Speculative and Precautionary Demand for Liquidity in Competitive Banking Markets*

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Abstract

We demonstrate that the co-existence of different motives for liquidity preferences profoundly affects the efficiency of financial intermediation. Liquidity preferences arise because consumers wish to take precautions against sudden and unforeseen expenditure needs, and because investors want to speculate on future investment opportunities. Without further frictions, the co-existence of these motives enables banks to gain efficiencies from combining liquidity insurance and credit intermediation. With standard financial frictions, banks cannot reap such economies of scope. Indeed, the co-existence of a precautionary and a speculative motive can cause efficiency losses which would not occur if there were only a single motive. Specifically, if the arrival of profitable future investment opportunities is sufficiently likely, such co-existence implies inefficient separation, pooling, or even non-existence of pure strategy equilibria. This suggests that policy implications derived solely from a single motive for liquidity demand can be futile.

Keywords: expenditure needs · investment opportunities · liquidity insurance · penalty rates · competitive bank business models

JEL Classification: D11, D86, E21, E22, G21, L22

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1 Introduction

Based on the premise that preferences for holding liquid assets exist for various reasons, this paper addresses a fundamental question: How will the *diversity of liquidity preferences* affect the provision of liquidity in competitive banking markets?

We consider liquidity preferences that arise because consumers wish to take precautions against sudden and unforeseen consumption *needs*, and because investors want to speculate on future investment *opportunities* that require quick actions to realize their profits. Our primary interest lies in the implications of the *co-existence* of these two motives, which we study in a unified two-period model of a competitive banking sector with consumers and investors.[1] As liquidity preferences arising from consumption needs have been already widely studied – albeit only in isolation – we introduce enterprising investment opportunities to investors: they arise spontaneously at uncertain dates and only to a (small) fraction of investors; they are scalable; they are more profitable than existing long-term investments; and they are not contractible as their returns cannot be fully pledged to creditors. In the model, the initial endowments of consumers and investors can be stored or invested in a long-term project delivering safe returns in the second period. After the first period, some consumers learn that they need to consume immediately, and some investors learn about a short-term investment opportunity.

Banks provide for the consumers' and investors' desire to hold liquid assets. In absence of frictions, pooling the endowments of consumers and investors allows banks to realize efficiency gains that were unattainable if the two motives for liquidity preferences would not co-exist. These efficiency gains are the result of economies of scope that arise when liquidity insurance and credit intermediation are combined. Specifically, banks hold solely storage in the first period, until the short-term investment opportunities arrive and uncertainty about consumption needs is resolved. Using their stored reserves, banks provide consumers with the means to meet their early consumption needs and grant the remainder as loans to investors who acquire profitable investment

[1] While for expositional purposes it is convenient to relate the motives to specific agents, it should be clear that in general both types of motives can affect both, consumers and investors alike. It is the motives that matter.

opportunities. After the second period, banks collect these loans and redistribute their earnings to patient consumers and all investors. Accordingly, banks do not invest in the illiquid long-term project and hence do not engage in maturity transformation at all. In the first period, banks issue demand deposits to consumers and equity shares to investors, both backed entirely by stored goods. In the second period, the banks' assets comprise only single-period loans to investors, their liabilities are the revolving demand deposits held by patient consumers.

The efficiency gains arising from the co-existence of consumption needs and investment opportunities, however, are not feasible in the presence of widely researched frictions. Indeed, the co-existence of the two motives can then cause efficiency losses. The first friction is that the realization of individual liquidity events is private information. This is a standard friction and, on its own, does not impair a socially efficient market outcome here as the respective incentive constraints are not binding. The second friction is that the returns of enterprising investment opportunities are not contractible. This credit friction is standard too but precludes the socially efficient allocation already if considered on its own. However, the market outcome is still constrained-efficient and, importantly, independent from the co-existence of the two motives for liquidity preferences. The third and final friction is that the individual motive for liquidity preference is private information. This friction gives rise to a self-selection problem, which is well-known for markets plagued with asymmetric information. On its own, this friction has no implications because, if it was not for the other two frictions, the identity of consumers and investors could still be elicited indirectly.

With these frictions combined, however, the co-existence of the two liquidity motives does not allow for the realization of economies of scope anymore but can actually cause efficiency losses. We consider only equilibria in pure strategies. Equilibrium outcomes depend on the share of investors acquiring a profitable future investment opportunity, a key determinant of investors' liquidity preference. Abusing terminology slightly, we henceforth refer to this share as *propensity to speculate*. Provided the propensity is small, a competitive banking industry emerges where two different bank business models exist side by side, each offering exactly one contract aiming either at consumers or at investors. The optimal contract for investors is a *term deposit* contract which entails a penalty rate for early withdrawal and a long-run interest rate above the rate of return on the long-run project, while consumers are offered a *demand deposit* contract providing the standard liquidity insurance. Such an equilibrium outcome is constrained-efficient, i.e. Pareto-optimal given the credit friction. Demand deposits are more liquid than term deposits in that the discount one has to accept for accessing their bank deposits before the final date is lower for the demand deposit than for the term deposit.

If the propensity to speculate is large, however, the option value associated with investment opportunities gains in importance. In this case, optimal contracts for addressing the different motives interfere with each other and, depending on parameters, different types of equilibrium outcomes are possible.[2] Separating equilibria can emerge where liquidity provision is not even constrained-efficient anymore. Alternatively, pooling equilibria might occur, where all banks offer identical contracts to investors and consumers. Finally, pure-strategy equilibria may not exist altogether if, for example, one of the two motives is sufficiently over-represented.[3]

The precautionary motive has been widely studied and interpreted as the reason for banks to provide liquidity insurance. Seminal papers include Bryant (1980) and Diamond and Dybvig (1983) for consumers, and Holmström and Tirole (1998) for producers. The speculative motive, however, has been largely overlooked.[4] Our basic framework is borrowed from Diamond and Dybvig (1983) where the precautionary motive for liquidity demand arises because consumers face random early consumption needs. We introduce a private surprise option for investors to invest in a highly profitable project only after funds had to be already committed to illiquid long-term projects. This investment opportunity gives rise to the speculative motive for liquidity demand.

Our investment option differs from the perspective in Holmström and Tirole (1998). While those authors also stress the liquidity implications of limitations to pledge future returns, our framework emphasizes the sudden occurrence of new investment opportunities as an alternative

[2] In a trading context, Gehrig and Jackson (1998) also find that the micro-structure of liquidity motives decisively affects the strategic properties of securities demand and, hence, market outcomes.

[3] Equilibria with randomization across contracts may still exist.

[4] In Diamond and Rajan (2001), the motive for liquidity demand, and thus any potential interaction between motives, is inconsequential given the assumed universal risk-neutrality.

motive for early fund withdrawals. In Holmström and Tirole (1998), credit frictions are not absolute, though.[5] Therefore, banks can offer lines of credit to firms. In their model firms pay for credit lines, which is similar to the penalty rate that long-term investors pay in our optimal term deposit contract. Unlike credit lines, however, such a term deposit contract also provides for higher long-term returns.[6]

Credit frictions have also been identified to generate a demand for liquid, marketable financial assets, such as volatile bubbles (Martin and Ventura, 2012) and fiat money (Dietrich et al., 2020). In our model, credit frictions generate a maturity transformation banks typically engage in. A range of reasons has been identified for the credit friction utilized in the present paper. For example, only the investor may have the specific skills needed to successfully manage and complete the project (Hart and Moore, 1994), consumption may not be observable (Wallace, 1988), or penalties like future exclusion from financial markets may be ineffective for enforcing loans (Kehoe and Levine, 1993).

The potentially moderating role of more productive reinvestment technologies for the amount of incentive compatible liquidity insurance has been analysed by von Thadden (1997, 1998) in dynamic versions of the Diamond and Dybvig (1983) model. In these models, and unlike the present one, alternative investment opportunities are known throughout to market participants and the only shocks occur to private consumption needs. In our paper, the preference for liquidity stems from a desire to speculate on the arrival of private, enterprising investment opportunities with non-contractible returns.

Our analysis has a number of implications with some bearing on economic policy and financial regulation. Firstly, financial frictions imply that banks engage in more maturity transformation; are more leveraged; and may not facilitate even a constrained-efficient allocation. Secondly, demand deposits as well as term deposits are potentially subject to the coordination problem identified by Bryant (1980) and Diamond and Dybvig (1983). However, term deposits are less prone to co-

[5]In Donaldson et al. (2018), warehouses serve as financial intermediaries that provide liquidity services to producers by overcoming credit frictions more effectively than direct lenders.

[6]This is in line with empirical evidence which suggests only imperfect substitutability for corporations between bank deposits and lines of credit (e.g. Acharya et al., 2007; Campello et al., 2011; Acharya et al., 2013).

ordination failures than demand deposits. Accordingly, banks providing speculative liquidity to investors are less fragile than banks providing liquidity insurance to consumers. Moreover, the co-existence of both bank business models can increase the fragility of banks providing liquidity insurance. Thirdly, while in equilibrium both bank business models earn zero profits, the speculative model requires lower levels of reserves and earns higher rates of return. Finally, the scope for liquidity insurance is fading if returns on long-term projects decline. This can imply that pure-strategy equilibria do not exist and equilibrium outcomes are thus effectively indeterminate. If such a decline of returns on long-term projects can be related to an economy-wide (exogenous) fall in the level of interest rates, hitherto unnoticed – and unintended – consequences emerge from a zero-interest rate environment.

To conclude, liquidity preferences, and the role of banks in the provision of liquidity, have been central themes since Keynes (1930, 1936). Keynes' speculative motive is related to the liquidity preference of investors in our model, while his precautionary motive captures the liquidity preference of consumers.[7] In terms of liquidity provision, Keynes (1930, Chapter 2) explicitly refers to bank deposits but not to fiat money, which quantitatively comprises only a small portion of the overall provision of liquidity. Interestingly, in our model there are parametrizations in which pure-strategy equilibria do not exist. This result is akin to the instability of aggregate liquidity demand postulated by Keynes (1936).

The paper is organized as follows: Section 2 presents the model. It introduces the concept of speculative liquidity demand and compares it to the standard precautionary liquidity motive. Section 3 contains the key insights for the frictionless economy. Section 4 provides the analysis of equilibrium outcomes in the presence of frictions. Section 5 contains a short discussion of implications for bank business models and regulation, while Section 6 concludes. All technical proofs are relegated to the Appendix.

[7] For Keynes (1936), the speculative motive arises from a 'variety of opinions' about the future path of the interest rate, combined with an in-elasticity of interest rate expectations. Later, Tobin (1958) relates the speculative motive for liquidity preference, or 'investment balances' in his words, to the management of portfolio risks.

2 Liquidity preferences

2.1 Setup

Consider an economy which unfolds over three dates $t \in \{0,1,2\}$, and is populated by three types of agents: investors, consumers, and banks. There is one good at every date. The good can be consumed or used for production in three different technologies.

Technologies The technologies are: storage, long-term production and short-term production. Storage is one-for-one and can be used at dates $t \in \{0,1\}$. Long-term production has to be initiated at $t=0$, and takes two periods until $t=2$ to produce the good. Per-unit-returns are $R > 1$. Unless indicated otherwise, long-term production cannot be prematurely liquidated at date $t=1$. Henceforth, we refer to long-term production also as R-technology. Short-term production opportunities arise at date $t=1$, to produce $Q > R$ per unit of investment after one period at date $t = 2$. Accordingly, short-term production is called Q-technology.

Investors There are many groups of investors. Each group comprises a continuum of investors of mass one. Every investor is endowed with one unit of the good at $t=0$ and nothing thereafter. All investors value consumption $c \geq 0$ only at date $t=2$. They have access to the R-technology at date $t=0$, and to storage at dates $t=0$ and $t=1$. At date $t=1$, a share $\mu \in]0,1[$ of all investors in a group gain access to the Q-technology, while the other investors in that group do not get access to the Q-technology. The share of investors with access is deterministic, identical for all groups of investors, and common knowledge. Assuming that all investors have an equal chance, the probability for an individual investor of getting access to the Q-technology is μ and the probability of not getting access is $1-\mu$. Investors have identical risk preferences. Their Bernoulli utility function u is twice continuously differentiable, with $u'(c) > 0$, $u''(c) < 0$, $\lim_{c\to 0} u'(c) = \infty$, and $\lim_{c\to\infty} u'(c) = 0$.

Consumers There are many groups of consumers. Each group comprises a continuum of consumers of mass one. Every consumer is endowed with one unit of the good at date $t=0$ and nothing thereafter. All consumers have access to the R-technology at date $t=0$, and to storage at dates $t=0$ and $t=1$. At date $t=1$, a share $\lambda\in]0,1[$ of all consumers in a group become impatient and value consumption $c_1\geq 0$ at date $t=1$, while the other consumers are patient and value consumption $c_2\geq 0$ at date $t=2$. The share of impatient consumers is deterministic, identical for all groups of consumers, and common knowledge. Assuming that all consumers are equally likely to become impatient, the probability for an individual consumer of being impatient is λ and the probability of being patient is $1-\lambda$. A consumer values consumption with $u(c_1)$ if impatient, and with $u(c_2)$ if patient. Consumers' Bernoulli utility function u is identical to the investors' Bernoulli utility function.

Banks There is a continuum of banks. A bank has no own endowment. Banks have access to storage and to the R-technology, but not to the Q-technology. Banks are perfectly competitive and maximize expected profits. At date $t=0$, consumers and investors can exchange their own endowments for contracts offered by banks. A *contract* $\mathscr{D}=(r_1,r_2)$ is a sequence of payments $\{r_t\}_{t\in\{1,2\}}$ a bank makes to customers at $t=1$ and $t=2$, respectively. A *business model* $\mathscr{M}=(r_1,r_2,y)$ consists of a contract $\mathscr{D}$ and a portfolio share held in storage $y\in[0,1]$. A business model is *sustainable* if it is designed to earn non-negative profits.

Contractual environment Financial contracts are plagued with three types of frictions. Firstly, at date $t=0$, the ex-ante motive for the liquidity preference is private information. Accordingly, consumers and investors are free to choose between all contracts banks offer. Secondly, at date $t=1$, the realized consumption need is private information, i.e. only the individual consumer learns whether they are impatient and need to consume immediately, or patient and can wait until date $t=2$. Similarly, access to the Q-technology is private information as only the individual investor learns at date $t=1$ whether they are lucky and can invest in the profitable new opportunity or not. Therefore, contracts cannot be made contingent on the ex-post realization of liquidity needs of consumers or investors. Thirdly, while storage and the R-technology are available to investors, consumers, and banks alike, and are thus fully contractible, the Q-technology is specific to investors who are hence not able to credibly pledge the returns they realize with this superior technology at date $t = 2$.

