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Welfare Improving Taxation on Savings in a Growth Model*

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Abstract

We consider the optimal factor income taxation in a standard R&D model with technical change represented by an increase in the variety of intermediate goods. Redistributing the tax burden from labor to capital will increase the employment rate in equilibrium. This has opposite effects on two distortions in the model, one due to monopoly power, the second to the incomplete appropriability of the benefits of inventions. Their relative momentum determines the sign of the welfare effect. We show that, for parameter values consistent with available estimates, the optimal tax rate on capital will be sizable.

Keywords: *Capital Income Taxes, R&D, Growth Effect, Welfare Effect.*

JEL classification: *E62, H21, O41*

1 Introduction

This paper examines how the tax burden should be distributed between capital and labor income in a basic R&D model of endogenous growth.¹ The main message of the extensive literature on the optimal taxation of factor incomes,

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¹The taxation of capital involves many different kind of taxes, some on stocks (eg wealth tax, tax on bequests, property tax of capital), some on the income from savings (from the corporate income tax, the tax on interest and dividends, the taxation of capital gains). By a tax on capital income in our model we mean a tax on income from savings. In the model there is no capital in the physical sense, but wealth accumulates in the form of patents.

as summarised by Atkeson et al. (1999) is the following: taxing capital is a bad idea in the long-run.²

The result is surprisingly general and robust in a variety of settings, including models where capital-holders are distinct from workers (Judd 1985), overlapping generations models (Erosa and Gervais 2002) and models with human capital accumulation (Jones et al. 1997), while imperfect competition can even lead to the recommendation of a negative capital income tax (Judd 2002). We add that most quantitative investigations suggest that capital taxes should be zero or very small even in the short run (see Atkeson et al. 1999). The literature on endogenous growth tends to reinforce the message that capital income should not be taxed, as taxing it would have adverse effects on the rate of growth which would compound over time (see the survey in Jones and Manuelli 2005).

We check if the message also holds in a standard model of horizontal innovation, with an infinitely lived representative agent, originally proposed by Rivera-Batiz and Romer (1991) and known as the "lab-equipment model". Given its flexibility and simplicity this model has provided a tractable framework for analyzing a wide array of issues in economic growth, synthesized in Gancia and Zilibotti (2006). Entrepreneurs spend a fixed cost in order to develop new intermediate goods, over the production of which they then enjoy eternal monopoly power. Output in the final goods production sector is linear in the number of intermediate goods used so unbounded growth is possible. There are two inefficiencies in the model, a static one stemming from market power in the intermediate goods sector, and a dynamic one stemming from the uncomplete appropriability of the social surplus from innovating.

We extend this benchmark model by explicitly analysing the decision to supply labor as well as by introducing government spending. We assume that the only fiscal instruments are linear income taxes, that the government fixes the amount of revenue it wants to generate as a fixed fraction of income and that it balances the budget at all times. The tax rates (i.e. the labor income tax rate and the interest income tax rate) must adjust endogenously.

This gives what has become known as a "Ramsey Problem": maximize social welfare through the choice of taxes subject to the constraints that final allocations must be consistent with a competitive equilibrium with distortionary taxes and that the given tax system raises a pre-specified amount of revenue.

In a model with endogenous growth, the common trend between output and government expenditure cannot be ignored, so what we pre-specify here is the ratio between these variables and not the absolute amount of tax revenue.

²More precisely the Ramsey tax system advocates a high tax on initial capital stock (or on capital income in the initial period) and a zero tax on capital income in future times. However, the Ramsey results hinge on the assumption that the government can commit to zero tax in the future, as there is a problem of dynamic inconsistency (see Martin 2010).

Furthermore, to isolate the effects of taxation, rather than of more complex public action, we assume government revenues do not directly affect the marginal utility of private consumption and leisure or the marginal productivity of factors of production.

In this setting we derive an expression for the optimal tax rate on capital income, whose value will depend on the specifications of tastes and technology.

We then move to the analysis of calibrated versions of the model, and find that the tax rate on capital is never zero.

To understand intuitively our findings, consider that if the shift in the tax burden from capital to labor increases employment and as a consequence the productivity of each differentiated product, the demand for the product is increased. The production of each intermediate will then be more profitable, and the distortion due to monopoly power lower. Also, the invention activity, financed by household saving, is more rewarding the greater the prospective demand, and therefore profits from a new product. So a higher employment increases *ceteris paribus* the return to saving and linearly increases growth. However the increase in the tax on capital which is the counterpart to the reduction in the tax on labor directly discourages savings and growth, thus worsening the dynamic inefficiency. A third distortion in the model is created by government expenditure itself as agents do not internalize the fact that higher income will lead to extra public expenditure. Taxing both labor and capital income reduces this distortion (see Marrero and Novales 2007). For reasonable parameters' values the interplay between the various channels through which the tax program has effects means that the optimal tax on capital is not only positive but very sizable—given the levels of public spending observed in advanced economies.

Studies based on R&D models similar to ours have generally found that taxing savings is detrimental to growth and welfare (e.g. Lin and Russo 1999 and 2002 and Zeng and Zhang 2002). In particular our work complements Zeng and Zhang (2007), who study fiscal issues adopting our same specification of the horizontal innovation model but focus on a different issue. More specifically they compare the effects of subsidizing R&D investment to the effects of subsidizing final output or subsidizing the purchase of intermediate goods in terms of promoting growth. They consider distortionary taxation (i.e. taxes on labor income) but abstract from taxes on interest income.

Our findings also contribute to the literature exploring the circumstances under which optimal factor taxation may involve a non-zero tax rate on capital income, thus bridging the gap between economic theory prescriptions and the fact that in developed economies capital taxes are far from zero.³

A way in which taxing capital can be good is when government spending increases the marginal productivity of capital, as in Baier and Glomm (2001),

³See McDaniel (2007) for recent estimates of effective tax rates on capital.

Barro and Sala-i-Martin (1995), Guo and Lansing (1999), Turnovsky (1996, 2000), Chen (2007) and Zhang et al. (2008).⁴ More counter examples to the optimality of a zero tax on physical capital can be found in human capital models (see Ben-Gad 2003 and de Hek 2006). The presence of an informal sector the income from which cannot be taxed or of other restrictions on the taxation of factors are also grounds for the positive taxation of capital income (see Correia 1996, Penalosa and Turnovsky 2005 and Reis 2011). Chamley (2001), Ho and Wang (2007) and Imrohroglu (1998) among others have emphasized that if households face borrowing constraints and/or are subject to uninsurable idiosyncratic income risk, so that excessive savings arise, then the optimal tax system will in general include a positive capital income tax. Asea and Turnovsky (1998) and Kenc (2004) find that increasing the tax rate on capital income may increase growth in a stochastic environment. Many papers (eg Cremer et al. 2003, Hendricks 2003, 2004, Erosa and Gervais 2002, Song 2002, Uhlig and Yanagawa 1996 and Yakita 2003) show that in life cycle/OLG models the optimal capital income tax in general is different from zero, as such tax can facilitate the intergenerational transmission of wealth. Conesa et al. (2008) quantitatively characterize the optimal capital income tax in an overlapping generations model with idiosyncratic, uninsurable income shocks and find it to be significantly positive at 36 percent.

The arguments developed in these models as grounds for a positive rate of capital taxation are unrelated to ours as we model a perfect foresight closed economy with infinite lived agents, no effect of government expenditures on the rate of return of private factors of production, no human capital accumulation, no subsidies to investment. However, our paper like all in this literature, can be seen as an example of the argument that it is the presence of constraints (for the government or for the individuals) or suboptimal expenditure choices that makes capital income taxation desirable. In other words, ours are second-best results.

Often in the papers on taxation and growth, only the growth, not the welfare effect of the tax experiments are calculated, if in the market equilibrium growth is lower than optimal, because there is an implicit presumption that higher growth means more welfare as, through compounding, growth effects always prevail over level effects. However while, as we show, in our model growth is inefficiently low in the absence of taxes, even when the introduction of the tax lowers growth there might be a positive welfare effect. In our calibrated examples, this counterintuitive effect arises with parameter choice well within the range of selections studied in other settings, such as public finance, quantitative growth theory and business cycle analysis.

A complete assessment of the welfare effects of the tax program we consider has to include an analysis of its effect on the dynamic properties of the model.

⁴In Zhang et al. (2008) the government should tax net capital income more heavily than labor income, however this is because investment is subsidized at the same rate at which net capital income is taxed.

In fact it has recently been shown that factor taxes can affect the stability properties of the dynamic equilibrium of a market economy. In particular, Ben-Gad (2003), Chen and Lee (2006), Farmer and Guo (2004), Mino (2001), Park (2009), Palivos et al. (2005), Pelloni and Waldmann (1998) and (2000), Raurich (2001), Schmitt-Grohe and Uribe (1997) and Wong and Yip (2010) among others have shown that the introduction of taxes and government spending and /or of non separability of preferences over consumption and leisure (which we assume) may make the equilibrium exhibit local indeterminacy. However, this is not the case in this model, which, as we show, features a unique unstable balanced growth path.

The rest of the paper is organized as follows: in section 2 the model is presented, in section 3 the general equilibrium conditions of the model are described, section 4 analyzes the labor supply effect, the growth effect and the welfare effect of shifting the tax burden from labor to capital. Section 5 presents some calibrated examples and derives the optimal tax rates for various sets of parameters, section 6 presents the social planner's solution and section 7 concludes. Most proofs are relegated to the Appendices.

