# Fiscal Regimes and the Exchange Rate

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**Abstract**

In this paper we argue that the effect of monetary and fiscal policies on the exchange rate depends on the fiscal regime. A contractionary monetary (expansionary fiscal) shock can lead to a depreciation, rather than an appreciation, of the domestic currency if debt is not backed by future surpluses. We propose a model of sovereign default in which foreign investors endure higher haircuts compared to domestic ones and fiscal policy shifts between Ricardian and non-Ricardian regimes. In the latter, the default probability drives the currency risk premium and policy shocks have unconventional effects on the exchange rate. We look at daily movements of the BRL/USD exchange rate around policy announcements and find strong support for the existence of two regimes with opposite signs. Consistent with our model, the unconventional response of the exchange rate is more likely to occur when economic agents expect weaker fiscal policies and their concern about debt sustainability rises.

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# 1 Introduction

The aim of this paper is to study the response of the exchange rate to monetary and fiscal policy. In particular, we are interested in exploring how the backing of government bonds, or lack thereof, determines the effect of policies on the exchange rate and, ultimately, on domestic macroeconomic variables. Following the early literature we make the distinction between Ricardian and non-Ricardian fiscal regimes (see for example Sargent (1982) and Aiyagari and Gertler (1985)). In the former, the fiscal authority provides full backing for its debt; at each point in time it commits to levying a stream of future taxes with a present discounted value equal to the current value of its obligation. On the other hand, in a non-Ricardian regime the fiscal authority does not fully finance its debt. In this case, either debt is monetised, ie the central bank accommodates fiscal deficits with current and future money creation, or the fiscal authority is forced to default. While the literature has largely focused on the former case, in this paper we focus on the latter.

Our analysis is both theoretical and empirical. In the theoretical part, we develop a small open economy model of sovereign default in which foreign investors endure higher haircuts than domestic investors. This assumption implies that the credit spread on government bonds is not sufficient to compensate foreign investors for the overall risk they face. This has two important consequences. First, the excess return required to compensate them for their additional risk must be generated through exchange rate movements. Hence, the probability of a sovereign default enters into the uncovered interest parity condition of the model and drives the currency risk premium. Second, the effective interest rates used to discount future primary surpluses are decreasing in the probability of default. Therefore the expected path of future default risk is determined endogenously by the government intertemporal budget constraint.

We use the model to characterise the response of the exchange rate to fiscal and monetary policy shocks in Ricardian and non-Ricardian regimes. In a Ricardian regime, an increase in government expenditures or in the domestic policy rate unambiguously leads to an appreciation of the domestic currency. In both cases, the domestic interest rate increases vis-a-vis the foreign interest rate, appreciating the exchange rate. On the other hand, in a non-Ricardian regime the increase in debt and the associated default probability raises the currency expected excess return and tends to depreciate the exchange rate. The use of continuous-time techniques allows us to provide analytical results, even in the case in which fiscal policy switches stochastically between the two regimes. We also show that the different behaviour of the exchange rate in the two regimes arises even if in the non-Ricardian one the central bank monetises the fiscal deficit to avoid default.

In the empirical part of the paper we test these predictions using Brazil as a case study. First, we use the daily forecasts of the primary balance and the monetary policy rate around policy announcements to construct time series of fiscal and monetary policy surprises. For each series, we then estimate a Markov switching regression model of the daily movement of the BRL/USD exchange rate around policy announcements. For both shocks, the estimated coefficients in the two regimes have opposite and significant signs. In response to expansionary fiscal shocks and to contractionary monetary policy shocks, the exchange rate appreciates in one regime while it depreciates in the other.

While these results are consistent with the predictions of our theoretical model, the empirical one does not inform about the structural differences across regimes. In the last step, we link the estimated probabilities to various measures of expectations about future fiscal policies. The probability of being in the regime associated with a depreciation is positively correlated with upward revisions in the current year expected primary deficit, interest payments and net debt, and with the CDS-implied probability of default. These results suggest that, consistent with the mechanism of the model, the unconventional response of the exchange rate to policy shocks is more likely to occur when market participants expect weaker fiscal policies and their concern about debt sustainability rises, features more closely associated with a non-Ricardian regime. .

