$	$E^{NJ} = \frac{2(a-\bar{e}t)(9\gamma-4t^2)(9\gamma\bar{e}+2t(a-2\bar{e}t))}{3(9\gamma-2t^2)}$
C	$y^C = \frac{2t^2(1+\bar{\beta})(a-t\bar{e})^2\gamma}{(9\gamma-2(1+\bar{\beta})t^2)^2}$	$q^C = \frac{(a-t\bar{e})(9\gamma-4(1+\bar{\beta})t^2)}{3(9\gamma-2(1+\bar{\beta})t^2)}$	$E^C = \frac{2(a-\bar{e}t)(9\gamma-4t^2(1+\bar{\beta}))(9\bar{e}\gamma+2t(a-2\bar{e}t)(1+\bar{\beta}))}{3(9\gamma-2t^2(1+\bar{\beta}))}$
CJ	$y^{CJ} = \frac{4t^2(a-t\bar{e})^2\gamma}{(9\gamma-4t^2)^2}$	$q^{CJ} = \frac{(a-t\bar{e})(9\gamma-8t^2)}{3(9\gamma-4t^2)}$	$E^{CJ} = \frac{2(a-\bar{e}t)(9\gamma-8t^2)(9\bar{e}\gamma+4t(a-2\bar{e}t))}{3(9\gamma-4t^2)}$

Table II: E-R&D Stage solutions - KMZ Model

Cases N and NJ: Using equation (5) and solving the first order conditions yields the equilibrium solutions reported in Table II.[8]

The following proposition shows that, E-R&D expenditure decreases as spillovers increase. This again points to the pivotal role that RJVs have. In addition, it points out to the strategic effect of $\bar{\beta}$ on E-R&D expenditure dominating firms decisions.

---

[8]The second order necessary condition is $9\gamma > (2-\beta)^2t^2$.

Proposition 5 *For a given emission tax t, increasing the spillover $\bar{\beta}$ has the effect of decreasing E-R&D expenditure. This implies that $y^{NJ} \leq y^N$ for $0 \leq \bar{\beta} \leq 1$.*

Cases C and CJ: Under the research cartel, both firms choose their E-R&D expenditure y_i in order to maximize joint profit. In the case of a cartelized RJV $\bar{\beta} = 1$ (CJ) and in the case of a cartel $0 < \bar{\beta} < 1$ (C). Solving this maximization problem we obtain the equilibrium values reported in Table II.[9]

Although not provided here there is a similar result to Proposition 4, for the case of cartelized research establishing that E-R&D expenditure is an increasing function of the spillover so that $y^C > y^{CJ}$ for $0 < \bar{\beta} < 1$. As a consequence abatement will be higher and emissions taxes will be lower under C than under CJ.

5   The Effects of an Optimal Emissions Tax

In this section, we explore the optimal (second-best) emission tax. As stated earlier, the government makes an ex ante commitment at the beginning of the E-R&D stage to a particular emissions tax. The government selects this tax by maximizing total welfare (SW).[10] The social welfare function is given by:

$$SW(t; \beta, \gamma) = CS(t; \beta, \gamma) + PS(t; \beta, \gamma) + T(t; \beta, \gamma) - D(t; \beta, \gamma),$$

---

[9]The second order necessary condition for the existence of an equilibrium under research cartelization is $9\gamma > 2(5 - \beta(8 - 5\beta))t^2$.

[10]The optimal emission tax is optimal in a second best sense, which is consistent with the welfare analysis of R&D in Suzumura (1992).

where, consumer surplus is $CS = \frac{1}{2}Q^2$, producer surplus $PS = \pi_1 + \pi_2$ and T is the aggregate emissions tax revenue. Environmental damage D is a function of total emissions, $D(E)$, with $D(0) = 0, D' > 0, D'' > 0$ for $E > 0$. We use the following specific function to model damage[11]

$$D(t; \beta, \gamma) = \frac{1}{2}E^2,$$

where $E = e_1 + e_2$. Taxes are linear in emissions, $T = tE$, where $t \in [0, 1]$ is the tax rate. A uniform tax is used as we focus only on the symmetric equilibrium. Table III sets out the main components of the social welfare for both the AJ and the KMZ models. It includes only aggregate emissions E, as damages D and the total tax bill T are functions of E and can be computed with little effort.