2 The Model

2.1 Households

We assume that in the economy there is a continuum of length one of identical households.⁵ Each has utility U given by:

$$U = \int_{t=0}^{\infty} e^{-\rho t} \left(\frac{1}{1-\sigma} C^{1-\sigma} h(H) \right) dt \quad (1)$$

where C is consumption, H labor, $\rho > 0$ is the rate of time discount and $1/\sigma > 0$ is the intertemporal elasticity of substitution. As Zeng and Zhang (2007) note, normalizing the population to unity removes from the analysis of taxes the so called "scale effect". The following conditions ensure non satiation of consumption and leisure:

$$h(H) > 0 \quad (2)$$

and

$$(1 - \sigma)h'(H) < 0. \quad (3)$$

Strict concavity of instantaneous felicity imposes:

$$(1 - \sigma)h''(H) < 0 \quad (4)$$

and

$$\sigma \frac{h''h}{(\sigma - 1)} - h'^2 > 0. \quad (5)$$

⁵As Zeng and Zhang (2007) note, normalizing the population to unity removes from the analysis of taxes the "scale effect" discussed by Jones (1995).

The instantaneous budget constraint consumers face is given by:

$$\dot{F} = r(1 - \tau_r)F + \pi_n(1 - \tau_r)N + w(1 - \tau_w)H - C. \quad (6)$$

Households derive their income by loaning entrepreneurs their financial wealth F (of which all have the same initial endowment), by profits π_n (net of the interest payments) of the N firms and by supplying labor H to firms, taking the interest rate r and the wage rate w as given. Capital income is taxed at the rate τ_r while labor income is taxed at the rate τ_w . Optimization at an interior point implies that the marginal rate of substitution between leisure and consumption equals their relative price:

$$\frac{h'}{h} = \frac{w(1 - \tau_w)(\sigma - 1)}{C}. \quad (7)$$

Optimal consumption and leisure must also obey the intertemporal condition:

$$-\sigma \frac{\dot{C}}{C} + \frac{h'}{h} \dot{H} = \frac{\dot{\lambda}}{\lambda} = \rho - r(1 - \tau_r) \quad (8)$$

where $\lambda = c^{-\sigma}h$ is the shadow value of wealth. Given a no Ponzi game condition the transversality condition imposes:

$$\lim_{t \rightarrow \infty} \lambda F \exp(-\rho t) = 0. \quad (9)$$

2.2 Firms

In this economy there are a final goods sector and an intermediate goods sector. The former is perfectly competitive, whereas the latter is monopolistic. R&D activity leads to an expanding variety of intermediate goods. All patents have an infinitely economic life, that is, we assume no obsolescence of any type of intermediate goods.

The production function of firm i in the final goods sector is given by:

$$Y(i) = AL(i)^{1-\alpha} \int_0^N x(i, j)^\alpha dj \quad (10)$$

where $Y(i)$ is the amount of final goods produced and $L(i)$ is labor used by firm i and $x(i, j)$ is the quantity this firm uses of the intermediate goods indexed by j . For tractability both i and j are treated as continuous variables. We assume $0 < \alpha < 1$. The final goods sector is competitive and we assume a continuum of length one of identical firms. We can then suppress the index i to avoid notational clutter. Firms maximize profits given by

$$Y - wL - \int_0^N P(j)x(j)dj \quad (11)$$

where w is the wage rate and $P(j)$ is the price of the intermediate good j . By profit maximization, the demand for good j is given by:

$$x(j) = L \left(\frac{A\alpha}{P(j)} \right)^{\frac{1}{1-\alpha}} \quad (12)$$

and labor demand by:

$$w = (1 - \alpha) \frac{Y}{L}. \quad (13)$$

Since the firms in the final goods sector are competitive and there are constant returns to scale their profits are zero in equilibrium. In contrast the firms which produce intermediate goods with patent which they invent then earn monopoly profits for ever. The cost of production of the intermediate good j , once it has been invented, is given by one unit of the final good.

The present discounted value at time t of monopoly profits for firm j , or in other words the value of the patent for the j^{th} intermediate good $V(j, t)$ at time t is:

$$V(j, t) = \int_t^{\infty} (P(j) - 1)x(j)e^{-\bar{r}(s,t)(s-t)} ds \quad (14)$$

where $\bar{r}(s, t)$ is the average interest rate during the period of time from t to s . The inventor of the j^{th} intermediate good chooses $P(j)$ to maximize $(P(j) - 1)x(j)$ where $x(j)$ is given by (12), so for each j , the equilibrium price is and quantity are:

$$P(j) = P = \frac{1}{\alpha} \quad (15)$$

and

$$x(j) = x = LA^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}. \quad (16)$$

The price is higher than the marginal cost of producing good j , and the quantity produced, $x(j)$, is therefore lower than the socially optimal level. This is in fact the first inefficiency in the model, a straightforward consequence of market power in the intermediate sector.

Plugging equation (16) in equation (10) gives us equation

$$Y = NLA^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} \quad (17)$$

while plugging (17) in (13) we have:

$$w = N(1 - \alpha)A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}. \quad (18)$$

Profits are given, as a consequence of (16) and (15), by:

$$\pi = LA^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left(\frac{1}{\alpha} - 1 \right). \quad (19)$$

A higher labor supply implies a higher quantity of each intermediate goods and thus higher profits in equilibrium. This means there is an externality to labor

in the model, because when deciding labor supply workers will not take into account this positive effect on profits. So a tax program leading to increasing L can increase welfare by reducing the inefficiency due to monopolistic conditions. In section 6 we show formally that in this market economy employment is always lower than its efficient level.

The cost of development of new products is η and there is free entry in the market for inventions. Intermediate goods firms will push the price of a patent to equate its cost. Here a second inefficiency in the model appears, which is due to an appropriability problem: only the discounted value of profits, as opposed to all of social surplus originating from an invention, is taken into account when deciding whether to pay for research leading to innovation, so that its pace will be too low.

If we drop the j index in V , (14) can be written as the Hamilton-Jacobi-Bellman equation:

$$r = \frac{\pi}{V} + \frac{\dot{V}}{V} \quad (20)$$

which allows us to interpret it from an asset pricing perspective. The return on holding a blueprint, rV , is given by dividends π , plus the capital gains, i.e. the change in its value V . In the appendix, we show that, in a growing economy, we must have $V = \eta$ in equilibrium at all times, while $\pi_n = 0$.⁶ But if $V = \eta$ at all times, (20), given (19), implies that in equilibrium we will have:

$$r = C_1 L \quad (21)$$

with

$$C_1 \equiv \frac{1}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}} (1-\alpha).$$

The higher is labor supply the higher is the interest rate. As the sales of each intermediate good and therefore profits are increasing in labor supply, for their present discounted value to be equal to the given cost of an invention, the interest rate will have to increase.

2.3 Government

We assume government consumption G equals a fixed fraction, g , of gross output: $G = gY$. We rule out a market for government bonds and assume that the government runs a balanced budget. The revenue from income taxes is used for financing expenditures. In equilibrium:

$$r\tau_r F + \tau_w wL = gY \quad (22)$$

where on the left-hand side we have inflows and on the right-hand side we have outflows. Our assumption of a given g is made mainly for convenience but the public expenditure components that might be seen as exogenous in actual

⁶Our proof is an extension to the case of a variable L , to the one offered in Acemoglu (2009) for the case of a fixed L .

economies (from public wages, the payments of interest on public debt etc.) are far from zero and have remained fairly stable, as a percentage of output, over the last decades. Marrero and Novales (2007) document this and show that factor income taxes may be preferable to lump-sum taxes under the assumption of a given g , as they allow an internalization of the fact that higher income will lead to extra public spending. This simple effect is also at work in our model.

2.4 Market Equilibrium

In calculating the equilibrium in the final goods market, intermediate goods used in production, xN , are subtracted from final production Y to obtain total value added. All investment in the model is investment in research and development of new intermediate goods $\eta\dot{N}$. The economy-wide resource constraint is therefore given by:

$$Y - xN = C + \eta\dot{N} + gY. \quad (23)$$

We are now ready for the following:

Definition 1 *In a competitive equilibrium individual and aggregate variables are the same and prices and quantities are consistent with the (private) efficiency conditions for the households (6), (7), (8) and (9), the profit maximization conditions for firms in the final goods sector, (12) and (13) (or 18), and for firms in the intermediate goods sector, (15) (or 16) and (21), with the government budget constraint (22) and with the market clearing conditions for labor ($H = L$), for wealth ($F = VN$), and for the final good, (23).*

The following relationship between before-tax labor income and before-tax capital income holds in equilibrium:

$$wL/rF = \frac{1}{\alpha}. \quad (24)$$

From (22) and (24) we can then infer that:

$$\tau_w = \frac{g}{1-\alpha} - \alpha\tau_r. \quad (25)$$

Given this from the definition of equilibrium we can now arrive at the following:

Proposition 2 *The competitive equilibrium conditions in the model give rise to the following differential equation for labor:*

$$\dot{L} = \frac{B(L)}{A(L)} \quad (26)$$

where

$$A(L) \equiv \left(\frac{\sigma h''}{h'} + \frac{h'}{h}(1-\sigma) \right) \quad (27)$$

and

$$B(L) \equiv \frac{\sigma}{\alpha} C_1 \frac{h((1-\sigma))}{h'} \left(1 + \alpha \tau_r - \frac{g}{(1-\alpha)} \right) + \rho - C_1 L \left(1 - \tau_r - \sigma - \frac{\sigma}{\alpha} \left(1 - \frac{g}{(1-\alpha)} \right) \right). \quad (28)$$

Proof. See appendix. ■

If a balanced growth path (hence BGP) exists, variables grow at a constant rate along this path, and in particular employment is constant at a value \tilde{L} . Given (26) we have:

Proposition 3 *The condition for the existence of a BGP equilibrium in this model in which all variables grow at the same rate is that (26) has a fixed point \tilde{L} between 0 and 1, implicitly defined by $B(\tilde{L}) = 0$, consistent with the TVC and with a positive growth rate γ for capital and consumption given by:*

$$\gamma = \frac{C_1 \tilde{L} (1 - \tau_r) - \rho}{\sigma}. \quad (29)$$

Proof. From (7) and (18), in a BGP, i.e. when $\dot{L} = 0$, C and N will grow at the same rate. From (8) this is seen to be given by (29). ■

From (29) we see that BGP growth is linearly increasing in \tilde{L} , as is the interest rate, through (21). So if an increase in τ_r and a corresponding decrease in τ_w induces a rise in \tilde{L} , the effect on growth will not be proportional, and, at least in theory, the net interest rate $C_1 \tilde{L} (1 - \tau_r)$ may increase rather than decrease. Also, if the net interest rate decreases the effect on growth will be lower the higher is σ .