The key feature of our model, and its main departure from the rest of the literature, is the unequal treatment of foreign and domestic investors upon default. Gelpern and Setser (2006) document how the evolution of debt structures and their legal characteristics leaves ample space for sovereigns to treat various creditor groups differently based on who they are rather than the instrument they hold. Historically, unequal treatment has taken many different forms. In the 1998 Ukraine treasury bills (OVDPs) exchange, domestic commercial banks and nonresident holders were offered different exchange options.[1] By comparing the net present value of old and new debt, Sturzenegger and Zettelmeyer (2008) (SZ) estimate that domestic investors endured an average haircut of 7% while nonresident investors were treated significantly worse and endured an average haircut of 56%. On the other hand, in Russia's 1998 default the offer to exchange ruble-denominated debt for cash and new longer-term instruments was open to all investors.[2] However, unlike domestic investors, foreigners had to deposit all proceeds in restricted accounts preventing them from converting the proceeds into foreign currency and taking them abroad. SZ estimate that through this exchange residents recovered 54% of their credits while nonresidents only 41%. Furthermore, many domestic investors obtained much better deals. Russian banks and Russian depositors that had invested in the defaulted securities indirectly through the banking system, were able to exchange their ruble debt holdings for dollar-denominated bonds, central bank paper (KBOs), and cash in full (see Gelpern and Setser (2006)). Similarly, in the 2001 Argentinian default all investors were offered to tender their dollar-denominated bonds in exchange for longer-term dollar loans issued under Argentinian law. However, the exchange was uniquely attractive to domestic banks and institutions since they could value the new instrument at par instead of its market price. Nearly all of the bonds held by Argentine financial institutions were tendered in the exchange. The new loans were redenominated in local currency a few months later. Non-resident investors refused the exchange and tendered in 2005 for a different set of instruments (see Sturzenegger and Zettelmeyer (2008)). SZ estimate that investors who tendered in the Phase 1 of the exchange, including the "Pesification", endured an average haircut of 66%, while nonresident investors who exchanged in 2005 endured an average haircut of 73%. Overall, while not a systematic feature of all sovereign defaults, violation of intercreditor equity can occur and when it does it is often at the disadvantage of foreign investors.

The rest of the paper is organised as follows. In Section 2 we review the relevant literature. In Section 3 we describe the model and in Section 4 we derive its main predictions. Section 5 presents the empirical evidence and Section 6 concludes.

[1] OVDPs are domestic-currency securities issued under Ukrainian law. Domestic banks were offered to exchange T-bills into longer-term domestic currency bonds of 3-6 years maturity discounted at the prevailing T-bill rate of about 60%. The interest rate on the new bonds was set at 40% for the first year, and a floating coupon equal to the future six-month T-bill yield plus 1 percentage point for the remainder of the period. Nonresident holders, on the other hand, were given the chance to exchange their t-bills for a domestic currency bond with a 22% hedged annual yield, or to receive a two-year zero coupon dollar denominated Eurobond with a yield of 20% (see Sturzenegger and Zettelmeyer (2008)).

[2] The exchange included GKOs, short-term zero-coupon ruble-denominated treasury securities governed by Russian law and OFZs, coupon-bearing ruble-denominated bonds governed by Russian law (see Gelpern and Setser (2006) for further information on the 1998 Russia's default episode).

## 2 Literature review

Standard international macroeconomic models predict that a monetary policy tightening leads to an appreciation of the domestic currency. The empirical evidence for advanced economies largely supports this prediction (see for example Eichenbaum and Evans (1995) for the US and Kim and Roubini (2000) for the other G7 countries). For emerging markets, the evidence is mixed. Kohlscheen (2014) investigates the impact of monetary policy shocks on the exchange rates of Brazil, Mexico and Chile and finds no significant impact. Hnatkovska, Lahiri, and Vegh (2008) examine cross-country data for 72 countries and find that while developed country currencies appreciate in response to a monetary tightening, developing economy currencies depreciate. To rationalize their findings, they propose a model in which a fiscal and a real channels dampen the impact of monetary policy on the long-run level of prices and therefore on the level of the nominal exchange rate. Our contribute to this literature is twofold. On the empirical side, we show that Brazilian data supports the existence of two regimes with opposite responses of the exchange rate. This might explain why unconditional regressions deliver insignificant results. On the theoretical side, we provide an explanation based on default risk and deviations from the uncovered interest parity condition, rather than on the long run level of the exchange rate. The idea was originally proposed by Blanchard (2004) but never formalized in a stochastic general equilibrium model.

Regarding fiscal policy, the prediction of a large class of models is that an expansionary fiscal shock leads to an appreciation of the domestic currency. The empirical evidence for advanced economies provides little support for this prediction. Monacelli and Perotti (2008), Kim and Roubini (2008) and Enders, Müller, and Scholl (2011) find that a US expansionary fiscal policy shock decreases the relative price of imports and depreciates the real exchange rate. Ravn, Schmitt-Grohé, and Martín Uribe (2012) confirm this findings in a panel VAR from four industrialized countries and propose a model where, under deep habits, an increase in government spending provides an incentive for firms selling in the domestic market to lower markups relative to foreign markups. For emerging economies the empirical evidence is scarce. Ilzetzki, Mendoza, and Végh (2013) show that in response to an increase in government consumption, the real exchange rate appreciates on impact by a statistically significant margin in developing countries, while it is barely affected in high-income countries. As before, our contribution to this literature is both empirical and theoretical.