Let γ and $1-\gamma$ be the shares of consumer and investor groups, respectively, in the total population. Then, an *economy* $\mathscr{E}$ is a description of investors, consumers, and technologies, i.e. $\mathscr{E} = (u, \gamma, \lambda, \mu, Q, R)$.

2.2 Speculative demand for liquidity: Investors' problem

To look at the investors' problem in isolation, suppose for now there are no consumers. Investors demand liquidity in case they are lucky and spot investment opportunities at date $t = 1$ which are better than those originally available at date $t = 0$. We begin by showing that if a bank can enforce loans to investors, the bank can solve the investors' liquidity problem such that the first-best allocation obtains. The first-best is the solution to the investors' problem taking into account only the feasibility constraints. Let c_R and c_Q denote the consumption by an investor with and without investment opportunity, respectively. Similarly, x_R, x_Q and y are resources per investor directed to the R-technology, the Q-technology, and to storage, respectively.

As all investor groups are identical, we can focus on a representative group. The optimization problem then reads

$$\max_{(c_R, c_Q, x_R, x_Q, y) \in \mathbb{R}^4_+ \times [0,1]} \quad \mu u(c_Q) + (1-\mu) u(c_R) \tag{1}$$

$$\text{s.t.} \begin{cases} x_R + y & \leq & 1 \\ \mu x_Q & \leq & y \\ \mu c_Q + (1-\mu) c_R & \leq & \mu Q x_Q + R x_R + y - \mu x_Q \end{cases}$$

The first constraint is the resource constraint at date $t = 0$; the second constraint states that investment in the Q-technology at date $t = 1$ cannot be larger than what has been stored at date

$t = 0$; the third constraint states that total consumption at date $t = 2$ is limited by the amount of goods produced by either technology and what is left of the storage at date $t = 1$. By standard arguments, all constraints hold with equality, i.e. the problem becomes

$$\begin{aligned} &\max_{(c_R, c_Q, y) \in \mathbb{R}_+^2 \times [0,1]} \quad \mu u(c_Q) + (1-\mu) u(c_R) \\ &\text{s.t.} \quad c_R = \tfrac{Qy + R(1-y) - \mu c_Q}{1-\mu} \end{aligned} \tag{2}$$

A solution for this problem satisfies the first-order condition

$$\mu u'(c_Q) - \mu u' \left(\frac{Qy + R(1-y) - \mu c_Q}{1-\mu} \right) = 0, \tag{3}$$

which implies $c_Q = c_R = Qy + R(1-y)$, i.e. *full insurance*. The optimization problem thus simplifies further to

$$\max_{y \in [0,1]} \quad Qy + R(1-y). \tag{4}$$

Since $Q > R$, the solution to program (1) is $y = 1$ and $c_R = c_Q = Q$.

Accordingly, the first-best allocation is a corner solution. The intuition is straightforward. Investors care only about late consumption and hence they are interested only in maximizing the amount of the good available at date $t = 2$. As all technologies are constant returns-to-scale, this is achieved if all resources end up being invested into the Q-technology, i.e. there is no investment in the R-technology at date $t = 0$.

As investors gain access to the Q-technology only with some probability μ, the first-best does not obtain in autarky. However, banks are able to implement the first-best, provided that at date $t = 2$ the bank can collect principal and interest of any loans granted to investors at date $t = 1$. Suppose a bank operates a business model $\mathscr{M} = (0, Q, 1)$. That is, the bank accepts endowments from investors in exchange for promises to pay $r_1 = 0$ and $r_2 = Q$. A possible contract that delivers this sequence of payments are shares in the bank's equity. Moreover, the bank stores all endowments from date $t = 0$ to date $t = 1$. At this date, the bank lends out the stored goods at a lending rate of Q

to investors who wish to borrow. Provided these investors invest the loans into the Q-technology, the loan earnings collected at date $t=2$ are used to pay the initially promised amount of Q to every investor at date $t=2$. Investors with access to the Q-technology are thus indifferent between borrowing and not borrowing at date $t=1$, while investors without access are strictly better off by not borrowing. Hence, investors with access to the Q-technology, and only those, actually borrow from the bank at date $t=1$.

If a bank cannot enforce loan repayments from investors, however, they cannot be made to share the return on their investment in the Q-technology once they have gained access to it. We show next how the associated constrained-efficient solution to the investors' liquidity problem looks like and how a bank can implement it. Without loan enforcement, the problem of investors reads

$$\max_{(c_R,c_Q,x_R,x_Q,y)\in\mathbb{R}^4_+\times[0,1]} \quad \mu u(c_Q)+(1-\mu)u(c_R)$$
$$\text{s.t.} \begin{cases} x_R+y &= 1 \\ \mu x_Q &= y \\ (1-\mu)c_R &= Rx_R \\ c_Q &= Qx_Q \end{cases} \tag{PI}$$

Stating all constraints directly with equality is innocent but simplifies the exposition.[8] The first and second constraints are as before. The third constraint states that consumption by investors without access to the Q-technology is equal to what is generated with the R-technology. The fourth constraint states that consumption by investors with access to the Q-technology is equal to what is generated with this technology. A solution to this problem satisfies these constraints and the first-order condition

$$u'\left(\tfrac{R(1-y)}{1-\mu}\right)-\tfrac{Q}{R}u'\left(\tfrac{Qy}{\mu}\right)=0. \tag{5}$$

[8]In short, equality follows from non-satiation together with $Q>R>1$. The latter implies that it is neither efficient to keep any storage between date $t=1$ and $t=2$ nor to use the R-technology for the consumption by investors with access to the Q-technology.

Let y^d denote the solution to condition (5). Then, banks can implement the solution to the investors' problem (PI) with a business model $\mathscr{M} = (r_1^d, r_2^d, y^d)$, provided $r_1^d = x_Q = y^d/\mu$ and $r_2^d = Rx_R/(1-\mu) = R(1-y^d)/(1-\mu)$.

The contract (r_1^d, r_2^d) features certain characteristics worthy further elaboration. Let c_Q^d and c_R^d be the consumption by investors with and without access to the Q-technology, respectively, associated with the business model (r_1^d, r_2^d, y^d). Then, condition (5) implies $c_Q^d > c_R^d$. For constant relative risk aversion equal to one, i.e. $-cu''(c)/u'(c) = 1$, we obtain $xu'(x) = u'(1)$. Hence, $Ru'(R) = Qu'(Q)$, and the first-order condition (5) requires $c_Q^d = Q$ and $c_R^d = R$. Accordingly, the bank's business model satisfies $r_1^d = 1$, $r_2^d = R$ and $y^d = \mu$, i.e. investors are allowed to withdraw at date $t = 1$ exactly what they have deposited in the bank at date $t = 0$. For relative risk aversion greater one, i.e. $-cu''(c)/u'(c) > 1$, we obtain $Ru'(R) > Qu'(Q)$. Therefore, condition (5) requires $R < c_R^d < c_Q^d < Q$. Accordingly, the bank's business model satisfies $r_1^d < 1$, $r_2^d = R(1-\mu r_1^d)/(1-\mu) > R$, and $y^d < \mu$. That $r_1^d < 1$ implies that the contract entails a penalty for early withdrawals.

The contract (r_1^d, r_2^d) is also incentive compatible, i.e. investors of neither type have an incentive to misrepresent themselves, if relative risk aversion is greater than one. Consider investors without access to the Q-technology. Since $r_1^d < r_2^d$, they are better off withdrawing at date $t = 2$. Next consider investors with access to the Q-technology. If they withdraw at date $= 1$, they consume Qr_1^d, while their consumption is r_2^d if they withdraw at date $t = 2$. Since $Q/R > 1$, the first-order condition (5) implies $Qr_1^d > r_2^d$, such that these investors are better off withdrawing at date $t = 1$.

Figure 1 illustrates the constrained-efficient solution to the investors' problem. The contract that serves best the speculative demand for liquidity is characterized by a pair (r_1^d, r_2^d) for which the investors' indifference curve is tangent to the banks' intertemporal budget line $r_2 = R(1-\mu r_1)/(1-\mu)$, provided the banks' business model is targeted solely at investors. For relative risk aversion greater than one, this contract lies to the north-west of $(1, R)$. Lemma 1 summarizes these results.

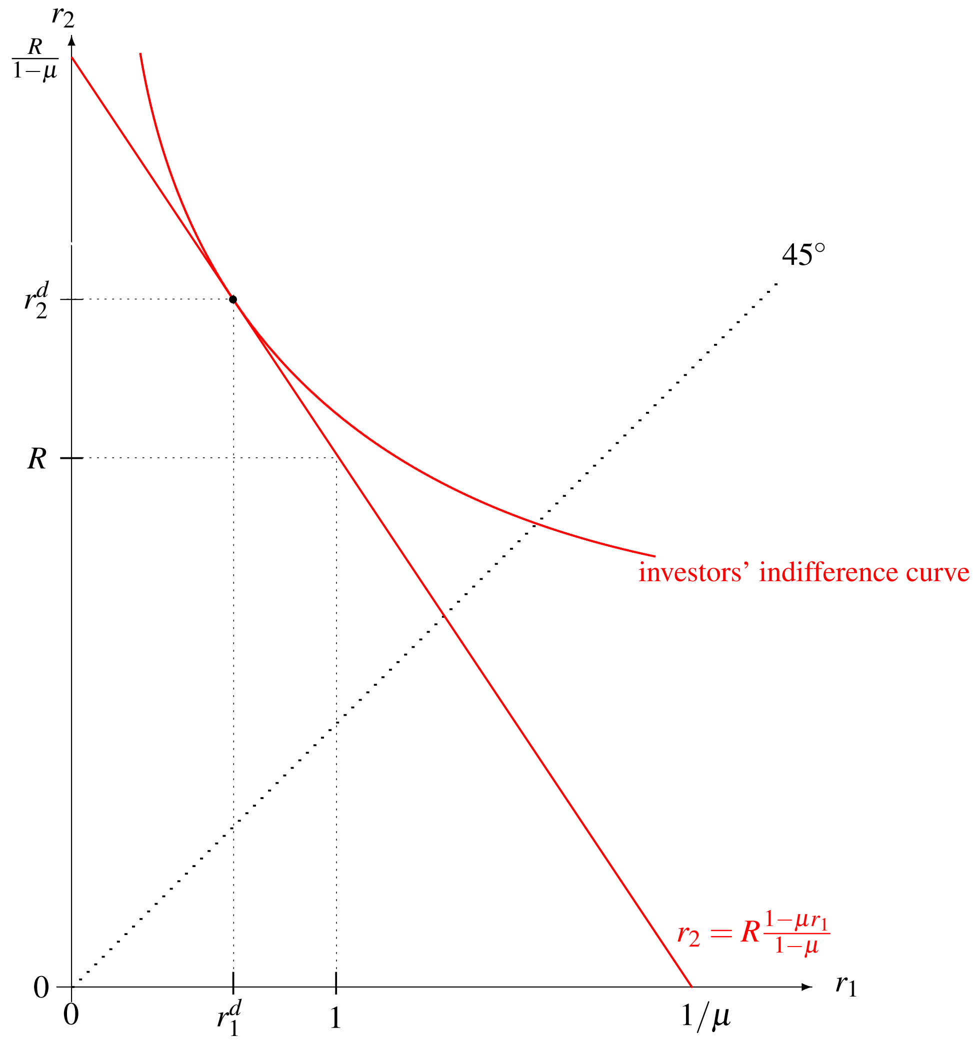

Figure 1: Constrained-efficient Investor Contract ($-cu''(c)/u'(c) > 1$).

Lemma 1 (Term Deposit Contract) *Suppose $-cu''(c)/u'(c) > 1$. Provided the returns of enterprising investment opportunities are not contractible, the optimal contract is a term deposit contract $\mathscr{D} = (r_1^d, r_2^d)$ with a penalty $1 - r_1^d > 0$ for early withdrawals.*

In conclusion, if loan enforcement is perfect, then all endowments will eventually end up in the projects with the highest productivity anyway. Accordingly, in the first-best, the probability of acquiring a short-term investment opportunity is irrelevant for the allocation. Without loan enforcement, in contrast, not all resources can be directed into the Q-technology, and the allocation very much depends on the probability of short-term investment opportunities, μ, henceforth also referred to as *propensity to speculate*.

2.3 Precautionary demand for liquidity: Consumers' problem

To look at the consumers' problem in isolation, suppose for now there are no investors. Consumers demand liquidity in case they need to consume at date $t=1$ rather than at date $t=2$. This is the standard liquidity insurance problem considered in Diamond and Dybvig (1983). Therefore, we keep it concise in showing that a bank can solve the consumers' problem such that the first-best allocation obtains. Enforceability poses no additional constraint on the consumers' problem.