Restrictions on parameters ensuring existence of a BGP equilibrium will be considered after introducing a specific form for the function h . However for the general case we can establish some interesting results on the uniqueness and stability of the BGP, assuming existence.

Proposition 4 *If \tilde{L} defined by $B(\tilde{L}) = 0$ exists, while either $\sigma > 1$, or $\sigma < 1$ and $\tau_w \leq 1 - \alpha(1 - \sigma)$ are true, then $B'(\tilde{L}) > 0$. The BGP equilibrium is then unique and locally determinate, and there is no transitional dynamics to it.*

Proof. See Appendix A. ■

As the necessary conditions for $B'(\tilde{L})$ negative require unrealistic parameters' values (in particular a very low σ or a very high τ_w), from now on we concentrate mainly on the case of a determinate and unique BGP equilibrium.

3 Effects of Taxes

3.1 Effect on labor

It is relatively simple to calculate the effect of taxes on employment in this model because the wage rate does not vary with it. As said above equilibrium labor supply \tilde{L} can be expressed as the solution to $B(\tilde{L}) = 0$. The effect of shifting the tax burden from labor to capital can be deduced by using the total derivative of $B(\tilde{L}) = 0$ with respect to labor and the tax (τ_r), keeping the ratio of government expenditure g fixed. This gives us:

$$\frac{dL}{d\tau_r} = \frac{C_1 \left(\frac{\sigma(\sigma-1)h}{h'} - \tilde{L} \right)}{B'(\tilde{L})}. \quad (30)$$

With $B'(\tilde{L}) > 0$, the case on which we focus, this derivative signs as the numerator of the fraction. In the appendix we show that the TVC can be rewritten as:

$$\tilde{L} < \frac{(\sigma-1)h}{h'}. \quad (31)$$

This is, in light of (7), the well known condition that consumption must be higher than labor income for dynamic efficiency. For $\sigma > 1$, we can easily see that we will always have $\frac{d\tilde{L}}{d\tau_r} > 0$. We are therefore ready to state the following:

Proposition 5 *An increase in the tax rate on capital income whose proceeds are used to reduce the tax on labor income will increase employment, given determinacy, if and only if $\frac{\sigma(\sigma-1)h}{h'} > \tilde{L}$. This condition is always satisfied if $\sigma > 1$.*

If $h' > 0$, i.e. $\sigma > 1$, then $U_{cL} > 0$, i.e. leisure and consumption are substitutes, so that taxing capital ie lowering the relative price of consumption today in terms of consumption tomorrow, making consumption more attractive makes leisure less attractive, helping to offset the labor-leisure distortion due to labor income taxation. For $\sigma < 1$, leisure and consumption are complements, so the increase in the tax on capital could in theory lead to more, rather than less leisure. The compensated (Frisch) elasticity of labor supply with respect to the wage, ε_F , is given by:

$$\varepsilon_F = \frac{1}{L \left(\frac{h''}{h'} + \left(\frac{1}{\sigma} - 1 \right) h^{-1} h' \right)}. \quad (32)$$

The partial derivative $\partial \varepsilon_F / \partial \sigma = \frac{L \varepsilon_F^2 h^{-1} h'}{\sigma^2}$ is positive if $h' > 0$, i.e. if $\sigma > 1$, so an increase in the net wage will produce a stronger effect on employment the higher is σ .

Since estimates tend to suggest for σ a value bigger than one we conclude that in the model shifting the tax burden from labor to capital will push employment up. It may be interesting to note that this is consistent with empirical

evidence: the data for developed countries tend to show that the higher the tax on labor income is the lower the yearly hours worked per adult are (see Ohanian et al. 2008 and references therein).

3.2 Effect on Growth

The growth effect of an increase of τ_r (and a corresponding decrease in t_w) is:

$$\begin{aligned} \frac{d\gamma}{d\tau_r} &= \frac{\partial\gamma}{\partial r} r'(\tilde{L}) \frac{d\tilde{L}}{d\tau_r} + \frac{\partial\gamma}{\partial\tau_r} \\ &= \frac{r}{\sigma} \left(\frac{(1-\tau_r)\tau_r d\tilde{L}}{\tau_r \tilde{L} d\tau_r} - 1 \right). \end{aligned}$$

Not surprisingly the condition for the tax change to be growth increasing is stricter than the condition for it to be employment increasing, because for growth to increase we need the net interest rate to increase not just the gross interest rate, which is a linear function of the employment rate. When $\tau_r > 0$, the condition for the policy to be growth increasing is that the elasticity of labor supply with respect to the tax $\frac{d\tilde{L}/\tilde{L}}{d\tau_r/\tau_r}$ is not only positive but bigger than $\tau_r/(1-\tau_r)$. In particular we have:

Proposition 6 *An increase in the tax rate on capital income whose proceeds are used to reduce the tax on labor income will increase growth, given determinacy, if and only if*

$$\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1 \right) - \frac{1 + \alpha\tau_r - \frac{g}{1-\alpha}}{\alpha(1-\tau_r)} \left(1 + (1-\sigma) \left(1 - \frac{hh''}{(h')^2} \right) \right) \geq 0. \quad (33)$$

This condition requires $\sigma > \left(\frac{1-\tau_w}{\alpha(1-\tau_r)} \right)^2$ and, regardless of the level of τ_w , is never satisfied if $\sigma \leq \left(\frac{1-\frac{g}{1-\alpha}}{\alpha} \right)^2$.

Proof. See Appendix B. ■

To understand intuitively the conditions we recall that the Frisch elasticity of labor supply is increasing in σ , for $\sigma > 1$, so the tax will provoke a stronger positive effect on employment and on the gross of tax interest rate if σ is higher. The condition is easier to satisfy the higher is α . In fact when the wage net of tax and \tilde{L} go up and therefore labor income goes up, the increase on profits is α times the increase in labor income. This means that the higher is α , coeteris paribus, the higher the increase in the rate of interest (and therefore growth), necessary to equate the PDV of profits from a new intermediate to the fixed cost of its development.

3.3 Effect on Welfare

Given γ , the BGP rate of growth, and \tilde{L} the BGP labor supply, it is possible to calculate maximum lifetime utility V along a balanced growth path:

$$V = \int_{t=0}^{\infty} e^{-[\rho-\gamma(1-\sigma)]t} \left(\frac{1}{1-\sigma} C(0)^{1-\sigma} h(\tilde{L}) \right) dt. \quad (34)$$

In the appendix B it is shown how to express V as a differentiable function of the tax rate τ_r and of equilibrium employment \tilde{L} (itself a function of τ_r). The effect on welfare of an increase in τ_r is then positive if $\frac{dV}{d\tau_r}$ is positive. To simplify calculations, we consider the following monotonically increasing transformation of V : $\frac{\log[(1-\sigma)V]}{1-\sigma}$. $\frac{d(\log[(1-\sigma)V])}{(1-\sigma)d\tau_r}$ signs as $\frac{dV}{d\tau_r}$ but is easier to manipulate algebraically so we will use it. We have:

$$\frac{d(\log[(1-\sigma)V])}{(1-\sigma)d\tau_r} = \frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial\tau_r} + \frac{d\tilde{L}}{d\tau_r} \cdot \frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial\tilde{L}}. \quad (35)$$

In Appendix B we show the following:

$$\frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial\tilde{L}} = \frac{\frac{h'}{h} - \frac{\sigma}{\tilde{L}}}{\sigma - 1} \cdot \frac{(\sigma - 1) \left(\frac{hh''}{(h')^2} - 1 \right) + 1}{\frac{(\sigma-1)h}{h'\tilde{L}} - 1} \quad (36)$$

and

$$\frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial\tau_r} = \frac{1}{\sigma - 1} \cdot \frac{\alpha\sigma}{1 + \alpha\tau_r - \frac{g}{1-\alpha}}. \quad (37)$$

Substituting (30), (36) and (37) in (35), we get:

$$\frac{d(\log[(1-\sigma)V])}{(1-\sigma)d\tau_r} = \frac{\alpha(1-\tau_r) \left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1 \right) - \frac{(1+\alpha\tau_r - \frac{g}{1-\alpha}) \left((1-\sigma) \left(1 - \frac{hh''}{(h')^2} \right) + 1 \right)}{\frac{\sigma(\sigma-1)h}{h'\tilde{L}}}}{\left(1 + \alpha\tau_r - \frac{g}{1-\alpha} \right) \left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1 \right) \frac{B'(\tilde{L})}{\sigma C_1}}. \quad (38)$$

The denominator of the expression on the RHS is always positive by 25 (and the positivity of tax rates) and by (31), given $B'(\tilde{L}) > 0$. So the derivative signs as the numerator of the expression on the RHS. Hence we arrive at the following:

Proposition 7 *If $B'(\tilde{L}) > 0$, i.e. if the BGP equilibrium is determinate, the sufficient and necessary condition for an increase in the tax rate on capital income whose revenue is used to reduce the tax on labor income to improve welfare is:*

$$\left(\frac{(\sigma-1)h}{h'\tilde{L}} - 1 \right) - \frac{1 + \alpha\tau_r - \frac{g}{1-\alpha}}{\alpha(1-\tau_r) \frac{\sigma(\sigma-1)h}{h'\tilde{L}}} \left(1 + (1-\sigma) \left(1 - \frac{hh''}{(h')^2} \right) \right) \geq 0. \quad (39)$$

If a value for τ_r exist such that for this value (39) holds as an equality, while it holds strictly for lower tax rates, (39) gives us an implicit expression for the optimal tax rate, given the tax program.⁷

In appendix B we prove the following:

Proposition 8 *If $\sigma > 1$, or $0 < \sigma < 1$ and $\frac{(\sigma-1)h}{h'\tilde{L}} > \frac{1}{\sigma}$, it is possible for a revenue neutral increase in the tax rate on capital income to increase welfare while decreasing growth.*

This result goes against the widely held belief, that when growth is suboptimal, further decreasing it cannot possibly be a Pareto improvement, no matter what static gains would go with the reduction, as the adverse growth effects always prevail by compounding over time. However, in the next section we will show that our surprising finding is more than a theoretical possibility and that for specifications of tastes and technology parameters commonly used in calibration exercises it is possible for the tax program to induce Pareto improvements but reduce growth. In fact while raising a tax on savings always induces lower growth in our simulations, not to tax them is generally inefficient. The examples we consider are also useful to confirm and further develop our intuitions on the interpretation of the various mechanisms at work.

3.4 Model Specification and Calibration

We consider here the following class of functions for the disutility of labor:

$$h(L) = (1 - L)^{1-\chi} \quad (40)$$

where $\chi > 1$ if $\sigma > 1$ or $\chi < 1 < \chi + \sigma$ if $0 < \sigma < 1$.

First we notice that when h is specified as in (40), (26) with $B(\tilde{L}) = 0$ gives us the following value for employment in equilibrium:

$$\tilde{L} = \frac{\frac{\sigma}{\alpha} (1 - \tau_w) \frac{\sigma-1}{\chi-1} - \frac{\rho}{C_1}}{\frac{\sigma}{\alpha} (1 - \tau_w) \frac{\sigma+\chi-2}{\chi-1} + (\sigma - 1) (1 - \tau_r)}. \quad (41)$$

To be more precise, \tilde{L} as defined in (41), will be equal to employment in a BGP equilibrium if it is positive, less than 1 and consistent with positive growth and with the TVC.

Proposition 9 *Conditions for the existence of a determinate equilibrium with positive growth are:*

$$-\frac{\sigma}{\alpha} (1 - \tau_w) + (1 - \sigma) (1 - \tau_r) < \frac{\rho}{C_1} < \frac{\frac{\sigma-1}{\chi-1} (1 - \tau_r)}{\frac{\sigma+\chi-2}{\chi-1} + \alpha \frac{(1-\tau_r)}{(1-\tau_w)}}. \quad (42)$$

⁷Solving the Ramsey problem by choosing the instrumental variables (here the tax rates) that maximize the indirect utility functions derived by the private agents reaction in a decentralized economy is known as the dual formulation.

When $\sigma > 1$, these conditions are sufficient as well as necessary, and in fact the first, as well as the TVC, will always hold. When $\sigma < 1$, a further condition (derived from the TVC) is:

$$\frac{\rho}{C_1} > \frac{(1-\sigma)^2(1-\tau_r)}{2-\sigma-\chi}. \quad (43)$$

Finally, the necessary and sufficient condition for determinacy is:

$$\tau_w < 1 - \frac{\alpha(1-\sigma)(1-\tau_r)(\chi-1)}{\sigma(\sigma+\chi-2)}. \quad (44)$$

Reverting all these inequalities we have necessary and sufficient conditions for an indeterminate BGP equilibrium with positive growth.

Proof. See appendix B. ■

We add that if (44) holds, then $\frac{\sigma-1}{\chi-1}(1-\tau_r) > \frac{(1-\sigma)^2(1-\tau_r)}{2-\sigma-\chi} \frac{\sigma+\chi-2}{\chi-1} + \alpha \frac{(1-\tau_r)}{(1-\tau_w)}$, so it is possible for both the second inequality in (42) and the inequality in (43) to hold. With indeterminacy, $\frac{\sigma-1}{\chi-1}(1-\tau_r) < \frac{(1-\sigma)^2(1-\tau_r)}{2-\sigma-\chi} \left(\frac{\sigma+\chi-2}{\chi-1} + \alpha \frac{(1-\tau_r)}{(1-\tau_w)} \right)$ so again the inverses of the second inequality in (42) and of the inequality in (43) will not be inconsistent.

By (29) and (41) the BGP growth rate is:

$$\gamma = \frac{\frac{C_1(\sigma-1)(1-\tau_r)(1+\alpha\tau_r - \frac{g}{1-\alpha})}{\chi-1} - \rho \left(1 + \alpha - \frac{g}{1-\alpha} + \left(1 + \alpha\tau_r - \frac{g}{1-\alpha} \right) \frac{(\sigma-1)}{\chi-1} \right)}{\sigma \left(1 + \alpha - \frac{g}{1-\alpha} + \left(1 + \alpha\tau_r - \frac{g}{1-\alpha} \right) \frac{\sigma-1}{\chi-1} \right) - \alpha(1-\tau_r)}. \quad (45)$$

Using (41) the effect of τ_r on BGP labor supply can be seen to be:

$$\frac{d\tilde{L}}{d\tau_r} = \frac{\left(1 + \frac{1-\frac{g}{1-\alpha}}{\alpha} \right) \frac{\sigma(\sigma-1)^2}{\chi-1} + \frac{\rho}{C_1} \left(1 + \frac{\sigma(\sigma-1)}{\chi-1} \right)}{\left[\frac{\sigma}{\alpha} \left(1 - \frac{g}{1-\alpha} \right) \frac{\sigma+\chi-2}{\chi-1} + \sigma - 1 + \tau_r \left(1 + \frac{\sigma(\sigma-1)}{\chi-1} \right) \right]^2}.$$

As we already now from the general case the effect on labor will be always positive for $\sigma > 1$.

By Proposition 7, a positive welfare effect, given $B'(\tilde{L}) > 0$, requires:

$$\alpha \left(\frac{\sigma-1}{\chi-1} \frac{1-\tilde{L}}{\tilde{L}} - 1 \right) (1-\tau_r) - \frac{\left(1 + \alpha\tau_r - \frac{g}{1-\alpha} \right) (\sigma+\chi-2) \tilde{L}}{\sigma(\sigma-1)(1-\tilde{L})} \geq 0. \quad (46)$$

To calculate the optimal asset income tax we plug in (46) the expression for \tilde{L} given by (41) and we equate it to zero:⁸

⁸When (41) is true, (31) will be true as well, so we do not have to check that it is respected

$$\frac{\left(\frac{(\sigma-1)^2(1-\tau_r)}{(\sigma+\chi-2)} + \frac{\rho}{C_1}\right) \alpha(1-\tau_r)}{\frac{\sigma}{\alpha} \left(1 + \alpha\tau_r - \frac{g}{1-\alpha}\right) (\sigma-1) - \frac{\rho(\chi-1)}{C_1}} - \frac{\frac{1+\alpha\tau_r - \frac{g}{1-\alpha}}{\alpha(\chi-1)} - \frac{\rho}{C_1\sigma(\sigma-1)}}{\frac{\sigma}{\alpha} + \frac{(\sigma-1)(1-\tau_r) + \frac{\rho}{C_1}}{1+\alpha\tau_r - \frac{g}{1-\alpha}}} = 0. \quad (47)$$

The root of this non linear equation in τ_r gives us the optimal value of the tax, for each six-tuple of parameters $\{\sigma, \alpha, g, \rho, \chi, C_1\}$. For all the parameterizations we consider, the expression on the LHS of the equation is always decreasing in τ_r for $0 \leq \tau_r \leq 1$, so the stationary point of the welfare function by equating it to zero we find does indeed correspond to a maximum.

We now use (47) to calculate the optimal tax rates for reasonable values of the parameters. We are completely aware that this model is not rich enough in number of variables to fit the data well. So the aim of our exercise cannot be the finding of precise quantitative results, but rather the understanding of possible mechanisms of action of policy not noticed before in the literature.

Several objects needed for the calculations have closed real-world counterparts so their calibration is relatively straightforward, while our other choices in feeding numbers to the model follow related studies, especially of R&D models (especially Comin and Gertler 2006, Jones and Williams 2000, Strulik 2007 and Zeng and Zhang 2007).

First, we set values for the the 7-tuple $\{\gamma, \rho, \tilde{L}, \sigma, \alpha, g, \tau_r\}$. These values imply values for r and C_1 (through 29), for τ_w (through 25), and for χ (through 41). We then solve (47), given the values $\{\rho, \sigma, \alpha, g, \chi, C_1\}$.

For the intertemporal elasticity of substitution and time preference parameter ρ , we follow Zeng and Zhang (2007) and set $\sigma = 1.5$ in our baseline economy. The former is closer to the value used in DSGE models of OECD economies than to the estimates of the parameter, which tend to much lower values than unity (see Alan and Browning 2010 for a recent study).⁹ As in most studies we set our central value for the rate of time discount ρ equal to 0.04 and alternatively consider 0.03 and 0.05. Coming labor supply, in 2005 the average US worker used 21 percent (24 percent) of her (his) time endowment to work, while the German one 13 percent.¹⁰ So we choose 0.17 as our benchmark value and use $\{0.13, 0.21\}$ for our sensitivity analysis.