Finally, the theoretical part of the paper is related to the sovereign default literature. The closest paper in this literature is Martín Uribe (2006). Like him, we propose a model in which the probability of default is determined endogenously by the government intertemporal budget constraint.[3] However, the mechanism developed in this paper is conceptually different. In Martín Uribe (2006), when the intertemporal budget constraint is violated, the government must immediately default on a fraction of its debt to reduce its current value. In our model default risk, rather than default itself, restores the equilibrium by changing the discount factor used to discount future primary surpluses. This implies that, while in Martín Uribe (2006) the equilibrium probability of default is unforecastable, in our model it is a predetermined variable known in advance to all creditors. Schabert and Wijnbergen (2014) follow a similar approach. However, in their model the central bank controls the interest rate on government bonds and an increase in the default probabilities reduce the implicit risk-free rate of the economy, boosting aggregate demand and prices. Thus, default restores debt sustainability through inflation, like in models of the fiscal theory of the price level (see for example Leeper (1991), Sims (1994) and Woodford (2001)). In fact, in their model an equilibrium with default is possible only if monetary policy is passive. As we shall see below, in

[3] This is in contrast with the strategic default approach, pioneered by Eaton and Gersovitz (1981), where default is a policy variable optimally chosen by the government.

our model an equilibrium with default can arise only if monetary policy is active.

## 3 A Small Open Economy Model

Consider a small open economy model with infinite horizon. The world economy is composed of a continuum of countries, indexed by $v \in [0,1]$. The focus of this paper is on the equilibrium of a single economy which we call "Home" and can be thought of as a particular value of $H \in [0,1]$. To simplify the analysis, we assume that all foreign countries are identical at all points in time.[4] We treat them as a unique country, which we call "Foreign", and denote its variables with a star superscript. Home is inhabited by a measure one of households that consume and work for domestic firms producing tradable goods. The public sector is composed of a monetary authority, which we call central bank, that sets the interest rate on the domestic-currency riskless bond and a fiscal authority, which we call government, that taxes, borrows and spends.

For analytical convenience, the model is developed in continuous time. In the next subsections, we describe the problems faced by households and firms located in Home. Unless noted otherwise, the problems faced by Foreign agents are symmetric. We then describe the decision of foreign investors and domestic policies. We conclude this section by characterizing the equilibrium of the model and its log-linear dynamics around the steady state.

### Households

Home is inhabited by a measure one of identical households. The representative household maximizes

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\ln C(t) - \frac{L(t)^{1+\varphi}}{1+\varphi}\right)dt\right] \tag{1}$$

where $C$ is consumption and $L$ is the amount of labor supplied. The parameter $\rho > 0$ is the time discount factor, and $\varphi$ the inverse of the Frisch elasticity of labor supply. The consumption index $C$ is a composite of Home and imported goods, given by $C(t) \equiv C_H(t)^{1-\alpha} C_F(t)^{\alpha}(1-\alpha)^{-1+\alpha}\alpha^{-\alpha}$, where $\alpha \in [0,1]$ is the degree of Home bias in consumption. The imported goods index $C_F$ is itself an aggregator of goods produced in different countries and it is defined by $C_F(t) \equiv \exp\int_0^1 \ln C_v(t)\,dv$. The optimal allocation of expenditure across domestic and foreign goods yields the following demand function

$$C_H(t) = (1-\alpha)\left(\frac{P_H(t)}{P(t)}\right)^{-1} C(t)$$

where $P_H$ is the domestic Producer Price Index (PPI). $P$ is the domestic Consumer Price Index (CPI), which is given by $P(t) \equiv P_H(t)\,\mathcal{S}(t)^{\alpha}$ where $\mathcal{S}(t) \equiv P_F(t)/P_H(t)$ denotes the Home terms of trade.

Home households have access to a zero-net-supply riskless bond that pays the Home monetary policy rate $i$.[5] Households can also save in domestic and foreign currency bonds issued by the Home government which pay the rate of return $i_H$ and $i_F$, respectively, but are subject to default

[4]The former assumption allows us to abstract from foreign disturbances, while the latter allows us to keep track of only one set of international prices rather than a continuum of bilateral prices.

[5]The central bank affects the interest rate on the riskless nominal bond by changing the growth rate of money supply, through a no-arbitrage condition. This can be modelled formally by introducing money in the utility function or through a cash-in-advance constraint. Here we directly focus on the cashless limit of such economies. To be clear, the central bank does not issue the riskless asset. This would be inconsistent with the assumption that debt issued by the fiscal authority is subject to default risk.

risk. Let $B_H$ denote the amount of Home-currency government bonds, in units of the Home good, and $B_F$ the amount of Foreign-currency government bonds, in units of the Foreign good, held by the representative Home household. Finally, we assume that Home households can hold bonds issued by Foreign households but they are subject to a friction that delays portfolio adjustments. Due to this friction, Home households holding of foreign assets has only second order effects on the equilibrium of the model and therefore disappears in its log-linearised version.[6] We denote with $A_F$ the value, in units of the Foreign good, of the portfolio of foreign bonds held by the representative Home household.