Similar to the investors' problem, the first-best is the solution to the consumers' problem taking into account only the feasibility constraints. Let c_1 and c_2 denote the consumption by a consumer if impatient and patient, respectively. As all consumer groups are identical, we can focus on a representative group and the problem reads

$$\max_{(c_1,c_2,x_R,y)\in\mathbb{R}^3_+\times[0,1]} \quad \lambda u(c_1)+(1-\lambda)u(c_2)$$
$$\text{s.t.}\begin{cases} x_R+y & \leq & 1 \\ \lambda c_1 & \leq & y \\ \lambda c_1+(1-\lambda)c_2 & \leq & Rx_R+y \end{cases} \tag{PC}$$

The constraints are the feasibility constraint at dates $t=0$, $t=1$, and $t=2$, respectively. The first line requires that the investment in the R-technology and the amount held in storage cannot exceed the endowment of consumers. According to the second line, total consumption by impatient consumers at date $t=1$ cannot be larger than the stored goods available at that date. The third line states that total consumption is limited by the total availability of stored and produced goods. As standard, all constraints hold with equality. Therefore, the solution to the consumers' problem

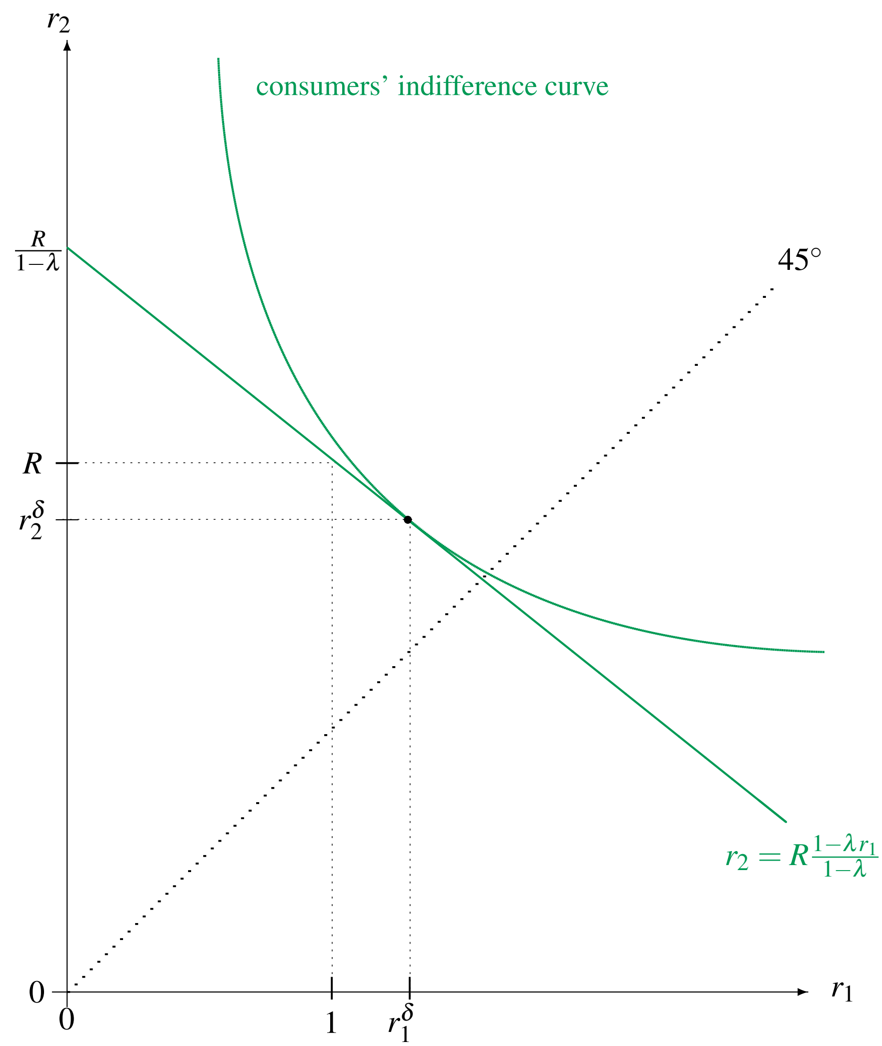

Figure 2: Efficient Consumer Contract ($-cu''(c)/u'(c) > 1$).

satisfies $c_1^\delta = y^\delta/\lambda$ and $c_2^\delta = R(1-y^\delta)/(1-\lambda))$ where y^δ solves the first-order condition

$$u'\left(\frac{y^\delta}{\lambda}\right) - Ru'\left(\frac{R(1-y^\delta)}{1-\lambda}\right) = 0. \tag{6}$$

As consumers need to consume early only with some probability λ, the first-best cannot be achieved in autarky. However, banks are able to implement the first-best by choosing the business model $\mathscr{M} = (r_1^\delta, r_2^\delta, y^\delta)$ with $r_1^\delta = y^\delta/\lambda$ and $r_2^\delta = R(1-y^\delta)/(1-\lambda)$. For relative risk aversion equal to one, the payment a bank offers to those who withdraw early is $r_1^\delta = 1$. Then, from the feasibility constraint for $t=2$, we obtain $r_2^\delta = R$. For relative risk aversion greater one, the bank pays an insurance benefit at date $t=1$, i.e. more than the consumer has deposited with the bank in the first place. Specifically, the bank pays $r_1^\delta = y^\delta/\lambda > 1$ and $r_2^\delta = R(1-\lambda r_1^\delta)/(1-\lambda) \in]r_1^\delta, R[$. With this demand deposit contract, consumers do not have incentives to misrepresent themselves. This is because impatient consumers have no choice but to withdraw at date $t=1$, while patient consumers are strictly better off by waiting until date $t=2$ since $r_2^\delta > r_1^\delta$.[9] Accordingly, precautionary liquidity demand receives a subsidized rate for early withdrawal, rather than a penalty as it is the case with speculative liquidity demand.

Figure 2 illustrates the solution to the consumers' problem. The contract that serves best the precautionary demand for liquidity is characterized by a pair (r_1^δ, r_2^δ) for which the consumers' indifference curve is tangent to the banks' intertemporal budget constraint $r_2 = R(1-\lambda r_1)/(1-\lambda)$, provided the banks' business model is targeted solely at consumers. For relative risk aversion greater than one, this contract point lies to the north-west of the 45° line, where consumers would get full insurance, and to the south-east of $(1, R)$.

3 Pareto-optimal liquidity provision

Investors and consumers exert a demand for liquidity, but for different reasons. Investors benefit from access to liquidity in case they find better investment opportunities at date $t=1$. Accordingly, investors are driven by a speculative motive to demand liquidity. Consumers benefit from access to liquidity in case they need to consume at date $t=1$. The consumers' liquidity demand thus arises from a precautionary motive. In this section we consider the implications of the co-existence of a speculative and a precautionary motive for the equilibrium outcome when there are no further frictions.

[9] A bank-run equilibrium is ruled out here as production cannot be liquidated at date $t=1$.

3.1 Characterization of a Pareto-optimum

Consider a social planner's allocation in absence of frictions, satisfying only the economy-wide feasibility constraints. Storage, the long-term R-technology, and the short-term Q-technology are all constant returns to scale. Hence, the Q technology dominates the R-technology in terms of producing consumption goods available at date $t=2$. On the other hand, storage dominates in terms of providing both, early consumption goods and funds for investment in the Q-technology. Accordingly, a planner would store all endowments from investors and consumers between dates $t=0$ and $t=1$, and then use this storage to fund the Q-technology and the consumption by impatient consumers. The returns on the Q-technology will then fund the consumption by patient consumers and by investors.

While the planner's allocation of funds across storage, long-term and short-term production is determinate, there are many Pareto-optimal allocations of consumption. We focus on the allocation which provide consumers with the same consumption profile as if investors would not exist. At date $t=0$, a planner collects the investors' and consumers' endowments and stores them until date $t=1$. Once the future investment opportunities arrive and uncertainty about consumption needs is resolved, this storage is partly used to provide for impatient consumers, $c_1^{\delta}=y^{\delta}/\lambda$, with y^{δ} satisfying the first-order condition (6). The remainder of the stored endowments, $1-\gamma y^{\delta}>1-\gamma$, is invested in the Q-technology.[10] At date $t=2$, the Q-technology will produce $Q(1-\gamma y^{\delta})$. A planner then distributes these returns to patient consumers and all investors. Each patient consumer gets $c_2^{\delta}=R(1-y^{\delta})/(1-\lambda)$, leaving for each investor an amount of $Q+\frac{\gamma}{1-\gamma}(Q-R)(1-y^{\delta}) > Q$. Therefore, consumers receive the consumption plan that corresponds to the first-best in case of isolation, and investors will be able to consume more than they could by providing for themselves. The reason is that by pooling the endowments of investors and consumers, the planner avoids the comparatively unproductive investment in the R-technology, in which consumers would have to invest if they were left to their own devices. Instead, all goods for consumption at date $t=2$ are produced with the comparatively more productive Q-technology.

[10]Recall, $\gamma\in]0,1[$ is the relative share of consumers in the economy.

3.2 Implementation of a Pareto-optimum

Provided banks know the individual motive for their customers' liquidity preference and can fully enforce loan repayments, a competitive banking sector can implement the optimal allocation. To see how, suppose consumers and investors deposit their endowments in banks at date $t=0$. Consumers do so in exchange for a demand deposit contract which allows them to withdraw $r_1^{\mathrm{C}}=c_1^{\delta}$ if they become impatient and $r_2^{\mathrm{C}}=c_2^{\delta}$ if they turn out to be patient. Investors receive shares in the bank's equity which allows them to share the value of the bank's assets net of payments to consumers at date $t=2$, i.e. $r_1^{\mathrm{I}}=0$ and $r_2^{\mathrm{I}}=Q+\frac{\gamma}{1-\gamma}(Q-R)(1-\lambda r_1^{\mathrm{C}})>Q$. At the middle date $t=1$, banks offer loans at a gross interest rate equal to Q. At this rate, investors with access to the Q-technology are just willing to borrow from banks all of their remaining storage $1-\gamma\lambda r_1^{\mathrm{C}}$. At the final date $t=2$, investors settle their loans and pay $Q(1-\gamma\lambda r_1^{\mathrm{C}})$ to banks. With these earnings, banks pay patient consumers r_2^{C} and investors $r_2^{\mathrm{I}}=Q+\frac{\gamma}{1-\gamma}(Q-R)(1-\lambda c_1^{\mathrm{C}})$. Accordingly, we conclude:

Lemma 2 (Economies of Scope) *The co-existence of a speculative and a precautionary demand for liquidity gives rise to efficiency gains from combining liquidity insurance and credit intermediation. In absence of frictions, banks can realize such economies of scope.*

Interestingly, under such ideal conditions banks would not have to engage in any maturity transformation at all to reap these economies of scope. At date $t=0$, banks issue demand deposits to consumers and equity shares to investors, both backed entirely by stored goods. From date $t=1$ onward, the banks' assets comprise the loans to investors and their liabilities are the demand deposits still held by patient consumers, with investors holding the residual claims on the banks' asset returns.

4 Frictions and equilibrium liquidity provision

In this section we consider economies where the motive for the liquidity preference and the individual liquidity event are private information and banks cannot enforce loan repayments. These frictions prevent the implementation of a Pareto-optimum. However, will the different motives simply co-exist in equilibrium with potentially different banks focusing on different clienteles? Or will the motives interact in equilibrium? And if they interact, how does their presence affect equilibrium outcomes? To address these questions, we first define our equilibrium concept.

4.1 Equilibrium concept

We consider competitive deposit markets and focus on pure-strategy equilibria, where each bank is limited to offering one deposit contract, and investors and consumers choose from all contracts offered by banks to maximize their expected utility but cannot randomize their choice. It is useful to begin with a definition of incentive compatible contracts.

Definition 1 (Incentive Compatible Contracts) *Let $\mathscr{D}^{\mathrm{I}}$ be the contract for investors, and $\mathscr{D}^{\mathrm{C}}$ the contract for consumers. An* incentive compatible *menu of contracts $\left\{\mathscr{D}^{\mathrm{I}}, \mathscr{D}^{\mathrm{C}}\right\}$ satisfies*

$$\mu u(Qr_1^{\mathrm{I}}) + (1-\mu)u(r_2^{\mathrm{I}}) \geq \mu u(Qr_1^{\mathrm{C}}) + (1-\mu)u(r_2^{\mathrm{C}}) \tag{7}$$

$$\lambda u(r_1^{\mathrm{C}}) + (1-\lambda)u(r_2^{\mathrm{C}}) \geq \lambda u(r_1^{\mathrm{I}}) + (1-\lambda)u(r_2^{\mathrm{I}}) \tag{8}$$

$$Qr_1^{\mathrm{I}} \geq r_2^{\mathrm{I}} \tag{9}$$

$$r_1^{\mathrm{I}} \leq r_2^{\mathrm{I}} \tag{10}$$

$$r_1^{\mathrm{C}} \leq r_2^{\mathrm{C}} \tag{11}$$

$$\mu u(Qr_1^{\mathrm{I}}) + (1-\mu)u(r_2^{\mathrm{I}}) \geq \sup\left\{\mu u\left(Qy + R(1-y)\right) + (1-\mu)u\left(R(1-y)+y\right) : y \in [0,1]\right\} \tag{12}$$

$$\lambda u(r_1^{\mathrm{C}}) + (1-\lambda)u(r_2^{\mathrm{C}}) \geq \sup\left\{\lambda u\left(y\right) + (1-\lambda)u\left(R(1-y)+y\right) : y \in [0,1]\right\} \tag{13}$$

Condition (7) requires that investors prefer the contract intended for investors over the contract intended for consumers, with strict inequality for $(r_1^{\mathrm{I}}, r_2^{\mathrm{I}}) \succ_{\mathrm{I}} (r_1^{\mathrm{C}}, r_2^{\mathrm{C}})$. Condition (8) requires that consumers prefer the contract intended for consumers, with strict inequality for $(r_1^{\mathrm{C}}, r_2^{\mathrm{C}}) \succ_{\mathrm{C}} (r_1^{\mathrm{I}}, r_2^{\mathrm{I}})$. These two incentive constraints need to be satisfied at date $t = 0$. For contracts to be incentive compatible, there are also incentive constraints to be observed at date $t = 1$ when consumers and investors have learnt about their status. Specifically, condition (9) requires that investors with access to the Q-technology must not be better off by pretending to have no access; condition (10) that investors without access to the Q-technology must not be better off by pretending to have access; and condition (11) that patient consumers must not be better off by pretending to be impatient. Finally, contracts must be such that depositing with banks makes investors and consumers better off than autarky. This holds provided the expected utility associated with their contract is at least as large as the expected utility a consumer and an investor achieve in autarky, respectively, i.e. if contracts satisfy the participation constraints (12) and (13).

We can now define a banking equilibrium.

Definition 2 (Banking Equilibrium) *A perfect-competition, pure-strategy* banking equilibrium *is an incentive compatible menu of contracts* $\{\mathscr{D}^{\mathrm{I}},\mathscr{D}^{\mathrm{C}}\}$ *such that the associated business models* $\{\mathscr{M}^{\mathrm{I}},\mathscr{M}^{\mathrm{C}}\}$ *are sustainable, while no bank can profitably enter the market with another contract* $\mathscr{D}' \notin \{\mathscr{D}^{\mathrm{I}},\mathscr{D}^{\mathrm{C}}\}$.

Banks' business models are sustainable if no operating bank makes a loss and would be strictly better off leaving the market. A business model $\mathscr{M}=(r_1^{\mathrm{I}},r_2^{\mathrm{I}},y^{\mathrm{I}})$ of offering contracts only to investors is sustainable if $\mu r_1^{\mathrm{I}} \leq y^{\mathrm{I}}$ and $(1-\mu)r_2^{\mathrm{I}} \leq R(1-y^{\mathrm{I}})$; a business model $\mathscr{M}=(r_1^{\mathrm{C}},r_2^{\mathrm{C}},y^{\mathrm{C}})$ of offering contracts only to consumers is sustainable if $\lambda r_1^{\mathrm{C}} \leq y^{\mathrm{C}}$ and $(1-\lambda)r_2^{\mathrm{C}} \leq R(1-y^{\mathrm{C}})$; and a business model $\mathscr{M}=(r_1^{\mathrm{P}},r_2^{\mathrm{P}},y^{\mathrm{P}})$ of offering the same contract, a pooling contract, to investors and to consumers alike, i.e. $\mathscr{D}^{\mathrm{I}}=\mathscr{D}^{\mathrm{C}}=(r_1^{\mathrm{P}},r_2^{\mathrm{P}})$, is sustainable if $(\gamma\lambda+(1-\gamma)\mu)r_1^{\mathrm{P}} \leq y^{\mathrm{P}}$ and $(1-(\gamma\lambda+(1-\gamma)\mu))r_2^{\mathrm{P}} \leq R(1-y^{\mathrm{P}})$. Provided either of these inequalities is strict, the respective business model is associated with strictly positive profits.