Coming to the the value of $1/\alpha$, which is the monopoly markup on intermediates, we infer it from the ratio of intermediate consumption to gross output, which is α^2 in our model. The US intermediate consumption takes up around 0.45 of gross output, hence the mark-up $1/\alpha$ is set at 1.49. This value exceeds the range $[1.05, 1.37]$ used by Jones and Williams (2000) but is lower than the 1.6 used by Comin and Gertler (2006). The latter note that while direct evidence is missing, given the specialized nature of these products an appropriate

⁹A logarithmic specification is often adopted for the period utility function, so as to match the observed variability of output, working hours, and investment observed in the US economy, while a value of 1.6 is estimated by Smets and Wouters (2003) for European economies.

¹⁰Source: US Bureau of Labor Statistics, current Population Survey, March 2005. For further discussion see chapter 2 of Borjas (2009).

number for $1/\alpha$ would be at the high range of the estimates of markups for other types of goods in the US.¹¹ The alternative values we consider for the ratio of intermediate consumption to gross output are $\{0.40, 0.50\}$.

For the initial growth rate, we use 2 percent, as the values used in related researches include 1.25 percent (Jones and Williams 2000), and 3 percent (Zeng and Zhang 2007).

The average ratio of consolidated government expenditure to GDP over the period 1995-2009 is 36.34 percent for the US, 47.47 percent for Germany and 53.21 percent for France.¹² We define this ratio as the variable $g_N \equiv g(1 - \alpha^2)$, and take 40 percent as our benchmark for it, using 45 and 35 percent in our alternative parameterizations.

For our baseline case we consider an initial capital income tax rate of 25 percent, close to the average tax rate on capital income estimated by McDaniel for the US in the period 1995-2007.

Our choices and results as regards the baseline economy are summarized in Table 1:

Table 1: Baseline Economy: Parameterization and Results

Parameters and Steady State Variables Set	Value
rate of time discount: ρ	0.04
initial labor: L	0.17
intermediate consumption to gross output ratio: α^2	0.45
intertemporal elasticity of substitution (inverse): σ	1.5
government expenditure to GDP ratio: g_N	0.40
initial capital income tax rate: τ_r	0.25
GDP per capita growth: γ	0.02
Steady State Variables under Optimal Taxation	
optimal capital income tax rate: $\hat{\tau}_r$	0.3041
optimal labor income tax rate: $\hat{\tau}_w$	0.4643
optimal labor: \hat{L}	0.1774
optimal growth: $\hat{\gamma}$	0.0185

A first comment is that the capital income tax rate associated with maximum utility $\hat{\tau}_r$, at 30.41 percent is even higher than the initial rate of 25 percent. So one can say that the prescription arising from our simple model is more or less in line with the levels of capital income taxation observed in the real world. Under our scheme, an increase in welfare is consistent with a negative growth effect. This is especially interesting because in this model the market equilibrium generates an inefficiently low growth rate (as shown in next section), while there

¹¹Zeng and Zhang (2007) assume a benchmark value of α at 0.3, leading to a mark-up as big as 3.33. Cross-country comparisons show that in some other OECD countries the estimated markup value is higher than in the US. Neiss (2001) estimates for 24 OECD countries the mean of the markup to be 2.03 with standard deviation 0.78.

¹²Data source: Consolidated Government Expenditure, OECD.

is a generally shared view that growth effects always tend to prevail over level effects, as regards their impact on welfare. This is definitely not the case here.

We now move to some sensitivity analysis, so as to clarify the role of the various parameters . Our alternative parameterizations and results are reported in Table 2.

Table 2: Sensitivity Analysis

	$\hat{\tau}_r$	$\hat{\tau}_w$	\hat{L}	$\hat{\gamma}$
$\rho=0.03$	0.2892	0.4743	0.1752	0.0191
$\rho=0.05$	0.3147	0.4572	0.1790	0.0180
$L=0.13$	0.3162	0.4652	0.1373	0.0183
$L=0.21$	0.2917	0.4726	0.2167	0.0188
$\alpha^2=0.40$	0.2783	0.477	0.1736	0.0192
$\alpha^2=0.50$	0.3257	0.4525	0.1811	0.0180
$g_N=0.35$	0.2443	0.4209	0.1693	0.0202
$g_N=0.45$	0.3617	0.5092	0.1886	0.0174
$\sigma=1.1$	0.2404	0.5071	0.1688	0.0203
$\sigma=2$	0.3597	0.4270	0.1858	0.0173

We can now draw a detailed map of the effects at work in delivering our results. We can see that the optimal τ_r is increasing in ρ , g_N , and σ and decreasing in initial L and the markup $1/\alpha$.

On impact, lowering the tax on the wage while increasing the tax on interest will cause labor supply to increase, because of the positive substitution effect, in the absence of an income effect, and assuming, as is empirically plausible, that the effect of the complementarity between consumption and leisure is not too strong. The increased labor supply induces a higher demand for the intermediate goods. Since the price of intermediate goods is greater than their marginal cost, increased demand for an intermediate good has a first order benefit for its inventor. As previously seen this spillover from labor to profits is increasing in α . The increase in profits induces a higher demand for investment in R&D so the interest rate will rise. But the after-tax interest rate will generally (but not always) be smaller than the interest rate with a zero tax on capital income. The BGP growth rate, as a monotonically increasing function of the after-tax interest rate, also decreases. As in the model a positive externality is associated with the invention activity driving growth, this decrease lowers welfare.

The parameter α also has an effect on the second externality in the model. The effect of an invention on the present discounted value of income is given by the cost of inventing divided by the income share of capital, that is $\eta(1+\alpha)\alpha^{-1}$, while the inventor only considers the part of the contribution to production that goes to capital income, that is η . The spillover here is represented by $\eta\alpha^{-1}$. Clearly this is decreasing in α : the higher the share of profits the lower the dynamic externality.

Since the tax shift from labor to capital helps to internalize the static spillover (positively related to α), while worsening the dynamic spillover (negatively related to α), a higher α makes for a higher optimal tax on capital income, through this double action.

To explain the role of σ in determining the optimal tax rates, again we must bear in mind that the advantage of pushing up the tax on capital and down the tax on labor is contingent on the increase in labor. A bigger increase in labor will make for a bigger reduction in the monopoly distortion and a relatively less

important worsening of the appropriability failure. The increase depends on the Frisch elasticity of labor supply, whose value is increasing in σ (when $\sigma > 1$) as shown in (32). Similarly, the Frisch elasticity of labor supply is decreasing in L . Most of the values for ε_F implied by our calibrations are located around 2. In particular, in the benchmark parametric space, the Frisch elasticity is 2.38. No consensus exists on a single number for the Frisch elasticity, as values used in macroeconomic calibrations to be consistent with observed fluctuations in employment over the business cycle are much larger than microeconomic studies would suggest. The values arising in our examples are at the lower end of the values used in the macro studies.¹³

Moreover, for a given effect of the tax program on the net interest rate, the higher is σ the lower will be the effect on the growth rate and therefore the less important the worsening of the dynamic inefficiency: a lower intertemporal substitution elasticity of consumption, means consumers weigh more the current increase in consumption (which is lower than future consumption in a growing economy) than the decrease in future consumption (which is higher). So, when the instantaneous consumption is increased along with employment this increment is given more weight than the future loss.

With higher subjective discount rate ρ , although consumption will grow at a lower rate with a higher tax on capital, this dynamic loss is discounted more heavily thus making for a higher tax on capital income.

As to g_N , the ratio of government consumption to GDP, intuitively, the higher it is, the higher the factor income taxes should be, so as to internalize the externality that both working and saving create in this model, by inducing more public spending.

3.5 Comparison between the market economy and the social planner's economy

In this subsection we compare the social planner's equilibrium with the market equilibrium. Our main aim is to rule out that our result on the possibility that welfare is improved while the growth rate is reduced is due to the fact that the BGP growth rate in the market economy is higher than the social optimum.

