Monetary and Fiscal Policy Interactions in a Frictional Model of Fiat Money, Nominal Public Debt and Banking*

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March 2, 2021

Abstract

In this paper, we study the interactions between fiscal and monetary policy in a frictional economy where fiat money, bank deposits and short and long-term nominal government bonds coexist. Since agents face information frictions and bankers have limited commitment, bank deposits need to be collateralized with nominal public debt. These bank deposits can only be used as payment instruments in some states of the world, while fiat money is always accepted. Within this frictional environment, we study monetary and fiscal policy interactions under different policy stances. When the monetary authority follows an active policy regime, a unique stationary equilibria exists regardless of how the supply of the various nominal government bonds is specified. Under this policy regime, we also find that consumption inequality increases when the central bank pursues an expansionary monetary policy. In contrast, when the fiscal authority pursues active policies, real indeterminacies can exist. However, when the fiscal authority issues sufficiently few long (or short) term bonds, a unique steady state exists. We also identify cases where an expansionary fiscal policy lead to a decline in consumption inequality between money transactions and deposit-backed transactions. Finally, regardless of the policy regime chosen by the government, financial innovations alter the relative demand for public debt and do not always lead to an increase in welfare.

JEL Codes: E40, E61, E62, H21.
Keywords: taxes; inflation; liquidity premium.

1 Introduction

The seminal work of Sargent et al. (1981) analyzed monetary and fiscal policy interactions by considering two different policy regimes for the central bank and the fiscal authority. In one regime, the central bank independently chooses a constant money growth rate, while the fiscal authority reacts by setting the path for current and future primary surpluses to balance the budget. In the other regime, the fiscal authority sets all current and future deficits, while the central bank adjusts the money growth rate so as to balance the government budget constraint.[1] Since then, there has been a large body of work studying monetary and fiscal policy interactions under a variety of economic environments that emphasize different frictions, various government liabilities and private assets.

*The authors wish to thank Hugo van Buggenum, the associate editor and the two anonymous referees for the comments and feedback we received.

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[1] Sargent et al. (1981) used the term dominance to characterize a policy regime in which either the fiscal authority or the central bank implements the policy of its choosing. Here, we adopt the terminology used by Leeper of active policies rather than dominance.

In addition, when analyzing such interactions, researchers have also considered various operating procedures for monetary and fiscal policies and have studied their consequences for inflation and debt.

In this paper, we add to the previous literature by studying the long-run consequences for inflation, total government debt, interest rate premia and consumption inequality in an environment where agents have access to fiat money, public debt of different maturities and bank deposits. To do so, we build on the frictional and incomplete framework of Williamson (2016). In this environment, the government can commit to either of the two policy regimes: one where the central bank is active and the fiscal authority is passive, and vice versa.[2] Agents trading in this economy are of fixed types: buyers, sellers and bankers. These agents face limited commitment and trade in sequential markets that are characterized by different frictions and trading protocols. In particular, in the first sub-period, a buyer bilaterally trades with a seller. A seller can be of two types: unconnected or connected. An unconnected seller is unable to verify the buyer's public debt nor deposit holdings. As a result, fiat money is the only medium of exchange that is accepted in such trades. Alternatively, a buyer can be matched with a connected seller. Such a seller is able to verify a buyer's bank deposits. As a result, trades in such matches can be settled either with fiat money or claims against their bank deposits. In second sub-period, all agents make consumption, labor and asset portfolio decisions in competitive markets. In contrast to sellers and bankers, buyers face trading uncertainty. In particular, after making optimal decisions in the second sub-period, buyers learn the type of seller they are going to trade with in the next period. As in Diamond and Dybvig (1983) and Williamson (2016), banks can offer some insurance through state-contingent one period deposit contracts. This type of contract allows buyers to undo part of their savings decision. In particular, after the realization of the shock, buyers can withdraw some or all of their deposits before the end of the period, or keep all of their deposits with the bank until maturity. However, since bankers face limited commitment, buyers and sellers require collateral to secure their deposits. Nominal government bonds of different maturities can serve such a collateral role. However, short-term public debt is more pledgeable than long-term bonds.[3]

It is important to highlight, as Bassetto and Sargent (2020) argue, that the lines between monetary and fiscal policy are blurred whenever the economy is Ricardian. However, once government bonds carry a premium, as is the case in our environment, the economy is non-Ricardian.[4] Thus, the stance of fiscal and monetary policy are quite different in terms of allocations and prices. As a result, there are different monetary and fiscal interactions depending on what policy regime is followed. Within this non-Ricardian environment, we address the following questions: Which policy regime is more likely to deliver real indeterminacies? For a given policy regime, are there any combinations of monetary and fiscal policies that can rule out multiple steady states? For a given policy regime, does the maturity composition of public debt affect the nature of the long-run equilibria? How do the different policy regimes affect consumption inequality in the long-run? How do financial innovations alter

[2]In particular, we define active monetary policy as case in which the monetary authority sets a constant money growth rate subject to a constant money to total government liabilities ratio. With this restriction, the fiscal authority passively sets the primary budget surplus to satisfy the government budget constraint. In contrast, active fiscal policy sets the path for the primary budget surplus while the monetary authority passively sets the money growth to satisfy the government budget constraint.

[3]The differential pledgeability assumption among U.S. Government Treasury Securities, which include Bills, Notes and Bonds, is consistent with the haircuts observed by clearing houses. For instance, the Chicago Mercantile Exchange group imposes a haircut of 1% for U.S. Treasury bonds that have a maturity less than one year, while it charges a 8% haircut for U.S. Treasury bonds with maturities between 10 to 30 years. Similarly, ICE Clear US imposes a haircut of 1.75% for Treasury securities that have a maturity less than one year, while it charges a 11.75% haircut for Treasury securities with maturities between 20 to 30 years. We refer to Rocheteau et al. (2018) and Dong and Xiao (2019), Domínguez and Gomis-Porqueras (2019), among others authors for more on this type of assumption.

[4]We refer to Andolfatto and Martín (2018), Domínguez and Gomis-Porqueras (2019), Gomis-Porqueras (2020) and Carli and Gomis-Porqueras (2021), among others, for discussions on public debt premium and non-Ricardian economies.

fiscal-monetary policy interactions and consumption inequality?

Under an active monetary policy regime, where the central bank has a constant money growth rate and a constant money-to-government liabilities ratio, we find that at most one stationary equilibrum can exist. Moreover, consumption in connected and unconnected meetings are independent of each other. Finally, the maturity composition of public debt does not affect DM consumption allocations in the money-backed trades. These results are observed regardless of whether collateral is plentiful (efficient consumption in connected trades is possible) or collateral is scarce, and whether fiscal authority follows a constant value of short or long-term government debt. Since our economy is non-Ricardian, a one-time change in the money growth rate has real effects. In particular, an increase in the money growth rate results in larger consumption inequality. When collateral is scarce, a larger money growth rate increases nominal government debt. As a result, the amount of collateral available to bankers increases, changing its value. This expansion of collateral expands the consumption opportunities that are backed with bank deposits. Thus, buyers with connected sellers consume more. In contrast, buyers trading with unconnected sellers consume less. This is the case as the purchasing power of fiat money is reduced when the money growth rate increases. Here, consumption inequality increases in response to a combination of limited commitment and portfolio channels as identified by Coibion et al. (2017).[5] It is worth highlighting that in our environment, the maturity composition of government debt has real effect on deposit claim backed consumption and consumption inequality.

The equilibrium properties under an active fiscal policy depend on whether collateral is plentiful or scarce. In an economy with plentiful collateral, there exists a unique stationary equilibria. Moreover, consumption in connected and unconnected meetings are independent of each other. Finally, the maturity composition of public debt does affect DM consumption in unconnected trades. These are observed regardless of the bond supply process followed by the fiscal authority. These properties are in sharp contrast when collateral is scarce. In such scenario multiple stationary equilibria can exist. Moreover, when collateral is scarce, DM consumption across the different states of the world are not independent of each other. Furthermore, fiscal policy affects the consumption in connected and unconnected trades, which in turn affect consumption inequality. Real indeterminacies in the scarce collateral equilibria are a consequence of having two different money growth rates that can satisfy the government's budget constraint. However, there are policies that can rule out real multiple steady states. In particular, when the amount of long-term or short-term government debt is sufficiently *small*, a unique steady state exists. In the stationary equilibrium, risk aversion and limited commitment are commingled in such a way that only one money growth rate satisfies the government budget constraint with small fractions of either long-term or short-term government debt.

Finally, we consider how financial innovation affects the fiscal-monetary policy interactions and consumption inequality. We do so as financial innovation changes the relative demand of the different government liabilities: fiat money and public debt of different maturities. Since our environment is non-Ricardian, financial innovations are going to alter the underlying monetary and fiscal policy interactions and consumption inequality. In this paper, we consider two different types of financial innovations. The first one we consider affects the intensive margin of frictional trades, which can be thought as capturing financial deepening. In particular, we consider innovations

[5]Coibion et al. (2017) identify four channels through which monetary policy can affect inequality. The portfolio channel is operational in our setup because it applies to households with different financial assets. Unconnected and connected trades in the first sub-period will determine how households will pay for things and thus how inflation rates affect consumption in each type of trade. Our setup also has the financial-segmentation channel. This is the case as households have access to different financial markets and instruments. Here, expansionary monetary policy also affects the collateral available to banks.

that improve the pledgeability of long-term government debt.[6] We also consider innovations that reflect changes in the extensive margin on frictional market trades through the fraction of buyers trading with connected sellers, representing financial inclusion. The experiment is designed to capture changes in information and transaction technologies resulting a smaller fraction of transactions settled with fiat money. We find that financial innovation amplifies the effects of an active monetary and an active fiscal policy when there is an increase in the pledgeability of long-term bonds. In contrast, we find that the effects of fiscal and monetary policy are dampened when financial innovation occurs through an increased measure of buyers in connected meetings. In addition, active fiscal policy is relatively more effective in increasing deposit claim backed consumption when the innovation takes the form of improved pledgeability of long-term bonds. In contrast, active monetary policy is relatively more effective when the fraction of connected trades increases. Finally, we find that financial innovations do not always increase long-run welfare.

The remainder of the paper is as follows. The related literature can be found in Section 2. The model economy is characterized in Section 3. Section 4 defines the monetary equilibrium. We consider existence and policy impacts in an active monetary policy regime in Section 5. We followup with our presentation of existence conditions, policy impacts, and equilibrium selection in an active fiscal regime in Section 6. In Section 7, we consider how changes in financial technology can affect equilibrium properties. Section 8 offers a brief summary and conclusion.

2 Related Literature

This paper connects with two different strands of literature. The first strand is one that studies fiscal and monetary policy interactions. The other literature we relate to is one that explores the links between monetary policy and consumption inequality.

The seminal work of Sargent et al. (1981) emphasized that in order to satisfy the government's budget constraint, monetary and fiscal policies have to be either coordinated or consolidated. When real resources fully back debt, Sargent et al. (1981) find that the price level adjusts to clear the money market, while the fiscal authority adjusts its future taxes. As a result, fiscal policy is inflationary only if the central bank monetizes deficits. However, when nominal debt is not backed by real resources, Leeper (1991), Woodford (1994), Sims (1994), Cochrane (2001), Schmitt-Grohé and Uribe (2000), among others, argue that fiscal policy creates a direct link between current and expected deficits and inflation. In fact, it is the coordination of monetary and fiscal policy that is needed for price and debt determination. According to Leeper (1991), in order to deliver unique and determinate equilibrium, when fiscal policies are passive (active), monetary policy needs to be active (passive). However, these active/passive policy prescriptions may not always deliver desirable equilibria: for example, (i) when agents are boundedly rational, as in Evans and Honkapohja (2007) or Eusepi and Preston (2018); (ii) when government bonds provide liquidity services, as in Canzoneri and Diba (2005), Andolfatto and Williamson (2015)

[6]In this paper, Treasury securities are the sole means of collateral. After the 2007-08 financial shock, researchers began studying how collateral changes in response to a variety of factors. Gorton and Metrick (2012) for example, provide an overview of the changes in haircuts that occurred in late 2007. Anderson and Joeveer (2014) analyze the effects that regulatory reform and changing market conditions have on collateral demand. More recently, Lee and Neuhann (2019) builds a model economy in which hidden information affects the quality of collateral. The variation in haircuts on Treasury securities is less than the variation observed as the financial crisis unfolded. Our comparative static analysis is aimed at analyzing how financial innovations–akin to Anderson and Joeveer's regulatory reform–could impact the haircuts on Treasury securities.

and Domínguez and Gomis-Porqueras (2019); (iii) when there is fiscal dominance, as in Bianchi and Ilut (2017), (iv) when an economy randomly switches between active and passive policies, as in Davig and Leeper (2011),; (v) when financial markets are not complete, as in Bassetto and Cui (2018), Andolfatto and Martín (2018) and Gomis-Porqueras (2020) among others. When these various frictions and imperfections are considered, different wealth and substitution effects emerge when revaluing public debt. As a result, the policy prescriptions that deliver unique and determinate equilibrium are quite different from those of Leeper (1991). We contribute to this literature by considering an environment that offers private and public assets that can be used as a store of value and a medium of exchange.

Within the context of consumption inequality, Coibion et al. (2017) present evidence that monetary policy contractions –which are increases in the nominal interest rate– temporally precede persistent increases in consumption inequality. Cui and Stek (2019) study a New Keynesian environment with heterogeneous households and assets differentiated by their liquidity. They focus on experiments in which the central bank's Quantitative Easing transforms the household's portfolio by trading more liquid reserves for less long-term bonds. They find that Quantitative Easing reduces insurance provided by deposit accumulation, thus resulting in greater consumption inequality.[7] In addition, Kaplan et al. (2018) study monetary policy implemented through a Taylor Rule to examine the relationship between conventional expansionary policy and inequality at business-cycle frequencies. The equilibrium distribution of consumption becomes more uneven under a temporary expansionary policy. Finally, Aït Lahcen and Gomis-Porqueras (2021) consider a monetary framework with endogenous credit market participation, where financial inclusion is an equilibrium outcome. Within this framework, we analyze various policies aimed at increasing financial inclusion and find that a direct transfer to bank account holders yields the highest welfare and lowest consumption inequality. We contribute to this literature by considering an environment that offers private and public assets that can be used as a store of value and a medium of exchange, while considering fiscal and monetary policy regimes.

3 Economic Environment

Our model builds on the frictional and incomplete market framework of Williamson (2016). Time is discrete, and indexed by t. Each period has two sub-periods that correspond to a decentralized and frictional specialized goods market (DM) and a frictionless and competitive market (CM). The CM is marked by a sequence of actions that occur before and after the realization of an idiosyncratic trade shock. The economy has four types of agents: unconnected DM-sellers, connected DM-sellers, DM-buyers and bankers. These types are permanent. DM-buyers and bankers are of unit measure and unconnected (connected) DM-sellers are of measure $\rho \quad (1-\rho)$. All private agents are infinitely-lived and discount the future at a rate $\beta \in (0,1)$. As in Lagos and Wright (2005) and Rocheteau and Wright (2005), DM-buyers face stochastic trading opportunities in the frictional goods market, while bankers only trade in CM. All private agents face limited commitment. Finally, this economy has a government that can tax CM activities and issue nominal liabilities of different maturities as well as fiat money.

[7]Cui and Stek (2019) share some similar structures to our model economy where banks operate and money is an explicit asset. However, money is not essential as a means of payment, and hence does not drive any of their primary results. They need differences in liquidity, but do not treat payments.

Preferences: An individual DM-buyer derives utility from consuming the DM perishable good and obtains disutility from CM effort. An individual buyer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(\mathbf{q}_t) - H_t \right]$$

where $\mathbf{q}$ stands for the quantity consumed by DM-buyers such that $\mathbf{q}_t \in \{q_t, q_t^u\}$ where q_t is consumption by DM-buyers in connected trades and q_t^u is in unconnected trades, H_t denotes effort exerted in CM, and E_0 is the expectation operator. DM-sellers, on the other hand, derive utility from consuming the CM perishable good and obtain disutility from DM effort. Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[-h_t + X_t^p \right]$$

where h_t denotes DM effort and X_t^p represents consumption of the CM good. We use the superscript p to identify DM-sellers.

Bankers derive payoffs from CM consumption and effort. Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[X_t^b - H_t^b \right].$$

where X_t^b represents consumption of the CM good by bankers and H_t^b denotes their CM effort.

We assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u'(0) = \infty$, $u'(\infty)$= 0.

Technology: All perishable goods in the economy are produced according to a linear technology, where labor is the only input. The production function is such that one unit of labor yields one unit of output. DM goods are produced by DM-sellers, while CM goods can be produced by DM-buyers and bankers. Here, we assume that all DM-buyers and DM-sellers are matched in each DM. Finally, bankers have access to a costless record-keeping technology that allows them to register the identity of agents.[8] Since the government is able to partially enforce CM deposits and loans, bankers can then accept deposits and acquire assets. The one-period deposit contract specifies a state-contingent return per unit of CM good deposited until the end of the period, while providing the possibility of withdrawing fiat money early.

Timing: Each period t is divided into two sub-periods (DM and CM). Each sub-period is characterized by different markets, differing in terms of trading protocols and frictions. Agents trade first in DM and then in CM. Actions in the CM are comprised of those before the trading shock and those after. Before the trading shock is revealed, consumption, effort, savings in period t occur. For DM-buyers, a shock is realized, informing them which type of trade they will be facing in the ensuing DM at $t + 1$. To be clear, DM buyers write new deposit contracts with a banker before the trading shock is realized. Further, the bank chooses its asset holdings before the trading shock is realized.

[8] Similar technologies are found in Berentsen et al. (2007), Williamson (2012), Williamson (2016), Aït Lahcen and Gomis-Porqueras (2021) among others.

Asset and goods markets close after the realization of the shock. Thus, consumption, effort and savings decisions can not be undone. Banks can purchase fiat money and public debt so that they can offer state-contingent deposit contracts. This type of contract allows DM-buyers to undo part of their savings decision after the trading shock is realized. In particular, after the idiosyncratic trading shocks, DM-buyers can withdraw some or all of their deposits before the end of the period, or keep all of their deposits with the bank for one period. The withdrawal decision depends on the type of shock they have received.[9]

Shocks: Idiosyncratic and independently distributed shocks are observed in CM by DM-buyers before the end of each period. In particular, with probability ρ, a DM-buyer receives a shock that informs him that he will be bilaterally matched with an unconnected DM-seller in the ensuing DM. With complementary probability, a DM-buyer receives a shock that informs him that he will be matched with a connected DM-seller in the next DM.