Let $A$ denote the value of the portfolio of assets held by Home households, $W$ the wage rate and $\Upsilon$ profits received from domestic firms, all in units of the domestic good. Then, the dynamic budget constraint of the representative household is

$$
\begin{aligned}
dA(t) = {} & [A(t)(i(t) - \pi_H(t)) + W(t)L(t) + \Upsilon(t) - C(t)\mathcal{S}(t)^{\alpha} - T(t)]\,dt \\
& + B_H(t)[dB_H(t)/B_H(t) - (i(t) - \pi_H(t))\,dt] \\
& + B_F(t)\mathcal{S}(t)[d(B_F(t)\mathcal{S}(t))/(B_F(t)\mathcal{S}(t)) - (i(t) - \pi_H(t))\,dt] \\
& + A_F(t)\mathcal{S}(t)[d(A_F(t)\mathcal{S}(t))/(A_F(t)\mathcal{S}(t)) - (i(t) - \pi_H(t))\,dt]
\end{aligned}
\tag{2}
$$

where $\pi_H(t) \equiv dP_H(t)/P_H(t)$ is PPI inflation and $T$ are lump-sum taxes. The second and third lines describe the excess return of the households portfolio of government bonds, where $dB_H(t)/B_H(t)$ is the return of the domestic-currency bond and $d(B_F(t)\mathcal{S}(t))/(B_F(t)\mathcal{S}(t))$ is the domestic-currency return of the foreign-currency bond. Their laws of motion together with the optimal portfolio decision of the households will be described later.

The problem of the representative household is to choose consumption, savings, and labor to maximize (1) subject to the budget constraint (2) and the no-Ponzi game condition $\lim_{k\to+\infty}\mathbb{E}\left[e^{\int_0^k (i(t)-\pi_H(t))dt}A(k)\right] \geq 0$. Her optimal consumption/saving policy is described by the Euler equation

$$
\mathbb{E}\left[\frac{dC(t)}{C(t)}\right] = (i(t) - \pi(t) - \rho + h.o.t.)\,dt \tag{3}
$$

where $\pi(t) \equiv dP(t)/P(t)$ is CPI inflation and $h.o.t.$ denotes higher-order terms which vanish in the log-linearisation and are therefore omitted for simplicity. The complete equation is reported in the appendix. Finally, her labor supply schedule is $W(t) = L(t)^{\varphi}C(t)\mathcal{S}(t)^{\alpha}$.

Foreign households have identical preferences and solve a symmetric problem. Their Euler equation is $dC^*(t)/C^*(t) = (i^*(t) - \pi^*(t) - \rho^* + h.o.t.)\,dt$ while their demand function for the Home good is $C_H^*(t) = \alpha(P^*(t)/P_H^*(t))C^*(t)$, where $P_H^*$ is the Foreign-currency price of the Home good. We assume that there is full exchange rate pass-through to both import and export prices such that $P_F(t) = \mathcal{E}(t)P^*$ and $P_H^*(t) = P_H(t)/\mathcal{E}(t)$, where $\mathcal{E}$ is the nominal exchange rate

[6] This friction can take the form of an adjustment cost or infrequent adjustments. It is meant to capture the attrition involved in trading in international financial markets. To simplify the algebra, we directly assume that the strength of the friction is maximal and Home households hold a fixed portfolio of Foreign bonds which is equal, both in size and composition, to the steady-state portfolio of Home government bonds held by Foreign investors. This assumption allows us to solve for a symmetric steady state, that is with a zero net foreign asset position, in which a fraction of the Home government debt is held by foreign investors. Furthermore, it prevents unexpected time-zero shocks to have first-order country-wide wealth effects. While solving the model around a symmetric steady state is not necessary for our results, it allows us to directly compare our model with standard ones in the literature which are typically solved around a symmetric steady state (see for example Galí and Monacelli (2005)). Similarly, preventing first-order wealth effects is not crucial and it actually weakens our results. For example, assume that all foreign assets held by Home households are denominated in Foreign currency. Then, a depreciation (appreciation) of the exchange rate would generate a negative (positive) wealth effect for the Home country which would further depreciate (appreciate) the exchange rate.

between the Home country and the rest of the world defined as the Home currency price of one unit of Foreign currency. A decrease in $\mathcal{E}$ corresponds to an appreciation of the domestic currency. The real exchange rate is defined as $\mathcal{Q}(t) \equiv \mathcal{E}(t) P^{*}(t) / P(t)$.

### Firms

The Home production sector is composed of intermediate firms and retailers. Intermediate firms hire labor from domestic households to produce a continuum of differentiated goods, indexed by $j \in [0,1]$. Retailers combine intermediate goods to produce the Home good purchased by domestic and foreign households.

The retail sector is competitive and is composed of a measure one of homogeneous firms. Their aggregate production function is described by the constant elasticity of substitution aggregator $Y(t) \equiv \left[\int_0^1 Y_j(t)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$, where the parameter $\epsilon > 1$ measures the elasticity of substitution across intermediate goods. Thus, their demand function for variety $j \in [0,1]$ is given by

$$Y_j(t) = \left(\frac{P_{H,j}(t)}{P_H(t)}\right)^{-\epsilon} Y(t) \tag{4}$$

while the domestic PPI is $P_H(t) \equiv \left(\int_0^1 P_{H,j}(t)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$.