Cases	Consumer Surplus (CS)	Producer Surplus (PS)	Emissions (E)
AJ Model			
N	$\dfrac{18\gamma^2(a-\bar{e}t)^2}{\left(9\gamma-2(2-\tilde{\beta})(1+\tilde{\beta})t^2\right)^2}$	$\dfrac{\gamma\left(a-\bar{e}t\right)^2\left(9\gamma-2(2-\tilde{\beta})^2t^2\right)}{\left(9\gamma-2(2-\tilde{\beta})(1+\tilde{\beta})t^2\right)^2}$	$\dfrac{6\gamma\left(9\gamma\bar{e}-2a(2-\tilde{\beta})(1+\tilde{\beta})t\right)(a-\bar{e}t)}{\left(9\gamma-2(2-\tilde{\beta})(1+\tilde{\beta})t^2\right)^2}$
NJ	$\dfrac{18\gamma^2(a-\bar{e}t)^2}{(9\gamma-4t^2)^2}$	$\dfrac{\gamma\left(a-\bar{e}t\right)^2\left(9\gamma-2t^2\right)}{(9\gamma-4t^2)^2}$	$\dfrac{6\gamma(9\gamma\bar{e}-4at)(a-\bar{e}t)}{(9\gamma-4t^2)^2}$
C	$\dfrac{18\gamma^2(a-t\bar{e})^2}{\left(9\gamma-2(1+\tilde{\beta})^2t^2\right)^4}$	$\dfrac{2\gamma(a-t\bar{e})^2}{9\gamma-2(1+\tilde{\beta})^2t^2}$	$\dfrac{6\gamma\left(9\gamma\bar{e}-2at(1+\tilde{\beta})^2\right)(a-t\bar{e})}{\left(9\gamma-2(1+\tilde{\beta})^2t^2\right)}$
CJ	$\dfrac{18\gamma^2(a-t\bar{e})^2}{(9\gamma-8t^2)^4}$	$\dfrac{2\gamma(a-t\bar{e})^2}{9\gamma-8t^2}$	$\dfrac{6\gamma(9\gamma\bar{e}-8at)(a-t\bar{e})}{(9\gamma-8t^2)}$
KMZ Model			
N	$\dfrac{2(a-\bar{e}t)^2\left(9\gamma-4(2-\bar{\beta})t^2\right)^2}{9\left(9\gamma-2(2-\bar{\beta})t^2\right)^2}$	$\dfrac{2(a-\bar{e}t)^2\left(\left(1+\bar{\beta}\right)\left(9\gamma-4(2-\bar{\beta})t^2\right)+18\gamma(2-\bar{\beta})^2t^2\right)}{9(1+\bar{\beta})\left(9\gamma-2(2-\bar{\beta})t^2\right)}$	$\dfrac{2(a-\bar{e}t)(9\gamma-4t^2(2-\bar{\beta}))(9\gamma\bar{e}-2t(2-\bar{\beta})(a-\bar{e}t))}{243\gamma^2+12(2-\bar{\beta})^2t^4}$
NJ	$\dfrac{2(a-\bar{e}t)^2\left(9\gamma-4t^2\right)^2}{9(9\gamma-2t^2)^2}$	$\dfrac{2(a-\bar{e}t)^2\left(\left(2(9\gamma-4t^2)+18\gamma t^2\right)\right)}{18(9\gamma-2t^2)}$	$\dfrac{2(a-\bar{e}t)(9\gamma-4t^2)(9\gamma\bar{e}-2t(a-\bar{e}t))}{243\gamma^2+12t^4}$
C	$\dfrac{2(a-t\bar{e})^2(9\gamma-4(1+\bar{\beta})t^2)}{(9\gamma-2(1+\bar{\beta})t^2)^2}$	$\dfrac{2(a-\bar{e}t)^2(9\gamma-8t^2(1+\bar{\beta}))}{9(9\gamma-2t^2(1+\bar{\beta}))}$	$\dfrac{2(a-t\bar{e})^2(9\gamma-4(1+\bar{\beta})t^2)(9\gamma\bar{e}+2t(1+\bar{\beta})(a-2\bar{e}t))}{3(9\gamma-2(1+\bar{\beta})t^2)^2}$
CJ	$\dfrac{2(a-t\bar{e})^2(9\gamma-8t^2)}{(9\gamma-4t^2)^2}$	$\dfrac{2(a-\bar{e}t)^2(9\gamma-16t^2)}{9(9\gamma-4t^2)}$	$\dfrac{2(a-t\bar{e})^2(9\gamma-8t^2)(9\gamma\bar{e}+4t(a-2\bar{e}t))}{3(9\gamma-4t^2)^2}$

Table III: Components of Social Welfare

---

[11]This specific function is chosen in order to present the basic point on the difference between the two modelling approaches as simply as possible.

As is often the case in this type of analysis, a closed-form solution (for the maximization of the social welfare equation) cannot be obtained. Nevertheless we have solved this numerically and have run extensive simulations.[12] Below we report representative results. In the simulations, we chose parameter values that are allowed by the restrictions imposed by the analysis; these restrictions refer to the E-R&D efficiency parameter γ (specifically we report here results for $\gamma = 8$ to satisfy the second order and stability conditions for both non-cooperative and cooperative cases in the AJ and KMZ mdoels), the market size a (which is a scale factor and set at $a = 20$) and initial marginal emissions, $\bar{e}$. The latter must be $\bar{e} \leq 1$. If $\bar{e} > 1$, the optimal tax rate would be set at $t = 1$ and there would never be any interaction or offset from the spillover. Hence, to avoid examining this (trivial) case we focus on $\bar{e} \leq 1$, where the optimal tax rate is $t < 1$.

5.1 The Optimal Tax and Emissions

Figures 1,2 and 3 show the relationship between the spillover β and the emission tax t, while Figure 4 illustrates agreggate emissions for the AJ and KMZ models.

For case N in the AJ model there is a non-monotonic relationship between the optimal emissions tax rate and the spillover (figure 1a). This is due to the two opposing effects (direct and strategic) reported in Proposition 1. When $\tilde{\beta} \geq 0.5$ the strategic effect of an increase in the spillover, creates an incentive for firms to free-ride on abatement effort. The second effect occurs because as $\tilde{\beta}$ increases, it also increases the effectiveness of aggregate abatement. When $\tilde{\beta} < 0.5$, as the spillover increases the optimal emissions