Unconnected DM-sellers cannot observe the DM-buyer's asset holdings nor his identity. As a result, unsecured credit is not extended by unconnected DM-sellers. In contrast, a connected DM-seller has access to (costless) information regarding the DM-buyer's deposits with financial intermediaries. Thus DM-sellers can accept claims to DM-buyer's deposits in exchange for DM goods.

Assets: There are four fundamental stores of value in the economy. One is a divisible and intrinsically useless asset that pays no dividend; i.e, fiat money, M_t. The real price of money, in terms of the CM good, is ϕ_t. There are also two public debt instruments with different maturities. The short-term government liability is a one-period nominal bond that promises to pay one unit of fiat money if held for one period. The price of this short-term instrument is z_t^s. The long-maturity bond is a promise to pay one unit of fiat money in the next and all future CM. The price of this perpetuity is z_t^l. In contrast to fiat money, public debt is not a *physical object*. As in Berentsen and Waller (2011), Martín (2011), Domínguez and Gomis-Porqueras (2019), Carli and Gomis-Porqueras (2021), among others, government bonds are book-entries in the government's record. These records are only available in CM. Other than having different coupon payments, long-maturity bonds are also less pledgeable than short-term public debt. Finally, agents have access to deposit contracts. However, since bankers face limited commitment, depositors require that these be collateralized.

Settlement: In states of the world where the DM-buyer is matched with an unconnected DM-seller, the only means of payment is fiat money. This is the case as the DM-seller cannot verify the DM-buyer's assets. In contrast, when the DM-buyer is matched with a connected DM-seller, claims to the DM-buyer's deposits can be used as a medium of exchange. This is the case as the DM-seller can verify them. Note however, that claims to public debt are not accepted by DM-sellers as they cannot verify their ownership. This is the case as DM-sellers do not have access to government records in DM. As a result, claims to DM-buyers' bond holdings cannot be used to facilitate trade in DM.

Given the timing of shocks and the type of deposit contract offered by bankers, a DM-buyer facing an unconnected DM-seller in period $t+1$ has the option to withdraw his deposit for a quantity of currency specified in the deposit contract at the end of the date-t in CM. In contrast, DM-buyers assigned to connected DM-sellers

[9] Thus, this type of bank deposit is then an option for DM-buyers. As in Diamond and Dybvig (1983), the deposit contract provides insurance against the DM-buyer's shock.

can exchange deposit claims to DM-sellers in the ensuing DM. After $t+1$ DM trades are completed, DM-sellers can redeem the DM-buyer's deposit claims for a specified quantity of goods in CM in period $t+1$.

Government: The government operates only in CM, where the fiscal authority can collect lump sum taxes τ_t. In addition, it can issue short (B_t^s) and long-term (B_t^l) nominal bonds. Finally, the central bank issues fiat money. The corresponding government budget constraint is then given by

$$\phi_0\Big(M_0 + z_0^s B_0^s + z_0^l B_0^l\Big) - \tau_0 = 0 \tag{1}$$

$$\phi_t\Big(M_t - M_{t-1} + z_t^s B_t^s - B_{t-1}^s + z_t^l B_t^l - (z_t^l + 1)B_{t-1}^l\Big) - \tau_t = 0 \tag{2}$$

where ϕ_t is the real value of fiat money in terms of CM goods at time t, z_t^s (z_t^l) represents the price of short (long) term bonds at time t. Implicit in this formulation is the assumption that the government has no assets or liabilities at the beginning of period 0.

Throughout the rest of the paper, we focus on monetary equilibria where the central bank does not implement the Friedman rule. Thus, positive nominal interest rates are observed in equilibrium; i.e, $\frac{\beta\phi_{t+1}}{\phi_t} < 1$. Finally, in contrast to Williamson (2016), we consider two different policy regimes and different operating procedures for monetary and fiscal policies.[10] In particular, we consider an active monetary policy regime and an active fiscal one. In addition, we study different operating procedures for monetary and fiscal policy, which will be described later on in the paper.

3.1 Households' Problem

Given the sequential nature of the environment, we solve agents' optimal decisions backwards. In what follows, we first analyze the decisions by DM-buyers and DM-sellers. Finally, we characterize the bankers' problem.

Note that depending on the shock DM-buyers have experienced, these agents may have traded with a connected or unconnected DM-seller. Throughout our analysis, the superscript u denotes the state of the world where the DM-buyer has traded with an unconnected DM-seller. In contrast, the absence of a superscripts represents the state in which the DM-buyer has traded with a connected DM-seller.

3.1.1 CM problem

In period t, depending on the shock that they have experienced, DM-buyers enter CM with some combination of currency holdings (c_{t-1}), deposits (d_{t-1}) and claims to their bank deposits (n_{t-1}), which were used to pay DM consumption in t. These differ depending on the shock that DM-buyers received in $t-1$.

In this competitive market, DM-buyers choose labor effort (H_t), fiat money holdings (c_t), public debt holdings of different maturities $(\mathcal{B}_t^s, \mathcal{B}_t^l)$ and deposits (d_t). DM-buyers also pay taxes. To fund $t+1$ DM consumption, DM-buyers must save. Fiat money, public debt of different maturities and deposits are the only savings instruments available to these private agents.

[10]Williamson (2016) assumes that, in a stationary equilibrium, the fiscal authority fixes exogenously the real value of the consolidated government debt. Moreover, the central bank can trade currency for government debt (it can conduct open market operations), and any profit it makes is rebated lump-sum to the fiscal authority.

Given the timing of CM asset markets, DM-buyers must make the deposit decision (and hence, asset-holding decision) before the realization of the shock. However, given the informational frictions and limited commitment in DM, claims to government bonds are not accepted by either unconnected or connected DM-sellers. As a result, public debt is not a useful DM consumption funding instrument. Thus, DM-buyers do not directly hold any public debt, $\mathcal{B}_t^s = \mathcal{B}_t^l = 0$. On the other hand, when DM-buyers purchase deposit contracts in CM, they can at least partly undo their savings decision after the shock is realized. In particular, once DM-buyers know they will trade with an unconnected DM-seller in the ensuing DM, they can withdraw fiat money before the end of the period.[11] In contrast, once DM-buyers know they will trade with a connected DM-seller, they can use claims to their deposits to fund DM consumption.[12]

In this paper, we study economies in which the state-contingent return (if held until maturity) on banks deposits, R_t, dominates fiat money. As a result, risk-averse DM-buyers prefer the deposit contract offered by competitive banks, partly insuring them against the shocks they experience. Thus, there is no need for DM-buyers to simultaneously hold both currency and deposits. As a result, we have that $c_t = 0$ and $d_t > 0$. Let $V(c_t, d_t)$ and $V^u(c_t, d_t)$ represent the value function of entering the next DM and trading with a connected or unconnected DM-seller, respectively. Then the resulting date-t CM value function before a DM-buyer before the shock is realized is given by

$$W(c_{t-1}, d_{t-1}, n_{t-1}) = -H_t + \beta\, [\, (1-\rho)\, V(0, d_t) + \;\rho\, V^u(0, d_t)] \tag{3}$$

where H_t is CM effort by DM-buyers which is given by $H_t = d_t - R_{t-1}(d_{t-1} - n_{t-1}) - \tau_t$, where τ_t denotes CM taxes. It is important to emphasize that the decision on how much of the new deposits d_t to withdraw is made after shocks are revealed. Given that banks face free entry and trade in a competitive market, the optimal deposit is found in the bank's problem.

DM-sellers do not face any uncertainty nor hold assets across periods. To pay for CM consumption, they can simply use the proceeds of their previous DM production. Thus, these agents do not carry fiat money across periods, nor deposit with private banks. As a result, we have that $c_t^p = d_t^p = 0$. Thus, the resulting date-t CM value function of a connected DM-seller is given by

$$W^p(c_{t-1}, d_{t-1}, n_{t-1}) = X_t^p + \beta\, V^p(0, 0) \tag{4}$$

where CM consumption is given by $X_t^p = R_{t-1} n_{t-1} - \tau_t$. Similarly, the date-$t$ CM value function of an unconnected DM-seller is given by

$$W^p(c_{t-1}, d_{t-1}, n_{t-1}) = X_t^p + \beta\, V_p(0, 0) \tag{5}$$

where CM consumption is given by $X_t^p = \phi_t l_{t-1}^u - \tau_t$, where l_{t-1}^u denotes the cash payment he receives for producing DM goods in the previous sub-period.

[11]This is the case as fiat money is the only means of payment that is accepted by unconnected DM-sellers.

[12]This is the case as claims to deposits, together with fiat money, are the only means of payment accepted by connected DM-sellers.

3.1.2 DM problem

When matched, the DM-buyer makes a take it or leave it offer to the unconnected DM-seller that specifies the DM quantity to be exchanged and the corresponding payment in this frictional market. The DM-buyer that knows he will trade with an unconnected DM-seller withdraws all his deposits in the form of fiat money. This is the case as fiat money is costly to carry across periods and it is the only medium of exchange that will be accepted when trading with an unconnected DM-seller. Let us denote all the fiat money available to trade in DM at period t by c^w_{t-1}. Note that this is the quantity of currency withdrawn from the bank at $t-1$ before the end of CM.

At the beginning of period t, the optimal terms of trade when DM-buyer is matched with an unconnected DM-seller solve the following problem

$$V^u(c^w_{t-1}, 0) = \max_{q^u_t, l^u_t} \left\{ u(q^u_t) + W(c^w_{t-1} - l^u_t, 0, 0) \right\} \quad \text{s. t.} \tag{6}$$

$$-q^u_t + W^p(l^u_t, 0, 0) \geq W^p(0, 0, 0)$$

$$l^u_t \leq c^w_{t-1}$$

$$c^w_{t-1} = d_{t-1}$$

where q^u_t denotes the quantity of DM goods that is exchanged for l^u_t units of currency with an unconnected DM-seller. Note that the first constraint represents the DM-seller's incentive compatibility constraint, which is required to induce DM production. The second one highlights the fact that the DM-buyer cannot hand in more fiat money than what he has brought into the match. Finally, the third constraint takes into account the optimal withdraw decision of the DM-buyer in the previous CM at $t-1$.

It is easy to show that the optimal terms of trade imply the following DM consumption schedule

$$q^u_t(m_{t-1}) = \begin{cases} q^* & \text{if } \phi_t c^w_{t-1} \geq q^* \\ \phi_t c^w_{t-1} & \text{if } \phi_t c^w_{t-1} < q^* \end{cases} \tag{7}$$

where $m_{t-1} = \phi_{t-1} c_{t-1}$ is the real money balance in term of $t-1$ CM goods, q^* is the efficient DM allocation, which is implicitly defined by $u'(q^*) = 1$.

When the DM-buyer has received a shock informing him that he will trade with a connected DM-seller, he can use his deposits to fund DM consumption. In particular, the DM-buyer does not need to transfer his deposits to the DM-seller. Instead he can offer claims to them. Let n_{t-1} denote the quantity of claims to the DM-buyer's deposits. Under a take it or leave offer made by the DM-buyer, the optimal terms of trade at the beginning of period t are given by

$$V(0, d_{t-1}) = \max_{q_t, n_{t-1}} \{ u(q_t) + W(0, d_{t-1} - n_{t-1}, n_{t-1}) \} \tag{8}$$

$$\text{s.t} - q_t + W^p(0, 0, n_{t-1}) \geq W^p(0, 0, 0)$$

$$n_{t-1} \leq d_{t-1}$$

where the first constraint represents the DM-seller's incentive compatibility constraint. The second one reflects the fact that the DM-buyer may not offer more claims than his deposits.

It is easy to show that the optimal terms of trade imply the following DM consumption schedule

$$q_t(n_{t-1}) = \begin{cases} q^* & \text{if } R_{t-1}n_{t-1} \geq q^* \\ R_{t-1}n_{t-1} & \text{if } R_{t-1}n_{t-1} < q^*. \end{cases} \tag{9}$$

3.2 Bankers

Bankers operate in a free-entry and competitive environment and only trade in CM. Without loss of generality, given perfect competition among bankers, we consider the banking industry operating as a single bank.[13] While DM-buyers can trade at most with one banker, a banker can contract with all DM-buyers and DM-sellers. As in Williamson (2012), banks maximize the expected utility of depositors, offering state-contingent deposit contracts subject to their balance sheet constraints. Since bankers face limited commitment, the incentive compatibility constraint has to be considered by depositors. As in Diamond and Dybvig (1983), a deposit contract provides insurance against DM-buyer shocks.

After deposits are made and before shocks are realized, the representative bank purchases fiat money and public debt in CM. These deposits are one-period contracts that allow the possibility of early withdrawal after the shocks have been revealed and before the ensuing DM. Because of free entry and prefect competition, the deposit contract maximizes the expected utility of a representative DM-buyer. The deposit contract specifies a state-contingent return, R, per unit of CM good deposited if held until maturity. Claims to deposit are redeemed by the bank in CM for R units of CM goods per unit claim. The deposit contract also allows currency withdrawals before the end of the CM period. Thus, if a DM-buyer deposits with a bank, after he receives his shock and before the end of the CM period, he can either withdraw currency or leave his deposit in the bank until the next CM, depending on the shock he has received. In particular, if matched with a connected DM-seller in the ensuing DM, the DM-buyer can use claims to his deposits to fund DM consumption. In contrast to Diamond and Dybvig (1983), bankers face limited commitment. As a result, when offering deposit contracts, DM-buyers require that bank deposits (loans) be collateralized, as in Kiyotaki and Moore (1997), among others. Long-term government bonds are less pledgeable forms of collateral than short-term public debt; i.e, $(1-\theta_s) > (1-\theta_l)$.[14] Using the law of large numbers, the banker's objective function in period t is given by

$$\mathcal{U}_t = -d_t + \beta\left[\rho\, u\Big(q^u_{t+1}(m_t)\Big) + (1-\rho)\, u\big(q_{t+1}(n_t)\big)\right] \tag{10}$$

where d_t represents the quantity of goods deposited by the representative depositor, which is funded by exerting effort in CM. $q^u_{t+1}(m_t)$ and $q_{t+1}(n_t)$ are the DM consumption in the two different states of the world, which are given by optimal terms of trade; i.e, (7) and (9), respectively.

In period t, the banker uses the DM-buyers' deposits to acquire fiat money and government debt. At date $t+1$, the banker receives proceeds from all the holdings of public debt to fund the payments to his depositors. The banker's balance sheet and incentive compatibility constraint are given by

$$d_t - \rho m_t - z^s_t b^s_t - z^l_t b^l_t - (1-\rho)R_t n_t + \frac{\phi_{t+1}}{\phi_t} b^s_t + \frac{\phi_{t+1}}{\phi_t} b^l_t(1+z^l_{t+1}) = 0 \tag{11}$$

[13] A similar insurance scheme is found in the random-relocation models of Bencivenga and Smith (1991); Schreft and Smith (1998) and Gomis-Porqueras (2000), among others.

[14] θ_s and θ_l represent haircuts on short and long-term bonds respectively.

$$- (1-\rho)R_t n_t + \frac{\phi_{t+1}}{\phi_t} b_t^s (1-\theta_s) + \frac{\phi_{t+1}}{\phi_t} b_t^l (1+z_{t+1}^l)(1-\theta_l) \geq 0 \tag{12}$$

where d_t is the quantity of goods deposited by the representative depositor, n_t represent claims to deposits, $m_t = \phi_t c_t$ denotes real balances expressed in terms of CM goods that the banker holds, and $b_t^s = \phi_t B_t^s$ ($b_t^l = \phi_t B_t^l$) represents real short (long) term public debt that the banker holds in his portfolio.

We can write the banker's problem as follows

$$\max_{d_t, n_t, m_t, b_t^s, b_t^l} \mathcal{U}_t \quad \text{s.t.} \quad (11) \text{ and } (12)\ .$$

It is worth noting that away from the Friedman rule, DM consumption in unconnected trades is not the first best; i.e, $q_{t+1}^u(m_t) = \frac{\phi_{t+1}}{\phi_t} m_t < q^*$. Substituting deposits from the balance sheet condition into the objective function, the banker's first-order conditions are given by

$$n_t: \quad -R_t + \beta\, u'\big(q_{t+1}(n_t)\big)\, \frac{\partial q_{t+1}(n_t)}{\partial n_t} - \Lambda_t R_t \leq 0 \tag{13}$$

$$m_t: \quad -1 + \beta\, u'\left(\frac{\phi_{t+1} m_t}{\phi_t}\right) \frac{\phi_{t+1}}{\phi_t} \leq 0 \tag{14}$$

$$b_t^s: \quad -z_t^s + \frac{\phi_{t+1}}{\phi_t} - \Lambda_t \frac{\phi_{t+1}}{\phi_t}(1-\theta_s) \leq 0 \tag{15}$$

$$b_t^l: \quad -z_t^l + \frac{\phi_{t+1}}{\phi_t}(1+z_{t+1}^l) + \Lambda_t \frac{\phi_{t+1}}{\phi_t}(1-\theta_l)(1+z_{t+1}^l) \leq 0 \tag{16}$$

where Λ_t is the Lagrange multiplier associated with the bank's incentive constraint given by equation (12). Finally, the DM consumption in connected trade is given by

$$q_{t+1}(n_t) = \begin{cases} q^* & \text{if } R_t n_t \geq q^* \\ R_t n_t & \text{if } R_t n_t < q^*. \end{cases}$$

The solution to the banker's problem depends on whether the consumption of DM-buyers in connected trades is first best or not. Below we analyze these two scenarios.

Plentiful Collateral

In this case, the state contingent deposit return is large enough that connected DM consumption is efficient so that $q_t(n_{t-1}) = q^*$. Since such return is backed by government debt of different maturities, having a large deposit return implies that public debt is plentiful. The DM consumption for unconnected trades, on the other hand, is given by

$$1 = \beta\, u'(q_{t+1}^u)\, \frac{\phi_{t+1}}{\phi_t}.$$

It is worth highlighting that when there is plentiful collateral, the optimal deposit claim is the lowest possible; i.e, $n_t = q^*/R_t$. This is the case as this quantity allows the DM-buyer to exert sufficiently less CM effort to obtain funds that are going to be deposited into the bank.

Finally, the returns on short- and long-term government bonds are equal whenever the banker's incentive constraint does not bind, $\Lambda_t = 0$. In other words, all government debt are priced fundamentally as it does not expand DM consumption in connected trades. Throughout the rest of the paper, we focus on the case where the banker's incentive constraint binds.