Variables keep the same meaning as in the market economy, but the index s is used to show they characterize the social optimum. Let $X_s \equiv \int_0^{N_s} X_s(i) di$, where $X_s(i)$ is the amount of each type of the intermediate goods in the social planner's economy and X_s is the total amount produced of such goods. Then

¹³In King and Rebelo (1999) the needed elasticity is 4. This is also the value used by Prescott (2004) to explain differences in hours worked across OECD due to taxes. One explanation for this divergence between micro and macro estimates is that indivisible labor generates extensive margin responses that are not captured in micro studies of hours choices (e.g. Rogerson and Wallenius 2009). Imai and Keane (2004) find that the Frisch elasticity of labor supply may be as high as four, when taking into account that measured wages are less than the shadow wage because the second also reflects the value of on-the-job human capital accumulation. Finally Domeij and Floden (2006) point out that ignoring borrowing constraints will induce a (50%) downward bias in elasticity estimates.

the final output in equilibrium can be expressed as

$$Y = AL_s^{1-\alpha} \int_0^{N_s} X_s(i)^\alpha di. \quad (48)$$

The Hamiltonian for the social planner's problem is:

$$J = \frac{C_s^{1-\sigma} h(L_s) e^{-\rho t}}{1-\sigma} + \frac{\mu}{\eta} \left(A(1-g)L_s^{1-\alpha} \int_0^{N_s} X_s(i)^\alpha di - C_s - \int_0^{N_s} X_s(i) di \right) \quad (49)$$

where μ is the Lagrangian multiplier attached to the social budget constraint. The social planner decides on the optimal path of the control variables L_s , C_s , and $X_s(i)$, and that of the state variable N_s . The key optimality conditions are:

$$X_s(i) = (A(1-g))^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} L_s; \quad (50)$$

$$C_s = \frac{(\sigma-1)h(L_s)}{h'(L_s)} (A(1-g))^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha)N_s; \quad (51)$$

and

$$-\sigma \frac{\dot{C}_s}{C_s} + \frac{h'(L_s)}{h(L_s)} \dot{L}_s - \rho = \frac{\dot{\mu}}{\mu} = -\frac{1-\alpha}{\eta} (A(1-g))^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} L_s. \quad (52)$$

In the balanced growth path, L_s is constant so $\dot{L}_s = 0$. From (52) we get:

$$\frac{\dot{C}_s}{C_s} = \frac{\frac{1-\alpha}{\eta} (A(1-g))^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} L_s - \rho}{\sigma}. \quad (53)$$

In equilibrium, the rate of return used by the social planner r_s is then:

$$r_s = \frac{1-\alpha}{\eta} (A(1-g))^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} L_s. \quad (54)$$

Substituting (50) into (48) we get

$$Y_s = A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} L_s N_s. \quad (55)$$

The resource constraint can be expressed as:

$$\frac{\dot{N}_s}{N_s} = \frac{Y_s(1-g) - C_s - X_s}{\eta N_s} = \frac{(1-\alpha)(A(1-g))^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} L_s \left(1 - \frac{(\sigma-1)h(L_s)}{h'(L_s)L_s}\right)}{\eta} \quad (56)$$

where the second equality uses equations (50), (51) and (55). We use γ_s to denote the BGP growth rate in the centralized economy. In the BGP,

$$\frac{\dot{C}_s}{C_s} = \frac{\dot{N}_s}{N_s} = \gamma_s.$$

The transversality condition requires $0 < \gamma_s < r_s$, which, from (54) and (56) is equivalent to:

$$0 < \frac{(\sigma - 1)h(L_s)}{h'(L_s)L_s} < 1. \quad (57)$$

This is different from the analogous condition (31) in the market equilibrium. We exploit this difference to compare the steady state labor supply in the social planner's economy and that in the decentralized economy. Given our specification of the utility function in (40), $\frac{(\sigma-1)h(L)}{h'(L)L}$ equals $\frac{\sigma-1}{\chi-1} \frac{1-L}{L}$, which is a strictly decreasing function of L . But then $\frac{\sigma-1}{\chi-1} \frac{1-L_s}{L_s} < 1 < \frac{\sigma-1}{\chi-1} \frac{1-\tilde{L}}{\tilde{L}}$ (by 31 and 57), where \tilde{L} is equilibrium employment in the decentralized economy. We deduce that the steady state labor supply in the social planner's economy is larger than in the market economy.

For optimal growth to be lower than growth in a market economy we would need $C_1 \tilde{L}(1 - \tau_r) > r_s$, and a fortiori, since $L_s > \tilde{L}$, $C_1 L_s(1 - \tau_r) > r_s$, or using the definition of C_1 and (54) $\frac{1}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}} (1-\alpha)(1-\tau_r) > \frac{1-\alpha}{\eta} (A(1-g))^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}}$ or $\tau_r < 1 - \left(\frac{1-g}{\alpha}\right)^{\frac{1}{1-\alpha}}$. For realistic α and g this would require a negative τ_r .

4 Conclusions

This study analyses how the tax burden should be distributed between factor incomes in the "lab equipment" model of endogenous technological progress, thus complementing the study of fiscal policy in this same model by Zeng and Zhang (2007). Plausible calibrations of our model imply that the optimal tax rate on capital will not in general be zero. In fact, in many of the cases we consider the optimal tax rate is higher than 25 percent. Our analysis helps to make sense of the fact that in advanced economy tax rates on capital are generally well above the zero level generally recommended by the literature. In the model there are two inefficiencies, one related to the market power of firms, the second related to the appropriability problem related to the invention of new products. Shifting the tax burden from labor to capital has opposite effects on these two distortions. The increase in the interest income tax and the corresponding decrease in the labor income tax changes the opportunity cost of leisure without any change in disposable income, so labor supply will increase due to the substitution effect. Raising labor supply increases the quantity of goods produced by monopolistic firms so that the welfare cost of monopoly is reduced. For plausible calibrations of the model, the after-tax interest rate is decreasing in the tax rate on capital and so the growth rate goes down, ie the second distortion (which consists in an inefficiently low rate of growth even with a zero capital income tax) is worsened. We have shown that the optimal tax on capital income is higher the higher the elasticity of labor supply, the lower the elasticity of intertemporal substitution in consumption, the lower the income share of labor, the higher the rate of time discount and the higher the ratio between government spending and income.

We find that the sign of the growth effect of a tax program is not necessarily the same as that of the welfare effect and that the two effects should be analysed separately, even in models when growth is sub-optimal.

In future research we plan to explore the generality of the result along two main directions: ie considering a richer tax structure that includes consumption taxes, and considering a model of vertical rather than horizontal innovation. Further developments would be considering home production and the dependence of the marginal utility of leisure on its economy-wide average level.

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A Proofs for Section 2

A.1 Proof that $V = \eta$ in a growing economy.

$V > \eta$ is never possible because of the free entry assumption in the research market. On the other hand if $V < \eta$, no research would be done so that $\dot{N} = 0$, and from the economy-wide resource constraint we would have $Y - xN = C + gY$, or, using (16) and (17),

$$C = (1 - \alpha^2 - g) N L \alpha^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}. \quad (58)$$

Plugging this, together with (13), in (7), the equilibrium level of employment would be implicitly given by:

$$\frac{h}{h'} = \frac{L(1 - \alpha^2 - g)}{(1 - \alpha)(1 - \tau_w)(\sigma - 1)}. \quad (59)$$

So if this equation had a solution for L between 0 and 1, this solution would define the equilibrium level of employment in a growthless economy, L_{ng} . Plugging L_{ng} in (58) and (19), the consumption level and the profit level in this growthless economy would also be given. With labor and consumption fixed over time, the Euler equation (8) implies an interest rate equal to $\frac{\rho}{1 - \tau_r}$. Now suppose that $V = V_0 < \eta$. If $\frac{\rho}{1 - \tau_r} - \frac{L_{ng} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} (\frac{1}{\alpha} - 1)}{V_0} > 0$, or if, in other words i.e. $r - \frac{\pi}{V_0} > 0$, then, by (20), $\dot{V} > 0$. So V will increase and, since π and r will stay the same, $r - \frac{\pi}{V}$ will increase as well, ie \dot{V} will be increasing. This implies that in finite time V will get to η , but then $\dot{V} > 0$ will be no longer possible. It would then become profitable to invest in inventions and growth would start. However this would require a jump in C and L (no longer dictated by 58 and 59) which would violate the equilibrium conditions of agents. In analogous fashion, if $\frac{\rho}{1 - \tau_r} - \frac{L_{ng} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} (\frac{1}{\alpha} - 1)}{V_0} < 0$ that is if $r - \frac{\pi}{V} < 0$, V would be decreasing at an increasing rate, reaching the value 0 in finite time. If that happened (20) could not hold any longer. So again we would have a contradiction. Finally if $\frac{\rho}{1 - \tau_r} = \frac{L_{ng} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} (\frac{1}{\alpha} - 1)}{V_0}$, then $V_0 < \eta$ would be the equilibrium price of existing patents and the economy would never grow.

Summing up we can say that in a growing economy we must have $V = \eta$ at all times.

A.2 Proof of Proposition 2

Using the factor exhaustion condition that the wage bill plus total interest payments is equal to GDP, and the fact just established that growth requires $V = \eta$, we have $Y - xN = wL + r\eta N$, while substituting for C using equation (7), given (24) and (25) we can write (23) as:

$$\frac{\dot{N}}{N} = \left(\frac{1}{\alpha} + 1 \right) r - g \frac{r}{\alpha(1 - \alpha)} + \frac{h(1 - \sigma)}{h'} \left(1 + \alpha\tau_r - \frac{g}{(1 - \alpha)} \right) \frac{r}{\alpha L}. \quad (60)$$

Differentiating (7) with respect to time we obtain:

$$\frac{\dot{C}}{C} = \frac{\dot{N}}{N} + (h'/h - h''/h')\dot{L}. \quad (61)$$

Plugging this expression for $\frac{\dot{C}}{C}$ in (8) we obtain:

$$\frac{\frac{h'}{h}\dot{L} - \rho + r(1 - \tau_r)}{\sigma} - (h'/h - h''/h')\dot{L} = \frac{\dot{N}}{N}. \quad (62)$$

Finally if we substitute in (62) the expression for $\frac{\dot{N}}{N}$ given by (60) we obtain:

$$\dot{L} = \frac{\rho - r(1 - \tau_r) + \sigma \left(\frac{1}{\alpha} + 1\right) r - g \frac{\sigma r}{\alpha(1-\alpha)} + \frac{\sigma h((1-\sigma)}{h'} \left(1 + \alpha \tau_r - \frac{g}{(1-\alpha)}\right) \frac{r}{\alpha L}}{\frac{h'}{h} - \sigma(h'/h - h''/h')}$$

and using (21) we get (26) in the text.

A.3 Proof of Proposition 4

The proof is divided into two parts First we prove that $B'(\tilde{L}) > 0$ implies uniqueness and determinacy of the BGP, with no transitional dynamics to it. Second we prove that if $\sigma > 1$ or if $\sigma < 1$ and $\tau_w < 1 - \alpha(1 - \tau_r)(1 - \sigma)$, then $B'(L) > 0$, hence $B'(\tilde{L}) > 0$.