Intermediate good firms are monopolistically competitive. While each of them produce a differentiated good, they all use the same technology described by the production function

$$Y_j(t) = L_j(t) \tag{5}$$

Each firm faces an identical isoelastic demand schedule for its own good, given by (4), and set prices infrequently a la Calvo (1983). Each firm is allowed to reset its price only at stochastic dates determined by a Poisson process with intensity $\theta$. A firm that resets at time $t$ chooses its price $\hat{P}_{H,j}$ to maximize the present discounted value of its stream of profits

$$\mathbb{E}\left[\int_t^{\infty} \theta e^{-(\rho+\theta)(k-t)} \frac{C(t) P(t)}{C(k) P(k)} \left\{\hat{P}_{H,j}(t) - (1-\tau(t)) W(k) P_H(k)\right\} Y_j(k|t)\, dk\right]$$

where $Y_j(k|t) = \left(\hat{P}_{H,j}(t) / P_H(k)\right)^{-\epsilon} Y(k)$ and $\tau$ is a labor subsidy which is set by the policymaker to maximize welfare in the flexible prices equilibrium.[7] The firms optimal price-setting behaviour implies that PPI inflation evolves as

$$\mathbb{E}[d\pi_H(t)] = [(\epsilon-1)\pi_H(t) - \theta]\left\{\pi_H(t) + \frac{P_H(t) Y(t)}{P(t) C(t)}\left[\mathcal{M}(1-\tau(t))\frac{W(t)}{\mathcal{U}(t)} - \frac{1}{\mathcal{V}(t)}\right] + h.o.t.\right\} dt$$

where $\mathcal{U}$ and $\mathcal{V}$ are the present discounted values of future costs and revenues, respectively. Their equations and laws of motion are reported in the appendix.

[7] The optimal flexible-prices labor subsidy is $\tau(t) = 1 - \frac{\epsilon}{\epsilon-1}\left(1 + \frac{\alpha}{1-\alpha}\frac{\mathcal{Q}(t)C^{*}(t)}{C(t)}\right)$. To reduce notation, we assume that the tax needed to finance it is levied lump-sum directly from firms.

### Foreign investors and no-arbitrage conditions

Foreign investors, like domestic households, can invest in both bonds issued by the Home government.[8] Let $B_H^*$ denote the amount of Home-currency bonds, in units of the Home good, and $B_F^*$ the amount of Foreign-currency bonds, in units of the Foreign good, held by Foreign investors.

Government bonds are subject to default risk. We model sovereign default as a random event with endogenous probability. Formally, we assume that the time of default is stochastic and distributed according to a Poisson process $\mathcal{P}$ with time-varying intensity $\eta(t)$. This implies that over the interval of time $[t, t+dt)$ the government defaults with probability $\eta(t)\,dt$. The equilibrium default intensity is determined endogenously by the intertemporal budget constraint of the fiscal authority, as will be described in the next subsection.

We assume that default is non-selective, that is it involves all securities issued by the government,[9] but creditors are ex-post treated unequally. In line with the narrative evidence provided in the introduction, we assume that Foreign investors endure higher haircuts. Upon default Home households are able to recover a fraction $\chi$ of their credit while Foreign investors can only recover a fraction $\chi^* < \chi$. This assumption implies that the returns of the assets are different for domestic households and foreign investors. Home households face the following return processes

$$
\begin{aligned}
dB_H(t)/B_H(t) &= (i_H(t) - \pi_H(t))\,dt - (1-\chi)\,d\mathcal{P}(t) \\
dB_F(t)/B_F(t) &= (i_F(t) - \pi^*(t))\,dt - (1-\chi)\,d\mathcal{P}(t)
\end{aligned}
$$

while Foreign investors face

$$
\begin{aligned}
dB_H^*(t)/B_H^*(t) &= (i_H(t) - \pi_H(t))\,dt - (1-\chi^*)\,d\mathcal{P}(t) \\
dB_F^*(t)/B_F^*(t) &= (i_F(t) - \pi^*(t))\,dt - (1-\chi^*)\,d\mathcal{P}(t)
\end{aligned}
$$

The portfolio choices of Home households and Foreign investors give rise to the no-arbitrage conditions

$$i_H(t) - i(t) = (1-\chi)\,\eta(t) + h.o.t. \tag{6}$$

$$i_F(t) + \mathbb{E}\left[\frac{d\mathcal{E}(t)}{\mathcal{E}(t)}\right] - i(t) = (1-\chi)\,\eta(t) + h.o.t. \tag{7}$$

and

$$i_H(t) - \mathbb{E}\left[\frac{d\mathcal{E}(t)}{\mathcal{E}(t)}\right] - i^*(t) = (1-\chi^*)\,\eta(t) + h.o.t. \tag{8}$$

$$i_F(t) - i^*(t) = (1-\chi^*)\,\eta(t) + h.o.t. \tag{9}$$

Equation (6) is the Home household's no-arbitrage condition between the domestic riskless asset and the Home-currency government bond. Households require a default premium over the riskless rate to compensate for the risk of default, captured by the term $(1-\chi)\,\eta(t)$, and a risk premium which

[8] Similar to what we assume for Home households, Foreign investors can hold bonds issued by Home households and they are also subject to a friction that delays portfolio adjustment. Due to this friction, Foreign investors holding of the Home risk-free asset has only second order effects on the equilibrium and therefore disappears in the log-linearization. To simplify the algebra, we directly assume that the strength of the friction is maximal and Foreign investors do not hold any Home risk-free bond.