In equilibrium, banks with business models associated with contracts other than $\mathscr{D}^{\mathrm{I}}$ and $\mathscr{D}^{\mathrm{C}}$ cannot profitably enter the market. Therefore, equilibrium requires that a business model associated with a contract only for investors necessarily satisfies $(1-\mu)r_2^{\mathrm{I}}=R(1-\mu r_1^{\mathrm{I}})$; a business model associated with a contract only for consumers necessarily satisfies $(1-\lambda)r_2^{\mathrm{C}}=R(1-\lambda r_1^{\mathrm{C}})$; and a business model associated with one contract for both, consumers and investors, necessarily satisfies $(1-(\gamma\lambda+(1-\gamma)\mu))r_2^{\mathrm{P}}=R(1-(\gamma\lambda+(1-\gamma)\mu)r_1^{\mathrm{P}})$.

If an equilibrium is *constrained-efficient*, two distinct business models need to be present, i.e. the *equilibrium needs to be separating*. One is associated with a contract $\mathscr{D}^{\mathrm{I}}$ that maximizes the expected utility of investors subject only to the feasibility constraint $(1-\mu)r_2 = R(1-\mu r_1)$, and the other business model is associated with a contract $\mathscr{D}^{\mathrm{C}}$ that maximizes the expected utility of consumers subject only to the feasibility constraint $(1-\lambda)r_2 = R(1-\lambda r_1)$ (see the investors' problem (PI) and the consumers' problem (PC)). In constrained-efficient equilibria, neither the incentive constraints (7) and (8) nor the participation constraints (12) and (13) can be binding.

In a *separating equilibrium that is not constrained-efficient*, there are banks which offer contracts that are located on the investors' intertemporal budget line $(1-\mu)r_2 = R(1-\mu r_1)$, and other banks which offer contracts located on the consumers' intertemporal budget line $(1-\lambda)r_2 = R(1-\lambda r_1)$. However, in contrast to constrained-efficient equilibria, at least one of the incentive constraints (7) and (8) is binding. For example, if (7) is binding, then banks cannot profitably stay in, or enter, the market with a business model $(r_1^{\mathrm{C}}, r_2^{\mathrm{C}}, y^{\mathrm{C}}) = (r_1^{\delta}, r_2^{\delta}, y^{\delta})$ because they would attract not only all consumers but also all investors, which renders such business model unviable.

In a *pooling equilibrium* investors and consumers obtain one and the same contract, i.e. $\mathscr{D}^{\mathrm{I}} = \mathscr{D}^{\mathrm{C}}$, and this contract satisfies the joint intertemporal budget constraint that obtains if banks pool the resources of consumers and investors. Banks offering separating contracts cannot profitably enter the market in such pooling equilibria. This is because either consumers and investors would both prefer the pooling contract over the separating contracts, or consumers prefer the investors' contract, investors prefer the consumers' contract, or both.

4.2 A special case

For the special case of constant relative risk aversion equal to one, the equilibrium outcome is straightforward, since the optimal deposit contracts are identical for both liquidity motives. In this case, no insurance benefit is offered for early consumers, nor is there any compensation for investors for not getting access to the higher-yielding speculative project.

Lemma 3 (Logarithmic Utility) *Suppose* $-cu''(c)/u'(c) = 1$. *Then, the banking equilibrium is a menu of contracts* $\left\{\mathscr{D}^{\mathrm{I}}, \mathscr{D}^{\mathrm{C}}\right\}$ *with* $\mathscr{D}^{\mathrm{I}} = \mathscr{D}^{\mathrm{C}} = (1, R)$.

Proof: See Appendix A. □

If relative risk-aversion is equal to one, the optimum contracts for investors and consumers satisfy $(r_1^d, r_2^d) = (1, R)$ and $(r_1^\delta, r_2^\delta) = (1, R)$. Even though the contracts are identical, the underlying business models can be different as the contract for consumers requires a business model with reserves $y^\delta = \lambda$ and for investors $y^d = \mu$. As the speculative demand for liquidity is thus best met with a contract that also best meets the precautionary demand for liquidity, there is no incentive for investors or consumers to misrepresent themselves. Moreover, being on every banks' budget line, contract $(1, R)$ is an allocation any bank can offer, regardless the respective shares of impatient depositors, μ and λ. Therefore, the equilibrium is constrained-efficient.

The situation is quite different, however, when relative risk aversion differs from one. Henceforth, we focus on the case of relative risk aversion above one.[11]

Assumption 1 (Relative Risk Aversion)
The coefficient of relative risk aversion exceeds one, i.e. $-cu''(c)/u'(c) > 1$.

For the further analysis, we will also assume the single crossing property for the indifference curves of consumers and investors.

Assumption 2 (Single Crossing Condition)
In the (r_1, r_2) *space, indifference curves of consumers and of investors cross only once.*

Single crossing is a standard assumption in mechanism design theory. It is satisfied, for example, if relative risk aversion is constant. In our context, this assumption ensures that the set of propensities to speculate μ, for which a constrained- efficient allocation obtains, is convex.

In what follows, we first consider economies where the propensity to speculate μ is not too high and investment opportunities thus rare. Whilst bearing in mind the stylized nature of the model,

[11] Many results will be just reversed for the case of $-cu''(c)/u'(c) < 1$.

examples could be linked to economies with a low research intensity, or innovation trajectory. Following this, we consider more dynamic economies where investment opportunities are frequent as the propensity to speculate μ is comparatively high.

4.3 Rare investment opportunities

It is easy to observe that for $\mu \leq \lambda$ there will always be a constrained-efficient separation of the business models. To see why, recall the intertemporal budget constraints for banks. For banks meeting the investors' speculative demand for liquidity, this constraint requires $r_2 = R(1-\mu r_1)/(1-\mu)$, and for banks meeting the consumers' precautionary demand for liquidity it reads $r_2 = R(1-\lambda r_1)/(1-\lambda)$. The associated budget lines are linear in a (r_1, r_2) space and go through $(1,R)$ for both bank types, regardless of the value for λ and μ. We have also established that for relative risk aversion greater one, the optimal provision of liquidity for speculative purposes is a point on the respective budget line to the north-west of $(1,R)$, while the precautionary demand for liquidity is best met in a point to the south-east of $(1,R)$ on the respective budget line (see Figures 1 and 2). Finally, a bank's budget line is steeper for larger proportions of impatient depositors.

If the proportion of impatient consumers is not smaller than the proportion of impatient investors, $\mu \leq \lambda$, the budget line for consumers is steeper than the respective budget line for investors. Since the efficient consumers' contract (r_1^δ, r_2^δ) is to the south-west of $(1,R)$, it lies below the budget line associated with the investors' problem, i.e. inside the set of feasible contracts for investors. Therefore, investors prefer their own constrained-efficient contract (r_1^d, r_2^d) over the consumers' efficient contract. Intuitively, from an investor's perspective, the insurance benefit of a consumer contract to those withdrawing early is small relative to what one has to give up when turning out patient. This makes the consumers' contract sufficiently unattractive to investors. A similar argument can be made for the incentives of consumers. The contract intended for investors is unattractive to consumers because, as consumers are more likely to withdraw early, the penalty associated with an investors' contract is particularly costly for consumers.

Equilibria with constrained-efficient separation not only exist for $\mu \leq \lambda$, but even for $\mu > \lambda$ up to a critical level $\bar{\mu} < 1$, above which constrained-efficient equilibria fail to exist.

Proposition 1 (Constrained-efficient Separation)

Consider economies $\mathscr{E} = (u, \gamma, \lambda, \mu, Q, R)$, where u satisfies Assumption 1 *and* Assumption 2. *Then, for every $R > 1$, $Q > R$, and $\lambda \in]0,1[$, there is $\bar{\mu} \in]\lambda, 1[$ such that a constrained-efficient banking equilibrium exists* if and only if $\mu \leq \bar{\mu}$. *In constrained-efficient equilibria, the marginal rate of substitution between r_1 and r_2 is lower for investors than for consumers, i.e.* $-\frac{\mu}{1-\mu}\frac{u'(Qr_1^{\mathrm{I}})}{u'(r_2^{\mathrm{I}})}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^{\mathrm{C}})}{u'(r_2^{\mathrm{C}})}$.

Proof: See Appendix B. □

Figure 3 illustrates equilibria which involve constrained-efficient separation. Investors strictly prefer the solution (r_1^d, r_2^d) to their problem (PI) over the solution (r_1^δ, r_2^δ) to the consumers' problem (PC), as (r_1^δ, r_2^δ) lies below the investors' indifference curve going through their own contract (r_1^d, r_2^d). Similarly, consumers strictly prefer (r_1^δ, r_2^δ) over (r_1^d, r_2^d). Under Assumptions 1 and 2, these preference relations imply that the indifference curve of investors is flatter than the indifference curve of consumers, as can be seen at the intersection of both curves in Figure 3. It is not possible for any bank to profitably enter the market by offering a contract designated either exclusively to investors or exclusively to consumers, because investors as well as consumers already enjoy the best possible allocation. Also, a bank cannot profitably enter the market with a pooling contract. This is because the budget constraint associated with pooling, $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)}$, does not facilitate any contracts that are Pareto-improvements to the separating contracts (r_1^δ, r_2^δ) and (r_1^d, r_2^d).

To sum up, provided the propensity to speculate μ is not too large, a banking equilibrium (constrained) efficiently provides for the liquidity needs of consumers and investors. Consumers are insured against the risk of the need to consume early, while investors are insured against the risk of missing a better investment opportunity. Both motives require some liquidity management, but

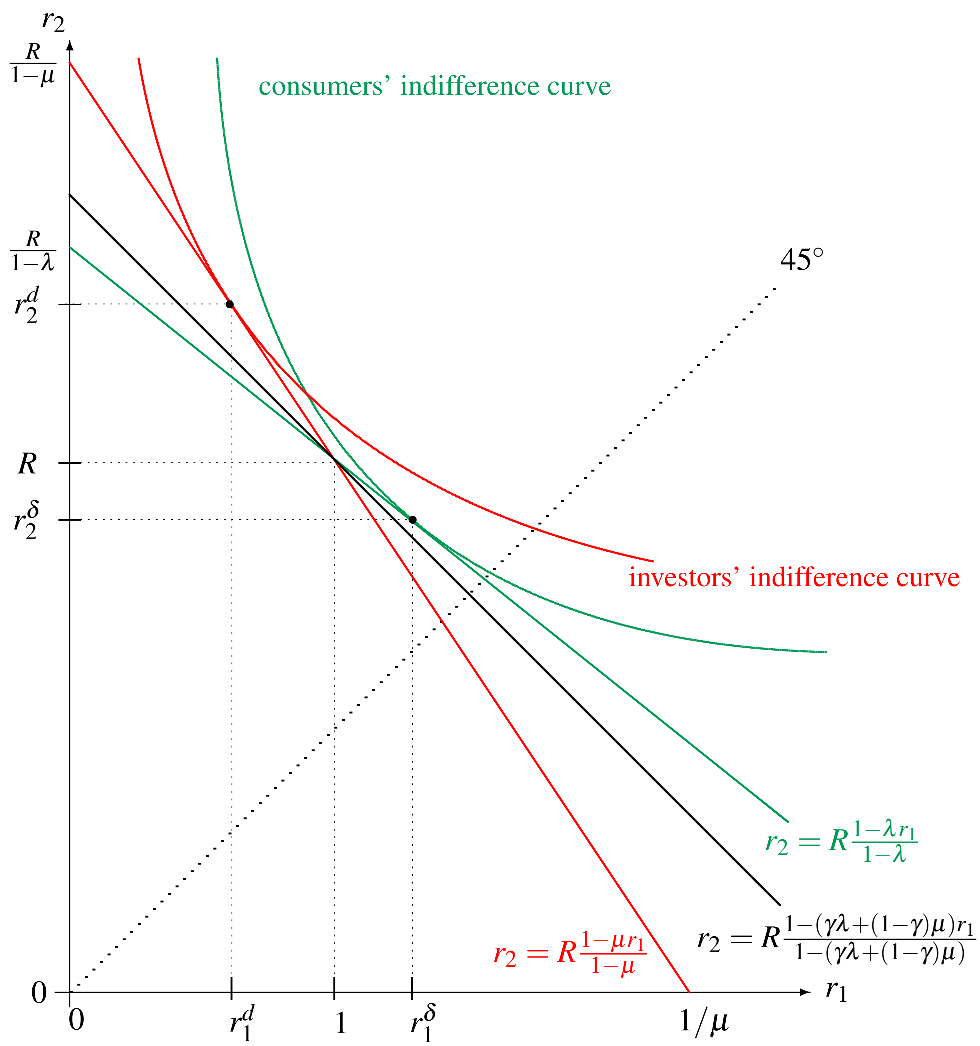

Figure 3: Efficient Separation.

optimal contracts stipulate different solutions. While the insurance payment is front-loaded in the contract with consumers, and back-loaded in the contract with investors, nobody has an incentive to hide their own motive for their liquidity preference.

Corollary 1 (Bank Reserves)

For $\mu \leq \lambda$ the reserve holdings of the consumer bank are larger than the investor bank, i.e. $y^C > y^I$. Accordingly, expected returns of the investor bank exceed those of the consumer bank.

Business models $(r_1^{\mathrm{I}}, r_2^{\mathrm{I}}, y^{\mathrm{I}})$ associated with term deposits for investors thus require lower reserve holdings than business models $(r_1^{\mathrm{C}}, r_2^{\mathrm{C}}, y^{\mathrm{C}})$ associated with demand deposits for consumers.

This is because the investor bank requires reserves below the propensity to speculate μ, i.e. $y^{\mathrm{I}} = y^d < \mu$, while consumer bank requires reserves in excess of the probability to consume early λ, i.e. $y^{\mathrm{C}} = y^{\delta} > \lambda$. Therefore, $y^{\mathrm{I}} < y^{\mathrm{C}}$ for $\mu \leq \lambda$. The differences in bank portfolios have direct implications for the returns on bank assets. As those are determined by $y + R(1 - y)$, returns on assets are higher for an investor bank than for a consumer bank provided $y^d < y^{\delta}$.