First part Given the definition of B in (28) we can write

$$B(L) \equiv m(L) - f(L)$$

with

$$m(L) \equiv \frac{\sigma}{\alpha} C_1 \frac{h((1-\sigma)}{h'} \left(1 + \alpha \tau_r - \frac{g}{(1-\alpha)}\right)$$

and

$$f(L) \equiv -\rho + C_1 L \left[1 - \tau_r - \sigma - \frac{\sigma}{\alpha} \left(1 - \frac{g}{(1-\alpha)}\right)\right].$$

Any point of intersection, assuming it exists, between the two curves m and f , both continuous and differentiable, defines a BGP equilibrium \tilde{L} . If $B'(L) > 0$ the $m(L)$ curve always crosses the $f(L)$ curve from below. But a continuous function cannot cross another continuous function from below twice in a row. This establishes uniqueness of equilibrium given its existence if $B'(L) > 0$. We define the unique BGP equilibrium labor supply as \tilde{L} .

To study the dynamic nature of a fixed point of (26), i.e. of BGP labor supply, we have to sign $d\dot{L}(\tilde{L})/d\tilde{L}$. If this derivative is positive the fixed point \tilde{L} is a repeller and the BGP is locally determinate. If $d\dot{L}(\tilde{L})/d\tilde{L}$ is negative then \tilde{L} is an attractor, i.e. there is local indeterminacy. $A(L)$ as defined in (27), is always strictly positive for all values of L , by the negative definiteness condition of the hessian of the utility function (4), so the differential equation (26) is defined for all values of L between 0 and 1. We have: $\frac{d\dot{L}}{dL}(\tilde{L}) = \frac{B'(\tilde{L})}{A(\tilde{L})} - \frac{A'(\tilde{L})B(\tilde{L})}{A^2(\tilde{L})} = \frac{B'(\tilde{L})}{A(\tilde{L})}$ (since $B(\tilde{L}) = 0$). So $B'(\tilde{L}) > 0$ implies $d\dot{L}(\tilde{L})/d\tilde{L} > 0$.

We have therefore established that if $B'(L) > 0$, the equilibrium value of L , \tilde{L} , will be unique and unstable. This implies that no other value of L is consistent with the general equilibrium conditions. Since for a given L the ratio between C and N is given, from (7) and (18), this means that in this model the economy will always be on a BGP.

Second Part We now show that if $\sigma > 1$, or if $\sigma < 1$ and $\tau_w \leq 1 - \alpha(1 - \sigma)$, we will have $B'(L) > 0$.

Given the definition of B in (28), taking the derivative and grouping terms in τ_r we have:

$$B'(L) = C_1 \frac{\sigma}{\alpha} \left(1 - \frac{g}{1 - \alpha}\right) \left(1 + (1 - \sigma) \left(1 - \frac{hh''}{(h')^2}\right)\right) + \quad (63)$$

$$C_1 \left(\sigma - 1 + \tau_r \left(1 + \sigma(1 - \sigma) \left(1 - \frac{hh''}{(h')^2}\right)\right)\right)$$

The upper bound for g is $1 - \alpha^2$, which corresponds to the case in which all net income $Y - xN = (1 - \alpha^2)Y$, is confiscated by the government, so that $\tau_r = 1$ and $\tau_w = 1$. However this upper bound for g is not a maximum, because for any economic activity to take place we need $g = 1 - \alpha^2 - \varepsilon_g$, for some real number ε_g in $(0, 1 - \alpha^2]$, as production will not happen with a confiscatory tax rate on labor income, while there will be no growth with a confiscatory tax rate on interest income. So growth requires $\tau_r = 1 - \varepsilon_{\tau_r}$, with $\varepsilon_{\tau_r} \in R$, $0 < \varepsilon_{\tau_r} \leq 1$ and $\tau_w = 1 - \varepsilon_{\tau_w}$, with $\varepsilon_{\tau_w} \in R^+ / 0$. From $g = 1 - \alpha^2 - \varepsilon_g$, from $\tau_w = 1 - \varepsilon_{\tau_w}$ and from $\tau_w = \frac{g}{1 - \alpha} - \alpha\tau_r$ (by 25) we deduce: $0 \leq \tau_r = 1 - \varepsilon_g \frac{1}{\alpha(1 - \alpha)} + \frac{\varepsilon_{\tau_w}}{\alpha}$ and $\varepsilon_g \frac{1}{(1 - \alpha)} > \varepsilon_{\tau_w} > 0$.

We can then rewrite (63) as:

$$\begin{aligned} \frac{B'(L)}{C_1} &= \frac{\sigma}{\alpha} \left(1 - \frac{1 - \alpha^2 - \varepsilon_g}{1 - \alpha}\right) \left(1 + (1 - \sigma) \left(1 - \frac{hh''}{(h')^2}\right)\right) + \sigma - 1 \\ &\quad + \left(1 - \varepsilon_g \frac{1}{\alpha(1 - \alpha)} + \frac{\varepsilon_{\tau_w}}{\alpha}\right) \left(1 + \sigma(1 - \sigma) \left(1 - \frac{hh''}{(h')^2}\right)\right) \\ &= (\sigma - 1) \frac{\varepsilon_g}{\alpha(1 - \alpha)} + \frac{\varepsilon_{\tau_w}}{\alpha} + \frac{\varepsilon_{\tau_w}}{\alpha} \sigma(1 - \sigma) \left(1 - \frac{hh''}{(h')^2}\right) \\ &> (1 - \sigma) \left(-\frac{\varepsilon_g}{\alpha(1 - \alpha)} + \frac{\varepsilon_{\tau_w}}{\alpha}\right) + \frac{\varepsilon_{\tau_w}}{\alpha}. \end{aligned}$$

For the inequality we have used condition (5). Since $\varepsilon_g \frac{1}{\alpha(1 - \alpha)} - \frac{\varepsilon_{\tau_w}}{\alpha} = 1 - \tau_r > 0$, if $\sigma > 1$ the last expression is always positive so indeterminacy never obtains.

If $0 < \sigma < 1$ we write

$$(1 - \sigma) \left(-\frac{\varepsilon_g}{\alpha(1 - \alpha)} + \frac{\varepsilon_{\tau_w}}{\alpha}\right) + \frac{\varepsilon_{\tau_w}}{\alpha} = -(1 - \sigma)(1 - \tau_r) + \frac{1 - \tau_w}{\alpha}.$$

So a necessary condition for $B'(L) < 0$ is $\tau_w > 1 - \alpha(1 - \sigma)(1 - \tau_r)$.

A.4 Proof that the TVC can be written as $\left(1 + \frac{(1 - \sigma)h}{h'\tilde{L}}\right) < 0$.

The condition (9) implies that the BGP rate of growth, γ , is lower than $r(1 - \tau_r)$. (60) gives us:

$$\gamma = r + \frac{r}{\alpha} \left(1 - \frac{g}{1 - \alpha}\right) \left(1 + \frac{(1 - \sigma)h}{h'\tilde{L}}\right) + r\tau_r \frac{(1 - \sigma)h}{h'\tilde{L}},$$

so

$$0 > \gamma - r(1 - \tau_r) = r \left(1 + \frac{(1 - \sigma)h}{h'\tilde{L}} \right) \left(\frac{1}{\alpha} \left(1 - \frac{g}{1 - \alpha} \right) + \tau_r \right).$$

Notice that: $\left(\frac{1}{\alpha} \left(1 - \frac{g}{1 - \alpha} \right) + \tau_r \right) > 0$, since $\left(1 - \frac{g}{1 - \alpha} + \alpha\tau_r \right) = 1 - \tau_w > 0$.
So $\left(1 + \frac{(1 - \sigma)h}{h'\tilde{L}} \right) < 0$ or $\frac{(\sigma - 1)h}{h'\tilde{L}} > 1$.

B Proofs for Section 3

B.1 Tax effect on growth

Using the derivative of labor with respect to the tax program (21) and (30) we get:

$$\frac{d\gamma}{d\tau_r} = \frac{r}{B'(\tilde{L})\sigma} \left[(1 - \tau_r)C_1 \left(\frac{\sigma(\sigma - 1)h}{h'\tilde{L}} - 1 \right) - B'(\tilde{L}) \right].$$

As we focus on the case $B'(\tilde{L}) > 0$, we need just to consider the sign of the expression inside the square brackets. The expression can be written, using (63), rearranging and dividing by $C_1\sigma$ as:

$$\begin{aligned} & \left(\frac{(\sigma - 1)h}{h'\tilde{L}} - 1 \right) (1 - \tau_r) - \frac{\left(1 + \alpha\tau_r - \frac{g}{1 - \alpha} \right) \left(1 + (1 - \sigma) \left(1 - \frac{hh''}{(h')^2} \right) \right)}{\alpha} \\ < & \frac{\alpha(1 - \tau_r)^2}{1 + \alpha\tau_r - \frac{g}{1 - \alpha}} - \frac{1}{\alpha} \left(1 + \alpha\tau_r - \frac{g}{1 - \alpha} \right) \left(1 + (1 - \sigma) \left(1 - \frac{hh''}{(h')^2} \right) \right) \\ < & \frac{\alpha(1 - \tau_r)^2}{1 + \alpha\tau_r - \frac{g}{1 - \alpha}} - \frac{1}{\alpha\sigma} \left(1 + \alpha\tau_r - \frac{g}{1 - \alpha} \right). \end{aligned} \quad (64)$$