[9] While we could allow the government to default independently on Home- and Foreign-currency bonds without altering our results, the assumption of non-selective default is closer to the data. As reported by Mallucci (2015), non-selective defaults are the norm and represents 55% of the sovereign default episodes between 1990 and 2005.

is given by the covariance between their stochastic discount factor and the default process. The risk premium is second order and therefore it is included in the $h$(igher) $o$(rder) $t$(erm) term. The complete equations are reported in the appendix. Equation (7) is the no-arbitrage condition between the domestic riskless asset and the Foreign-currency government bond, which again contains a default and a risk premium. However, since the bond is denominated in foreign currency, its return also depends on the behaviour of the exchange rate. Similarly, equations (8) and (9) are the Foreign investors' no-arbitrage conditions between the Foreign riskless asset and the Home government bonds. By combining (6) and (8), or (7) and (9), we obtain the Uncovered Interest rate Parity (UIP) of the model

$$\mathbb{E}\left[\frac{d\mathcal{E}(t)}{\mathcal{E}(t)}\right] = i(t) - i^*(t) - (\chi - \chi^*)\,\eta(t) + h.o.t. \tag{10}$$

This equation highlights the link between default risk and exchange rate which will be at the core of the analysis. Bonds issued by the Home government must pay a premium to compensate both Home and Foreign holders for the probability of default. However, since the default risk faced by domestic and foreign creditors is different, part of this premium is generated through a spread over the domestic riskless rate, while part of it is generated by movements of the exchange rate. Thus, default risk drives the currency risk premium. Since Foreign investors have lower recovery rates, they demand a higher default premium. When the probability of default increases the domestic currency depreciates on the spot in order to generate an expected appreciation, that is $\mathbb{E}\left[d\mathcal{E}(t)/\mathcal{E}(t)\right] < 0$. The expected appreciation increases the foreign-currency expected return of the Home-currency government bond and compensate Foreign investors for the increased default risk.

This mechanism is consistent with the empirical evidence on the dynamics of international portfolios in response to changes in sovereign risk. Converse and Mallucci (2019) show that changes in yields do not fully compensate foreign investors for additional sovereign risk, and that international bond mutual funds reduce their exposure to a country's assets when the sovereign default risk increases. Andritzky (2012) and Broner et al. (2014) document how foreign investors reduced their holding of government securities during the global financial crisis and the European debt crisis, while domestic investors increased them.

## Public sector

The Home public sector is composed of a monetary authority and a fiscal authority. The fiscal authority must finance a stream of expenditure given by $G(t) = \varepsilon_g(t)$, where $\varepsilon_g$ follows an autoregressive stochastic process. To finance its expenditure, the government levies taxes on domestic households and borrows. Let $B(t)$ denote the total amount of government debt outstanding expressed in units of domestic output. Market clearing requires $B(t) = B_H(t) + B_H^*(t) + \mathcal{S}(t)\left(B_F(t) + B_F^*(t)\right)$. Therefore, the budget constraint of the fiscal authority is:

$$dB(t) = (G(t) - T(t))\,dt + dB_H(t) + dB_H^*(t) + d\left(\mathcal{S}(t)\,B_F(t)\right) + d\left(\mathcal{S}(t)\,B_F^*(t)\right) \tag{11}$$

By using the no-arbitrage conditions (6)-(8) we can rewrite the budget constraint of the government as:

$$\mathbb{E}\left[dB(t)\right] = \left[B(t)\left(i(t) - \pi_H(t) - \xi(t)(\chi - \chi^*)\,\eta(t) + h.o.t.\right) + G(t) - T(t)\right]dt \tag{12}$$

where $\xi(t) \equiv \left(B_H^*(t) + \mathcal{S}(t)\,B_F^*(t)\right)/B(t)$ is the share of debt held by foreign investors. By iterating it forward and using the transversality condition

$\lim_{k\to+\infty}\mathbb{E}\left[e^{-\int_t^k (i(z)-\pi_H(z)-(\chi-\chi^*)\xi(z)\eta(z)+h.o.t.)dz}B\left(k\right)\right] = 0$ we obtain the intertemporal budget constraint:

$$B\left(t\right)=\mathbb{E}\left[\int_t^{\infty} e^{-\int_t^k (i(z)-\pi_H(z)-\xi(z)(\chi-\chi^*)\eta(z)+h.o.t.)dz}\left(T\left(k\right)-G\left(k\right)\right)dk\right] \tag{13}$$