Even though not literally true it is tempting to view the investor bank as an investment bank and the consumer bank as traditional retail bank. Corollary 1 suggest that due to their different business models the investor and the consumer bank cannot be meaningfully compared (and ranked) according to their return properties, nor according to their reserve holdings. In equilibrium both earn zero (excess) profits and share the same fundamental risk.

4.4 Frequent investment opportunities

Let us now consider economies with a relatively high propensity to speculate, i.e. $\mu > \bar{\mu}$. As those are economies where highly productive investment opportunities arrive frequently, we can think of them as economies with high R&D activity, or simply high growth economies. How will equilibrium outcomes be affected under such conditions? It turns out that the outcomes can vary substantially, depending on the specific characteristic of the economy at hand: there can be separating equilibria with inflated consumer insurance, or pooling equilibria, or it can even be that no pure-strategy equilibria exist altogether.

Separating contracts with inflated consumer insurance Suppose the marginal rate of substitution for consumers exceeds the rate for investors for all realizations of (r_1, r_2), yet the propensity to speculate μ is sufficiently large such that investors prefer the efficient contract $(r_1^{\delta}, r_2^{\delta})$ for consumers over the constrained-efficient contract (r_1^d, r_2^d) for investors. Constrained-efficient separa-

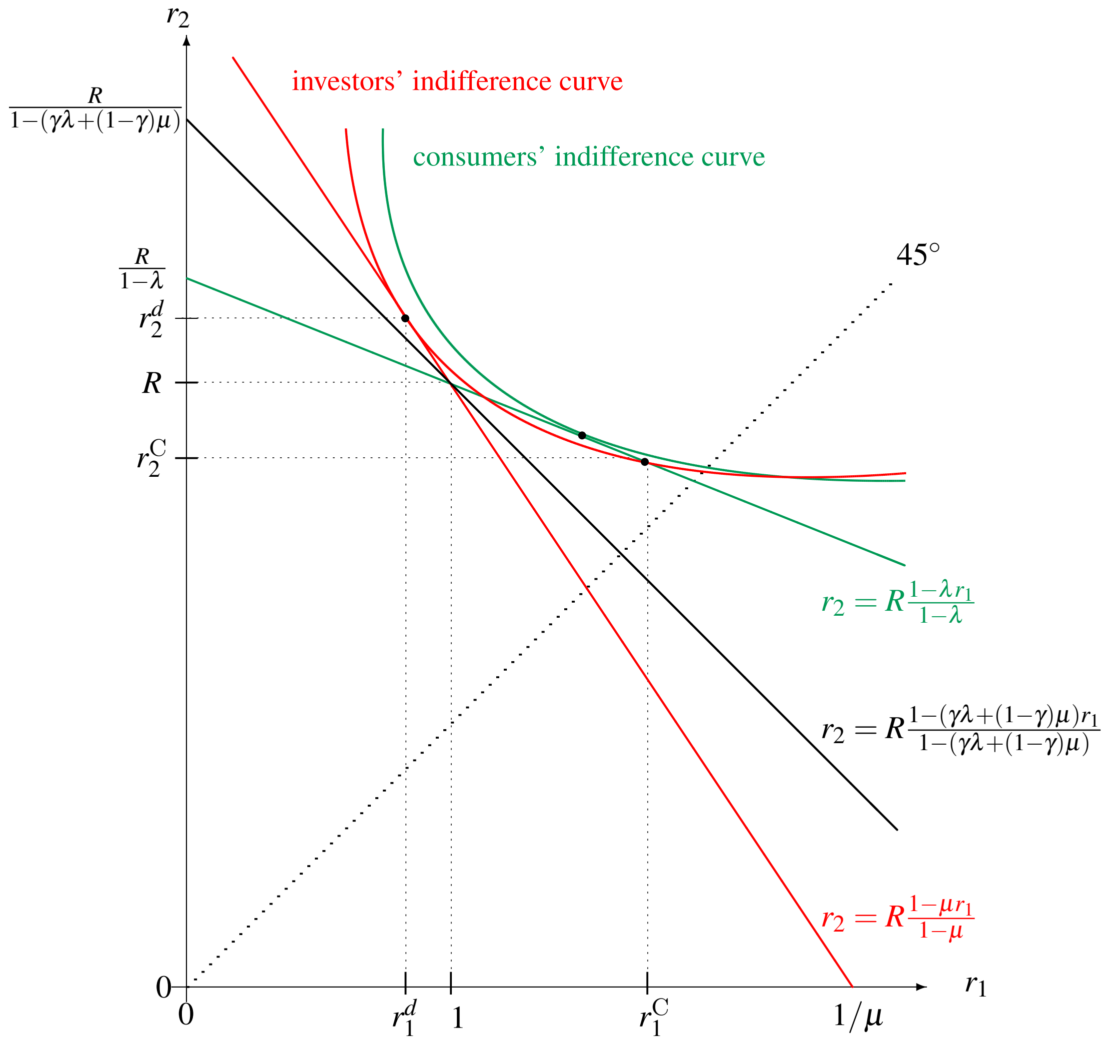

Figure 4: Inefficient Separation with Inflated Consumer Insurance.

tion thus breaks down, as the incentive constraint for investors (7) is violated for $(r_1^I, r_2^I) = (r_1^d, r_2^d)$ and $(r_1^C, r_2^C) = (r_1^\delta, r_2^\delta)$.[12]

Figure 4 illustrates a possible scenario for this case. If both contracts are on the same indifference curve for investors, they weakly prefer their own contract, (r_1^d, r_2^d), over the contracts offered to consumers, (r_1^C, r_2^C). Banks make zero-profits with consumer contracts, if (r_1^C, r_2^C) is on the consumers' budget line. In Figure 4, there are thus two potential contracts, characterized by the intersection of the investors' indifference curve and the consumers' budget line. One contract is to

[12]If the consumers' incentive constraint (8) is violated but not the investors' incentive constraint (7), then the marginal rate of substitution for consumers cannot exceed the respective rate for investors.

the north-west of (r_1^δ, r_2^δ), and the other to the south-east. Investors are indifferent between these two. However, as long as the contract to the south-east of (r_1^δ, r_2^δ) satisfies $r_1^{\mathrm{C}} < r_2^{\mathrm{C}}$, consumers strictly prefer this one because the marginal rate of substitution for consumers exceeds the rate for investors.[13]

Such equilibrium thus implies an even larger insurance benefit to consumers relative to the case where a speculative motive for liquidity demand is absent. Given the incentive constraint of investors, $(r_1^{\mathrm{C}}, r_2^{\mathrm{C}})$ is the best separating contract consumers can get. Also, a bank cannot profitably enter the market with a pooling contract as the budget constraint associated with a pooling business model does not facilitate contracts that would be a Pareto-improvement to the two separating contracts, (r_1^d, r_2^d) and $(r_1^{\mathrm{C}}, r_2^{\mathrm{C}})$.

The following Proposition generalizes these insights.

Proposition 2 (Inflated Consumer Insurance)

Consider economies $\mathscr{E} = (u, \gamma, \lambda, \bar{\mu}, Q, R)$ for which $\bar{\mu}$ is such that $(r_1^\delta, r_2^\delta) \sim_{\mathrm{I}} (r_1^d, r_2^d)$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{C}} (r_1^d, r_2^d)$. Under Assumption 2*, for each such economy $\mathscr{E}$ there exist $\eta(\mathscr{E}) > 0$ such that there are economies $\mathscr{E}' = (u, \gamma, \lambda, \hat{\mu}, Q, R)$ with $\hat{\mu} \in]\bar{\mu}, \bar{\mu} + \eta(\mathscr{E})[$ where a* separating *equilibrium obtains in which the investors' contract $(r_1^{\mathrm{I}}, r_2^{\mathrm{I}})$ satisfies $r_1^{\mathrm{I}} = r_1^d$ and $r_2^{\mathrm{I}} = r_2^d$, and the consumers' contract $(r_1^{\mathrm{C}}, r_2^{\mathrm{C}})$ satisfies $r_1^{\mathrm{C}} > r_1^\delta$ and $r_2^{\mathrm{C}} < r_2^\delta$.*

Proof: See Appendix C. □

The next corollary states an interesting feature of the limits to inflated consumer insurance.

Corollary 2 (Consumer-dominated Populations)

The set of propensities to speculate, for which equilibria with inflated liquidity insurance for consumers obtain, converges to the empty set if the share of consumers in the population γ approaches one.

[13]If $r_1^{\mathrm{C}} > r_2^{\mathrm{C}}$, patient consumers are better off pretending to be impatient and withdraw at date $t = 1$, which renders this contract incentive incompatible.

Proof: See Appendix D. □

Intuitively, neither the indifference curves nor the budget lines associated with separating contracts depend on the composition of the population, but the slope of the pooling budget line, $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)}$, does (see Figure 4). As γ goes to one, it converges to the budget line for consumers, $r_2 = R\frac{1-\lambda r_1}{1-\lambda}$. Therefore, the pooling budget line eventually intersects a set of contracts enclosed by the consumers' indifference curve going through $(r_1^{\mathrm{C}}, r_2^{\mathrm{C}})$ and the consumers' budget line. With the consumers' indifference curves being steeper than the investors' indifference curves, a set of pooling contract becomes thus available that are Pareto-improvements to the separating contracts $(r_1^{\mathrm{C}}, r_2^{\mathrm{C}})$ and (r_1^d, r_2^d). Therefore, for any given propensity to speculate for which inflated consumer insurance is an equilibrium provided $\gamma = 0$, there is a $\bar{\gamma} < 1$ such that separating contracts with inflated consumer insurance cannot be an equilibrium for all $\gamma \in]\bar{\gamma}, 1[$.

Pooling contracts Suppose the marginal rate of substitution of investors exceeds the marginal rate of substitution for consumers. Then, an equilibrium with separating contracts cannot exist. To see how, consider first two contracts between which investors are just indifferent. Of these two contracts, let one contract satisfy the intertemporal budget constraint for consumers, $r_2 = R(1-\lambda r_1)/(1-\lambda)$, and the other the budget for investors, $r_2 = R(1-\mu r_1)/(1-\mu)$. Among these two contracts, consumers then strictly prefer the contract intended for investors if and only if the marginal rate of substitution between r_1 and r_2 is higher for investors than for consumers. Conversely, if we consider two contracts between which consumers are just indifferent, again one contract satisfying the budget constraint for consumers, $r_2 = R(1-\lambda r_1)/(1-\lambda)$, the other the budget for investors, $r_2 = R(1-\mu r_1)/(1-\mu)$, then investors will prefer the contract intended for consumers.

While equilibria with separating contracts are, therefore, not possible, equilibria in which banks offer *pooling contracts* may still exist. Such pooling contracts specify identical payment schedules, $\mathscr{D}^{\mathrm{I}} = \mathscr{D}^{\mathrm{C}} = (r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$, to consumers and to investors. Figure 5 illustrates this. Competitive banks with business models associated with pooling contracts offer payments satisfying

$r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)}$, i.e. they are located on the pooling budget constraint. Consider any contract on that line other than $(1,R)$, for example as in Point A. Given that the marginal rate of substitution of investors exceeds the marginal rate of substitution for consumers, there is a contract B such that investors are just indifferent between A and B, while consumers strictly prefer B. Hence, a bank could profitably enter the market by offering contract B, pulling away consumers from banks offering the pooling contract A. Left with only investors as clientele, contract A is no longer sustainable. Therefore, contract A cannot be an equilibrium. In turn, contract B as part of a separating equilibrium is not sustainable either, given the condition for the marginal rates of substitution. A similar argument can be made for pooling contracts to the north-west of $(1,R)$, ruling out pooling contracts on that upper branch of the pooling budget line.

Next, consider the only remaining contract, $(1,R)$. Any contract on the consumers' budget constraint to the south-east of $(1,R)$ would not only make consumers better off but also investors, and any contract on the investors' budget constraint to the north-west of $(1,R)$ would not only make investors better off but also consumers. Therefore, there are no separating contract offers which can break a pooling contract $(1,R)$. Indeed, as long as the slope of the pooling budget line is between the slope of the indifference curve of the consumers and the slope of the indifference curve of the investors, there are no other contracts on the pooling budget line that would be Pareto-improvements to $(1,R)$ and thus attract both, investors, and consumers.

The following proposition formalizes these insights.

Proposition 3 (Pooling Contracts)

Consider economies $\mathscr{E} = (u,\gamma,\lambda,\mu,Q,R)$ *with* $\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q > \frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}$ *for all* $(r_1,r_2) \in \mathbb{R}^2_+$*. If* $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)$ *the only equilibrium contract is a pooling contract. This contract is determined as* $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}) = (1,R)$.