Scarce Collateral

In an environment where collateral is scarce, the efficient DM consumption of connected trades is not feasible. When the banker's incentive constraint binds, the return on deposits carried over to the next period will dominate the return on fiat money. The shadow value of the bankers incentive constraint is given by (13), $\Lambda_t = \beta\ u'\big(R_t n_t\big) - 1$, and from equations (15) and (16), we can derive the gross nominal bond yields for the short and long-term government bonds. These are given by

$$\frac{1}{z_t^s} \equiv 1 + r_t^s = \frac{\phi_t}{\phi_{t+1}\Big(\beta\ u'\big(R_t n_t\big)(1-\theta_s) + \theta_s\Big)} \tag{17}$$

$$\frac{1 + z_{t+1}^l}{z_t^l} \equiv 1 + r_t^l = \frac{\phi_t}{\phi_{t+1}\Big(\beta\ u'\big(R_t n_t\big)(1-\theta_l) + \theta_l\Big)}. \tag{18}$$

where $r^s(r^l)$ are the net interest for short-term (long-term) government bonds.

We can now establish the following result. (Proofs can be found in Appendix A.)

Lemma 1. *The nominal term premium between the long- and short-term bonds is positive if and only if $\theta_l > \theta_s$.*

Lemma 1 highlights the consequences of having public debt of different maturity that have differential pledgeability. Anything that relaxes the scarcity of collateral will enlarge the DM consumption possibilities. What differentiates the long-term bonds is less pledgeability; i. e, $\theta_l > \theta_s$. This results in an upward sloping yield curve. Since the total value of assets is insufficient to support efficient trade for DM connected trades, any additional instrument that is collateralizable can help relax the borrowing constraint and deliver a consumption closer to the first best.[15]

Finally, the DM consumption for unconnected trades, on the other hand, is given by

$$1 = \beta\ u'(q_{t+1}^u)\ \frac{\phi_{t+1}}{\phi_t}$$

while that of connected trades is given by $q_{t+1} = R_t n_t$. Note that when the banker's incentive constraint binds, the banker's optimal asset allocation and optimal terms of trade satisfies the following condition

$$(1-\rho)R_t n_t\Big(\beta\ u'(R_t n_t)(1-\theta_s) + \theta_s\Big) + \frac{z_t^l b_t^l(\theta_l - \theta_s)}{u'\left(\beta R_t n_t\right)\left(1-\theta_l\right) + \theta_l} = (z_t^s b_t^s + z_t^l b_t^l)(1-\theta_s). \tag{19}$$

4 Monetary Equilibrium

Having solved for the optimal decisions of agents, we can now characterize the resulting monetary equilibrium. The corresponding dynamic monetary equilibria describing our economy satisfies the following system of non-

[15] This is consistent with the rationale found in Kocherlakota (2003) and Andolfatto (2011).

linear equations

$$(1-\rho)R_tn_t\Big(\beta u'(R_tn_t)(1-\theta_s)+\theta_s\Big)+\frac{z_t^lb_t^l(\theta_l-\theta_s)}{\beta u'\left(R_tn_t\right)\left(1-\theta_l\right)+\theta_l}=(z_t^sb_t^s+z_t^lb_t^l)(1-\theta_s) \tag{20}$$

$$z_t^s=\frac{\beta u'\left(R_tn_t\right)\left(1-\theta_s\right)+\theta_s}{\beta u'\left(\frac{\phi_{t+1}m_t}{\phi_t}\right)} \tag{21}$$

$$\frac{1+z_{t+1}^l}{z_t^l}=\frac{\beta u'\left(\frac{\phi_{t+1}m_t}{\phi_t}\right)}{\beta\ u'\big(R_tn_t\big)(1-\theta_l)+\theta_l} \tag{22}$$

$$\frac{\phi_t}{\phi_{t+1}}=\beta u'\left(\frac{\phi_{t+1}m_t}{\phi_t}\right) \tag{23}$$

$$q_{t+1}(n_t)=\begin{cases}q^* & \text{if } R_tn_t\geq q^*\\ R_tn_t & \text{if } R_tn_t<q^*.\end{cases} \tag{24}$$

$$q_{t+1}^u(m_t)=\frac{\phi_{t+1}m_t}{\phi_t} \tag{25}$$

$$\phi_tM_t=\rho m_t \tag{26}$$

$$\phi_tB_t^s=b_t^s \tag{27}$$

$$\phi_tB_t^l=b_t^l \tag{28}$$

$$\phi_0\Big(M_0+z_0^sB_0^s+z_0^lB_0^l\Big)-\tau_0=0 \tag{29}$$

$$\phi_t\Big(M_t-M_{t-1}+z_t^sB_t^s-B_{t-1}^s+z_t^lB_t^l-(z_t^l+1)B_{t-1}^l\Big)-\tau_t=0. \tag{30}$$

Definition 1. *Given monetary and fiscal policies, a stationary monetary equilibria is a set of CM and DM consumption bundles $\{q^u,q\}$, real balances and real bond holdings $\left\{\phi M,\phi B^s,\phi B^l\right\}$, deposits, claims to deposits and prices $\left\{d,n,z^s,z^l,R\right\}$ that do not evolve over time, satisfy agents' optimization problems and markets clear, which are given by equations (20)-(30).*

Next we specify government policies that are going to be invariant across the monetary and fiscal regime.

Government Policies: Regardless of which policy regime is considered, throughout our analysis, we impose that the money-to-total-value-of-nominal-government-liabilities ratio (M-L ratio) is constant. In addition, given that the fiscal authority has liabilities of different maturities, we make additional assumptions regarding the supply of government debt. In particular, we consider the case where the fiscal authority fixes either the real value of short-term (Policy $\mathcal{S}$) or long-term (Policy $\mathcal{L}$) public debt. For robustness, we also consider a policy of constant total government liabilities, which allows for open market operations (including quantitative easing). In addition, we consider a policy with supply process of government debt to be represented by a constant quantity of long or short-term debt.[16]

[16]See Martín (2011) for constant money-to-bond ratio and Williamson (2012) for money-to-government liability ratio.

In the next sections, we characterize the resulting stationary equilibria under an active monetary policy regime and an active fiscal policy stance. It is important to note that our economy is non-Ricardian. This is the case as the banker's incentive constraint binds. This results in government debt exhibiting premia. This indicates that households are willing to pay a price above its fundamental value when purchasing debt of different maturities. Thus, the revenue implications of issuing debt of different maturities or taxing are quite different in this environment. In this non-Ricardian world, meaningful characterizations of monetary and fiscal stabilization policies are going to be conditional on the prevailing policy regime as well as composition and size of government debt. Throughout the rest of the paper, we focus on stationary equilibria.

5 Active Monetary Policy Regime

The central bank sets a money growth rate, μ, while the fiscal authority adjusts taxes (or transfers) so as to satisfy the government budget constraint. Let δ be the constant M-L ratio. For a money growth rate μ, a constant M-L ratio implies the following

$$M_{t+1} = \mu M_t = \mu\delta(M_t + z_t^s B_t^s + z_t^l B_t^l) \tag{31}$$

which implies that in stationary equilibrium, we have $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$. Finally, the resulting steady state taxes that ensures the government budget constraint is satisfied is given by

$$\tau = \phi M\left(\frac{1}{\delta} - \frac{1}{\mu} - \frac{(1-\delta)}{\delta\mu z^s}\right) - \frac{\phi B^l}{\mu}\left(z^l + 1 - \frac{z^l}{z^s}\right). \tag{32}$$

As we saw from the banker's problem, the optimal DM consumption in the different state of the world depends on whether the collateral is scarce or not. Next we analyze these cases. We study stationary equilibria where the banker's incentive constraint binds, so that no default is observed in equilibrium and public debt exhibits a premia.

5.1 Plentiful Collateral

When collateral is plentiful so that DM consumption in connected trades is efficient, $q = q^*$, the stationary equilibria satisfies the following system of equations

$$\beta u'(q^u) = \mu \tag{33}$$

$$z^s(\mu) = \frac{\beta(1-\theta_s) + \theta_s}{\beta u'(q^u)} \tag{34}$$

$$\frac{1}{z^l(\mu)} + 1 = \frac{\beta u'(q^u)}{\beta(1-\theta_l) + \theta_l} \tag{35}$$

$$(1-\rho)q^*\Big(\beta(1-\theta_s) + \theta_s\Big) + \frac{z^l(\mu)\, b^l(\theta_l - \theta_s)}{\beta(1-\theta_l) + \theta_l} = (z^s(\mu)\, b^s + z^l(\mu)\, b^l)(1-\theta_s) \tag{36}$$

$$\tau = q^u\mu\left(\frac{1}{\delta} - \frac{1}{\mu} - \frac{(1-\delta)}{\delta\mu\, z^s(\mu)}\right) - \frac{b^l}{\mu}\left(z^l(\mu) + 1 - \frac{z^l(\mu)}{z^s(\mu)}\right). \tag{37}$$

With an active monetary policy, an increase in μ results in a decrease in consumption of unconnected trades,

q^u. This is the case as the real price of money falls. In contrast, an expansion of the monetary base does not affect consumption of connected DM-buyers. Moreover, in order to support q^*, the government must supply enough assets such that the banker's incentive compatibility is satisfied.

It is important to note that given a money growth rate μ, DM consumption in all states of the world is pinned down and these different consumption patterns are independent of each other. The price of short (z^s) and long-term (z^s) public debt are fully determined as they depend on q^u, as shown by equations (34) and (35), respectively. The equilibrium real public debt of different maturities, b^s and b^l, will depend on the type of bond supply process the fiscal authority follows. As we previously mentioned we consider two such processes. Policy $\mathcal{L}$ sets the *value* of long-term government debt to be constant over time. In contrast, Policy $\mathcal{S}$ fixes the *value* of short-term debt to be a constant over time. Given the evolution of debt, equilibrium taxes will adjust so as to balance the budget. Next, we explore how these two different bond supply processes affect the nature of the stationary equilibria.

5.1.1 Policy $\mathcal{L}$

The fiscal authority sets the value of long-term government debt to be a constant, $\phi_t z_t^l B_t^l = A \ \ \forall t$. The resulting unique steady state for short-term real bonds is given by

$$b^s(\mu) = \frac{1}{(1-\theta_s)z^s(\mu)}\left[(1-\rho)q^*\Big(\beta(1-\theta_s)+\theta_s\Big) + \frac{A(\theta_l-\theta_s)}{\beta(1-\theta_l)+\theta_l}\right] - A$$

while equilibrium taxes are given by

$$\tau(\mu) = q^u\mu\bigg(\frac{1}{\delta} - \frac{1}{\mu} - \frac{(1-\delta)}{\delta\mu z^s(\mu)}\bigg) - \frac{A}{\mu\, z^l(\mu)}\left(z^l(\mu) + 1 - \frac{z^l(\mu)}{z^s(\mu)}\right).$$

We can then conclude that an active monetary policy rules out real indeterminacies. Moreover, the DM consumption in connected and unconnected trades does not depend on the amount of long-term debt that the fiscal authority issues.

5.1.2 Policy $\mathcal{S}$

The fiscal authority sets the value of short-term government debt to be a constant, $\phi_t z_t^s B_t^s = B \ \ \forall t$. The resulting equilibrium short-term real bonds is a solution to the following equation

$$(1-\rho)q^*\Big(\beta(1-\theta_s)+\theta_s\Big) + \frac{z^l(\mu)\; b^l(\mu)\; (\theta_l-\theta_s)}{\beta(1-\theta_l)+\theta_l} = (B + z^l(\mu)\; b^l(\mu))(1-\theta_s).$$

Since this expression is linear in $b^l(\mu)$, there is a unique steady state for real long-term bonds. Finally, equilibrium taxes are given by

$$\tau(\mu) = q^u\mu\bigg(\frac{1}{\delta} - \frac{1}{\mu} - \frac{(1-\delta)}{\delta\mu\; z^s(\mu)}\bigg) - \frac{b^l(\mu)}{\mu}\left(z^l(\mu) + 1 - \frac{z^l(\mu)}{z^s(\mu)}\right).$$

Here, we also find that an active monetary policy delivers a unique stationary equilibrium. We also find that supply of short-term public debt does not affect DM consumption in connected and unconnected trades.

Thus, with plentiful collateral, an active monetary policy always delivers a unique steady state and that stationary equilibrium is invariant to the bond supply process. The quantities consumed in the DM are independent of each other; that is, q and q^u do not depend on each other. Moreover, the maturity composition of public debt does not affect DM consumption allocations.

5.2 Scarce Collateral

When collateral is scarce, DM consumption in all states of the world is inefficient. In such equilibrium, a change in government bonds can help expand the consumption opportunities of connected trades. As a result, public debt of different maturities are not priced fundamentally. Ricardian equivalence then does not hold in this environment. Households are willing to pay a price above the public debt's fundamental value. Thus, the revenues generated by issuing nominal debt, when compared to taxing, are quite different from a situation when government bonds are priced fundamentally. As pointed out by Bassetto and Sargent (2020), the lines between monetary and fiscal policy are no longer blurred. The underlying monetary and fiscal interactions may deliver quite different equilibrium properties depending on which policy regime is followed.

As in the plentiful collateral case, next we examine the resulting stationary equilibria under two different bond supply processes.

5.2.1 Policy $\mathcal{L}$

The fiscal authority sets the value of long-term government debt to be a constant, $\phi_t z_t^l B_t^l = A \ \ \forall t$. After imposing stationarity on equations (20)-(30) and after repeated substitution, the stationary equilibria can be summarized by DM consumption, q and q^u. In particular, the stationary equilibria satisfies the following equations

$$(1-\rho)\ q\ (\beta u'(q)(1-\theta_s)+\theta_s)+\frac{A(\theta_l-\theta_s)}{\beta u'(q)(1-\theta_l)+\theta_l}=\frac{\rho(1-\delta)q^u\ u'(q^u)(1-\theta_s)}{\delta} \tag{38}$$

$$\beta u'(q^u)=\mu \tag{39}$$

where equation (39) pins down the DM-buyer's consumption in unconnected meetings, while equation (38) then determines consumption in connected trades. It is through the binding banker's incentive compatibility constraint, equation (38), that the DM consumption in the two states of the world are linked to each other. In general, there are two opposing forces that link these two different consumption allocations. To provide more intuition, suppose there is an increase in q^u. The money growth rate needed to satisfy the government budget constraint declines, reducing the future price level. As a result, the real value of government debt obligations increases. This in turn loosens the bank's incentive compatibility constraint. As a result, DM-buyers in connected trades can increase their consumption. The size of such an increase depends on how risk averse agents are. This is the case as claims to deposit contracts are effectively claims to a financial instrument that acts like insurance. To illustrate the other opposing effect, the collateral channel, suppose there is a decrease in the money growth rate that causes a decrease in the supply of short-term debt. Because the quantity of collateral is reduced, the incentive compatibility constraint tightens and buyers in connected trades will consume less.

It is worth pointing out that if the collateral (risk-aversion) effect dominates, the locus is downward (upward) sloping. In other words, trades in connected and unconnected meetings are substitutes (complements). Hence, in general, the locus can be downward sloping or upward sloping or non-monotonic, depending on the underlying primitives and policies that describe our economy. For instance, the haircut on long-term debt relative to short-term debt, the value of long-term debt outstanding and the measure of connected and unconnected sellers, impacts the shape of the consumption locus.[17] More precisely, if the difference in haircut approaches zero ($\theta_l \to \theta_s$) it follows that q and q^u are complements. In this new scenario, the collateral effect is diminished and the risk-aversion effect dominates, resulting in DM consumption complementarity across the two states of the world.

We can fully characterize the net returns for short and long-term bonds as well as taxes once DM consumptions in connected trades are determined. In particular, we have that these equilibrium objects are given by

$$r^s = \frac{\mu}{\Big(\beta u'(q)(1-\theta_s)+\theta_s\Big)} - 1 \tag{40}$$

$$r^l = \frac{\mu}{\Big(\beta u'(q)(1-\theta_l)+\theta_l\Big)} - 1 \tag{41}$$

$$\tau = \frac{\rho\mu q^u}{\beta}\left(\frac{1}{\delta} - \frac{1}{\mu} - \frac{(1+r^s)(1-\delta)}{\delta\mu}\right) - (1+r^l-r^s)A. \tag{42}$$

We can now establish the following result.

Proposition 1. *There exists a unique stationary monetary equilibrium whenever the following condition is satisfied*

$$(1-\rho)q^*(\beta(1-\theta_s)+\theta_s) + \frac{A(\theta_l-\theta_s)}{(\beta(1-\theta_s)+\theta_s)} \geq \frac{\rho(1-\delta)q^u\ u'(q^u)(1-\theta_s)}{\delta}.$$

To gain some intuition, let us graph equations (38) and (39) in the (q^u, q) space, as illustrated in Figure 1.[18] Let $q = F(q^u)$ represent the locus of consumption quantities that satisfy (38) and $q = G(q^u)$ represent the locus of consumption that satisfies (39). We plot the two loci for a given set of policies. The intersection between the two loci determines the resulting stationary equilibria. Note that the second term of equation (38) is positive and the first term changes sign from positive to negative as q increases. This observation is illustrated by the hump-shaped of the $F(.)$ locus. For lower values of q^u (higher values of μ), the risk-averse effect dominates the collateral one and the two types of consumption are complements. For higher values of q^u (lower values of μ), the collateral effect dominates and the consumption allocations in the two different states of the world are substitutes. Since the $G(.)$ locus is a vertical line, there is a unique stationary equilibrium. Point D represents an equilibrium DM consumption values q^u and q in economies where the price of short-term bonds is $z^s < 1$.

Let us now analyze how changes in the maturity composition of debt affects the nature of the equilibrium. Figure 2 illustrates the effect of a permanent increase in the value of long-term bonds. This situation is similar in spirit to the "Operation Twist" conducted by the Federal Reserve, where the monetary authority increases its position in long-term debt. This effectively reduces the short-term government debt in order to keep δ constant. With a higher value of long-term government debt outstanding, $A_1 > A$, the $F(.)$ locus shifts down to $F1(.)$. This represents a tightening of the bank's incentive compatibility constraint (IC) constraint. This is the case as long-

[17] We analyze each of the changes later in the paper.

[18] For illustrative purposes we use the functional form $u(x) = -e^{-\sigma x}$ with $\sigma = 2$, $\delta = 0.5$, $\rho = 0.5$, $\theta_s = 0.05$ and $\theta_l = 0.1$. However, our results are not restricted to this particular parameterization. Appendix B provides a different choice of parameter values that match closely USA data.