The intertemporal budget constraint requires that, at any point in time, the value of debt outstanding is equal to the present discounted value of future primary surpluses. Whenever the budget constraint is violated one of three things must occur. The fiscal authority can adjust taxes or expenditures to raise expected future surpluses. This is the conventional equilibrium featured in most models. Alternatively, the central bank might cut its policy rate to reduce future real rates and increase the present value of future surpluses. This is the equilibrium analyzed in the fiscal theory of the price level (FTPL) literature. Finally, if neither fiscal policy nor monetary policy can ensure intertemporal solvency then the current value of debt must fall. That is, the government is forced to default instantaneously on a fraction of its debt. This is the equilibrium developed by Martín Uribe (2006) is his fiscal theory of sovereign risk (FTSR) and then used in the fiscal limit literature.

Our model features a fourth mechanism that is distinct from the others analyzed in the literature. As it is clear from equation (13), the default probability $\eta$ affects the effective expected real interest rate used to discount future surpluses. An increase in the default probability increases the discount factor and raises the present value of future primary surpluses. Hence, whenever the budget constraint is violated and policies do not ensure intertemporal solvency, the expected path of the probability of default adjust to make (13) hold. Note that this mechanism is conceptually different from the FTSR approach. In Martín Uribe (2006) default restores the equilibrium by reducing the left-hand side of equation (13). In our model default risk, rather than default itself, restores the equilibrium by increasing the discount factor.[10] The FTPL works in a similar way. A passive monetary policy restores the equilibrium by reducing real rates and increasing the discount factors. Unlike the FTPL, however, our model does not require monetary policy to be passive. Indeed, as we shall see below, in our model an equilibrium with default can arise only if monetary policy is active. When debt is inflated away, the probability of default is always zero.

Note that the default probability enters equation (13) proportionally to the fraction of debt held by foreigners and the gap in the recovery rates of domestic and foreign investors. This is because the default probability affect the discount factor associated with external debt, while it does not affect the one associated with debt held domestically. In other words, the default probability affects the effective interest rate, ie the return on debt minus the probability of default, paid by the government on debt held by foreigners, but not the one paid on debt held by domestic agents. When the probability of default rises, the yield on government bonds rises proportionally to compensate domestic investors for the increased risk of default. The two effects offset each other, leaving the effective interest rate on domestic debt unchanged. On the other hand, the additional default risk faced by foreign investors is generated through an expected appreciation of the Home currency, rather than through an increase in yield. Hence, the effective interest rate paid on external debt falls.

[10] A similar distinction between initial value and discount rates arises in the FTPL literature, but the comparison is misleading. When monetary policy is passive, in discrete-time models both the initial price level and expected inflation rise in response to an increase in debt. However, in a continuous time models only the discount factor effect is present since the price level cannot jump. The different role played by default in our model and in the FTSR literature is not due to the timing convention but, rather, on the assumption of different recovery rates between domestic and foreign bond holders.

To simplify the analysis, we assume that the probability of default depends only on the amount of debt outstanding. This feature is largely supported by the data, as documented by Edwards (1984), Eichengreen and Mody (2000), Martin Uribe and V. Z. Yue (2006) and Aizenman, Jinjarak, and Park (2013), among others. Furthermore, it is consistent with many models of strategic default which predicts that the likelihood of a default is increasing in the real level of deb (see, for example, Aguiar and Gopinath (2006), Arellano (2008) and V. Yue (2010)). Finally, since $\eta$ drives the currency expected excess return, our assumption is consistent with the empirical evidence on country imbalances and currency risk premia documented by Corte, Riddiough, and Sarno (2016). Formally, we assume

$$\eta(t) = \max\left\{0, \bar{\eta} + \eta^x \frac{B(t) - \bar{B}}{\bar{B}}\right\} \tag{14}$$

with $\eta^x \geq 0$ where $x \in \{R, N\}$ denotes the fiscal regime, and $\bar{B}$ denotes the steady-state level of government debt. The parameter $\bar{\eta}$ is assumed to be strictly positive, but negligible, and is introduced to facilitate the linearization (14). This functional form simplifies the determination of the equilibrium default probability since the problem of finding a process $\eta(t)$ that makes (13) hold boils down to a single parameter, $\eta^x$. The elasticity of the default probability with respect to debt is the endogenous object that adjusts to satisfy the government intertemporal budget constraint. As such, it depends on the fiscal regime considered.

We close the model by specifying monetary and fiscal policies. The monetary authority sets the interest rate on the domestic-currency risk-free nominal bond which is assumed to follow the simple Taylor rule

$$i(t) = [\rho + (1 + \phi_\pi)\pi_H(t)] + \varepsilon_i(t) \tag{15}$$

where the parameter $\phi_\pi > 0$ measures the responsiveness of the policy rate to inflation, and $\varepsilon_i$ is an exogenous component of monetary policy that follows an autoregressive stochastic process. When $\phi_\pi > 0$ monetary policy reacts to inflation by rising the real interest rate and, in the characterisation by Leeper (1991), is labelled as 'active'. In what follows we assume that the central bank sticks to its inflation stabilisation mandate, regardless of the fiscal regime.