Proof: See Appendix E. □

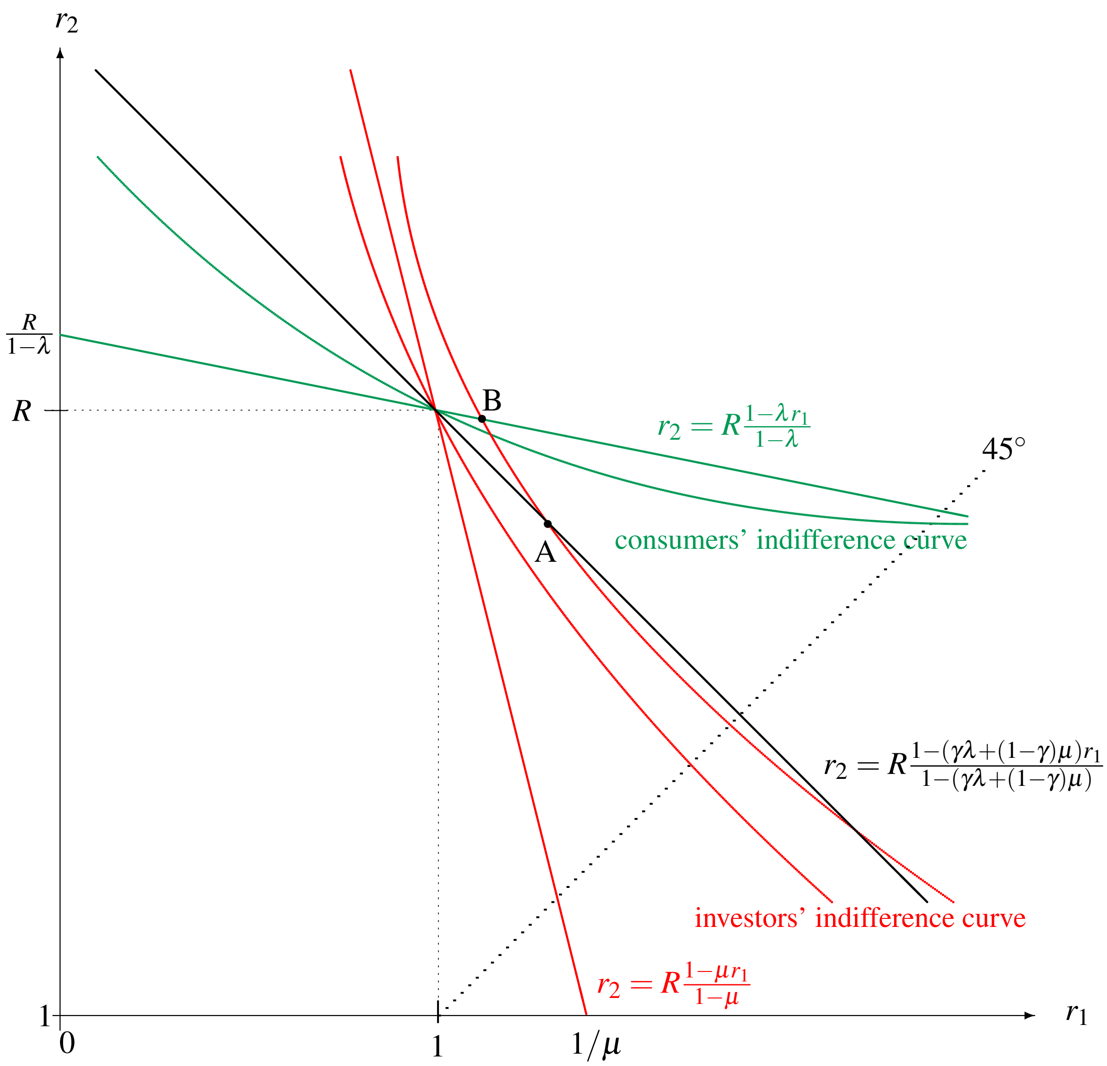

Figure 5: Pooling
$\left(\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)\right)$.

Note that a payment schedule $(1,R)$ also obtains in economies without banks but with asset markets. There, consumers and investors choose their own portfolio allocation between storage and the R-technology at date $t=0$, and then trade storage for R-projects in an asset market at date $t=1$ depending on their liquidity needs. For the asset market equilibrium to be arbitrage-free, equilibrium requires that the asset price equals one as only then storage and R-technology generate the same return between dates $t=0$ and $t=1$. With asset prices equal to one, impatient consumers will sell their R-projects and consume one unit, and patient consumers will use all their storage to buy R-projects and consume R units of the good. As for investors, those with access to the Q-technology will sell their holdings of R-projects and invest one unit in the new opportunity. Investors without access use their storage to buy additional R-projects.

Non-existence of pure-strategy equilibria Economies can also be such that there is no contract that cannot be dominated by another contract.[14]

Proposition 4 (Non-existence of Equilibrium)
Consider economies $\mathscr{E} = (u, \gamma, \lambda, \mu, Q, R)$ *with* $\mu > \lambda$ *and* $\frac{\mu}{1-\mu} \frac{u'(Qr_1)}{u'(r_2)} Q > \frac{\lambda}{1-\lambda} \frac{u'(r_1)}{u'(r_2)}$ *for all* $(r_1, r_2) \in \mathbb{R}^2_+$. *There is no equilibrium in pure strategies, provided the following condition* $\frac{\mu}{1-\mu} u'(Q)Q > \frac{\gamma\lambda + (1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)} u'(R)R > \frac{\lambda}{1-\lambda} u'(1)$ *is violated.*

Proof: See Appendix F. □

Under the conditions of this Proposition there is no viable contract that is not dominated by another contract. Figure 6 illustrates such case. Since the marginal rate of substitution of investors exceeds the marginal rate of substitution for consumers, neither separating contracts nor pooling contracts other than $(1, R)$ are feasible in equilibrium by the arguments already made above. However, a pooling contract $(1, R)$ cannot be an equilibrium either. To see why, suppose banks were offering a pooling contract $(1, R)$. Then, another bank could profitably enter the market by offering another pooling contract, for there are Pareto-improvements to $(1, R)$ along the pooling budget line — to the north-west of $(1, R)$ in Figure 6. As argued before, those contracts cannot be an equilibrium either given these marginal rates of substitution.

A pure-strategy choice of equilibrium contracts, i.e. one which does not apply lotteries over contracts, fails to exist here. Therefore, there is no stable market outcome. Interestingly, pure-strategy equilibria do not exist, if the population is highly unbalanced in either direction, i.e. if the

[14] While mixed strategy equilibria may exist when randomization across contracts is allowed for, we do not pursue this possibility in this paper. By their very nature mixed strategy equilibria will induce added strategic uncertainty, and, hence, instability in market outcomes (see Gehrig and Ritzberger, 2020).

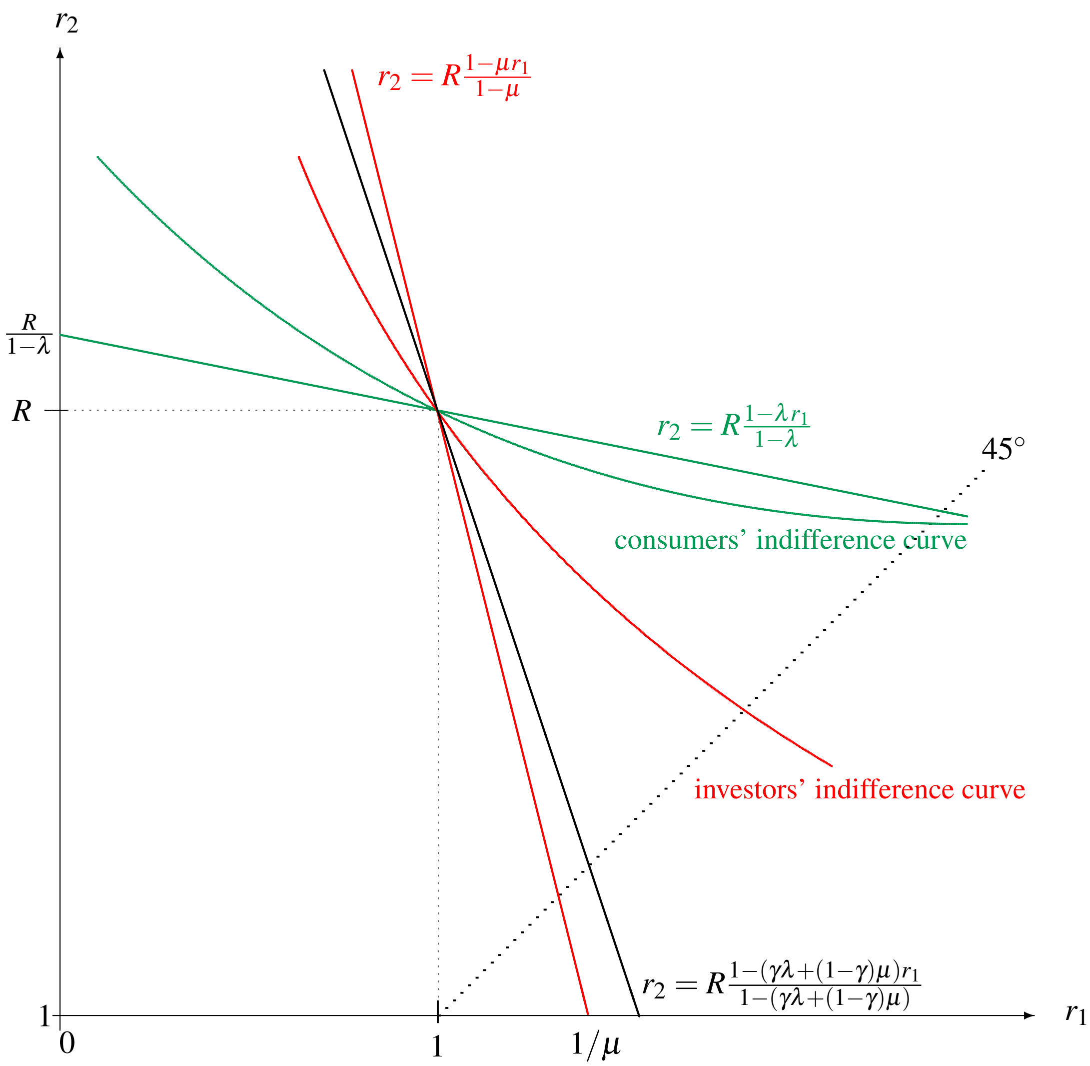

Figure 6: Non-existence of Equilibrium in Pure Strategies $\left(\frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\mu}{1-\mu}u'(Q)Q > \frac{\lambda}{1-\lambda}u'(1)\right)$.

proportion of consumers, γ, is either very close to zero or to unity. The value of γ determines only the slope of the pooling budget line. It converges to the investors' budget line for $\gamma \to 0$ and to the consumers' budget line for $\gamma \to 1$. Corollary 3 summarizes the implications for the limiting cases.

Corollary 3 (Unbalanced Populations)

Consider economies $\mathscr{E} = (u, \gamma, \lambda, \mu, Q, R)$ *with* $\mu > \lambda$ *and* $\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q > \frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}$ *for all* $(r_1, r_2) \in$

$\mathbb{R}^2_+$. *There is no equilibrium in pure strategies if the proportion of consumers in the population, γ, is either very large or very low.*

Proof: See Appendix G. □

Having characterized the various equilibrium outcomes for different economic constellations we will discuss implications for bank business models and economic policy in the next section.

5 Implications

Bank runs The present paper adds to the vast literature on bank runs and fragility of the banking system.[15] While term deposits are, in principle, prone to the same type of coordination failure as demand deposits, an important difference is that they are less likely to occur with term deposits.

Without going into the details of a fully fledged model of bank runs as a result of coordination failures, our analysis lends itself to some preliminary conclusions. Suppose that the R-technology can be liquidated at date $t=1$ for a per-unit scrap value equal to one; that depositors withdrawing at date $t=1$ are served sequentially; and that neither the bank nor the banking supervisory authority can precommit to suspend convertibility if a bank run is underway (see, e.g., Ennis and Keister, 2009). Suppose next that a patient depositor believes that the share of depositors actually withdrawing from their own bank at date $t=1$ is at least ν. Then, if $r_1 > (1-\nu r_1)R/(1-\nu)$, or $\nu > \tilde{\nu} := (R-r_1)/(r_1(R-1))$, the patient depositor is better off withdrawing at date $t=1$. Accordingly, $\tilde{\nu}$ can be seen as a measure of a bank's susceptibility to bank runs.[16] A lower $\tilde{\nu}$ indicates a higher susceptibility, and banks are not prone to runs at all if $\tilde{\nu} > 1$.

In pooling equilibria with a pooling contract $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}) = (1, R)$ for investors and consumers alike, we obtain $\tilde{\nu} = 1$ such that banks are not prone to bank runs. In separating equilibria where banks serve the speculative motive for liquidity demand (constrained) efficiently, those banks are not

[15] See e.g. Allen and Gale (2004), Bucher et al. (2018), Cooper and Ross (1998), Ennis and Keister (2006), Matutes and Vives (1996) and Rochet and Vives (2004).

[16] He and Manela (2016) refer to the mass of depositors it takes to run down the bank as *bank liquidity*.

prone to bank runs either since $r_1^{\mathrm{I}} < 1$ and, therefore, $\tilde{\nu} > 1$. However, banks that serve the precautionary motive are prone to bank runs since $r_1^{\mathrm{C}} > 1$ and, therefore, $\tilde{\nu} < 1$.[17] Interestingly, by this measure, these banks can be considered even more prone to bank runs in equilibria with inefficient separation than in equilibria with constrained-efficient separation because $r_1^{\mathrm{C}} > r_1^{\delta}$ (see Proposition 2). Therefore, the co-existence of a precautionary motive and a speculative motive for liquidity demand can add further risk to the stability of banks, but it is banks providing liquidity insurance to consumers which are affected.

Low interest rate environment Since the long-term production generates safe returns R, they can be expected to be linked to the return on long-term government debt. How would equilibrium be affected in a low interest rate environment, i.e. if the long-term rate R converges to one? It is readily verified that in such an environment the demand deposit contract converges to a contract merely repaying consumers their initial endowment, i.e. $\lim_{R\to 1}(r_1^{\delta}, r_2^{\delta}) = (1,1)$. In other words, the insurance motive looses relevance, while the speculative motive remains active as long as $Q > 1$.

Interestingly, a low interest rate environment can contribute to instability as equilibria in pure strategies may cease to exist when the returns on the long-term production fall. The following example illustrates this. Suppose that the initial return with long-term production is $R = R_0$, and that for this value a pooling equilibrium just obtains, i.e. there is a small $\varepsilon > 0$ such that

$$\frac{\mu}{1-\mu}u'(Q)Q - \varepsilon = \frac{\gamma\lambda + (1-\gamma)\mu}{1-(\gamma\lambda + (1-\gamma)\mu)}u'(R_0)R_0 > \frac{\lambda}{1-\lambda}u'(1).$$

Suppose next that the return on the long-term technology, R, falls to one. For relative risk aversion larger one we obtain $\frac{\mathrm{d}}{\mathrm{d}R}(u'(R)R) < 0$, with $\lim_{R\to 1} u'(R)R = u'(1) > u'(Q)Q$. Therefore, a fall of R to one will lead to a violation of $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R$ for sufficiently small ε, i.e. a pooling equilibrium, which exists and is the only equilibrium for $R = R_0$, ultimately fails to exist as $R \to 1$. In other words, a decrease in the long-term interest rate, as measured by R, increases

[17] Taking a global games perspective as put forward by Goldstein and Pauzner (2005), the threshold for bad fundamental news needed to trigger a run would thus be significantly higher for term deposit contracts that serve the speculative motive, compared to demand deposit contracts that serve the precautionary motive.

the range of unstable outcomes. Clearly, this type of instability will not arise in a world with only a single motive for liquidity demand.

However, by the argument developed above, run-related concerns for systemic risk would be of declining relevance in a low interest rate environment. Hence, the focus in business models shifts from "front-loaded" demand deposits to "back-loaded" term deposits. This regime shift should also be reflected in the supervisory and regulatory framework for banks.