---

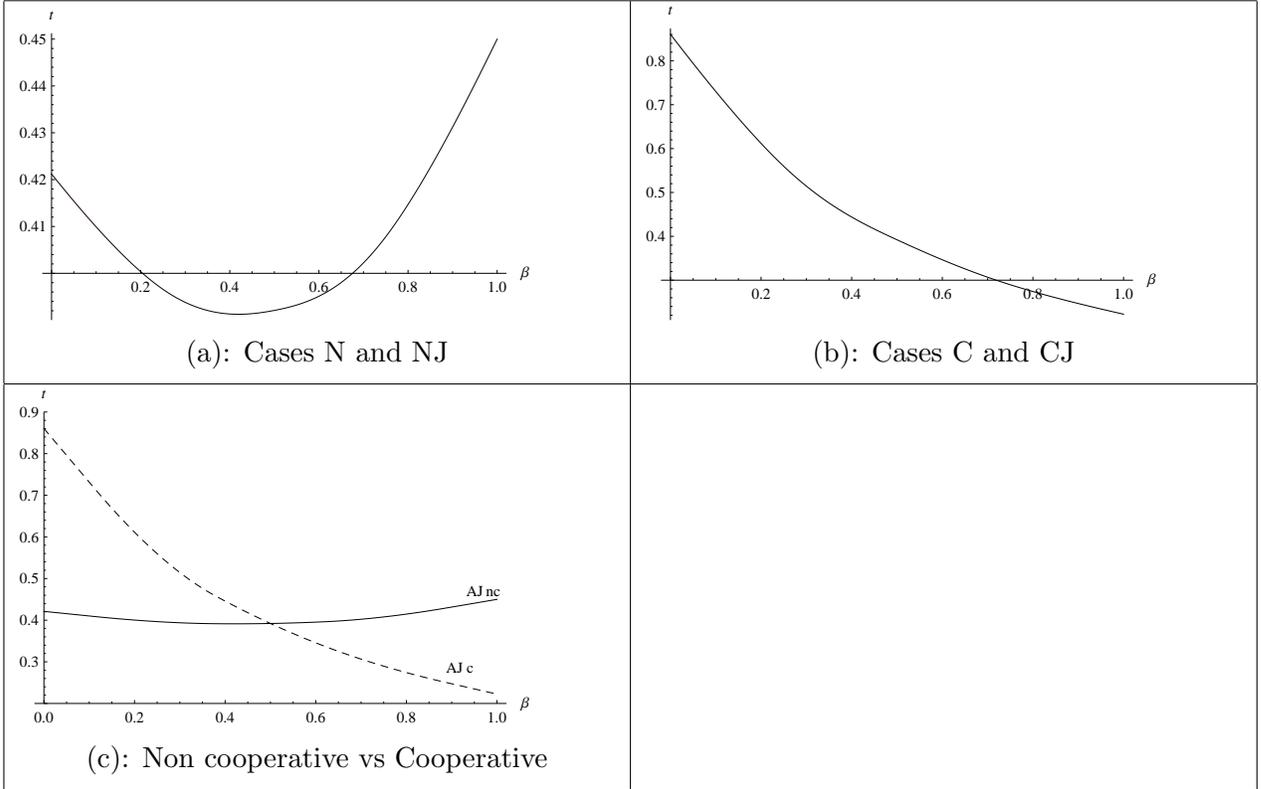[12]The results are available from the authors upon request.

Figure 1: Optimal emissions tax - AJ Model ($a = 20, e = 0.5, \gamma = 8$)

tax decreases. As emissions are decreasing in β (figure 4a), this implies that emissions will be lowest when the spillover $\beta = 1$. This confirms Proposition 3 and the discussion following it.

For case N in the KMZ model, the optimal emissions tax is always increasing with $\bar{\beta}$ (figure 2a). Recall that is the KMZ model, R&D (input) spillovers embody the positive knowledge externality created by R&D. There is no (output) spillover associated with abatement. As a consequence, the strategic free-riding effect associated with knowledge is dominant in the KMZ model under the non-cooperative R&D regimes. The underlying reason for this is that in the KMZ model there is an implicit assumption that E-R&D is
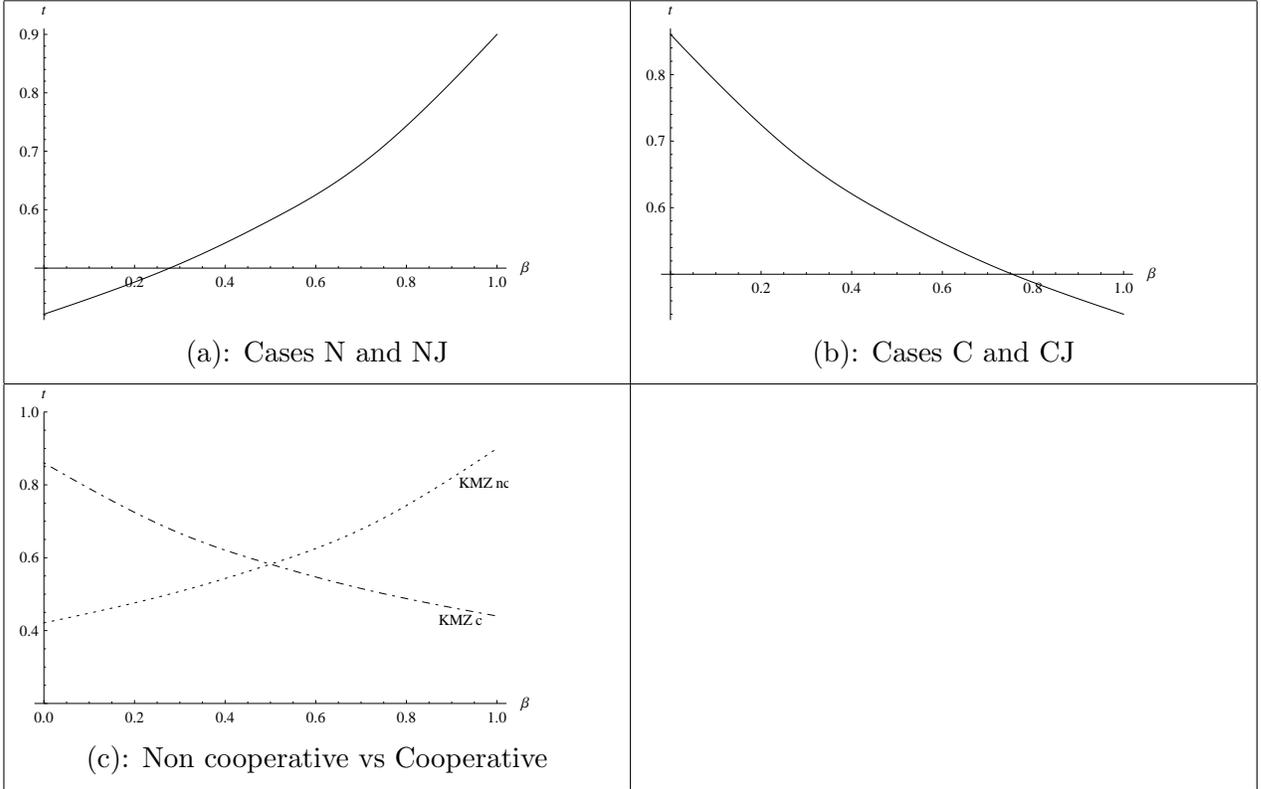
17