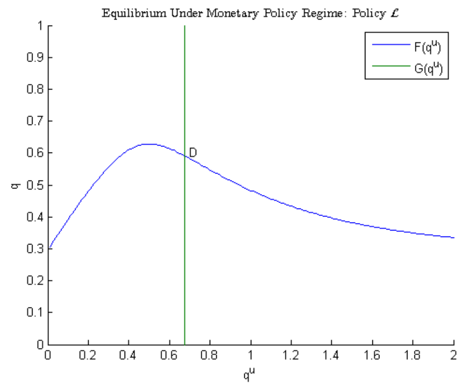


Figure 1: Equilibrium under an Active Monetary Regime

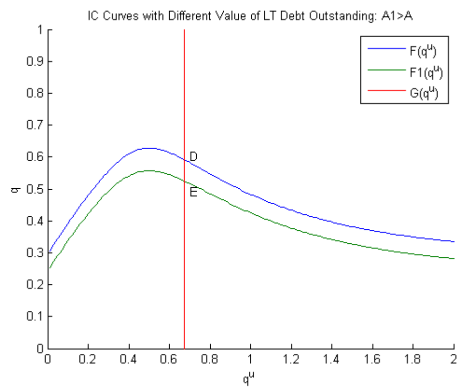


Figure 2: Effect of Increasing the Value of Long-term Debt

term debt is less pledgeable than short-term public debt. As a result, q declines, while q^u remains unchanged. This finding suggests that an increase in the average maturity structure of government debt also *twists* the relative DM consumption. In particular, those DM-buyers in connected trades have their consumption reduced, while those in unconnected trades see no change in their consumption.

Finally, it is important to emphasize that when the economy is at the zero lower bound, then it is not feasible to issue any short-term bonds to support an expansionary policy. In such case, long-maturity bonds, and hence policy $\mathcal{S}$, might be useful. This is the case as such policy reduces the term premium without violating the zero lower bound on the short-term interest rate.[19]

5.2.2 Policy $\mathcal{S}$

The fiscal authority sets the value of short-term government debt to a constant so that $\phi_t z_t^s B_t^s{=}B \;\; \forall t$. After imposing stationarity on equations (20)-(30) and after repeated substitution, the equilibrium can be summarized

[19]Perhaps our results are dependent on fixing the value of long-term government debt. To determine the robustness of the results, we consider a case in which the level of long-term government debt is fixed. Formally, instead of $\phi_t z_t^l B_t^l = A \;\; \forall t$, we consider $B_t^l = \mathcal{A} \;\; \forall t$. The results are qualitatively similar for these experiments. We refer the reader to Appendix B.

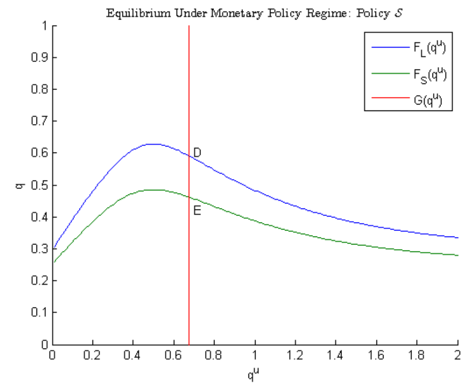


Figure 3: Equilibrium under Monetary Regime with Constant Value of Short-maturity Debt

by DM quantities, q and q^u. In particular, the stationary equilibrium satisfies the following equations

$$(1-\rho)q\Big(\beta u'(q)(1-\theta_s)+\theta_s\Big)+\frac{\Big(\frac{\rho(1-\delta)q^u\ u'(q^u)}{\delta}-B\Big)(\theta_l-\theta_s)}{\beta u'(q)(1-\theta_l)+\theta_l}=\Big(\frac{\rho(1-\delta)q^u u'(q^u)}{\delta}\Big)(1-\theta_s) \tag{43}$$

$$\beta u'(q^u)=\mu. \tag{44}$$

As with the previous equilibrium, bond prices and taxes are given by equations (40), (41) and (42), respectively. We can now establish the following result.

Proposition 2. *When the following condition is satisfied*

$$(1-\rho)q^*(\beta(1-\theta_s)+\theta_s)+\frac{(\Gamma-B)(\theta_l-\theta_s)}{(\beta(1-\theta_l)+\theta_l)}\geq\frac{\rho(1-\delta)q^u\ u'(q^u)(1-\theta_s)}{\delta}.$$

then there exists a unique stationary monetary equilibrium, where $\Gamma=\frac{\rho(1-\delta)q^u\ u'(q^u)}{\delta}$.

As with Policy $\mathcal{L}$, we illustrate the stationary equilibrium in the (q^u,q) space as depicted in Figure 3. We define $F_L(q^u)$ as the locus of consumption quantities that satisfy (38) when Policy $\mathcal{L}$ is enacted and $F_S(q^u)$ as the locus that satisfies (43) when Policy $\mathcal{S}$ is implemented. We also denote $q=G(q^u)$ as the locus of consumption in unconnected trades. Note that for a given money growth rate, consumption in connected trades under Policy $\mathcal{L}$ dominates that of Policy $\mathcal{S}$. This is the case as the locus $F_L(q^u)$ lies above $F_S(q^u)$, resulting in higher consumption in connected matches. The intuition is tied to the nature of the two different forms of collateral. Because short-term government debt provides better collateral services than long-term, the incentive compatibility constraint is looser in Policy $\mathcal{L}$ than in Policy $\mathcal{S}$. Hence, the consumption opportunities in connected trades is better in Policy $\mathcal{L}$, which is preferred to Policy $\mathcal{S}$ away from zero lower bound. As in Williamson (2016), we show that the maturity structure of government debt matters for real allocations when collateral is scarce and a variety of different policies are considered.

5.2.3 Monetary Policy and Consumption Inequality

Since the financial crisis and the Great Recession, there has been a renewed interest in analyzing the effect of monetary policy on inequality. Coibion et al. (2017) examine the effects of monetary policy shocks on consumption inequality. The authors present evidence at business cycle frequency and find that contractionary monetary policy –an increase in the nominal interest rate– results in a persistent increase in consumption inequality.

In contrast to Coibion et al. (2017), our focus is on the long-run consequences of monetary policy on consumption inequality. In particular, we analyze how consumption inequality changes as a result of a permanent increase in the money growth rate under Policies $\mathcal{L}$ and $\mathcal{S}$. Recall that the consumption of unconnected DM-buyers (q^u) is directly impacted by the money growth rate. Moreover, the consumption of connected and unconnected DM-buyers are linked through the banker's incentive compatibility constraint, when collateral is scarce. Thus, a change in the long-run inflation rate will alter the set of feasible DM consumption allocations (q^u, q) that are consistent with the banker not defaulting on his deposits. Taking these insights into account, we can then establish the following result.

Proposition 3. *When collateral is scarce, a permanent increase in the inflation rate results in greater consumption inequality, irrespective of the bond supply process, if the slope of $q = F(q^u)$ is less than one, or equivalently,*

$$\frac{\partial q}{\partial q^u} = \frac{\partial F(q^u)}{\partial q^u} = \frac{-\frac{(1-\delta)}{\delta}\rho(1-\theta_s)(u'(q^u) + q^u u''(q^u))}{(1-\rho)\Big((\beta u'(q)(1-\theta_s) + \theta_s) + \beta q u''(q)(1-\theta_s)\Big) - \frac{\beta A(\theta_l - \theta_s)u''(q)(1-\theta_l)}{\big(\beta u'(q)(1-\theta_l)+\theta_l\big)^2}} < .1$$

Figure 4 is useful for developing intuition. Consider the effect of a change in the money growth rate on equations (38) and (39). The left panel shows the effects under Policy $\mathcal{L}$.[20] Note that the locus $q = F(q^u)$ is invariant to the money growth rate. However, the locus $G(q^u)$ is inversely related to the money growth rate. Thus, an increase in the money growth rate shifts the $G(q^u)$ locus to the left, decreasing q^u. The effect on consumption of connected trades, q, depends on the nature of the equilibrium. More precisely, if the equilibrium lies on the downward sloping part of the $F(q^u)$ locus (range where collateral effect dominates), q increases as a result of an increase in μ. Such change in equilibrium is illustrated in Figure 4 by moving from point D to E.[21] Thus, expansionary monetary policy in this case results in greater consumption inequality. With an increase in the money growth rate under Policy $\mathcal{L}$, the value of short-term government bonds increases in order to maintain a constant M-L ratio (δ). Hence, collateral scarcity is loosened, the price of short-term government bonds declines, the return on short-term debt increases and depositors in connected trades are able to purchase more DM goods. Meanwhile, depositors in unconnected trades suffer a decline in the return on money as a result of a higher inflation rate.[22] Similar results are obtained when the fiscal authority follows Policy $\mathcal{S}$ as shown in the right panel of Figure 4. The consumption inequality findings for both bond supply processes are in line with those of Coibion et al. (2017).[23]

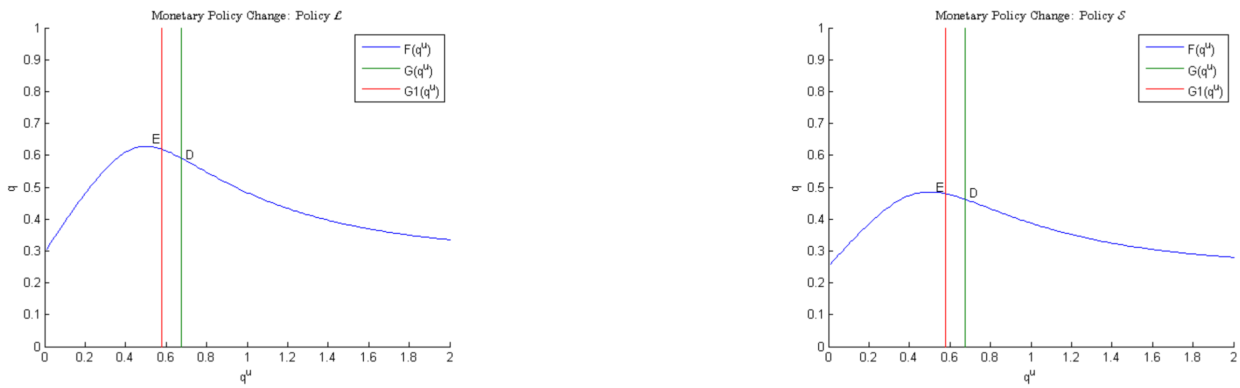
In contrast, if the equilibrium lies on the upward sloping part of the $F(q^u)$ locus, an increase in μ results in a

[20] We drop the superscripts since we are not comparing $\mathcal{L}$ to $\mathcal{S}$.

[21] See Williamson (2016) for a similar experiment. The difference here is that the shift in locus representing IC constraint does not occur when short-maturity government debt holding is increased. In our setting, there is a shift in the locus only when operation twist type policy is implemented. The reason is that we consider policies that take into account total government liabilities, whereas Williamson (2016) does not.

[22] To assess the welfare impact of an increase in the money growth rate, we need to consider the expected payoff to the connected DM-buyers relative to the unconnected ones. Formally, with $(1-\rho)u'(q) > \rho u'(q^u)$, an increase in μ, for example, leads to higher stationary ex-ante expected welfare in the range where collateral effect dominates. To further unpack this sufficiency condition, note that with $q^u \leq q$ the strict concavity implies that $u'(q^u) > u'(q)$. By the sufficiency condition, it follows that if ρ is small enough, i. e. if the fraction of buyers in unconnected trades is small enough, stationary ex ante expected welfare is positively related to the money growth rate in this range.

[23] In our environment, a faster money growth rate results in a higher nominal interest rate on government bonds in the long-run.



(a) Constant Value of Long-maturity Debt

(b) Constant Value of Short-maturity Debt

Figure 4: Equilibrium under Monetary Regime

decline in both types of DM consumption. In this range of $F(q^u)$, the money growth rate is comparatively higher, which results in the collateral effect being smaller compared to the risk-averse effect. In such circumstances, a larger money growth rate lowers welfare. Effect on inequality depends on the slope of the $F(q^u)$ locus. In particular, if the slope is less than 1, a decrease in q is relatively smaller, which implies higher inequality. Otherwise, the inequality decreases due to an increase in money growth rate. These properties are observed both when the fiscal authority implements Policy $\mathcal{L}$ or Policy $\mathcal{S}$. It is worth emphasizing that any expansionary policy in the range where risk-averse effect dominates is not optimal. Comparing the consumption inequality findings for both bond supply processes to those of Coibion et al. (2017), it may suggest that the relevant parameter configuration describing the U.S. is one where the equilibrium lies on the downward sloping part of the $F(q^u)$ locus or where the slope of the locus is less than 1.

5.2.4 Quantitative Easing

Since the economy has incomplete markets and collateral is scarce, an increase in the money growth rate alters the overall public debt maturity structure in the economy. The exact composition of public debt observed in equilibrium depends on which policy the fiscal authority has followed; i.e, Policy $\mathcal{L}$ or Policy $\mathcal{S}$. In addition, since the economy is non-Ricardian, a change in the average maturity of nominal government bonds has real effects. Our experiments can potentially shed additional light on the mechanisms through which recent policy actions implemented by the Federal Reserve during the Great Recession affect long-run equilibrium. In particular, we study the implications associated with Quantitative Easing (QE) policies.

Consider a central bank that purchases long-term government bonds. In order to study QE within the context of our environment, we need to modify the supply processes for government assets of our benchmark model. In particular, instead of fixing the money-to-total government liability ratio, we now fix the total nominal government liability (or transfer).[24] Under this new rule, the central bank is able to trade long-term government bonds in exchange for money, while holding the total nominal government liability constant.[25] In such a setting, an increase in money growth rate lowers consumption in unconnected trades, q^u. The effect on consumption

[24] In our baseline assumption, an increase in money growth rate also increases supply of long-term bonds, making it difficult to study the QE policy.

[25] The detailed analysis can be found in Appendix B.

in connected trades, q, depends on the nature of the equilibrium. In particular, when the equilibrium is in the downward sloping range of $F(q^u)$ locus (range where collateral effect dominates), an increase in money growth rate shifts the $G(q^u)$ locus to the left and increases q. By reducing the supply of long-term bonds (bad collateral), the average quality of the existing collateral is raised and the incentive compatibility constraint is loosened. Consequently, consumption in connected trades will increase. In the upward sloping range of $F(q^u)$ locus (range where risk-averse effect dominates), findings are again similar to those of baseline case.

As we can see, the findings regarding consumption inequality when QE is considered are similar to those found in the baseline model where an expansionary monetary policy is studied.

6 Active Fiscal Policy Regime

In this regime, the fiscal authority sets taxes and primary surpluses, while specifying the supply of short or long-term public debt. Given this fiscal policy stance, the central bank passively adjusts its money supply in order to satisfy the government's budget constraint. As in the active monetary policy regime, the nature of the equilibrium depends on whether DM consumption in the different state of the world is first best or not. Next we analyze these cases.

6.1 Plentiful Collateral

When collateral is plentiful so that consumption in connected trades is efficient ($q = q^*$), the stationary equilibria satisfies the following system of equations

$$\beta u'(q^u) = \mu \tag{45}$$

$$z^s = \frac{\beta(1-\theta_s)+\theta_s}{\beta u'\left(q^u\right)} \tag{46}$$

$$\frac{1}{z^l} + 1 = \frac{\beta u'\left(q^u\right)}{\beta(1-\theta_l)+\theta_l} \tag{47}$$

$$(1-\rho)q^*\Big(\beta(1-\theta_s)+\theta_s\Big) + \frac{z^l\, b^l(\theta_l-\theta_s)}{\beta(1-\theta_l)+\theta_l} = (z^s\, b^s + z^l\, b^l)(1-\theta_s) \tag{48}$$

$$\mu = \frac{\beta\delta}{\rho q^u}\left[\tau + z^l b^l + \delta + \frac{1-\delta}{\beta}\right] \tag{49}$$

where the money growth rate, μ, is now endogenous and depends on the taxes set by the fiscal authority.

It is important to note that in order to support q^*, the government must supply enough assets such that the banker's incentive compatibility is satisfied. Next we explore how the two different bond supply processes affect the nature of the stationary equilibria.

6.1.1 Policy $\mathcal{L}$

The fiscal authority sets the value of long-term government debt to be a constant, $\phi_t z_t^l B_t^l = A \;\; \forall t$. It is easy to show that the stationary equilibrium can be represented by DM consumption for unconnected trades that satisfies the following equation

$$\rho q^u u'(q^u) = \delta\left[\tau + A + \delta + \frac{1-\delta}{\beta}\right].$$

Given the concavity of the utility function, the resulting steady state for consumption in unconnected trades is unique. However, in contrast to the active monetary regime, this DM consumption depends on the amount of long-term debt that the fiscal authority issues.

6.1.2 Policy $\mathcal{S}$

The fiscal authority now sets the value of short-term government debt to be a constant, $\phi_t z_t^s B_t^s = B \ \ \forall t$. The resulting equilibrium short-term real bonds is a solution to the following equation

$$\rho q^u u'(q^u) = \delta \left[\tau + z^l b^l + \delta + \frac{1-\delta}{\beta} \right]$$

where the equilibrium real value of long-term public debt, $z^l b^l$, solves the following linear equation

$$(1-\rho) q^* \Big(\beta(1-\theta_s) + \theta_s \Big) + \frac{z^l \ b^l \ (\theta_l - \theta_s)}{\beta(1-\theta_l) + \theta_l} = (B + z^l \ b^l)(1-\theta_s).$$

Given the concavity of the utility function, the resulting steady state consumption in unconnected trades is also unique. As with Policy $\mathcal{L}$, this DM consumption depends on the amount of short-term debt that the fiscal authority issues.

As we can see, irrespective of the bond supply process, an active fiscal policy regime when collateral is plentiful always delivers a unique steady state. Consumption in connected and unconnected meetings is independent of each other. However, in contrast to the active monetary policy regime, the maturity composition of public debt does affect consumption in unconnected trades.

6.2 Scarce Collateral

When collateral is scarce, DM consumption in all states of the world is inefficient. In such equilibrium, a change in government bonds can help expand the consumption opportunities of connected trades. As in the plentiful collateral case, next we examine the resulting stationary equilibria under two different bond supply processes.