Bank regulation In equilibria with constrained-efficient separation, serving the speculative motive requires a different bank business model than addressing the precautionary motive.[18] However, there is no rationale for liquidity regulation other than possibly preventing runs. Our considerations above suggest, though, that the precautionary liquidity demand is prone to coordination failures, while speculative liquidity demand is not. Therefore, the motive of the liquidity demand matters for the design of bank regulations. Regulation should particularly focus on banks catering to the precautionary motive, typically retail banks, while others serving the speculative motive would seem to require less regulation.

Universal banks Interesting implications arise for the governance and the regulation of universal banks that typically serve both liquidity motives within the same house. In this case our analysis suggests that under competitive conditions, in a separating equilibrium the precautionary and the speculative motive should be served with independent business models. Moreover the internal rate of return associated with a business model targeted at speculative liquidity demand typically exceeds the rate of return associated with a business model targeted at precautionary liquidity demand. If shareholders insist on the same, highest rate of return across all activities within a universal bank, they are effectively interfering with the precautionary motive and requesting a lower level of reserves. On the other hand, prudential regulation would optimally treat different business models differently. This does not necessarily require the separation of ownership into

[18] See Corollary 2 for sufficient conditions.

several banking units but it does require to treat, and manage, separate business models separately from an organizational point of view.

Mutual funds Mutual funds have been seen to implement the constrained-efficient allocation without being prone to coordination failures (Jacklin, 1987). Our model implies that this may no longer hold if a speculative motive for liquidity demand is added to the traditional precautionary motive. Mutual funds imply that allocations are pooling outcomes as investors and consumers trade mutual fund shares for the same prices. In our model, however, a separating equilibrium dominates any pooling outcome, whenever it exists.[19]

Growth The speculative motive provides a new link to the empirical literature about the relation between liquidity provision and economic growth (e.g., Berger and Bowman, 2009). Beck et al. (2020) find that industries with mainly tangible capital are positively associated with bank liquidity provision while the opposite is true for knowledge-intensive sectors with limited pledgeable assets. In order to explain their findings they extend the model of Diamond and Dybvig (1983) by adding asset liquidity risk and credit risk for a given set of technologies. Adding the speculative model directly, as we do, in principle, should allow to address the link between bank liquidity and the propensity to speculate more directly. In the framework of Bencivenga and Smith (1991) it adds a complementary growth-enhancing liquidity option. Exploiting the richness of the underlying micro-structure of liquidity sources suggests a rich agenda for empirical testing and verification.

6 Concluding remarks

Our analysis reveals that the nature of liquidity demand crucially matters for the best institutional response. This does not only hold for different motives in isolation but in particular for the co-existence of several motives at any point in time. In a simple framework we have shown that the co-existence of need-based liquidity demand with an option-based motive has the potential to

[19] See Hellwig (1994) for another economic setting where a competitive banking system implements the second best allocation of liquidity.

benefit from economies of scope in a frictionless world. Likewise, in the presence of frictions, the precise nature of these frictions as well as their interplay will affect the nature of market outcomes, and, therefore, potential policy implications. For example, focusing on one motive and one friction only is likely to direct policy debate on bank runs, even for constellations, when their occurrence is not likely because maturity transformation does not take place in equilibrium. But also, as we show, constellations may arise, where the severity of the bank-run problem is underestimated because of the ignorance of other motives.

While our analysis focuses on competitive banking systems it is tempting to extend our framework to the imperfectly competitive case, when frictions will also be the source of market power. Another promising topic for research is the allocation of risk across depositor types when aggregate risk is introduced into the analysis.

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Appendix

A Proof of Lemma 3

For $-cu''(c)/u'(c) = 1$, we obtain $r_1^\delta = r_1^d = 1$. Hence, both incentive constraints hold with equality. The consumers' participation constraints are satisfied with strict inequality for all $\lambda \in]0,1[$ because $u(1) \geq u(y)$ and $u(R) \geq u(R(1-y)+y)$ for all $y \in [0,1]$ since $R > 1$, with $u(1) > u(y)$ for $y < 1$ and $u(R) > u(R(1-y)+y)$ for $y > 0$. The investors' participation constraints are satisfied with strict inequality for all $\mu \in]0,1[$ because $u(Q) \geq u(Qy+R(1-y))$ since $Q > R$ and $u(R) \geq u(R(1-y)+y)$ for all $y \in [0,1]$ since $R > 1$, with $u(Q) > u(Qy+R(1-y))$ for $y < 1$ and $u(R) > u(R(1-y)+y)$ for $y > 0$.

B Proof of Proposition 1

The proof is by establishing six claims consecutively.

Claim 1: (r_1^d, r_2^d) and (r_1^δ, r_2^δ) satisfy the participation constraints for investors and consumers, respectively.

The participation constraints are satisfied with strict inequality:

- For consumers:

$$\begin{aligned} &\lambda u\left(r_1^\delta\right) + (1-\lambda)u\left(\tfrac{R(1-\lambda r_1^\delta)}{1-\lambda}\right) \\ &\quad > \lambda u(1) + (1-\lambda)u(R) \\ &\qquad > \sup\{\lambda u(y) + (1-\lambda)u(R(1-y)+y) \mid y \in [0,1]\} \end{aligned} \tag{14}$$

  The first inequality obtains since $r_1^\delta \in \arg\max\{\lambda u(r_1)+(1-\lambda)u\left(\frac{R(1-\lambda r_1)}{1-\lambda}\right) \mid r_1 \in [0,\lambda^{-1}]\}$. The second inequality obtains since $R > 1$ implies for all $y \in [0,1]$ that $u(1) \geq u(y)$ and $u(R) \geq u(R(1-y)+y)$, with $u(1) > u(y)$ for $y < 1$ and $u(R) > u(R(1-y)+y)$ for $y > 0$.

- For investors:

$$
\begin{aligned}
&\lambda u\left(Qr_1^d\right)+(1-\lambda)u\left(\tfrac{R(1-\lambda r_1^d)}{1-\lambda}\right)\\
&\qquad > \lambda u(Q)+(1-\lambda)u(R)\\
&\qquad\qquad > \sup\{\mu u(Qy+R(1-y))+(1-\mu)u(R(1-y)+y)\,|\,y\in[0,1]\}
\end{aligned}
\tag{15}
$$

The first inequality obtains since $r_1^d \in \arg\max\{\lambda u(Qr_1)+(1-\lambda)u\left(\frac{R(1-\lambda r_1)}{1-\lambda}\right) \mid r_1 \in [0,\lambda^{-1}]\}$. The second inequality obtains since $Q > R > 1$ implies for all $y \in [0,1]$ that $u(Q) \geq u(Qy+R(1-y))$ and $u(R) \geq u(R(1-y)+y)$, with $u(Q) > u(Qy+R(1-y))$ for $y < 1$ and $u(R) > u(R(1-y)+y)$ for $y > 0$.

Claim 2: $(r_1^d, r_2^d) \succ_{\mathrm{I}} (r_1^\delta, r_2^\delta)$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{C}} (r_1^d, r_2^d)$ for all $\lambda \geq \mu$.

1. For $\mu = \lambda$, the incentive constraints are satisfied for investors and consumers:

   - investors: $\lambda u\left(Qr_1^d\right)+(1-\lambda)u\left(\frac{R(1-\lambda r_1^d)}{1-\lambda}\right) \geq \lambda u\left(Qr_1^\delta\right)+(1-\lambda)u\left(\frac{R(1-\lambda r_1^\delta)}{1-\lambda}\right)$ for all $r_1^\delta \in [0,\lambda^{-1}]$, with strict inequality if $-\frac{u''(c)}{u'(c)}c \neq 1$, since $r_1^d \in \arg\max\{\lambda u(Qr_1)+(1-\lambda)u\left(\frac{R(1-\lambda r_1)}{1-\lambda}\right) \mid r_1 \in [0,\lambda^{-1}]\}$.
   - consumers: $\lambda u(r_1^\delta)+(1-\lambda)u\left(\frac{R(1-\lambda r_1^\delta)}{1-\lambda}\right) \geq \lambda u(r_1^d)+(1-\lambda)u\left(\frac{R(1-\lambda r_1^d)}{1-\lambda}\right)$ for all $r_1^d \in [0,\lambda^{-1}]$, with strict inequality if $-\frac{u''(c)}{u'(c)}c \neq 1$, since $r_1^\delta \in \arg\max\{\lambda u(r_1)+(1-\lambda)u\left(\frac{R(1-\lambda r_1)}{1-\lambda}\right) \mid r_1 \in [0,\lambda^{-1}]\}$.

   Therefore, $(r_1^d, r_2^d) \succ_{\mathrm{I}} (r_1^\delta, r_2^\delta)$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{C}} (r_1^d, r_2^d)$ for $\lambda = \mu$ and $-\frac{u''(c)}{u'(c)}c > 1$.

2. For $\lambda > \mu$, it suffices to consider the incentive constraints for investors and consumers, respectively, letting λ increase for a given μ, starting from $\lambda = \mu$.

   - investors: The LHS of condition (7) is not affected by changes in λ. Hence, the total effect on the differential of expected utilities is positive as the RHS of condition (7) changes according to

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\mu u(Qr_1^\delta)+(1-\mu)u(r_2^\delta)\right)=\mu Qu'(Qr_1^\delta)\frac{\mathrm{d}r_1^\delta}{\mathrm{d}\lambda}+(1-\mu)u'(r_2^\delta)\frac{\mathrm{d}r_2^\delta}{\mathrm{d}\lambda}<0 \qquad (16)$$

as $\frac{\mathrm{d}r_1^\delta}{\mathrm{d}\lambda},\frac{\mathrm{d}r_2^\delta}{\mathrm{d}\lambda}<0$. The latter follows from applying the implicit function theorem to the consumers' first-order condition (6). If written as

$$u'\left(r_1^\delta\right)-Ru'\left(\frac{R(1-\lambda r_1^\delta)}{1-\lambda}\right)=0$$

we have

$$\frac{\mathrm{d}r_1^\delta}{\mathrm{d}\lambda}=-\frac{R^2u''(r_2^\delta)\frac{r_1^\delta-1}{(1-\lambda)^2}}{u''(r_1^\delta)+R^2u''(r_2^\delta)\frac{\lambda}{1-\lambda}}<0,$$

and if written as

$$u'\left(\frac{R-(1-\lambda)r_2^\delta}{\lambda R}\right)-Ru'(r_2^\delta)=0$$

we have

$$\frac{\mathrm{d}r_2^\delta}{\mathrm{d}\lambda}=-\frac{-u''(r_1^\delta)\frac{R-r_2^\delta}{\lambda^2R}}{-u''(r_1^\delta)\frac{1-\lambda}{\lambda R}-Ru''(r_2^\delta)}<0.$$

- consumers: By the Envelope theorem, the LHS in condition (8) changes in response to increases in λ by $u(r_1^\delta)-u(r_2^\delta)$. The RHS in condition (8) changes in response to increases in λ by $u(r_1^d)-u(r_2^d)$. Hence, the total effect on the differential of expected utilities is $\left(u(r_1^\delta)-u(r_1^d)\right)-\left(u(r_2^\delta)-u(r_2^d)\right)$ which is positive since $r_1^\delta>r_1^d$ and $r_2^\delta<r_2^d$.

Claim 3: There is $\tilde{\mu}>\lambda$ such that $(r_1^\delta,r_2^\delta)\succ_{\mathrm{I}}(r_1^d,r_2^d)$ for all $\mu\in]\tilde{\mu},1[$ and $(r_1^d,r_2^d)\succ_{\mathrm{I}}(r_1^\delta,r_2^\delta)$ for all $\mu\in]0,\tilde{\mu}[$.

From the investors' first-order condition (5), we obtain $\lim_{\mu\to1}y^d=\lim_{\mu\to1}r_1^d=1$. The LHS of condition (7) converges to $u(Q)$ and the RHS to $u(Qr_1^\delta)>u(Q)$ since $r_1^\delta>1$. By the intermediate value theorem, there is thus $\bar{\mu}>\lambda$ such that $(r_1^\delta,r_2^\delta)\succ_{\mathrm{I}}(r_1^d,r_2^d)$ for all $\mu\in]\bar{\mu},1[$. Since the utility differential $Z_{\mathrm{I}} = (\mu u(Qr_1^d) + (1-\mu)\,u(r_2^d)) - (\mu u(Qr_1^\delta) + (1-\mu)\,u(r_2^\delta))$ is monotone in μ with $\mathrm{d}Z_{\mathrm{I}}/\mathrm{d}\mu = \left(u(Qr_1^d) - u(Qr_1^\delta)\right) - \left(u(r_2^d - u(r_2^\delta))\right) < 0$, the claim is established.

Claim 4: If Q is large, and λ small, there is $\hat{\mu} \in]\lambda, 1[$ such that $(r_1^d, r_2^d) \succ_{\mathrm{C}} (r_1^\delta, r_2^\delta)$ for all $\mu \in]\hat{\mu}, 1[$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{C}} (r_1^d, r_2^d)$ for all $\mu \in]0, \hat{\mu}[$.

From the investors' first-order condition (5), we obtain $\lim_{\mu\to 1} y^d = \lim_{\mu \to 1} r_1^d = 1$. Therefore, $(r_1^d, r_2^d) \succ_{\mathrm{C}} (r_1^\delta, r_2^\delta)$ holds for $\mu \to 1$ provided

$$\lambda u(1) + (1-\lambda)u(r_2^d) > \lambda u(r_1^\delta) + (1-\lambda)u(r_2^\delta). \tag{17}$$

The consumers' contract (r_1^δ, r_2^δ) does not depend on μ or Q. The first-order condition (5), determining the investors' contract (r_1^d, r_2^d), implies $\mathrm{d}r_2^d/\mathrm{d}Q > 0$ for all μ if $-cu''(c)/u'(c) > 1$. Hence, condition (17) is more likely to hold if Q is large or λ is small.