Figure 2: Optimal emissions tax - KMZ Model ($a = 20, e = 0.5, \gamma = 8$)

always more cost efficient under a single laboratory, regardless of the extent of spillovers. Hence an increase in the spillover has a negative strategic effect (i.e. acts as a disincentive) on E-R&D, dominating the positive direct effect. The emission tax must increase to offset this (cf. proposition 5). As in the AJ model, emissions decrease as the spillover increases and are lowest when $\bar{\beta} = 1$ (figure 4b). However, this occurs because of the impact of higher emissions taxes, which rise to offset the impact $\bar{\beta}$.

For cases C and CJ, there is a clear trade-off between the E-R&D spillover β and the emission tax t across the AJ and KMZ models (figures 1b and 2b). Specifically, increases in the spillover translate into a lower optimal emission tax, with t is at its lowest when
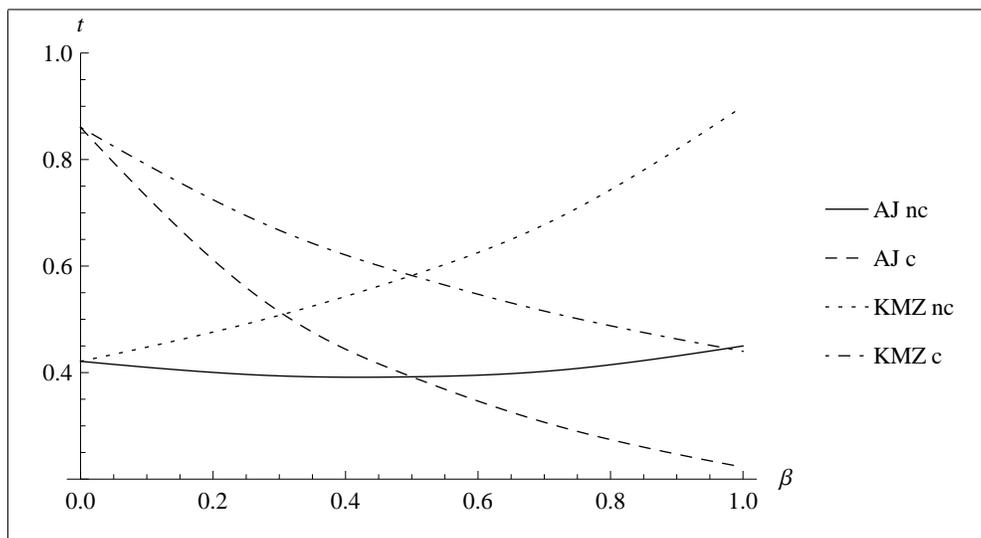
18

Figure 3: Optimal emissions tax - AJ and KMZ ($a = 20, e = 0.5, \gamma = 8$)