6.2.1 Policy $\mathcal{L}$

Given taxes, τ, the value of long-term government debt is fixed so that $\phi_t z_t^l B_t^l = A \ \ \forall t$. It is easy to check that the corresponding stationary equilibrium is characterized by the following system of equations

$$(1-\rho) \ q \ (\beta u'(q)(1-\theta_s) + \theta_s) + \frac{A(\theta_l - \theta_s)}{\beta u'(q)(1-\theta_l) + \theta_l} = \frac{\rho(1-\delta) q^u \ u'(q^u)(1-\theta_s)}{\delta} \tag{50}$$

$$\beta u'(q^u) = \mu \tag{51}$$

$$\mu = \frac{\beta\delta}{\rho q^u} \left[\tau + (1 + r^l - r^s) z^l b^l + \delta + (1+r^s)(1-\delta) \right] \tag{52}$$

while net returns for short and long-term public debt are given by

$$r^s = \frac{\mu}{\Big(\beta u'(q)(1-\theta_s)+\theta_s\Big)} - 1 \quad \& \quad r^l = \frac{\mu}{\Big(\beta u'(q)(1-\theta_l)+\theta_l\Big)} - 1.$$

After some substitutions, it is easy to show that the resulting stationary equilibrium is characterized by the following system of equations

$$(1-\rho)\; q\; (\beta u'(q)(1-\theta_s)+\theta_s) + \frac{A(\theta_l-\theta_s)}{\beta u'(q)(1-\theta_l)+\theta_l} = \frac{\rho(1-\delta)q^u\; u'(q^u)(1-\theta_s)}{\delta} \tag{53}$$

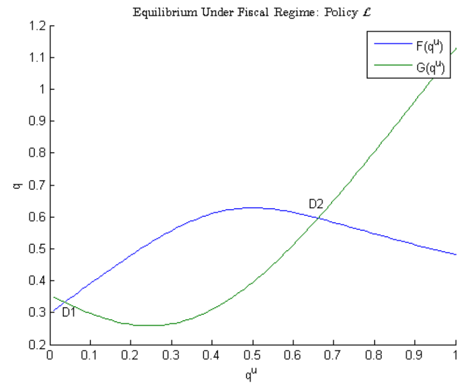
$$\beta u'(q^u) = \frac{\tau + A + \delta}{\frac{\rho q^u}{\beta\delta} - \left[\frac{A}{\Big(\beta u'(q)(1-\theta_l)+\theta_l\Big)} - \frac{A-(1-\delta)}{\Big(\beta u'(q)(1-\theta_s)+\theta_s\Big)}\right]} = \mu. \tag{54}$$

As in the active monetary policy regime, equation (53) reflects the banker's incentive compatibility constraint. Thus, the risk-averse and the collateral effects are also at play. The key differences relative to the active monetary policy regime are the combinations of q and q^u that define the endogenous money growth rate. Such endogenous rate is given by equation (54). In contrast to the active monetary policy regime, the DM consumption in both states of the world are not independent of each other. Thus, a new trade-off between quantities in connected and unconnected trades emerges. To illustrate this, let us consider an increase in the quantity of unconnected trades that coincides with a larger seigniorage base. Thus, a smaller money growth rate is needed to satisfy the government budget constraint, which affects quantity in connected trades. Similarly, an increase in q results in an increase in the returns on short-term and long-term bonds. As interest expenses increase, the primary surplus decreases. The quantity in unconnected trades is affected. The following proposition summarizes the findings for Policy $\mathcal{L}$.

Proposition 4. *When the fiscal authority sets taxes, τ, and the value of long-maturity government bonds (Policy $\mathcal{L}$), multiple steady states are possible.*

We graph equations (53) and (54) in the (q^u, q) space (see Figure 5). Let $q = F(q^u)$ and $q = G(q^u)$ represent the locus of consumption in connected trades, q, in terms of unconnected ones, q^u, respectively. The intersection between the two loci determines the resulting stationary equilibria. The key difference relative to the active monetary policy regime is the trade-off between q and q^u captured by locus $G(q^u)$, resulting in two steady states, D1 and D2. Thus, real indeterminacies are observed in this policy regime. These two stationary equilibria can be Pareto ranked. The steady state associated with D1 (the bad equilibrium) features relatively lower deposit-backed consumption and money-backed consumption compared to the equilibrium associated with D2 (the good equilibrium). Equilibrium D1 lies on the upward sloping part of the $F(q^u)$, which implies that the risk-averse effect dominates. On the other hand, collateral effect dominates in equilibrium D2.

With multiple equilibria, the active fiscal policy regime points to a regime marked by excessive economic volatility as sunspot equilibria can be easily constructed. The existence of multiple equilibria is consistent with the inflation-tax Laffer curve operating within the government budget constraint. The same level of seigniorage revenue can be collected with a low money growth rate and a high money growth rate. Thus, multiple money



(a) Equilibrium under Fiscal Regime: Policy $\mathcal{L}$

Figure 5: Equilibrium under Active Fiscal Policy Regime

growth rate satisfy the government budget given taxes and debt policy.

To determine the robustness of these results, we consider alternative fiscal rules that fix the amount of long-term bonds.[26] The findings are robust to different specifications of supply process of bonds. (See Appendix C for the proofs.)

Note that with an exogenous money growth rate (active monetary policy regime), we see different demands for government liabilities compared to that of an endogenous money growth rate (active fiscal policy regime). As a result, the resulting allocations and prices are typically going to be different across the different regimes. Moreover, since an active monetary policy regime delivers unique steady states, generically there is not going to be an equivalence between policy regimes. This is the case as multiple steady states are possible under an active fiscal regime.

Are there policies that can rule out real indeterminacies? There is a long tradition in economics emphasizing how government policies can be used as an equilibrium selection device.[27] The following results demonstrate how government debt can eliminate the real indeterminacies that generically emerge in our environment. In our context, we derive conditions on the value of government debt that guarantee unique stationary equilibrium.

Proposition 5. *In an active fiscal policy regime and Policy $\mathcal{L}$ is enacted, suppose the following condition is satisfied*

$$A < \frac{\frac{\delta}{1-\delta}(1-\rho)q^* - \big(\tau + \delta + (1-\delta)u'(q^u)\big)}{1 - \frac{\delta(\theta_l - \theta s)}{1-\delta}}$$

then there is a unique stationary equilibrium and real indeterminacies are ruled out.

To ensure that there is a unique stationary equilibrium in the active fiscal policy regime, there exists an upper bound on the value of long-term government debt. More precisely, an upper bound A that satisfies the condition in Proposition 5, ensures that the $G(.)$ loci cannot bend enough to intersect the $F(.)$ curve more than once. This result stresses the importance of considering the maturity structure of public debt as well as its relative size, when thinking of policies that can rule out real indeterminacies.

[26] Under this policy we have that $B^l_t = \mathcal{A} \;\; \forall t$.

[27] For instance, see Schreft and Smith (1998), Ennis and Keister (2005), Antinolfi et al. (2007), Domínguez and Gomis-Porqueras (2019) and Carli and Gomis-Porqueras (2021), among others, for more on this topic.

It is worth mentioning that an active monetary policy regime always delivers a unique stationary equilibrium. In contrast, with an active fiscal policy, multiple equilibria can exist because there are multiple money growth rates that can satisfy the government budget constraint for a given tax and debt policy set by the fiscal authority. By endowing the economy with a central bank with discretionary role, the money growth rate would result in unique equilibrium. It follows that any joint regime in which both the central bank and the fiscal authority having a discretionary role would result in a unique equilibrium.[28]

6.2.2 Policy $\mathcal{S}$

Given taxes, τ, the value of short-term government debt is fixed so that $\phi_t z_t^s B_t^s = B \;\; \forall t$, the monetary policy adjusts its money growth rate so as to satisfy the government budget constraint. After repeated substitution, it is easy to check that the stationary equilibria is given by

$$(1-\rho)q\Big(\beta u'(q)(1-\theta_s)+\theta_s\Big)+\frac{\Big(\frac{\rho(1-\delta)q^u\,u'(q^u)}{\delta}-B\Big)(\theta_l-\theta_s)}{\beta u'(q)(1-\theta_l)+\theta_l}=\Big(\frac{\rho(1-\delta)q^u u'(q^u)}{\delta}\Big)(1-\theta_s) \tag{55}$$

$$\begin{aligned} u'(q^u)-\left(1+\frac{u'(q^u)}{\Big(\beta u'(q)(1-\theta_l)+\theta_l\Big)}-\frac{u'(q^u)}{\Big(\beta u'(q)(1-\theta_s)+\theta_s\Big)}\right)\left(u'(q^u)(1-\delta)-\frac{\delta B}{\rho q^u}\right) \\ -\frac{\delta(1-\delta)}{\rho q^u}\frac{u'(q^u)}{\Big(\beta u'(q)(1-\theta_s)+\theta_s\Big)}=\frac{\delta(\tau+\delta)}{\rho q^u} \end{aligned} \tag{56}$$

where $\beta u'(q^u) = \mu$ and the money growth rate, μ, depends on fiscal policies so as to satisfy the government budget constraint.

Proposition 6. *When the fiscal authority sets taxes, τ, and the value of short-maturity government bonds (Policy $\mathcal{S}$) is enacted, multiple steady states cannot be ruled out.*

The intuition and graphical representation for the Policy $\mathcal{S}$ is similar to that of Policy $\mathcal{L}$. To determine the robustness of these results, we again consider alternative fiscal rules that fix the amount of short-term bonds.[29] The findings are robust to different specifications of supply process of bonds. (See Appendix C for the proofs.)

As in the case with Policy $\mathcal{L}$, there are specific fiscal policies that can be used as an equilibrium selection device and rule out multiple steady states.

Proposition 7. *When the fiscal authority follows an active Policy $\mathcal{S}$, the following conditions guarantee a unique stationary equilibrium*

$$B<\frac{\Gamma}{\delta(1-\theta_s)}+\frac{(1-\delta)(1-\theta_s)(\tau+\delta)-\delta(1-\rho)q^*}{(1-\delta)(1-\theta_s)-(\theta_l-\theta_s)}.$$

As we can see, there is an upper bound on the value of short-term government debt that eliminates real indeterminacies when the value of long-term debt is restricted.

[28] For instance, the joint regime considered by Martín (2011).

[29] Under this policy we have that $B_t^s = \mathcal{B} \;\; \forall t$.

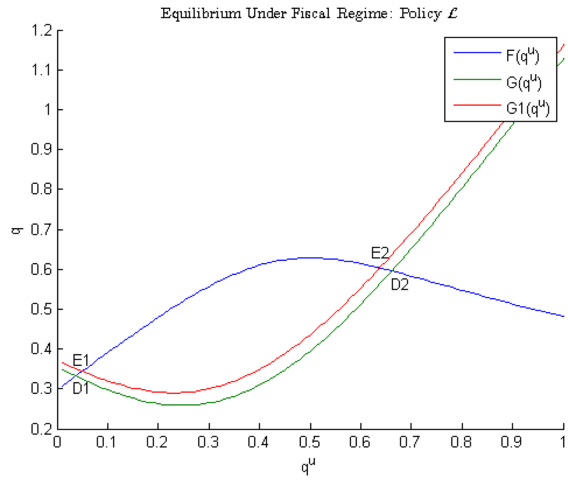


Figure 6: Effect of an Increase in Lump-sum Taxes.

6.3 Taxing Capacity

Since government debt is backed by taxes, we now explore the consequences of increasing lump-sum taxes when collateral is scarce. Note that the locus $q = F(q^u)$ is invariant to changes to the taxes levied to households. This is the case irrespective to which bond supply process is implemented; i.e, Policy $\mathcal{L}$ or Policy $\mathcal{S}$. This is in sharp contrast to that of locus $q = G(q^u)$. As multiple steady states exist, the effects of a tax change policy depends on the slope of the locus $F(.)$.

To simplify exposition, we focus on the Policy $\mathcal{L}$.[30] Using the previous loci representation, the effects of increasing taxes are depicted in Figure 6. An increase in taxes shifts locus $G(.)$ up for both Policy $\mathcal{L}$ and Policy $\mathcal{S}$. In the initial equilibrium D1, a tax increase results in an increase in consumption in unconnected trades, which is depicted by a shift to E1. In contrast, after a tax increase, the initial equilibrium D2 moves to E2. As a result, we observe a decrease in consumption in unconnected trades.

Since two different money growth rate satisfy the government budget constraint for a given tax policy, there is a version of the seigniorage Laffer curve at work. An increase in taxes induces a decline in demand for money. Hence, seigniorage revenue declines. Lost seigniorage is then recovered by increasing money growth rate in one equilibrium, while it is recovered by decreasing money growth rate in another equilibrium. In an equilibrium where collateral effect dominates (D2), an increase in q is driven by increased availability of collateral. This in turn is accompanied by an increase in money growth rate. As a result, consumption in unconnected trades, q^u, declines. Thus, an increase in taxes results in larger consumption inequality. In an equilibrium where the risk-averse effect dominates (D1), the lost seigniorage is recovered by lowering the money growth rate. This in turn results in higher consumption in the unconnected trades. The real value of future government debt increases, which loosens the bank's incentive constraint. This then allows higher consumption in connected trades.

The slope of $F(q^u)$ determines which type of DM consumption increases the most in the bad equilibrium. If the slope is greater (less) than one, the new equilibrium is evaluated at a point where the consumption in connected trades increases more (less) than that of unconnected trades. We call this case strong (weak) complementarity. Given an increase in taxes, strong (weak) complementarity results in an increase (decrease) in the consumption

[30] Findings for Policy $\mathcal{S}$ are similar and available upon request.

inequality.

In the bad equilibrium, an increase in lump-sum taxes is Pareto improving. However, there is a limit to how much the fiscal authority can increase their taxes. Once the peak of $F(.)$ is reached, a further increase in taxes induces a decline in q^u.[31] In the good equilibrium (D2), an increase in taxes can improve overall welfare if the share of unconnected buyers is relatively small. However, increasing taxes too much can result in a reduction in consumption in connected trades, q. This is the case as the collateral effect becomes smaller and the risk-averse effect starts to dominate. As a result, the equilibrium would lie on the upward sloping part of the $F(.)$ locus. Thus, when collateral is scarce, the fiscal government has an upper bound on how much the taxes can be increased in order to make collateral less scarce.

7 Financial Innovations

Over the last four decades, economies around the world have experienced various financial innovations that have increased financial deepening and financial inclusion. Such developments have changed the composition of households' portfolios and how trades are settled across different markets. In particular, innovations in financial markets (through the widespread use of new financial instruments such as derivatives, repurchase agreements, etc) have increased trade in secondary markets for government bonds. Indeed, the public debt has been increasingly used as collateral. The impact of these financial developments have not been fully studied in the literature.[32] Here we fill the gap.

It is important to note that when collateral is scarce, financial innovations affect allocations and consumption inequality. This is the case as these change the relative demand of the different government liabilities: fiat money and public debt of different maturities. In a Ricardian world, fiscal deficits and total public debt have no consequences for interest rates, as the private sector saves the full extent of discounted tax liability implied by a rise in the fiscal deficit. In a non-Ricardian world, however, changes in fiscal deficits can lead to changes in interest rates. Since our environment is non-Ricardian, as it exhibits a premium on government debt, financial innovations are going to alter the traditional monetary and fiscal interactions as well as consumption inequality. The non-neutrality of fiscal and monetary policies overturn the logic of separable monetary and fiscal dynamics, leading to a more symmetric treatment of monetary and fiscal policy aspects.

In this section we consider two types of financial innovation and explore how they affect fiscal and monetary policy interactions.[33] First, we focus on innovations that directly affect the *intensive* margin, which can be thought of as financial deepening. To do so, we consider innovations that increase the pledgeability of the long-term public debt. In our setting, this is captured by a decrease in θ_l.[34] To simplify exposition, we focus on Policy $\mathcal{S}$. Under an active monetary policy regime when collateral is scarce, it is important to note that an increase in pledgeability amplifies the effects of monetary policy and the collateral effects. Improved pledgeability implies that the long-term bonds are better collateral and hence, offer higher consumption opportunity for connected buyers.[35] In contrast, consumption in unconnected trade is unaffected. This implies that this type of financial

[31] Increasing taxes too much (shifting the $G(.)$ locus too much) can result in no equilibrium.

[32] A notable exception is Domínguez and Gomis-Porqueras (2019) who analyze the evolution of secondary markets for public debt and its impact on inflation and bond dynamics.

[33] Numerical analysis of the effects of the two financial innovations on the equilibrium outcome are presented in Appendix B.

[34] This type of financial innovation is trying to capture innovation in financial instruments where long-term debt is more valuable.

[35] To see this, note that the $F(q^u)$ locus shifts up when better collateral are used to back deposits.

innovation is welfare improving.

To illustrate how an increase in the pledgeability of long-term public debt amplifies monetary policy, let us analyze a situation where the collateral effect dominates. A smaller increase in the money growth rate, and hence inflation, will achieve a similar level of increase in deposit-backed consumption because of the financial innovation. In addition, a smaller increase in the money growth rate results in a smaller decline in money-backed consumption. Put differently, an increase in money growth rate results in a comparatively larger increase in consumption in connected trades when the pledgeability of long-term bonds is higher. The impact on consumption inequality, due to monetary policy, is also amplified. When collateral effect dominates, an expansionary monetary policy results in comparatively higher increase in q and the same level of decline in q^u. Thus, comparatively we observe higher inequality when the pledgeability of long-term bonds is higher.[36]

In order to understand the implications of the improved pledgeability of long-term debt in an active fiscal policy regime, we need to analyze the properties of $G(.)$ and $F(.)$. The properties of $F(.)$ are similar to the active monetary regime, but that is not the case for $G(.)$. The marginal effect of this innovation on $G(q^u)$ is given by

$$\frac{\partial q}{\partial \theta_l} = -\frac{u'(q^u)\big(-u'(q)+1\big)}{\left[\frac{u'(q^u)u''(q)(1-\theta_l)}{\left(u'(q)(1-\theta_l)+\theta_l\right)^2} - \frac{u'(q^u)u''(q)(1-\theta_l)}{\left(u'(q)(1-\theta_s)+\theta_s\right)^2}\right]\left[u'(q^u)(1-\delta) - \frac{\delta B}{\rho q^u}\right] + \frac{\delta(1-\delta)}{\rho q^u}\frac{u'(q^u)u''(q)(1-\theta_s)}{\left(u'(q)(1-\theta_s)+\theta_s\right)^2}}$$

where the denominator is negative.[37] With $-u'(q)+1 < 0$, the first best is not achieved, the derivative is negative. Hence, higher pledgeability of long-term bonds shifts both loci $F(.)$ and $G(.)$ upward. Consumption in connected trades increases as a result. The effect on q^u is indeterminate. If the upward shift in $F(.)$ locus is larger, q^u increases in the good equilibrium, while it decreases in bad equilibrium. On the other hand, if the upward shift in $G(.)$ locus is larger, q^u decreases in the good equilibrium ,while it increases in bad equilibrium. The overall impact on consumption inequality is then inconclusive.

Similar to the active monetary policy regime, the positive relationship between pledgeability and consumption in connected trades, q, implies that the impact of a tax increase is amplified. To understand the effect of a tax-increase policy, recall that an increase in taxes shifts locus $G(.)$ upward, while locus $F(.)$ is unaffected. Hence, an increase in lump-sum taxes results in a comparatively higher increase in q when pledgeability of long-term bond is higher. Hence, the collateral channel is present for both fiscal and monetary policies. The overall welfare effect again depends on the measure of sellers that are unconnected, ρ. One aspect, however, is clear. Welfare is more sensitive to expansionary active fiscal policy compared to an economy with less pledgeable long-term bonds.