The utility differential $Z_{\mathrm{C}} = (\lambda u(r_1^\delta) + (1-\lambda)\,u(r_2^\delta)) - (\lambda u(r_1^d) + (1-\lambda)\,u(r_2^d))$ is monotone in μ with

$$\frac{\mathrm{d}Z_{\mathrm{C}}}{\mathrm{d}\mu} = -\left(\lambda u'(r_1^d)\frac{\mathrm{d}r_1^d}{\mathrm{d}\mu} + (1-\lambda)u'(r_2^d)\frac{\mathrm{d}r_2^d}{\mathrm{d}\mu}\right) < 0 \tag{18}$$

as $\frac{\mathrm{d}r_1^d}{\mathrm{d}\mu}, \frac{\mathrm{d}r_2^d}{\mathrm{d}\mu} > 0$. The latter follows from applying the implicit function theorem to the investors' first-order condition (5). If written as

$$Qu'\left(r_1^d\right) - Ru'\left(\frac{R(1-\mu r_1^\delta)}{1-\mu}\right) = 0$$

we have

$$\frac{\mathrm{d}r_1^d}{\mathrm{d}\mu} = -\frac{-R^2 u''(r_2^d)\frac{1-r_1^d}{(1-\mu)^2}}{Q^2 u''(r_1^d) + R^2 u''(r_2^d)\frac{\mu}{1-\mu}} > 0,$$

and if written as

$$Qu'\left(Q\frac{R-(1-\mu)r_2^d}{\mu R}\right) - Ru'(r_2^d) = 0$$

we have

$$\frac{\mathrm{d}r_2^d}{\mathrm{d}\mu} = -\frac{u''(Qr_1^d)Q^2\frac{r_2^d-R}{\mu^2 R}}{-u''(Qr_1^d)Q^2\frac{1-\mu}{\mu R} - Ru''(r_2^d)} > 0.$$

By the intermediate value theorem, there is thus $\hat{\mu} \in]\lambda, 1[$ such that $(r_1^d, r_2^d) \succ_{\mathrm{C}} (r_1^\delta, r_2^\delta)$ for all $\mu \in]\hat{\mu}, 1[$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{C}} (r_1^d, r_2^d)$ for all $\mu \in]0, \hat{\mu}[$ if (17) holds and the claim is established. If (17) does not hold, then $(r_1^\delta, r_2^\delta) \succ_{\mathrm{C}} (r_1^d, r_2^d)$ for all $\mu \in]0, 1[$.

Claim 5 For $\mu \in]0, \bar{\mu}]$, there is no pooling contract which is a Pareto-improvement to $(r_1^d, r_2^d), (r_1^\delta, r_2^\delta)$.

The slope of the budget constraint for the pooling contract is between the slopes of the two budget constraints associated a separating equilibrium. A necessary condition for a pooling contract, which lies on the pooled budget constraint, to make consumers better off than (r_1^δ, r_2^δ) is thus that $r_1 < 1$, while a necessary condition for a pooling contract to make investors better off than (r_1^d, r_2^d) is that $r_1 > 1$. As these two condition rule each other out, there is no Pareto-improvement through pooling.

By claims 1 through 5, there is $\bar{\mu} = \min\left\{\mu \in]\lambda, 1[\middle| (r_1^\delta, r_2^\delta) \succsim_{\mathrm{I}} (r_1^d, r_2^d) \wedge (r_1^d, r_2^d) \succsim_{\mathrm{C}} (r_1^\delta, r_2^\delta)\right\}$ such that $(r_1^d, r_2^d) \succsim_{\mathrm{I}} (r_1^\delta, r_2^\delta)$ and $(r_1^\delta, r_2^\delta) \succsim_{\mathrm{C}} (r_1^d, r_2^d)$ if and only if $\mu \in]0, \bar{\mu}[$.

Claim 6 $-\frac{\mu}{1-\mu}\frac{u'(Qr_1^d)}{u'(r_2^d)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^d)}{u'(r_2^d)}$ and $-\frac{\mu}{1-\mu}\frac{u'(Qr_1^\delta)}{u'(r_2^\delta)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^\delta)}{u'(r_2^\delta)}$ obtains for all $\mu \in]0, \bar{\mu}]$.

The proof is by contradiction. Suppose μ is such that an equilibrium with constrained-efficient separating contracts exists, i.e. $(r_1^d, r_2^d) \succsim_{\mathrm{I}} (r_1^\delta, r_2^\delta)$ and $(r_1^\delta, r_2^\delta) \succsim_{\mathrm{C}} (r_1^d, r_2^d)$. If either $-\frac{\mu}{1-\mu}\frac{u'(Qr_1^d)}{u'(r_2^d)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^d)}{u'(r_2^d)}$ or $-\frac{\mu}{1-\mu}\frac{u'(Qr_1^\delta)}{u'(r_2^\delta)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^\delta)}{u'(r_2^\delta)}$ would not hold, **Assumption 2 implies** that either $(r_1^d, r_2^d) \succ_{\mathrm{C}} (r_1^\delta, r_2^\delta)$, or $(r_1^\delta, r_2^\delta) \succ_{\mathrm{I}} (r_1^d, r_2^d)$, or both, would necessarily hold.

C Proof of Proposition 2

For any given (r_1, r_2), the slope of the investors' indifference curve is

$$\frac{\mathrm{d}r_2}{\mathrm{d}r_1} = -\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q$$

and the slope of the consumers' indifference curve is

$$\frac{\mathrm{d}r_2}{\mathrm{d}r_1} = -\frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}.$$

By Assumption 2, if $(r_1^\delta, r_2^\delta) \sim_{\mathrm{I}} (r_1^d, r_2^d)$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{C}} (r_1^d, r_2^d)$, the consumers' indifference curve is steeper than the investors' indifference curve at $r_1 = r_1^\delta$ and $r_2 = r_2^\delta$, i.e.

$$-\frac{\bar{\mu}}{1-\bar{\mu}}\frac{u'(Qr_1^\delta)}{u'(r_2^\delta)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1^\delta)}{u'(r_2^\delta)}. \tag{19}$$

which together with (6) implies

$$-\frac{\bar{\mu}}{1-\bar{\mu}}\frac{u'(Qr_1^\delta)}{u'(r_2^\delta)}Q > -\frac{\lambda}{1-\lambda}R. \tag{20}$$

Let Z be defined by

$$Z{:=}(\mu u(Qr_1^d) + (1-\mu)\,u(r_2^d)) - (\mu u(Qr_1) + (1-\mu)\,u(r_2))$$

with $r_1^d = y^d/\mu$, $r_2^d = R(1-y^d)/(1-\mu)$, and y^d solves (5). By definition, $\mu = \bar{\mu}$ implies $Z = 0$ for $r_1 = r_1^\delta = y^\delta/\lambda$ and $r_2 = r_2^\delta = R(1-y^\delta)/(1-\lambda)$, with y^δ solving (6). Concavity of u thus implies that there is (r_1', r_2') with $r_1' < r_1^\delta = y^\delta/\lambda$ and $r_2' > r_2^\delta = R(1-y^\delta)/(1-\lambda)$, which are also feasible as they satisfy $r_2' = \frac{R(1-\lambda r_1')}{1-\lambda}$, and for which $Z = 0$ also holds. However, since (r_1^δ, r_2^δ) maximizes consumers' expected utility subject only to their feasibility constraint, $(r_1^\delta, r_2^\delta) \succ_{\mathrm{C}} (r_1', r_2')$. Hence, in response to a marginal increase in μ, starting from $\bar{\mu}$, consumers strictly prefer a marginal adjustment to a contract (r_1^δ, r_2^δ) over a marginal adjustment to a contract (r_1', r_2'). Therefore, banks offering marginal adjustment to a contract (r_1^δ, r_2^δ) will prevent other banks offering marginal adjustments to (r_1', r_2') from entering the market, even though both satisfy the investors' incentive constraint $Z = 0$.

Applying the implicit function theorem to $Z\!=\!0$ we obtain $\mathrm{d}r_2/\mathrm{d}\mu = -(\mathrm{d}Z/\mathrm{d}\mu)/(\mathrm{d}Z/\mathrm{d}r_2)$ with

$$\frac{\mathrm{d}Z}{\mathrm{d}\mu} = u(Qr_1^d) - u(Qr_1) + u(r_2) - u(r_2^d), \tag{21}$$

$$\frac{\mathrm{d}Z}{\mathrm{d}r_2} = \mu \frac{Q}{R} u'(Qr_1) \frac{1-\lambda}{\lambda} - (1-\mu) u'(r_2). \tag{22}$$

Equation (21) follows by taking into account the Envelope theorem, according to which the effects of changes in y^d, induced by changes in μ, have no effect as the first-order condition (5) applies. Equation (22) follows by taking into account the budget constraints, according to which $r_1 = (R - (1-\lambda)r_2)(\lambda R)^{-1}$. Evaluating (21) at $r_1 = r_1^\delta$ and $r_2 = r_2^\delta$ yields $\mathrm{d}Z/\mathrm{d}\mu < 0$ because $r_2^d > r_2^\delta$ and $r_1^d < r_1^\delta$. Evaluating (22) at $r_1 = r_1^\delta$ and $r_2 = r_2^\delta$ yields $\mathrm{d}Z/\mathrm{d}r_2 < 0$ because of (20). Hence, $\mathrm{d}r_2/\mathrm{d}\mu < 0$ and $\mathrm{d}r_1/\mathrm{d}\mu = -\left((1-\lambda)/\lambda R\right)(\mathrm{d}r_2/\mathrm{d}\mu) > 0$. By continuity, the result also applies to all $\mu > \bar{\mu}$ in some neighborhood of $\bar{\mu}$. Therefore, the Proposition obtains.

D Proof of Corollary 2

A necessary condition for equilibria with inflated consumer insurance to exist is $-\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q > -\frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}$ for all $(r_1, r_1) \in \mathbb{R}_+^2$. Consider two contracts (r_1^A, r_2^A) and (r_1^B, r_2^B) such that $(r_1^A, r_2^A) \sim_{\mathrm{I}} (r_1^B, r_2^B) \sim_{\mathrm{I}} (r_1^d, r_2^d)$, $r_2^A = R(1-\lambda r_1^A)/(1-\lambda)$, and $r_2^B = R(1-\lambda r_1^B)/(1-\lambda)$. As

$$\lim_{\gamma \to 1} R \frac{1-(\gamma\lambda + (1-\gamma)\mu)\, r_1}{1-(\gamma\lambda + (1-\gamma)\mu)} = R \frac{1-\lambda r_1}{1-\lambda}.$$

pooling contracts $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}})$ exist with $r_1^{\mathrm{P}} \in]r_1^A, r_1^B[$ and $r_2^{\mathrm{P}} = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1^{\mathrm{P}}}{1-(\gamma\lambda+(1-\gamma)\mu)}$ such that $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}) \succsim_{\mathrm{I}} (r_1^d, r_2^d)$ and $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}) \succsim_{\mathrm{C}} (r_1^A, r_2^A)$ as well as $(r_1^{\mathrm{P}}, r_2^{\mathrm{P}}) \succsim_{\mathrm{C}} (r_1^B, r_2^B)$.

E Proof of Proposition 3

The proof proceeds in five steps.

Step 1: Under the condition of Proposition 3, when $-\frac{\mu}{1-\mu}\frac{u'(Qr_1)}{u'(r_2)}Q < -\frac{\lambda}{1-\lambda}\frac{u'(r_1)}{u'(r_2)}$ for all (r_1, r_2) investors prefer switching their contracts across investor types, i.e. $(r_1^\delta, r_2^\delta) \succ_{\mathrm{I}} (r_1^d, r_2^d)$ and $(r_1^\delta, r_2^\delta) \succ_{\mathrm{C}} (r_1^d, r_2^d)$. Accordingly, a constrained-efficient separating equilibrium fails to exist.

Step 2: Pooling equilibria may occur only on the mixed inter-temporal budget line $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)}$. This intertemporal budget line intersect with the budget lines intended for each of the investor types only in $(1, R)$.

Step 3: If $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)$ the slope of the mixed inter-temporal budget line lies between the marginal rate of substitution of consumers at $(1, R)$ and investors. In this case for any contract $(\tilde{r}_1, \tilde{r}_2)$ on the mixed inter-temporal budget line with $\tilde{r}_1 < 1$ and $\tilde{r}_2 > R$, there exists a contract $(\hat{r}_1, \hat{r}_2)$ on the budget line for investors (i.e. with slope $\mu/(1-\mu)$) that is equivalent for consumer to $(\tilde{r}_1, \tilde{r}_2)$. Given the conditions on preferences $(\hat{r}_1, \hat{r}_2) \succ_{\mathrm{I}} (\tilde{r}_1, \tilde{r}_2)$. Hence $(\tilde{r}_1, \tilde{r}_2)$ cannot constitute an equilibrium contract.

Step 4: Analogously, if $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)$, then for any contract $(\tilde{r}_1, \tilde{r}_2)$ on the mixed-temporal budget line with $\tilde{r}_1 > 1$ and $\tilde{r}_2 < R$, there exists a contract $(\hat{r}_1, \hat{r}_2)$ on the budget line for consumers (i.e. with slope $\lambda/(1-\lambda)$) that is equivalent for investors to $(\tilde{r}_1, \tilde{r}_2)$. Given the conditions on preferences $(\hat{r}_1, \hat{r}_2) \succ_{\mathrm{C}} (\tilde{r}_1, \tilde{r}_2)$. Hence $(\tilde{r}_1, \tilde{r}_2)$ cannot constitute an equilibrium contract either.

Step 5: Accordingly, contract $(1,R)$ is the only contract that is feasible and not dominated by any other contract. This proves the claim of the Proposition.

F Proof of Proposition 4

The proof is similar to the Proof of Proposition 3. However, since in this case the condition $\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1)$ is violated, the only potential pooling contract $(1,R)$ is dominated by either (r_1^d, r_2^d) for investors (see Figure 6) or by (r_1^δ, r_2^δ) for consumers. Hence, no equilibrium contract obtains in this case. This proves the Proposition.

G Proof of Corollary 3

A necessary condition for pooling is

$$\frac{\mu}{1-\mu}u'(Q)Q > \frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R > \frac{\lambda}{1-\lambda}u'(1).$$

However,

$$\lim_{\gamma\to 0}\frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R = \frac{\mu}{1-\mu}u'(R)R > \frac{\mu}{1-\mu}u'(Q)Q$$

for $-cu''(c)/u'(c) > 1$. Therefore, for $\gamma \to 0$, $\gamma > 0$, Pareto-improving contracts to $(1,R)$ exist with $r_1 < 1$ and $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)} > R$.

Similarly,

$$\lim_{\gamma\to 1}\frac{\gamma\lambda+(1-\gamma)\mu}{1-(\gamma\lambda+(1-\gamma)\mu)}u'(R)R = \frac{\lambda}{1-\lambda}u'(R)R < \frac{\lambda}{1-\lambda}u'(1)$$

for $-cu''(c)/u'(c) > 1$. Therefore, for $\gamma \to 1$, $\gamma < 1$, Pareto-improving contracts to $(1,R)$ exist with $r_1 > 1$ and $r_2 = R\frac{1-(\gamma\lambda+(1-\gamma)\mu)r_1}{1-(\gamma\lambda+(1-\gamma)\mu)} < R$.