$\beta = 1$ (i.e., Case CJ). Only the direct effect operates here as there is no strategic effect. It is also evident (see figures 1c, 2c and 3) that the optimal emission tax under cooperative R&D is higher than under non-cooperative regimes whenever the spillover is relatively low ($\beta < 0.5$) and vice versa for relatively high spillover ($\beta \geq 0.5$). Note that when comparing non-cooperative research (Cases N and NJ), with cooperative research (C and CJ) in the AJ, that emissions reduction is higher under non-cooperative (cooperative) R&D when R&D spillovers are large (small), i.e. when R&D spillovers are greater (less) than 0.5 (figure 4c). This indicates that within the AJ model, as an additional policy instrument, when R&D spillovers are less than 0.5, the pursuit of R&D cooperation leads to greater reductions in emissions.
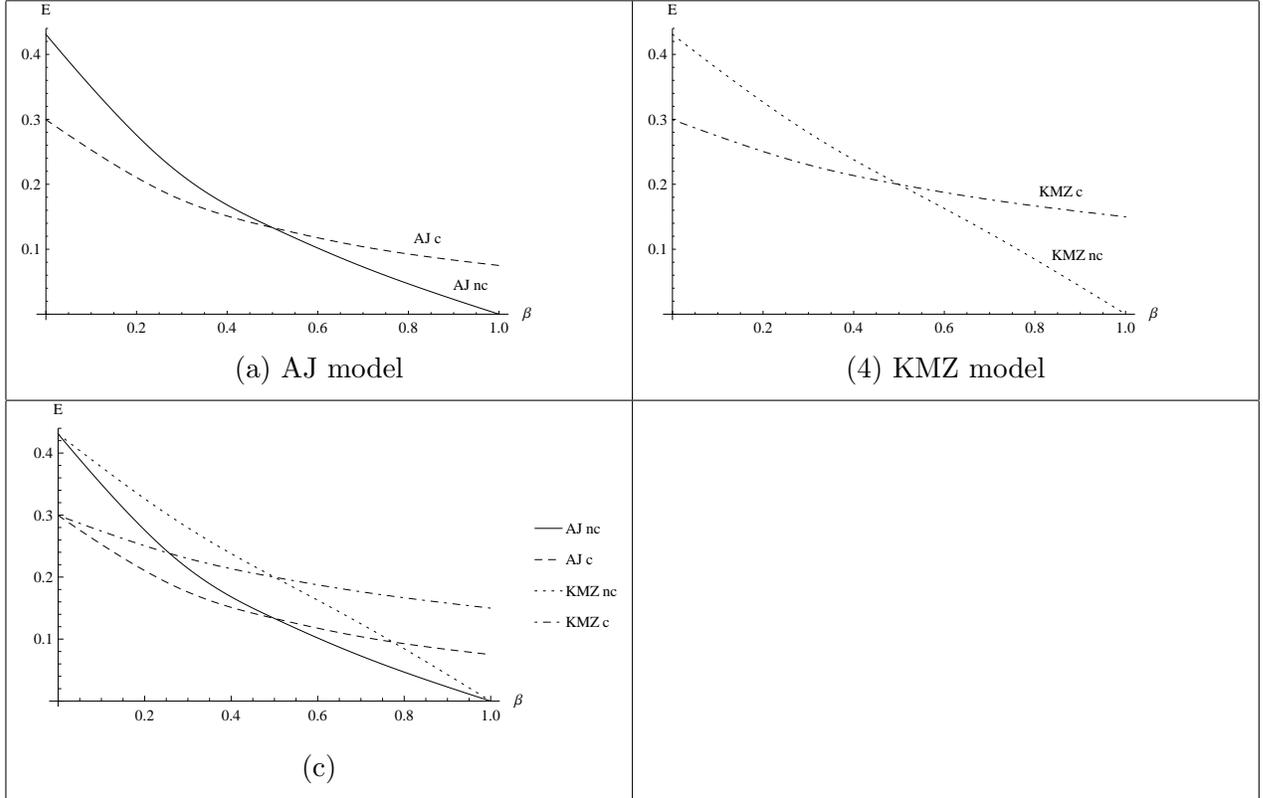
Figure 4: Equilibrium Emissions ($a = 20, e = 0.5, \gamma = 8$)

5.2   E-R&D - Abatement

Figures 5 and 6 show the relationship between E-R&D (abatement) and the spillover. The relationship between β and abatement activity is quite different in the two models. In the AJ model, an increase in $\tilde{\beta}$ always leads to a decrease in abatement activity, regardless of whether or not firms decide to cooperate in R&D (figures 5a and 5b). By comparison, in the KMZ model, an increase in $\bar{\beta}$ leads to higher investment in abatement (figures 5c and 5d). Figure 6 shows that abatement activity is always lower in the AJ model (except for a zero spillover when the models are equivalent).
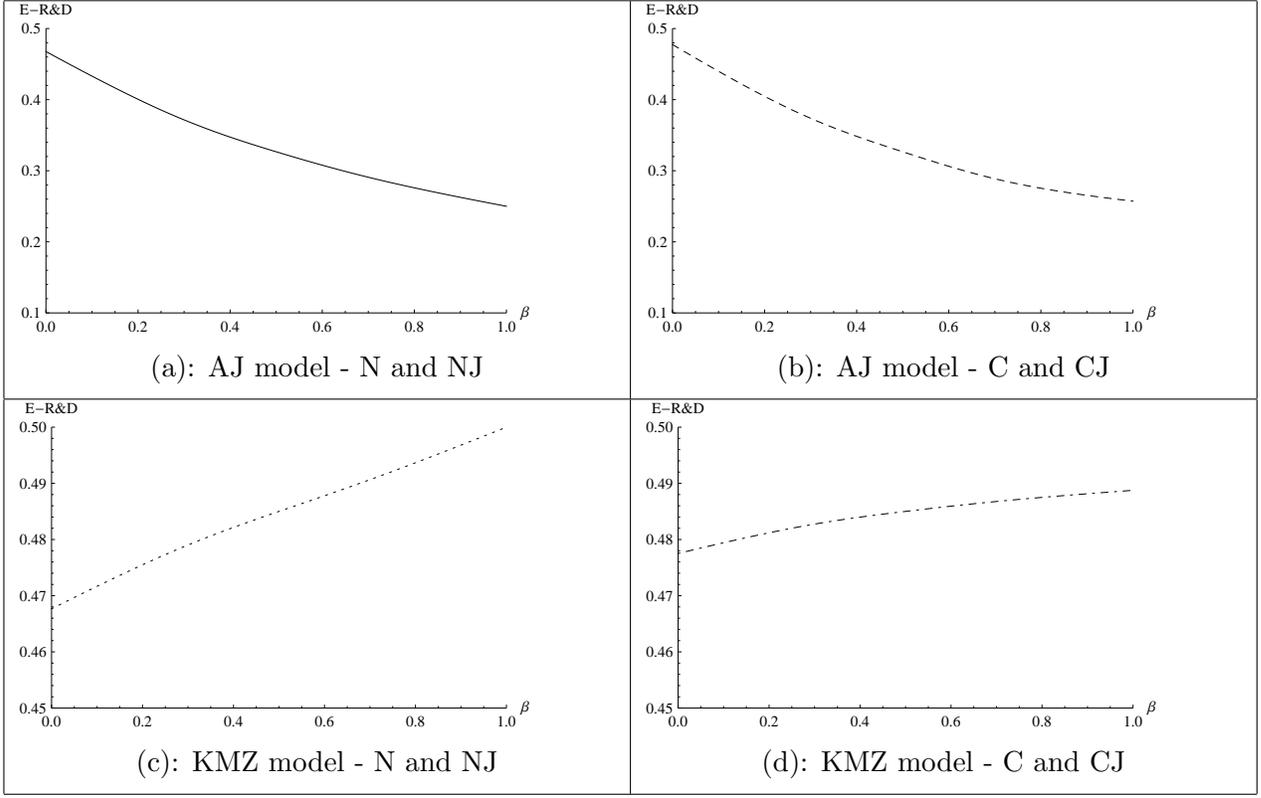
Figure 5: E-R&D (abatement activity) ($a = 20, e = 0.5, \gamma = 8$)