It is worth noting that since $G(.)$ and $F(.)$ increase when pledgeability of long-term bond improves, an increase in taxes in the active fiscal regime is comparatively more effective at increasing connected consumption than an increase in money growth rate in the active monetary regime. Moreover, as $\theta_l \to \theta_s$, maturity premium disappears and the two types of bonds become perfect substitutes.[38]

In addition, it is worth noting that the effect of such financial innovation is higher when the existing value of long-term debt is higher as improvement in the pledgeability improves the collateral property of the long-term

[36] Analysis on equilibrium where risk-averse effect dominates is similar.

[37] This is the case given the assumptions in Proposition 2.

[38] Then the corollary of Proposition 5 is that the unique stationary equilibrium is achieved.

debt.

The second type of innovation that we focus on affects the *extensive* margin, which captures financial inclusion.[39] In other words, this innovation increases the measure of sellers that are connected and can accept deposit claims as payment. Such innovation is captured by a reduction in ρ. In general, the effect of this innovation is to dampen the effects of active monetary policy. This is the case as consumption in unconnected trades is invariant to ρ. In contrast, consumption in connected trades, q, is not. In particular, a decrease in ρ shifts the locus $F(.)$ downwards. For those in connected trades, the existing collateral is spread across more depositors. Consequently, banks offer a lower return to depositors in connected trades. Consumption in connected trades decreases and so does the consumption inequality. In an active monetary policy regime where the money growth rate is increased, for example, consumption in unconnected trades declines. Because expanding the money supply results in more collateral, consumption in connected trades increases. However, as we have a larger measure of connected buyers, the increase in consumption is smaller.

In order to understand the implications of having greater access to deposit claims in active fiscal policy regime, we need to analyze the properties of $G(.)$ and $F(.)$. Note that the $F(.)$ locus shifts down due to a decrease in ρ. Derivative of $G(.)$ with respect to ρ is given by

$$\frac{\partial q}{\partial \rho} = \frac{(\tau + A + \delta)\frac{q^u}{\beta\delta}}{(\tau + A + \delta)u''(q)\left[-\frac{A(1-\theta_l)}{\left(u'(q)(1-\theta_l)+\theta_l\right)^2} + \frac{(A-(1-\delta))(1-\theta_s)}{\left(u'(q)(1-\theta_s)+\theta_s\right)^2}\right]}$$

where the bracketed term in the denominator is negative, hence the derivative is positive. This implies that $G(.)$ shifts downward when there is a decline in number of unconnected meetings. The overall effect results in lower consumption in connected trades in both types of equilibria. The intuition is similar to that of the active monetary policy case.

Next we examine the effect of an increased fraction of connected trades in an expansionary fiscal policy in an active fiscal policy regime. Such a policy generates an upward shift in locus $G(.)$, which results in an increase in connected consumption. For a given ρ, consumption in unconnected trades depends on the nature of the equilibrium. In particular, in the good equilibrium, q^u decreases while it increases in the bad equilibrium. As ρ decreases, the consumption gains are dampened. This effect is due to the fact that ρ affects both $F(.)$ and $G(.)$ loci negatively. Thus, expansionary fiscal policy is less effective in terms of consumption changes when the financial innovations affect the extensive margin. This is different to what we observe in an active monetary policy regime. This is the case as $G(.)$ is invariant to such financial innovations. So the dampening effect of an increase in the number of connected trades is smaller in an active monetary policy regime relative to the one observed in an active fiscal regime.

8 Conclusion

This paper contributes to the extensive literature that studies fiscal and monetary policy interactions in a frictional economy with various government liabilities. More precisely, we focus on the long-run consequences for inflation, government debt, interest rate premia and consumption inequality in economies where agents face limited

[39]This type of financial innovation is trying to capture improvements in the payment system.

commitment and multiple public assets coexist. Within this environment, we consider two scenarios: an active monetary and an active fiscal policy regimes. Because of limited commitment, loans are secured and government debt plays the role of collateral. The pledgeability of public debt differs across its maturity structure. In particular, short-term debt is more pledgeable. To prevent bankers from defaulting on their deposits, their incentive compatibility constraint binds. This implies that public debt exhibit premia. As a result, Ricardian equivalence breaks down and maturity structure of the government debt is relevant for real allocations and consumption inequality.

Under an active monetary policy regime, we find that at most one stationary equilibrium can exist. Moreover, consumption in connected and unconnected meetings are independent of each other. The maturity composition of public debt does not affect consumption in unconnected trades. These results are observed regardless of whether collateral is plentiful (efficient consumption in connected trades is possible) or collateral is scarce. The maturity composition does affect consumption in connected trades when collateral are scarce. An increase in the money growth rate results in larger consumption inequality when collateral is plentiful and when collateral is scarce and the collateral effect dominates the risk-averse one. This is the case regardless of how the fiscal authority sets the supply of short and long-term bonds.

Under an active fiscal policy regime, the equilibrium properties depend on whether collateral is plentiful or scarce. In an economy with plentiful collateral, there exists a unique stationary equilibrium. Moreover, consumption in connected and unconnected meetings are independent of each other. Finally, the maturity composition of public debt does affect consumption in unconnected trades. These properties are observed regardless of the bond supply process followed by the fiscal authority. In contrast, when collateral is scarce, multiple stationary equilibria can exist and consumption across the different states of the world are not independent of each other. Furthermore, fiscal policy affects the consumption of connected and unconnected trades, which in turn affects consumption inequality. Our findings are consistent with a passive Laffer curve present in the economy since two different money growth rates can satisfy the government budget constraint. These results stress the importance of the maturity structure of public debt as well as its composition in affecting allocations and consumption inequality.

Finally, we study how financial innovations alter monetary and fiscal policy interactions when an economy faces the two different policy regimes. We find that financial innovation that increases pledgeability, which can be thought as financial deepening, amplifies the effects of active monetary and active fiscal policy. In contrast, the effects of fiscal and monetary policy are dampened when financial innovation occurs through an increased measure of buyers in connected meetings, which can be thought as financial inclusion. In addition, active fiscal policy is relatively more effective in increasing consumption in connected trades when pledgeability of long-term bond has increased. On the other hand, active monetary policy is relatively more effective when the measure of connected trades has increased.

In our view, it is important to study how the interaction between fiscal and monetary policy affects the economy when various objects that can be used to settle payments are present, as this is the payment architecture that modern economies currently face. The next step in our research agenda is to study the dynamics where policy regime switching is possible.

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Appendix A

Proof of Lemma 1

From Equations (40) and (41) it is easy to show that

$$r^l_{t+1} - r^s_{t+1} = \frac{\phi_t\Big(\beta u'\big(R_{t+1}d_t\big) - 1\Big)(\theta_l - \theta_s)}{\phi_{t+1}\Big(\beta u'\big(R_{t+1}d_t\big)(1-\theta_s) + \theta_s\Big)\Big(\beta u'\big(R_{t+1}d_t\big)(1-\theta_l) + \theta_l\Big)}. \tag{57}$$

It is straightforward to see that the numerator in equation (57) is positive if the haircut on long-term government bonds is greater than the haircut on short-term government bonds.

Proof of Proposition 1

For a given money growth rate, it is straightforward to pin down the quantity consumed in the unconnected DM trades. We focus on checking that the incentive compatibiltiy (IC) constraint is satisfied. We define the following function

$$RHS(q) = \frac{\rho(1-\delta)q^u\ u'(q^u)(1-\theta_s)}{\delta}$$

as the right hand side of equation (38). Since the $RHS(q)$ is independent of q, it is invariant with respect to changes in consumption by those in connected meetings. Consequently, it is important to find values of q that satisfy the IC constraint. We define the function

$$LHS(q) = (1-\rho)\ q\ (\beta u'(q)(1-\theta_s) + \theta_s) + \frac{A(\theta_l - \theta_s)}{\beta u'(q)(1-\theta_l) + \theta_l}.$$

as the left-hand side of equation (38).

Two key properties will help us understand how $LHS(q)$ function behaves. First, $q \to 0$, $LHS \to 0$. Second, $q \to q^*$, $LHS \to (1-\rho)q^*(\beta(1-\theta_s)+\theta_s) + \frac{A(\theta_l-\theta_s)}{(\beta(1-\theta_s)+\theta_s)}$. Moreover, the derivative of $LHS(q)$ is given by

$$\frac{\partial LHS(q)}{\partial q} = (1-\rho)\Big((\beta u'(q)(1-\theta_s)+\theta_s) + \beta q u''(q)(1-\theta_s)\Big) - \frac{\beta A(\theta_l-\theta_s)u''(q)(1-\theta_l)}{\big(\beta u'(q)(1-\theta_l)+\theta_l\big)^2}. \tag{58}$$

For (58), the second term is positive for $A > 0$. Consequently, if the first term is positive, then the derivative is positive. The sign of the first term depends on the degree of relative risk aversion. Formally, the first term is positive if,

$$\frac{-qu''(q)}{u'(q)} \geq 1.$$

The implication is that if the coefficient of relative risk aversion values is between zero and one, the $LHS(q)$ is upward sloping in q while the $RHS(q)$ function is constant. Thus, for example with constant relative aversion that lies in the unit interval, the two curves intersect at most once if,

$$(1-\rho)q^*(\beta(1-\theta_s)+\theta_s) + \frac{A(\theta_l-\theta_s)}{(\beta(1-\theta_s)+\theta_s)} \geq \frac{\rho(1-\delta)q^u\ u'(q^u)(1-\theta_s)}{\delta}$$

If $u(.)$ exhibits increasing relative risk aversion, the coefficient of relative risk aversion is increasing in q. Suppose that when q is near zero, the coefficient of relative risk aversion is inside the unit interval and is greater than one as q approaches q^*. What we know is that the slope of the$LHS(q)$ is positive for lower values of q. With increasing relative risk aversion, we are interested in the behavior of the function $LHS(q)$ in the range $q \in (0, q^*)$. With increasing relative risk aversion, it is possible that the $LHS(q)$ function will slope downward.

As q increases, the first term of the Equation (58) switches sign from positive to negative. If the second term is larger than the first term, for A large enough, then the $LHS(q)$ is upward sloping. Otherwise, it is hump shaped. But as long as the sufficient condition is satisfied, the $LHS(q)$ intersects the $RHS(q)$ only once.

If A is negative, the $LHS(q)$ is either u-shaped or downward sloping. The analysis for this case is similar.

Thus, we have derived the sufficient condition for the existence of a unique stationary monetary equilibrium. Even with increasing relative risk aversion, the condition ensures that the $LHS(q)$ function intersects the $RHS(q)$ once. The sufficient condition will guarantee that a stationary equilibrium exists and guarantee that stationary equilibrium is unique.

Proof of Proposition 2

We again need to verify that the IC constraint is satisfied to ensure that the stationary monetary equilibrium exists. We rely on the fact that the right hand side of equation (43) is invariant to changes in q. From equation (43), with $q \to 0$, then $LHS^s(q) \to 0$. With $q \to q^*$, $LHS^s(q) \to (1-\rho)q^*(\beta(1-\theta_s)+\theta_s) + \frac{(\Gamma-B)(\theta_l-\theta_s)}{(\beta(1-\theta_l)+\theta_l)}$. Moreover, the derivative of $LHS^s(q)$ is given by

$$\frac{\partial LHS^s(q)}{\partial q} = (1-\rho)\Big((\beta u'(q)(1-\theta_s)+\theta_s) + \beta q u''(q)(1-\theta_s)\Big) - \frac{\beta(\Gamma-B)(\theta_l-\theta_s)u''(q)(1-\theta_l)}{\big(\beta u'(q)(1-\theta_l)+\theta_l\big)^2}. \quad (59)$$

In equation (59), the second term is positive if $B < \Gamma$, i. e. B not too large. Consider the case where the coefficient of relative risk aversion lies in the unit interval. The first term is positive as in policy $\mathcal{L}$. So the two curves intersect at most once if,

$$(1-\rho)q^*(\beta(1-\theta_s)+\theta_s) + \frac{(\Gamma-B)(\theta_l-\theta_s)}{(\beta(1-\theta_l)+\theta_l)} \geq \frac{\rho(1-\delta)q^u\ u'(q^u)(1-\theta_s)}{\delta}.$$

Now consider the case of increasing relative risk aversion. As q increases, the first of the Equation (58) switches sign from positive to negative. If the second term is larger than the first term, for B small enough, then the $LHS(q)$ is upward sloping. Otherwise, it is hump shaped. But as long as the sufficient condition is satisfied, the $LHS(q)$ intersects the $RHS(q)$ only once.

If $B > \Gamma$, the $LHS(q)$ is either u-shaped or downward sloping. The analysis for this case is similar.

Proof of Proposition 3

The result owes to two key conditions. First, with $\beta u'(q^u) = \mu$, it follows that q^u is decreasing in μ. This is not surprising as inflation is a tax on monetary transactions. Second, let locus $q = F(q^u)$ represent Equation (38). Derivative of the locus with respect to q^u is given by

$$\frac{\partial q}{\partial q^u} = \frac{-\frac{(1-\delta)}{\delta}\rho(1-\theta_s)(u'(q^u)+q^u u''(q^u))}{(1-\rho)\Big((\beta u'(q)(1-\theta_s)+\theta_s)+\beta q u''(q)(1-\theta_s)\Big) - \frac{\beta A(\theta_l-\theta_s)u''(q)(1-\theta_l)}{\big(\beta u'(q)(1-\theta_l)+\theta_l\big)^2}}$$

If $-\frac{qu''(.)}{u'(.)} < 1$, the denominator is positive and hence the derivative is negative. q increases with an increase in μ, resulting in a higher consumption inequality. Else, first term in the denominator is negative. If the second term is higher in absolute value than the first term, the derivative is still negative. Otherwise, it is positive (risk-averse effect dominates). When the derivative is positive, an increase in μ results in a decrease in q. Thus the effect on the inequality depends on the slope of the locus. A sufficient condition to ensure that the inequality is positively related to μ is that the slope of $F(q^u)$ locus is less than 1, or $\frac{\partial q}{\partial q^u}$ is less than 1.

Proof of Proposition 4

In compact form, the two equations for q and q^u are represented as follows

$$(1-\rho)q\Delta_0 + \frac{A(\theta_l - \theta_s)}{\Delta_1} = \frac{\Gamma}{\delta} \tag{60}$$

where Γ is defined above and $\Delta_0 = (\beta u'(q)(1-\theta_s) + \theta_s)$ and $\Delta_1 = (\beta u'(q)(1-\theta_l) + \theta_l)$ and equation (54) is written as, after rearranging and simplifying

$$\frac{\rho}{\delta} q^u u'(q^u) - \frac{Au'(q^u)}{\Delta_1} + \frac{(A-(1-\delta))u'(q^u)}{\Delta_0} = \tau + A + \delta. \tag{61}$$

Note that $\Delta_0, \Delta_1 > 0$, Both $\Delta_0', \Delta_1' < 0$. We differentiate (60) with respect to q and q^u, yielding

$$\Big[(1-\rho)\delta(\Delta_0 + q\Delta_0') - \frac{A\delta(\theta_l - \theta_s)\Delta_1'}{\Delta_1^2}\Big] dq = \Gamma' dq^u. \tag{62}$$

With increasing relative risk aversion, the left hand side of Equation (62) is positive when evaluated at values of q close to zero. It is negative if and only if $(1-\rho)\delta(\Delta_0 + q\Delta_0') - \frac{A\delta(\theta_l-\theta_s)\Delta_1'}{\Delta_1^2} > 0$, which can occur at higher values of q with an increasing relative risk aversion. Similarly, the left hand side of the equation is positive at lower values of q^u and can become negative at higher values of q^u with increasing relative risk aversion. In general, the sign of the slope of the locus is ambiguous and the locus can be upward sloping or hump shaped. The derivative of equation (61) with respect to q and q^u is

$$[\frac{\rho}{\delta}K_0 - \frac{Au''(q^u)}{\Delta_1} + \frac{(A-(1-\delta))u''(q^u)}{\Delta_0}]dq^u + [\frac{Au'(q^u)\Delta_1'}{(\Delta_1)^2} - \frac{(A-(1-\delta))u'(q^u)\Delta_0'}{(\Delta_0)^2}]dq = 0 \tag{63}$$

which, after rearranging, is,

$$\frac{dq}{dq^u} = -\frac{\frac{\rho}{\delta}K_0 - \frac{Au''(q^u)}{\Delta_1} + \frac{(A-(1-\delta))u''(q^u)}{\Delta_0}}{\frac{Au'(q^u)\Delta_1'}{(\Delta_1)^2} - \frac{(A-(1-\delta))u'(q^u)\Delta_0'}{(\Delta_0)^2}}. \tag{64}$$

where $K_0 = u'(q^u) + q^u u''(q^u)$. Here, the sign of K_0 is related to the coefficient of relative risk aversion. In addition, we know that $\Gamma > 0$, and that the sign of Γ' depends on whether the coefficient of relative risk aversion is greater than or less than one. If the coefficient of relative risk aversion lies in (outside) the unit interval, then $\Gamma' > (<)0$.

Let $q^u \to 0$ correspond to $K_0 \geq -1$. Given that $\Delta_1 < \Delta_0$ and $|\Delta_0'| > |\Delta_1'|$, it follows that the sum of second and third terms inside the first bracket is negative. Thus, the numerator is positive if the coefficient of relative risk aversion is greater than one, resulting in $K_0 > 0$ "big enough".

In general, the sign is ambiguous for the term in the second bracket of equation (63). If it is positive–specifically if $\frac{\Delta_1'}{(\Delta_1)^2} - \frac{\Delta_0'}{(\Delta_0)^2} > 0$, then the expression for $\frac{dq}{dq^u}$ in equation (64) will switch signs from negative to positive for economies in which preferences exhibit increasing relative risk aversion as q^u increases.

Under these conditions, the graph of equation (53) can be upward sloping or hump-shaped and the graph of equation (54) is U-shaped. It follows that the number of stationary values can be zero (no crossings), one (a single crossing), or two (a pair of crossings).

There are limiting conditions that can be written out such that $\frac{dq}{dq^u}$ in equation (63) combined with values of risk aversion close to zero will be negative. With increasing relative risk aversion, the numerator will be decreasing algebraically and eventually become negative so that the expression reaches its minimum and becomes positive.

Similarly, the expression for differentiated equation (62) is initially positive for values of relative risk aversion close to zero, reaches a maximum and becomes negative with increasing relative risk aversion.