To understand how the first inequality is obtained, notice the following. In a growing economy $\eta\dot{N}$ will be positive. From the resource constraint $\eta\dot{N} = Y - xN - C - G$, given $Y - xN = (1 - \alpha^2)Y$ (by 16 and 17), substituting for C its expression given by (7), after expressing the wage in terms of income by (13) and rearranging we get:

$$\eta\dot{N} = (1 - \alpha)Y \left[\alpha(1 - \tau_r) - \left(\frac{(\sigma - 1)h(\tilde{L})}{h'(L)\tilde{L}} - 1 \right) \left(1 + \alpha\tau_r - \frac{g}{1 - \alpha} \right) \right]$$

using also (25). So $\eta\dot{N} > 0$ implies, given $1 + \alpha\tau_r - \frac{g}{1 - \alpha} = 1 - \tau_w > 0$, that:

$$\frac{(\sigma - 1)h(L)}{h'(L)L} - 1 < \frac{\alpha(1 - \tau_r)}{1 + \alpha\tau_r - \frac{g}{1 - \alpha}}. \quad (65)$$

So the first inequality in (64) comes just by using (65). The second inequality in (64) is an immediate consequence of (5). Summing up a necessary condition

for $\frac{d\gamma}{d\tau_r} > 0$ is then:

$$\sigma > \left(\frac{1 + \alpha\tau_r - \frac{g}{1-\alpha}}{\alpha(1-\tau_r)} \right)^2 = \left(\frac{1 - \tau_w}{\alpha(1-\tau_r)} \right)^2. \quad (66)$$

As this lower bound on σ is a monotonically increasing function of τ_r , we can easily infer that, regardless of τ_w , for $\sigma \leq \left(\frac{1 - \frac{g}{1-\alpha}}{\alpha} \right)^2$, $\frac{d\gamma}{d\tau_r}$ is always negative.

B.2 Proof of equations 36 and 37

By solving the integral in (34) we obtain:

$$V = \frac{1}{1-\sigma} \frac{C(0)^{1-\sigma} h(\tilde{L})}{\rho - \gamma(1-\sigma)}.$$

By using (7), (21) and (25) we can write:

$$C(0) = \eta N(0) \frac{(\sigma-1)h(\tilde{L})}{h'(\tilde{L})} \frac{C_1 \left(1 + \alpha\tau_r - \frac{g}{1-\alpha} \right)}{\alpha}.$$

Using (29) we have:

$$\rho - \gamma(1-\sigma) = r(1-\tau_r) - \gamma,$$

while by using (60) to get an expression for γ , we obtain, again using (21):

$$r(1-\tau_r) - \gamma = \frac{C_1 \tilde{L}}{\alpha} \left(1 + \alpha\tau_r - \frac{g}{1-\alpha} \right) \left(\frac{(\sigma-1)h}{h' \tilde{L}} - 1 \right).$$

We can thus rewrite (34) as:

$$V = \frac{(\eta N(0))^{1-\sigma}}{1-\sigma} \frac{\left(\frac{\sigma-1}{h'(\tilde{L})} \frac{C_1(1+\alpha\tau_r - \frac{g}{1-\alpha})}{\alpha} \right)^{1-\sigma} h^{2-\sigma}}{\frac{C_1 \tilde{L}}{\alpha} \left(1 + \alpha\tau_r - \frac{g}{1-\alpha} \right) \left(\frac{(\sigma-1)h}{h' \tilde{L}} - 1 \right)}. \quad (67)$$

We have:

$$\begin{aligned} \frac{\log[(1-\sigma)V]}{1-\sigma} &= \log(\eta N(0)) + \log\left(\frac{\sigma-1}{h'}\right) + \log\left(\frac{C_1(1+\alpha\tau_r - \frac{g}{1-\alpha})}{\alpha}\right) + \\ &\frac{2-\sigma}{1-\sigma} \log(h) - \frac{1}{1-\sigma} \log\left(\frac{C_1}{\alpha} \left(1 + \alpha\tau_r - \frac{g}{1-\alpha} \right)\right) - \frac{1}{1-\sigma} \log\left(\frac{(\sigma-1)h}{h'} - \tilde{L}\right). \end{aligned}$$

From here we calculate:

$$\begin{aligned}
\frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial L} &= -\frac{h''}{h'} + \frac{(2-\sigma)h'}{(1-\sigma)h} + \frac{1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)}{(1-\sigma)\left(\frac{(\sigma-1)h}{h'}-\tilde{L}\right)} \\
&= \frac{\frac{h'}{h}-\frac{\sigma}{\tilde{L}}}{\sigma-1} \frac{1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)}{\frac{(\sigma-1)h}{h'\tilde{L}}-1},
\end{aligned}$$

which is (36) in the text. We also have:

$$\begin{aligned}
\frac{\partial(\log[(1-\sigma)V])}{(1-\sigma)\partial\tau_r} &= \frac{\alpha}{1+\alpha\tau_r-\frac{g}{1-\alpha}} - \frac{1}{1-\sigma} \frac{\alpha}{1+\alpha\tau_r-\frac{g}{1-\alpha}} \\
&= \frac{1}{\sigma-1} \cdot \frac{\alpha\sigma}{1+\alpha\tau_r-\frac{g}{1-\alpha}},
\end{aligned}$$

which is (37) in the text. Therefore:

$$\begin{aligned}
\frac{d(\log[(1-\sigma)V])}{(1-\sigma)d\tau_r} &= \frac{\alpha\sigma}{(\sigma-1)\left(1+\alpha\tau_r-\frac{g}{1-\alpha}\right)} \\
&+ \frac{\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right)\left(\frac{h'}{h}-\frac{\sigma}{\tilde{L}}\right)C_1\left(\frac{\sigma(\sigma-1)h}{h'}-\tilde{L}\right)}{(\sigma-1)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)B'(\tilde{L})}.
\end{aligned}$$

Using (63) and a common denominator this becomes:

$$\begin{aligned}
&\frac{\alpha\sigma\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)\frac{\sigma}{\alpha}\left(1-\frac{g}{1-\alpha}\right)\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right)}{(\sigma-1)\left(1+\alpha\tau_r-\frac{g}{1-\alpha}\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)\frac{B'(\tilde{L})}{C_1}} \\
&+ \frac{\sigma-1+\tau_r\left(1+\sigma(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right)}{(\sigma-1)\left(1+\alpha\tau_r-\frac{g}{1-\alpha}\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)\frac{B'(\tilde{L})}{C_1}} \\
&+ \frac{\left(1+\alpha\tau_r-\frac{g}{1-\alpha}\right)\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right)\sigma^2\left(-\frac{(\sigma-1)h}{h'\tilde{L}}+1-\frac{h'\tilde{L}}{h\sigma^2}\right)}{(\sigma-1)\left(1+\alpha\tau_r-\frac{g}{1-\alpha}\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)\frac{B'(\tilde{L})}{C_1}} = \\
&\frac{\alpha\sigma(\sigma-1)(1-\tau_r)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)-\left(1+\alpha\tau_r-\frac{g}{1-\alpha}\right)\frac{h'\tilde{L}}{h}\left(1+(1-\sigma)\left(1-\frac{hh''}{(h')^2}\right)\right)}{(\sigma-1)\left(1+\alpha\tau_r-\frac{g}{1-\alpha}\right)\left(\frac{(\sigma-1)h}{h'\tilde{L}}-1\right)\frac{B'(\tilde{L})}{C_1}}.
\end{aligned}$$

B.3 Proof of proposition 8

Let us just notice the difference between (33) and (39) is just in the second term of the expressions on the left of the inequality sign. In fact this term in (33) is equal to its analogous in (39) divided by $\frac{\sigma(\sigma-1)h}{h'\tilde{L}}$ (which is always positive by 2 and 3). The term is always negative, since $1 + \alpha\tau_r - \frac{g}{1-\alpha} > 0$ (by 25), and $1 + (1 - \sigma) \left(1 - \frac{hh''}{(h')^2}\right) > 0$ (by 5). So a positive growth effect will imply a positive welfare effect if $\frac{\sigma(\sigma-1)h}{h'\tilde{L}} > 1$. We know by (31) that this always the case if $\sigma > 1$.

B.4 Proof of Proposition 9

The first inequality in (42) just ensures that L , as given in (41), respects its upper bound of one, as can be seen by noticing that with h given by (40), the denominator of the fraction on the right-hand side of 41 $\frac{\sigma}{\alpha} \left(1 + \alpha\tau_k^l - \frac{g}{1-\alpha}\right) \frac{\sigma+\chi-2}{\chi-1} + (\sigma - 1) (1 - \tau_k^l)$ is equal to $\frac{B'(\tilde{L})}{C_1}$ (see 63), and therefore that under determinacy, ie when $B'(\tilde{L}) > 0$ (which immediately gives us 44), this denominator is positive. indeed, this inequality always holds for $\sigma > 1$. For positive growth we also need the net interest rate to be bigger than the rate of time discount or $C_1(1 - \tau_k^l)\tilde{L} > \rho$ by (29). Just by using (41) when the denominator of the fraction in (41) is positive (ie under determinacy) this second condition gives us the second inequality in (42). The TVC that $\gamma(1 - \sigma) - \rho < 0$ is always true for $\gamma \geq 0$ with $\sigma > 1$, however when $\sigma < 1$, by using (29) to express γ in terms of L and using 41, assuming determinacy, the TVC can be found to impose (43). The proof of the statement on the indeterminate equilibrium is obtained proceeding in a strictly analogous way but noticing that the denominator of the fraction on the left-hand side of 41 is negative under indeterminacy, ie when $\frac{B'(\tilde{L})}{C_1} < 0$.