This is purely the result of E-R&D having constant returns to scale under the AJ model: R&D spillovers have greater impact on the productivity of each firm's abatement effort leading to a lower emission tax relative to the KMZ model. A lower level of investment in E-R&D in the AJ model will always deliver relatively higher effective abatement effort and lower aggregate emissions, when compared to the KMZ model regardless of whether or not firms cooperate or not in R&D. This points to an important difference between the KMZ and AJ models, that in the AJ model spillovers occur because of abatement activity and R&D activity. Therefore emissions taxes and abatement are lower in the AJ model because emissions reduction occurs as a consequence of each firm's investment in
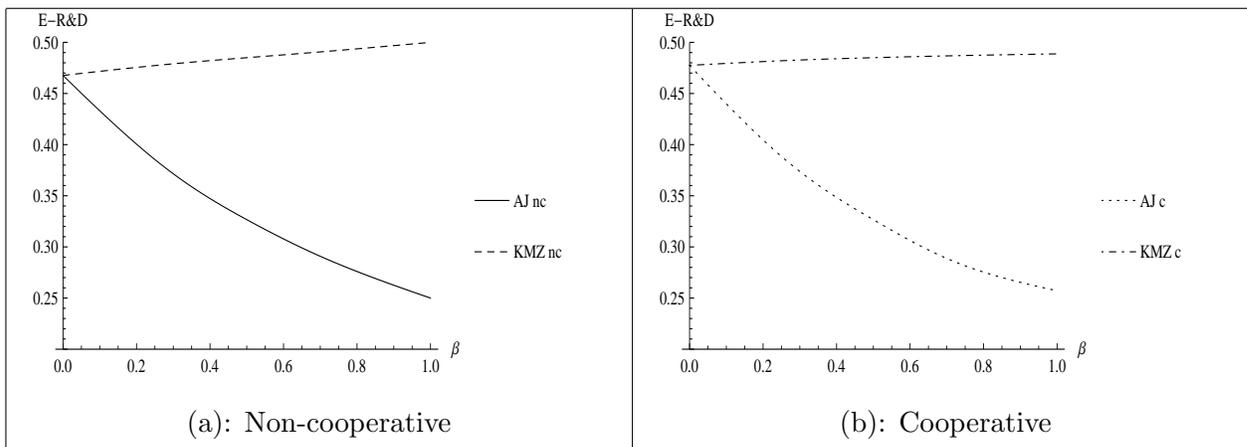
E-R&D and aggregate abatement activity.

Figure 6: E-R&D - Cooperative vs Non-cooperative ($a = 20, e = 0.5, \gamma = 8$)

From the perspective of a policy maker the following two implications can be drawn. First, governments need not set emission taxes at high levels to achieve reductions in emissions (pollution control) because of beneficial spillovers emanating from the abatement activities of firms in the economy. Second, the analysis has uncovered a novel trade-off between an optimal emissions tax and R&D cooperation in reducing pollution. When R&D spillovers are low ($\beta < 0.5$), R&D cooperation (research cartel or RJV) achieves greater reductions in emissions. By contrast when R&D spillovers are high ($\beta > 0.5$) non-cooperative R&D is more effective in conjunction with higher emissions taxes. This is a general rule that holds across both modelling specifications of the R&D spillover process.

22

6   Conclusion

This paper performs a comparison of two differing approaches to modelling spillovers associated with green technology (E-R&D) in a non-tournament R&D model. In summary:

1. There is a marked difference between the two models, in that the KMZ model (R&D input spillovers) leads to higher emission taxes, higher E-R&D and higher emissions, relative to the AJ model (R&D output spillovers).

2. There is a counter intuitive positive relationship between R&D input spillovers and emission taxes in the non-cooperative scenarios in the KMZ model. This reflects a stylised fact that R&D should be more cost efficient if generated in a single laboratory. These spillovers lead to an appropriation of E-R&D , which creates a disincentive for investment in E-R&D . Hence, the emission tax must increase to promote investment in E-R&D.

3. By contrast, there exists a U-shaped relationship between the optimal emissions tax rate and the R&D spillover rate in the non-cooperative scenarios in the AJ model, pointing to a tradeoff between R&D output spillovers and emissions taxes when spillovers are relatively small (less than 0.5). This trade-off between the optimal emissions tax and the R&D spillovers also exists for both input and output spillovers when firms cooperate in R&D.

4. In general the KMZ model requires higher taxes to achieve an equivalent reduction in emissions to that of the AJ model. This is because of the existence of abatement

and knowledge spillovers in the AJ model. Hence, the existence of knowledge and abatement spillovers implies that emissions taxes can be lower than what would be otherwise required.