Proof of Proposition 5

Consider Equation (60) and (61) in the proof of Proposition 4. Combining the two equations, we get

$$(1-\rho)q\Delta_0 + \frac{A(\theta_l - \theta_s)}{\Delta_1} = \frac{1-\delta}{\delta}\left[\tau + A + \delta + \frac{Au'(q^u)}{\Delta_1} - \frac{(A-(1-\delta))u'(q^u)}{\Delta_0}\right].$$

After rearranging, we obtain

$$\frac{\delta}{1-\delta}(1-\rho)q\Delta_0 = A\left[1 + \frac{u'(q^u)}{\Delta_1} - \frac{u'(q^u)}{\Delta_0} - \frac{(\theta_l-\theta_s)}{\Delta_1}\frac{\delta}{1-\delta}\right] + \tau + \delta + \frac{(1-\delta)u'(q^u)}{\Delta_0}.$$

Following the argument in the proof of Proposition 4, the left hand side is hump shaped. Next, derive the right hand side with respect to q, getting

$$A\left[u''(q^u)\frac{\partial q^u}{\partial q}\left(\frac{1}{\Delta_1} - \frac{1}{\Delta_0}\right) + u'(q^u)\left(-\frac{\Delta_1'}{\Delta_1^2} + \frac{\Delta_0'}{\Delta_0^2}\right) + \frac{(\theta_l-\theta_s)\Delta_1'}{\Delta_1^2}\frac{\delta}{1-\delta}\right] + (1-\delta)\left[\frac{u''(q^u)\frac{\partial q^u}{\partial q}\Delta_0 - u'(q^u)\Delta_0'}{\Delta_0^2}\right].$$

Following the assumptions in the proof of Proposition 4, the term in the first bracket is negative if $\frac{\partial q^u}{\partial q} > 0$, which holds for lower values of q^u. With an IRRA utility, the second term switches sign from negative to positive as q^u increases. In general, the RHS is either downward sloping or U shaped. Note that as $q \to 0$, $LHS \to 0$ and $RHS > 0$. A sufficient condition for a unique equilibrium is then to have the RHS lower than LHS when $q \to q^*$. It follows that the condition in the proposition is sufficient to ensure a single crossing between the F(.) loci and the G(.) loci.

Proof of Proposition 6

We proceed by writing equation (55) in compact form as follows

$$(1-\rho)q\Delta_0 + \frac{\left[\frac{\Gamma}{\delta(1-\theta_s)} - B\right](\theta_l - \theta_s)}{\Delta_1} = \frac{\Gamma}{\delta}. \tag{65}$$

Compared with Policy $\mathcal{L}$, we can see a key difference is present in Policy $\mathcal{S}$. Because the value of long-term bonds is not constant in $\mathcal{S}$, equation (65) indicates that the quantity of long-term bonds will respond to tax-policy changes. In particular, note that the second term on the left-hand-side of equation (65) is a function of the quantity of long-term debt. In addition, general equilibrium effects show up through changes in q and q^u. Indeed, the combination of direct and indirect effects can account for why the equation (65) includes additional terms compared with equation (60).

To further demonstrate that the comparison matters, recall that the derivative is

$$[(1-\rho)\delta(\Delta_0 + q\Delta_0') - \frac{[\frac{\Gamma}{(1-\theta_s)} - B](\theta_l - \theta_s)\Delta_1'}{\Delta_1^2}]dq = \Gamma'[1 - \frac{(\theta_l - \theta_s)}{(1-\theta_s)\Delta_1}]dq^u. \tag{66}$$

With the bracketed term on the right-hand-side of equation (66) as positive, the right-hand side is positive for lower values of q^u and negative for higher values of q^u. The second term on the left-hand side in equation (66) is positive, but with IRRA, the first term will change sign; it is positive for small values of q and turns negative as q increases. It follows that the left-hand side is positive for lower values of q. The sign is strictly positive if $\left|\frac{[\frac{\Gamma}{(1-\theta_s)} - B](\theta_l - \theta_s)\Delta_1'}{\Delta_1^2}\right| > |(1-\rho)\delta(\Delta_0 + q\Delta_0')|$. However, if this condition fails, the sign of the left-hand side switches to negative. In general the curve can be upward sloping or hump-shaped.

Equation (56) can be written as,

$$\rho q^u u'(q^u) - \Big(1 + \frac{u'(q^u)}{\Delta_0} - \frac{u'(q^u)}{\Delta_1}\Big)\big(\rho q^u u'(q^u)(1-\delta) - \delta B\big) - \frac{\delta(1-\delta)u'(q^u)}{\Delta_0} = \delta(\tau + \delta). \tag{67}$$

We derive the expression for equation (56). After collecting terms and writing in compact notation, we obtain

$$\frac{dq}{dq^u} = -\frac{\rho K_0 \left[\delta - \left(\frac{u'(q^u)}{\Delta_0} - \frac{u'(q^u)}{\Delta_1}\right)\right] - u''(q^u)\left[\left(\rho q^u u'(q^u)(1-\delta) - \delta B\right)\left(\frac{1}{\Delta_0} - \frac{1}{\Delta_1}\right) - \frac{\delta(1-\delta)}{\Delta_0}\right]}{-u'(q^u)\big(\rho q^u u'(q^u)(1-\delta) - \delta B\big)\left(\frac{\Delta_1'}{\Delta_1^2} - \frac{\Delta_0'}{\Delta_0^2}\right)}. \tag{68}$$

From equation (68), the denominator is negative if (C1): $\rho q^u u'(q^u)(1-\delta) > \delta B$ and (C2): $\frac{\Delta_1'}{\Delta_1^2} > \frac{\Delta_0'}{\Delta_0^2}$. We can confirm that condition (C2) is from our assumptions in policy $\mathcal{L}$. The numerator is more nuanced. K_0 changes sign from positive to negative with increasing relative risk aversion. In the numerator, the term in the first bracket is positive. If the second expression is negative and larger in magnitude than the first, then the equation is downward sloping always. In general, the locus can be u-shaped. The bottom line is that multiple equilibria cannot be ruled out because of the hump-shaped pattern in equation (56).

Proof of Proposition 7

Combining the equilibrium conditions in policy $\mathcal{S}$ and writing them in compact form, we get,

$$\delta(1-\rho)q\Delta_0 = B\left[\frac{\delta(\theta_l - \theta_s)}{\Delta_1} - \delta(1-\delta)(1-\theta_s)\left(1 + \frac{u'(q^u)}{\Delta_0} - \frac{u'(q^u)}{\Delta_1}\right)\right]$$
$$+(1-\delta)(1-\theta_s)\left[\frac{\Gamma}{1-\theta_s}\left(1 + \frac{u'(q^u)}{\Delta_0} - \frac{u'(q^u)}{\Delta_1}\right) + \frac{\delta(1-\delta)u'(q^u)}{\Delta_0} + \delta(\tau+\delta)\right] - \frac{\Gamma(\theta_l - \theta_s)}{(1-\theta_s)\Delta_1}$$

The left hand side is hump shaped. We represent the right-hand side with respect to q,

$$B\left[-\frac{\delta(\theta_l-\theta_s)\Delta_1'}{\Delta_1^2}-\delta(1-\delta)(1-\theta_s)\left(u''(q^u)\frac{\partial q^u}{\partial q}\left(\frac{1}{\Delta_0}-\frac{1}{\Delta_1}\right)-u'(q^u)\left(\frac{\Delta_0'}{\Delta_0^2}-\frac{\Delta_1'}{\Delta_1^2}\right)\right)\right]$$
$$+(1-\delta)(1-\theta_s)\left(\frac{\Gamma'}{1-\theta_s}\left(1+\frac{u'(q^u)}{\Delta_0}-\frac{u'(q^u)}{\Delta_1}\right)+\frac{\Gamma}{1-\theta_s}\left(u''(q^u)\frac{\partial q^u}{\partial q}\left(\frac{1}{\Delta_0}-\frac{1}{\Delta_1}\right)-u'(q^u)\left(\frac{\Delta_0'}{\Delta_0^2}-\frac{\Delta_1'}{\Delta_1^2}\right)\right)\right.$$
$$\left.+\delta(1-\delta)\left(\frac{u''(q^u)\frac{\partial q^u}{\partial q}}{\Delta_0}-\frac{u'(q^u)\Delta_0'}{\Delta_0^2}\right)\right)-\frac{\theta_l-\theta_s}{1-\theta_s}\left(\frac{\Gamma'}{\Delta_1}-\frac{\Gamma\Delta_1'}{\Delta_1^2}\right)$$

Following the assumptions in the proof of Proposition 6, expression $\left[u''(q^u)\frac{\partial q^u}{\partial q}\left(\frac{1}{\Delta_0}-\frac{1}{\Delta_1}\right)-u'(q^u)\left(\frac{\Delta_0'}{\Delta_0^2}-\frac{\Delta_1'}{\Delta_1^2}\right)\right]$ changes sign from negative to positive as q^u increases. Together with assumption on Γ', the last two expressions change sign from negative to positive as q^u increases. In the first bracketed term, the sign is always positive if the first expression inside the bracket is always larger than the second. Alternatively, the bracketed term changes sign from positive to negative if the second expression becomes larger in absolute value than the first. If the last two expressions are combined and dominate the first expression, then the RHS is U shaped. Otherwise, the RHS is upward sloping. As $q\to 0$, $LHS\to 0$ and $RHS>0$. So regardless of the shape of the RHS, sufficient condition for uniqueness is to prevent RHS from becoming larger than the LHS as $q\to q^*$. This condition is satisfied when

$$\delta(1-\rho)q^*>B\delta\left[(\theta_l-\theta_s)-(1-\delta)(1-\theta_s)\right]+\frac{\Gamma}{1-\theta_s}\left[(1-\delta)(1-\theta_s)-(\theta_l-\theta_s)\right]+\delta(1-\delta)(1-\theta_s)(\tau+\delta)$$

or equivalently,

$$B<\frac{\Gamma}{\delta(1-\theta_s)}+\frac{(1-\delta)(1-\theta_s)(\tau+\delta)-\delta(1-\rho)q^*}{(1-\delta)(1-\theta_s)-(\theta_l-\theta_s)}$$

The denominator is satisfied for reasonable parameter values. So we need $\delta(1-\rho)q^*<(1-\delta)(1-\theta_s)(\tau+\delta)$ in addition.

Appendix B

Central Bank's Purchase of Long-term Government Bonds

In this section, we consider Quantitative Easing (QE). Here, QE is implemented by the monetary authority purchasing long-term government bonds. The implication is that we are studying an active monetary policy $\mathcal{S}$. There is a modification to the setting studied previously. Here, we assume the total value of money and government debt is fixed. Let the rule be represented as follows:

$$\phi M_t + \phi z_t^s B_t^s + \phi z_t^l B_t^l = L. \tag{69}$$

With $\phi z_t^s B_t^s = B$, the active monetary policy setting is an open market operation, exchanging, for example, long-term government bonds for money. Substitute the equation on total value of money and bonds into Equation 19 and plug in the quantities and prices. The equilibrium is represented by the pair of equations

$$(1-\rho)q\Big(u'(q)(1-\theta_s)+\theta_s\Big) + \frac{(L-\rho q^u u'(q^u)-B)(\theta_l-\theta_s)}{u'(q)(1-\theta_l)+\theta_l} = (1-\theta_s)(L-\rho q^u u'(q^u)) \tag{70}$$

$$\beta u'(q^u) = \mu. \tag{71}$$

Second equation implies that q^u is inversely related to the money growth rate as before. To see how consumption in connected trades is affected, we follow a similar approach to the baseline case. Totally differentiate the IC equation to obtain.

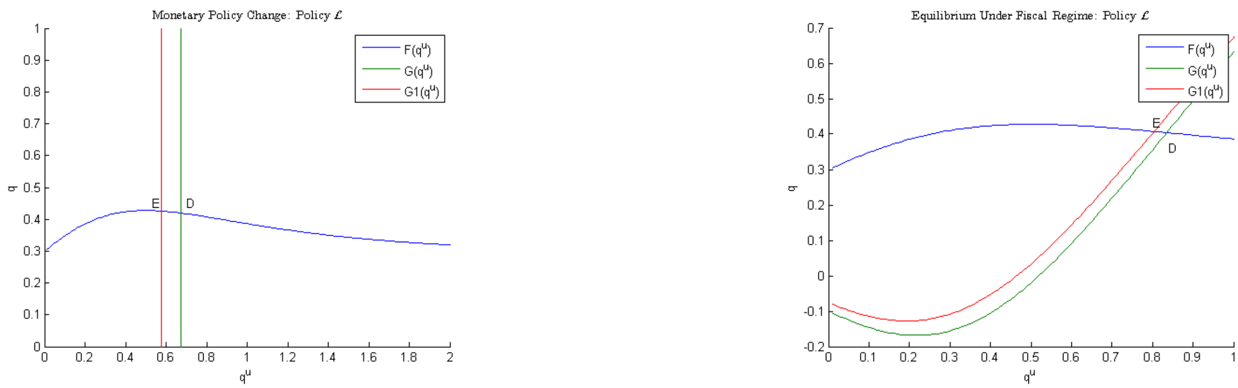
$$\begin{gathered}\left[(1-\rho)\left(u'(q)(1-\theta_s)+\theta_s+qu''(q)(1-\theta_s)\right) - \frac{(L-\rho q^u u'(q^u)-B)(\theta_l-\theta_s)u''(q)(1-\theta_l)}{(u'(q)(1-\theta_l)+\theta_l)^2}\right]\partial q = \\ \left[\rho\left(u'(q^u)+q^u u''(q^u)\right)\left(\frac{\theta_1-\theta_s}{u'(q)(1-\theta_l)+\theta_l}-(1-\theta_s)\right)\right]\partial q^u\end{gathered} \tag{72}$$

The second term on the LHS is positive. The first term changes sign from positive to negative. Following the similar argument to the baseline case, the LHS changes sign from positive to negative in general as q increases. Similarly, the RHS changes sign from positive to negative. Hence, the $q = F(q^u)$ locus is hump-shaped as in the baseline case.

Now consider an increase in money growth rate supported by a decrease in long-term bonds (QE policy). Any monetary policy that increases money growth rate lowers q^u. Effect on q depends on the nature of the equilibrium. In particular, in the downward sloping range of $F(q^u)$ locus (range where collateral effect dominates), an increase in money growth rate shifts the $G(q^u)$ locus to the left and increases q. By reducing the supply of long-term bonds (bad collateral), the average quality of the existing collateral is raised and the incentive to abscond is reduced. Consequently, consumption in connected trades will increase. If the equilibrium is on the upward sloping part of the $F(q^u)$ locus (where risk-averse effect dominates), a QE policy decreases q. Note that any increase in money growth rate in the range is sub-optimal. Our results indicate that a QE policy has the same qualitative impact as we found in the model economy without any constraint on the value of money and bonds outstanding.

Numerical Example

We explore whether our qualitative results are sensitive to the parameter values. In particular, we study how



(a) Equilibrium under Monetary Regime $\mathcal{L}$

(b) Equilibrium under Fiscal Regime $\mathcal{L}$

Figure 7: Equilibria under different regimes

changes in different structural parameters affect the stationary equilibrium. In the baseline analysis, preferences are such that $u(x) = -e^{-\sigma x}$ with $\beta = 0.99$ and $\rho = 0.5$. Over the period of 1970 to present, the average ratio of M1 money stock to the sum of M1 stock and Federal Debt held by public in the US is about 0.3. The ratio is higher in European countries, with the ratio of about 0.4 for the EU and about 0.5 for the UK. For the baseline model we use the European ratios as our guide and specify $\delta = 0.5$ as money-to-government liability ratio. We use $\theta_s = 0.05$ and $\theta_l = 0.1$ for the plegdeability of different types of bonds. Different pledgeability owes to the fact that the central bank generally applies different haircuts to debt with different maturities. In the United States, the Federal Reserve has historically applied one percent haircuts on government bonds of maturity less than five years and four percent on longer term bonds. Pledgeability of government assets are likely higher in other developed and developing countries. For the baseline numerical analyses, we specify values slightly larger than those observed in the United States.

Our first check is to specify parts of the parameter space that more closely correspond to values observed in the United States. Therefore, let $\delta = 0.3$, $\theta_s = 0.01$ and $\theta_l = 0.04$. In addition, let $\rho = 0.17$ for the measure of agents in unconnected trades, which is lower than the baseline case. The measure of the unbanked households in the US is about 6.5 percent and that of the underbanked is up to 17 percent in 2017.[40]

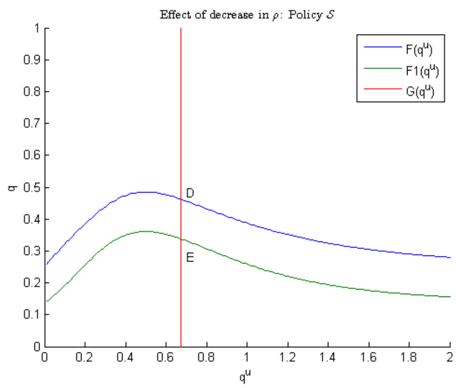
Figure 7 reports the results from the model economy for this part of the parameter space. Without loss of generality, we present results from monetary policy $\mathcal{L}$ and fiscal policy $\mathcal{L}$ only.[41] Panel (a) shows the graph for monetary policy $\mathcal{L}$ and Panel (b) shows the graph for fiscal policy $\mathcal{L}$. The shifts in the $G(.)$ locus are due to the expansionary policy change experiments. The results are qualitatively the same as in the baseline case. Note that the parameter choices here eliminates the bad equilibrium. More importantly, the shape of the $F(.)$ and $G(.)$ locus that results in indeterminacy is still present here. Predictions for changes in policies are qualitatively the same for equilibria that exist.

Illustration of Financial Innovation

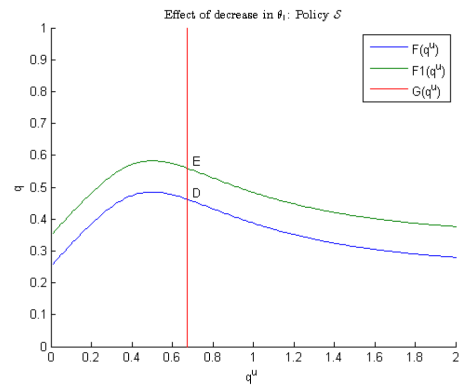
Figure 8 provides an example of the effect of financial developments on the equilibrium outcomes and inequalities for different policy regimes. Without loss of generality, we provide graphs for active monetary policy

[40]See https://www.fdic.gov/householdsurvey/

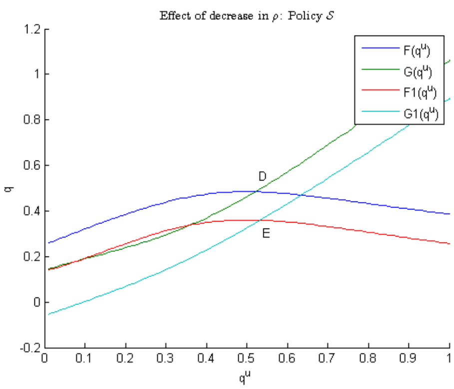
[41]Findings for policy $\mathcal{S}$ are similar and hence omitted. It is available upon request.