Another important observation is the relative effectiveness of combining cooperative R&D with the emissions tax as a means of reducing pollution. The numerical results show that (for both the KMZ and AJ models) cooperative R&D is more effective when R&D spillovers are low (less than 0.5). If R&D spillovers are relatively large (exceed 0.5), then emissions taxes coupled with competitive R&D become more effective in reducing pollution. This points to a trade-off for policy makers between R&D polices designed to promote cooperative R&D and emissions taxes. The results in this paper suggest that RJVs and similar research sharing agreements will be more effective at reducing pollution when R&D spillovers are low and that emissions taxes will be more effective when spillovers are high.

As Amir et al. (2008) indicate the AJ model provides a useful insight: that it is R&D effort, rather than funds invested in R&D, that spillover from an innovating firm to its rivals. In models of green technology (E-R&D), which are driven by emissions taxes, this is an important distinction. It can be argued that it is the impact of R&D on abatement effort and emission reduction that is of crucial importance in designing environmental policy. For this reason, the AJ model may be more suitable for modelling green technology.

The next step in this line of research is to ascertain empirically the type of E-R&D production function that drives environmental innovation. It might be that either mod-

elling approach is valid and this could depend on the type of product, technology and industry under investigation or it might be that neither of the modelling approaches is a good representation of reality. Apart from this mainly empirical data question, further questions to explore in future research include: (i) the importance of quadratic versus linear damage costs, (ii) the use of alternative environmental policy instruments, e.g., tradable permits or standards and (iii) issues of time consistency.

References

Amir, R. (2000) Modelling imperfectly appropriable R&D via spillovers. *International Journal of Industrial Organization* 18, 1013 - 1032.

Amir, R., Jin, J.Y. and Troege, M. (2008) On additive spillovers and returns to scale in R&D. *International Journal of Industrial Organization* 26, 695 - 703.

Burr, C., Knauff, M. and Stepanova, A. (2013) On the prisoner's dilemma in R&D with input spillovers and incentives for R&D cooperation. *Mathematical Social Sciences*. In Press.

Chiou, J. and Hu, J. (2001) Environmental research joint Ventures under emission taxes. *Environmental and Resource Economics*, 20: 129-146.

d'Aspremont, C., Jacquemin, A. (1988) Cooperative and noncooperative R&D in duopoly with spillovers. *American Economic Review* 78, 1133 - 1137.

Griliches, Z. (1984) *R&D, Patents and Productivity.* (University of Chicago Press, Chicago, Il.)

Griliches, Z. (1992) The search for R&D spillovers. *Scandinavian Journal of Economics* 94, S29-S47.

Hauenschild, N. (2003) On the role of input and output spillovers when R&D projects are risky. *International Journal of Industrial Organization* 21(8), 1065-1089

Hinloopen, J. and Vendekerchhove, J. (2009) Dynamic efficiency of Cournot and Bertrand competition: input versus output spillovers. *Journal of Economics* 98(2), 119-136.

Jaffe, A.B. (1986) 'Technological opportunity and spillovers of R&D: Evidence from firm's patents, profits and market value. *American Economic Review* 76, 984-1001.

Jaffe, A.B. (1998) Measurement issues. In: L.M. Branscomb and J.H. Keller, eds., *Investing in Innovation.* (MIT Press, Cambridge, MA).

Jaffe, A.B., Newell, R. and Stavins. R. (2003) Technological change and the environment. In: Maler, K-G. & Vincent, J. eds. *Handbook of Environmental Economics.* (North-Holland, Amsterdam).

Jung, C. , K. Krutilla and R. Boyd (1996) Incentives for advanced pollution abatement technology at the industry level: An evaluation of policy alternatives. *Journal of Environmental Economics and Management* 30, 95-111

Kamien, M., Muller, E., Zang, I. (1992) Research joint ventures and R&D cartels. *American Economic Review* 82, 1293-1306.

Kneese, A. and C. Schultze (1975) *Pollution, Prices, and Public Policy.* (Brookings Institute, Washington, D.C.)

OECD (2010) Climate policy and technological innovation and transfer: An overview of trends and recent empirical results, Working Party on Global and Structural Policies, OECD.

Petrakis, E. and Xepapadeas, A. (2001) To commit or not to commit: Environmental policy in imperfectly competitive markets. *University of Crete Working Paper 0110.*

Petrakis, E. and Xepapadeas, A. (2003) Location decisions of a polluting firm and the time consistency of environmental policy. *Resource and Energy Economics* 25, pp. 197-214.

Petrakis, E. and Poyago-Theotoky, J. (2002) R&D subsidies versus R&D cooperation in a duopoly with spillovers and pollution. *Australian Economic Papers*, 41, 37 - 52.

Poyago-Theotoky, J. (2007) The organization of R&D and environmental policy". *Journal of Economic Behavior and Organization*, 62(1), 63 - 75.

Poyago-Theotoky, J. (2010) Corrigendum to The Organization of R&D and Environmental Policy. [J. Econ. Behav. Org. 62 (2007) 63 75]. *Journal of Economic Behavior and Organization*, 76, 449.

Scott, J. (1996) Environmental research joint ventures among manufacturers. *Review of Industrial Organization* 11, 655-679.

Stepanova, A. and Tesoriere, A. (2011) R&D with spillovers: Monopoly versus non-Cooperative and cooperative duopoly. *Manchester School* 79(1), 125-144.

Suzumura, K. (1992) Cooperative and noncooperative R&D in an oligopoly with spillovers. *American Economic Review* 82, 1307-1320.