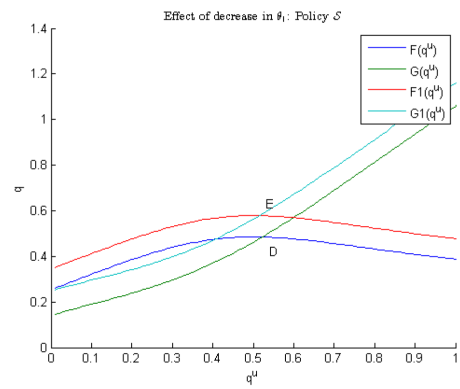
(a) Increase in Number of Connected Trades: Monetary policy $\mathcal{S}$



(b) Increase in Pledgeability of Long-term Bonds: Monetary Policy $\mathcal{S}$



(c) Increase in Number of Connected Trades: Fiscal policy $\mathcal{S}$



(d) Increase in Pledgeability of Long-term Bonds: Fiscal Policy $\mathcal{S}$

Figure 8: Effects of Financial Developments

$\mathcal{S}$ and active fiscal policy $\mathcal{S}$ only.[42] The effects of an increase in access to banking under active monetary policy are presented in panel (a) and the effects of an increase in pledgeability of long-term bonds are presented in panel (b). The effects of an increase in access to banking under active fiscal policy are presented in panel (c) and the effects of an increase in pledgeability of long-term bonds under fiscal policy regime are presented in panel (d).

Findings are as illustrated in the analytical section. Equilibrium in all the cases shifts from D to E as a result of the financial innovation under consideration. Under active monetary policy regime, a decrease in ρ (increase in number of connected trades) results in lower q, decreasing the consumption inequality. The effect of a decrease in θ_l (increase in pledgeability of long-term bond) increases q, resulting in higher consumption inequality.

For an active fiscal policy regime, an increase in number of connected trades results in lower q in a good equilibrium. An increase in pledgeability, on the other hand, increases q. Effect on q^u is ambiguous in both cases. What is clear is that consumption inequality declines when the number of connected trades increase while it gets wider when pledgeability of long-term bond increases.

[42]Findings for policy $\mathcal{L}$ are similar and hence omitted. It is available upon request. We chose to illustrate $\mathcal{S}$ because that is the case we focus on our analytical exercise.

Appendix C

Here, we consider the policies of the government where it fixes quantities of bonds rather than the value. Perhaps our results are dependent on fixing the values of the government debts. To determine the robustness of the results, we consider cases in which the level of government debts are fixed. Formally, instead of $\phi_t z_t^i B_t^i = X \;\; \forall t$, we consider $B_t^i = \mathcal{X} \;\; \forall t$, where $i = \{L, S\}$, $X = \{A, B\}$ and $\mathcal{X} = \{\mathcal{A}, \mathcal{B}\}$.

Active Monetary Policy

First, we consider the case where the monetary authority actively changes the money growth rate and the fiscal authority passively determines the quantity of bonds and taxes to balance the government budget. First, we consider the case where the quantity of long-maturity bonds are fixed, $B_t^l = \mathcal{A} \;\; \forall t$. Let us call this Policy $\mathcal{L}$. Proposition below summarizes the determinacy of the equilibrium for Policy $\mathcal{L}$.

Proposition 8. *If the following condition is satisfied*

$$(1-\rho)q^*(\beta(1-\theta_s)+\theta_s) + \frac{\phi\mathcal{A}(\theta_l-\theta_s)}{u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l)+\theta_l\Big)} \geq \frac{\rho(1-\delta)q^u \; u'(q^u)(1-\theta_s)}{\delta}$$

*then there exist a unique stationary equilibrium under Policy **L**, where q^* implicitly defined by $u'(q^*) = 1$.*

Proof. In equilibrium, the binding incentive compatibility becomes,

$$(1-\rho)\; q\; (\beta u'(q)(1-\theta_s)+\theta_s) + \frac{\phi\mathcal{A}(\theta_l-\theta_s)}{u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l)+\theta_l\Big)} = \frac{\rho(1-\delta)q^u \; u'(q^u)(1-\theta_s)}{\delta} \tag{73}$$

The right hand side of the equation is independent of q as it is pinned down by the money growth rate. We define the function

$$LHS(q) = (1-\rho)\; q\; (\beta u'(q)(1-\theta_s)+\theta_s) + \frac{\phi\mathcal{A}(\theta_l-\theta_s)}{u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l)+\theta_l\Big)}.$$

Two key properties will help us understand how the $LHS(q)$ function behaves. First, $q \to 0$, $LHS \to \frac{\phi\mathcal{A}(\theta_l-\theta_s)}{u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l)+\theta_l\Big)}$. Second, $q \to q^*$, $LHS \to (1-\rho)q^*(\beta(1-\theta_l)+\theta_l) + \frac{\phi\mathcal{A}(\theta_l-\theta_s)}{u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l)+\theta_l\Big)}$. Moreover, the derivative of $LHS(q)$ is given by

$$\frac{\partial LHS(q)}{\partial q} = (1-\rho)\Big((\beta u'(q)(1-\theta_s)+\theta_s) + \beta q u''(q)(1-\theta_s)\Big)$$

The sign of the derivative depends on the degree of relative risk aversion. Formally, the first term is positive if

$$\frac{qu''(q)}{u'(q)} \geq -1.$$

The implication is that if the coefficient of relative risk aversion values is between zero and one, the $LHS(q)$ is upward sloping in q while the $RHS(q)$ function is constant. If $u(.)$ exhibits increasing relative risk aversion, the derivative changes sign from positive to negative as q increases, similar to the baseline case. If the sufficient condition satisfies, the LHS and RHS intersect at most once.

□

Suppose $q = F(q^u)$ is the locus defining the IC constraint above. The proof of the proposition above shows that the locus is hump-shaped as in the baseline case. Hence, the rest of the analyses for policy **L** are similar to the baseline case.

Next, we consider the case where the quantity of short-maturity bonds are fixed, $B_t^s = \mathcal{B}\ \ \forall t$. Let us call this Policy **S**. Proposition below summarizes the determinacy of the equilibrium for Policy **S**.

Proposition 9. *If the following conditions are satisfied*

$$(1-\rho)q^*(\beta(1-\theta_s)+\theta_s) + \frac{\Big(\frac{\rho(1-\delta)q^u\ u'(q^u)}{\delta} - \phi\mathcal{B}\Big)(\theta_l-\theta_s)}{u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l)+\theta_l\Big)} \geq \frac{\rho(1-\delta)q^u\ u'(q^u)(1-\theta_s)}{\delta}$$

and

$$\mathcal{B} \geq \frac{\rho(1-\delta)q^u\ u'(q^u)}{\delta\phi(\theta_l-\theta_s)}\left[(\theta_l-\theta_s) - (1-\theta_s)\Big(u'(q^u) - \Big(u'(q^u)(1-\theta_l)+\theta_l\Big)\Big)\right]$$

then there exist a unique stationary equilibrium under Policy $\mathcal{S}$.

Proof. In equilibrium, the binding incentive compatibility becomes

$$(1-\rho)q\Big(\beta u'(q)(1-\theta_s)+\theta_s\Big) + \frac{\Big(\frac{\rho(1-\delta)q^u\ u'(q^u)}{\delta} - \phi\mathcal{B}\Big)(\theta_l-\theta_s)}{u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l)+\theta_l\Big)} = \Big(\frac{\rho(1-\delta)q^u u'(q^u)}{\delta}\Big)(1-\theta_s) \quad (74)$$

The right hand side of the equation is independent of q as it is pinned down by the money growth rate. We define the function as

$$LHS(q) = (1-\rho)q\Big(\beta u'(q)(1-\theta_s)+\theta_s\Big) + \frac{\Big(\frac{\rho(1-\delta)q^u\ u'(q^u)}{\delta} - \phi\mathcal{B}\Big)(\theta_l-\theta_s)}{u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l)+\theta_l\Big)}.$$

As $q \to 0$, $LHS \to \frac{\Big(\frac{\rho(1-\delta)q^u\ u'(q^u)}{\delta} - \phi\mathcal{B}\Big)(\theta_l-\theta_s)}{u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l)+\theta_l\Big)}$. As $q \to q^*$, $LHS \to (1-\rho)q^*(1-\theta_s)+\theta_s) + \frac{\Big(\frac{\rho(1-\delta)q^u\ u'(q^u)}{\delta} - \phi\mathcal{B}\Big)(\theta_l-\theta_s)}{u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l)+\theta_l\Big)}$.
The derivative of $LHS(q)$ is given by

$$\frac{\partial LHS(q)}{\partial q} = (1-\rho)\Big((\beta u'(q)(1-\theta_s)+\theta_s) + qu''(q)(1-\theta_s)\Big)$$

Similar to the Policy $\mathcal{L}$ case, the derivative above changes sign from positive to negative with IRRA utility function. Uniqueness follows from the sufficient condition in the proposition.

□

Again, the hump-shaped $F(.)$ locus implies that the rest of the analyses are similar to the baseline case. The results here are qualitatively similar to those with policies fixing the values of government bonds under the active monetary regime. Hence, our results are not dependent on fixing the values of government debt.

Active Fiscal Policy

Now we consider the case where the fiscal authority actively changes the lump-sum taxes and issues government bonds to support the tax policy. The monetary authority in this case passively determines the money growth rate necessary to balance the government budget. First, we consider the case where the quantity of long-maturity bonds are fixed, $B_t^l = \mathcal{A} \ \ \forall t$. Again, let us call this policy **L**. Proposition below summarizes the determinacy of the equilibrium for Policy $\mathcal{L}$ under an active fiscal policy regime.

Proposition 10. *In an economy in which fiscal policy is active and the government fixes tax collections, τ and fixes the quantity of long-maturity bonds (Policy $\mathcal{L}$), multiple steady states are possible.*

Proof. The stationary equilibrium in this case can be written compactly as

$$(1-\rho)q\Delta_0 + \frac{\phi\mathcal{A}(\theta_l - \theta_s)}{\Delta_2} = \frac{\Gamma}{\delta}$$

$$\frac{\rho}{\delta}q^u u'(q^u) - \frac{\phi\mathcal{A}u'(q^u)}{\Delta_2} + \frac{\frac{\phi\mathcal{A}\Delta_1}{\Delta_2} - (1-\delta)}{\Delta_0} = \tau + \frac{\phi\mathcal{A}}{\Delta_2} + \delta$$

where $\Delta_0 = (\beta u'(q)(1-\theta_s) + \theta_s)$, $\Delta_1 = (\beta u'(q)(1-\theta_l) + \theta_l)$, $\Delta_2 = u'(q^u) - \Big(\beta u'(q^u)(1-\theta_l) + \theta_l\Big)$ and $\Gamma = \rho(1-\delta)q^u \ u'(q^u)(1-\theta_s)$. Note that $\Delta_0, \Delta_1, \Delta_2$ are all positive with negative derivative. We differentiate these two equations with respect to q and q^u, yielding

$$(1-\rho)\delta(\Delta_0 + q\Delta_0')dq = \left[\frac{\phi\mathcal{A}(\theta_l - \theta_s)\Delta_2'}{\Delta_2^2} + \Gamma'\right] dq^u$$

which implies

$$\frac{dq}{dq^u} = \frac{\frac{\phi\mathcal{A}(\theta_l-\theta_s)\Delta_2'}{\Delta_2^2} + \Gamma'}{(1-\rho)\delta(\Delta_0 + q\Delta_0')}$$

If the coefficient of relative risk aversion is inside the unit interval, the sign of the denominator and Γ' are positive. The derivative is positive if $\Gamma' > \frac{\phi\mathcal{A}(\theta_l-\theta_s)\Delta_2'}{\Delta_2^2}$. When the coefficient of relative risk aversion is outside the unit circle (with higher valus of q and q^u under increasing relative risk aversion), the sign of the denominator and Γ' become negative. Hence, the curve is hump shaped.

Note that

$$\left[\frac{\rho}{\delta}K_0 - \frac{\phi\mathcal{A}(u''(q^u)\Delta_2 - u'(q^u)\Delta_2')}{\Delta_2^2} - \frac{\phi\mathcal{A}\Delta_1\Delta_2'}{\Delta_0\Delta_2^2} + \frac{\phi\mathcal{A}\Delta_2'}{\Delta_2^2}\right] dq^u + \left[\frac{\frac{\phi\mathcal{A}\Delta_1'\Delta_0}{\Delta_2} - \Delta_0'\Big(\frac{\phi\mathcal{A}\Delta_1}{\Delta_2} - (1-\delta)\Big)}{\Delta_0^2}\right] dq = 0$$

which implies,

$$\frac{dq}{dq^u} = -\frac{\frac{\rho}{\delta}K_0 - \frac{\phi\mathcal{A}(u''(q^u)\Delta_2 - u'(q^u)\Delta_2')}{\Delta_2^2} - \frac{\phi\mathcal{A}\Delta_1\Delta_2'}{\Delta_0\Delta_2^2} + \frac{\phi\mathcal{A}\Delta_2'}{\Delta_2^2}}{\frac{\frac{\phi\mathcal{A}\Delta_1'\Delta_0}{\Delta_2} - \Delta_0'\Big(\frac{\phi\mathcal{A}\Delta_1}{\Delta_2} - (1-\delta)\Big)}{\Delta_0^2}}$$

Note that the last two terms of the numerator combined are positive. The first two terms are of the same sign and change from positive to negative as q^u increases. So as before, if K_0 is big enough, the numerator changes sign from positive to negative. The denominator is positive if $\Delta_1'\Delta_0 > \Delta_0'\Delta_1$. This is quantitatively the same assumption that we made in the baseline case. Hence, the curve is U-shaped. Similar to the baseline case, the hump-shaped $F(q^u)$ and U-shaped $G(q^u)$ implies that multiple equilibria cannot be ruled out. □

Again, the hump-shaped $F(q^u)$ and U-shaped $G(q^u)$ implies that the rest of the analyses for fiscal Policy $\mathcal{L}$ are similar to those of the baseline case.

Now, we consider the case where the quantity of short-maturity bonds are fixed, $B_t^s = \mathcal{B} \;\; \forall t$. Again, let us call this Policy $\mathcal{S}$. The Proposition below summarizes the determinacy of the equilibrium for Policy $\mathcal{S}$ under active fiscal policy regime.

Proposition 11. *In an economy in which fiscal policy is active and the government fixes tax collections, τ and fixes the quantity of short-maturity bonds (Policy* ***S****), multiple steady states are possible.*

Proof. We proceed by writing the equilibrium in compact form as follows:

$$(1-\rho)q\Delta_0 + \frac{\left[\frac{\Gamma}{\delta(1-\theta_s)} - \frac{\phi B\Delta_0}{u'(q^u)}\right](\theta_l - \theta_s)}{\Delta_1} = \frac{\Gamma}{\delta} \tag{75}$$

and

$$\rho q^u u'(q^u) - \Big(1 + \frac{u'(q^u)}{\Delta_0} - \frac{u'(q^u)}{\Delta_1}\Big)\Big(\rho q^u u'(q^u)(1-\delta) - \delta\frac{\phi B\Delta_0}{u'(q^u)}\Big) - \frac{\delta(1-\delta)u'(q^u)}{\Delta_0} = \delta(\tau+\delta)$$

Derivatives of the first equation is given by

$$\left[(1-\rho)\delta(\Delta_0 + q\Delta_0') - \frac{\frac{\delta\phi B\Delta_0'(\theta_l-\theta_s)\Delta_1}{u'(q^u)} + \left[\frac{\Gamma}{(1-\theta_s)} - \frac{\delta\phi B\Delta_0}{u'(q^u)}\right](\theta_l-\theta_s)\Delta_1'}{\Delta_1^2}\right] dq = \left[\Gamma' - \frac{(\theta_l-\theta_s)}{\Delta_1}\Big(\frac{\Gamma'}{1-\theta_s} + \frac{\delta\phi B\Delta_0 u''(q^u)}{u'(q^u)^2}\Big)\right] dq^u$$

Similar to the baseline case, the sign of the derivative is strictly positive if $\left|\frac{[\frac{\Gamma}{(1-\theta_s)} - \frac{\delta\phi B\Delta_0}{u'(q^u)}](\theta_l-\theta_s)\Delta_1'}{\Delta_1^2}\right| > |(1-\rho)\delta(\Delta_0 + q\Delta_0')|$. Otherwise, the curve is hump-shaped. Derivative of the second term is

$$\frac{dq}{dq^u} = -\frac{\rho K_0 - \Big(1 + \frac{u'(q^u)}{\Delta_0} - \frac{u'(q^u)}{\Delta_1}\Big)\Big(\rho K_0(1-\delta) - \frac{\delta\phi B\Delta_0 u''(q^u)}{u'(q^u)^2}\Big) - u''(q^u)\left[\Big(\rho q^u u'(q^u)(1-\delta) - \delta\frac{\phi B\Delta_0}{u'(q^u)}\Big)\Big(\frac{1}{\Delta_0} - \frac{1}{\Delta_1}\Big) - \frac{\delta(1-\delta)}{\Delta_0}\right]}{-u'(q^u)\Big(\rho q^u u'(q^u)(1-\delta) - \delta\frac{\phi B\Delta_0}{-u'(q^u)}\Big)\Big(\frac{\Delta_1'}{\Delta_1^2} - \frac{\Delta_0'}{\Delta_0^2}\Big) + \Big(1 + \frac{u'(q^u)}{\Delta_0} - \frac{u'(q^u)}{\Delta_1}\Big)\delta\frac{\phi B\Delta_0'}{u'(q^u)}}$$

The denominator is negative if (C1): $\rho q^u u'(q^u)(1-\delta) > \delta\frac{\phi B\Delta_0}{u'(q^u)}$ and (C2): $\frac{\Delta_1'}{\Delta_1^2} > \frac{\Delta_0'}{\Delta_0^2}$. We can confirm that condition (C2) is from our assumptions in policy $\mathcal{L}$. The last expression in the numerator is positive. The first two expressions can be written as $\rho K_0\Bigg(\delta - \Big(\frac{u'(q^u)}{\Delta_0} - \frac{u'(q^u)}{\Delta_1}\Big)\Bigg) + \Big(1 + \frac{u'(q^u)}{\Delta_0} - \frac{u'(q^u)}{\Delta_1}\Big)\frac{\delta\phi B\Delta_0 u''(q^u)}{u'(q^u)^2}$. The second expression is negative. Moreover, the expression in the parenthesis of the first expression is positive. So the shape of the curve again depends on the sign of K_0. K_0 changes sign from positive to negative as q^u increases. So if K_0 is big enough, the curve is U shaped. □

The shape of $F(q^u)$ and $G(q^u)$ in this case again is similar to the baseline case and hence, analyses are similar. This section shows that the findings in the baseline case is robust to alternative specification of fixing the quantity of bonds.