Appendix

Proof of proposition 1

Proof. The equilibrium output for each firm in the product market stage game is

$$q_i = \frac{1}{3}\left(a - t\left(\bar{e} - r_i\left(2 - \tilde{\beta}\right) + r_j\left(1 - 2\tilde{\beta}\right)\right)\right), \quad i,j = 1,2, i \neq j.$$

Taking first partial derivatives of q_i with respect to r_i and, q_j with respect to r_i gives

$$\frac{\partial q_i}{\partial r_i} = \frac{t\left(2 - \tilde{\beta}\right)}{3} > 0, \quad 0 \leq \tilde{\beta} < 1,$$

$$\frac{\partial q_j}{\partial r_i} = -\frac{t\left(1 - 2\tilde{\beta}\right)}{3} \gtreqless 0, \quad \tilde{\beta} \lesseqgtr 1/2.$$

Taking the first partial derivative of q_i with respect to $\tilde{\beta}$,

$$\frac{\partial q_i}{\partial \tilde{\beta}} = -\frac{1}{3}t(r_i - 2r_j) > 0, \quad i,j = 1,2, i \neq j.$$

if $r_i \leq 2r_j$, and it will be negative otherwise. ∎

Proof of proposition 2

Proof. The equilibrium output for firm i in the product market stage game is given by

$$q_i = \frac{1}{3}\left(a - t\left(\bar{e} - \sqrt{2}\sqrt{\frac{y_j + \bar{\beta}y_i}{\gamma}} + 2\sqrt{2}\sqrt{\frac{y_i + \bar{\beta}y_j}{\gamma}}\right)\right), \quad i,j = 1,2$$

Taking first partial derivatives with respect to y_i and y_j yields:

$$\frac{\partial q_i}{\partial y_i} = \frac{t}{3\sqrt{2}\gamma} \left[\frac{2}{\sqrt{\frac{y_i + \bar{\beta}y_j}{\gamma}}} - \frac{\bar{\beta}}{\sqrt{\frac{y_j + \bar{\beta}y_i}{\gamma}}} \right] > 0, \quad 0 < \bar{\beta} \le 1$$

and

$$\frac{\partial q_i}{\partial y_j} = \frac{t}{3\sqrt{2}\gamma} \left[-\frac{1}{\sqrt{\frac{y_j + \bar{\beta}y_i}{\gamma}}} + \frac{2\bar{\beta}}{\sqrt{\frac{y_i + \bar{\beta}y_j}{\gamma}}} \right] \gtreqless 0, \quad \bar{\beta} \lesseqgtr \frac{1}{2} \sqrt{\frac{y_i + \bar{\beta}y_j}{y_j + \bar{\beta}y_i}}.$$

Note that the conclusions are identical to the AJ model, if expenditures are identical for each firm, so that $y_i = y_j$ for all firms i, j. Taking the first partial derivative of q_i with respect to $\bar{\beta}$,

$$\frac{\partial q_i}{\partial \bar{\beta}} = -\frac{t}{3\sqrt{2}\gamma} \left(-\frac{y_i}{\sqrt{\frac{y_j + \bar{\beta}y_i}{\gamma}}} + \frac{2y_j}{\sqrt{\frac{y_i + \bar{\beta}y_j}{\gamma}}} \right) > (<) \, 0, \quad i, j = 1, 2,$$

only if

$$\frac{y_i}{y_j} < (>) \frac{1}{2} \sqrt{\frac{y_i + \bar{\beta}y_j}{y_j + \bar{\beta}y_i}}, \quad 0 < \beta \le 1.$$

■

Proof of proposition 3

Proof. Differentiating $r^N = \frac{(2 - \tilde{\beta})(a - \bar{e}t)t}{9\gamma - 2(2 - \tilde{\beta})(1 + \tilde{\beta})t^2}$ with respect to $\tilde{\beta}$ and applying the second order condition gives

$$\frac{\partial r^N}{\partial \tilde{\beta}} = \frac{-2t(a - t\bar{e})}{(9\gamma - 2(2 - \tilde{\beta})(1 + \tilde{\beta})t^2)^2} [9\gamma - 2(2 - \tilde{\beta})^2 t^2] < 0, \quad 0 \le \tilde{\beta} < 1.$$

As $\tilde{\beta} \to 1$, E-R&D decreases monotonically. This implies that $r^{NJ} \leq r^N$ for $0 < \tilde{\beta} \leq 1$.

∎

Proof of proposition 4

Proof. Differentiating $r^C = \frac{2t(1+\tilde{\beta})(a-t\bar{e})}{9\gamma - 2(1+\tilde{\beta})^2 t^2}$ with respect to $\tilde{\beta}$ and applying the second order condition,

$$\frac{2t(a - \bar{e}t)(9\gamma + 2(1 + \tilde{\beta})^2 t^2)}{(9\gamma - 2(1 + \tilde{\beta})^2 t^2)^2} > 0, \quad 0 \leq \tilde{\beta} < 1.$$

This indicates that as $\tilde{\beta} \to 1$, investment in E-R&D increases monotonically. This implies that in equilibrium, $r^C \leq r^{CJ}$ for $0 < \tilde{\beta} \leq 1$. ∎

Proof of proposition 5

Proof. Differentiating $y^N = \frac{2t^2(a-t\bar{e})^2(2-\bar{\beta})^2\gamma}{(1+\bar{\beta})\left(9\gamma - 2(2-\bar{\beta})t^2\right)^2}$ and assuming that the second order condition holds,

$$\frac{-2t(a - t\bar{e})}{(9\gamma - 2(2 - \bar{\beta})(1 + \beta)t^2)^2}[9\gamma - 2(2 - \bar{\beta})^2 t^2] < 0, \quad 0 \leq \bar{\beta} < 1.$$

Note that as $\bar{\beta}$ increases the costs of abatement decrease. ∎