Finally, in the spirit of Bohn (1998), we assume that the government follows a simple tax policy described by the fiscal rule

$$T(t) - \bar{T} = \psi_b^x \left(B(t) - \bar{B}\right) + \psi_\pi^x \phi_\pi \pi_H(t)\bar{B} \tag{16}$$

where $\bar{T}$ denotes the steady-state level of taxes. The parameter $\psi_b^x \geq 0$ measures the responsiveness of taxes to changes in government debt. If $\psi_b^x$ is sufficiently high, an increase in debt leads the government to raise enough taxes to cover the higher debt servicing cost and repay part of the principal. When this is the case, debt tends to return to its steady state level and its dynamic is stable. If on the other hand $\psi_b^x$ is low, the increase in taxes is not sufficient to cover the higher debt servicing cost which might cause debt to grow unboundedly. Following the language set forth by Leeper (1991), in the former case fiscal policy is said to be 'passive' while in the latter it is said to be 'active'. Proposition (2) below derives the threshold above (below) which fiscal policy is passive (active) and characterises the possible fiscal regimes of the model. The parameter $\psi_\pi^x \in \{0, 1\}$ marks inflation indexation. Note that an increase in expected inflation raises the debt servicing cost at rate $(1 + \phi_\pi)\pi_H$ while it reduces the real burden of the nominal stock of debt at rate $\pi_H$. Hence, the net effect of inflation on the dynamics of real debt is $\phi_\pi \pi_H$. When $\psi_\pi^x = 1$ this effect is neutralized. We will set $\psi_\pi^x = 0$ or $\psi_\pi^x = 1$ in order to better separate the effects of default and inflation on the dynamics of debt.

### Equilibrium

To reduce the number of equilibrium variables, let $\Lambda(t) \equiv C(t) / (\mathcal{Q}(t) C^*(t))$ denote the wedge between the marginal rate of substitution between Home and Foreign consumption and their marginal rate of transformation, ie the real exchange rate. Using equations (3) and (10) we can derive its law of motion:

$$\mathbb{E}[d\Lambda(t)] = \Lambda(t)[\rho^* - \rho + (\chi - \chi^*)\eta(t) + h.o.t.]\,dt \tag{17}$$

Let $Y(t) \equiv \left[\int_0^1 Y_j(t)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ be aggregate domestic output. Market clearing in the goods market requires $Y(t) = C_H(t) + C_H^*(t) + G(t)$. Therefore, output evolves as

$$\mathbb{E}[dY(t)] = (Y(t) - G(t))\left[i(t) - \pi_H(t) - \rho - \alpha\frac{\rho^* - \rho + (\chi - \chi^*)\eta(t)}{(1-\alpha)\Lambda(t) + \alpha} + h.o.t.\right]dt + \mathbb{E}[d\varepsilon_g(t)] \tag{18}$$

and the terms of trades are given by $\mathcal{S}(t) = [\alpha + (1-\alpha)\Lambda(t)]^{-1}(Y(t) - G(t))/C^*(t)$. The labor market clearing condition is $L(t) = \int_0^1 L_j(t)\,dj = \Delta(t)Y(t)$ where $\Delta$ is and index of price dispersion whose equation and law of motion are reported in the appendix. Hence, the law of motion of PPI inflation can be rewritten as

$$\mathbb{E}[d\pi_H(t)] = [(\epsilon-1)\pi_H(t) - \theta]\left\{\pi_H(t) + \left(\frac{\alpha}{\Lambda(t)} + 1 - \alpha\right)\left[\frac{\Delta(t)^{\varphi}Y(t)^{1+\varphi}}{(1-\alpha)\mathcal{U}(t)} - \frac{Y(t)/\mathcal{V}(t)}{Y(t) - G(t)}\right] + h.o.t.\right\}dt \tag{19}$$

Finally, let $Z(t) \equiv (B(t) - A(t)) / (\mathcal{S}(t) C^*(t))$ be the value of Home net foreign debt, in units of the Foreign good. By combining (2), (12), (10) we obtain the Home country's dynamic budget constraint

$$\mathbb{E}[dZ(t)] = [Z(t)\rho^* + \alpha(\Lambda(t) - 1)]\,dt \tag{20}$$

which is subject to the transversality condition $\lim_{k\to+\infty}\mathbb{E}\left[e^{-\int_0^k(i^*(t)-\pi^*(t))dt}Z(k)\right] = 0$.

We close the model by specifying the laws of motion of the exogenous variables. Let $\mathcal{B}^g$ and $\mathcal{B}^i$ two independent standard Brownian motions.[11] Then, $\varepsilon_g$ and $\varepsilon_i$ evolve according to the Ornstein-Uhlenbeck processes

$$d\varepsilon_g(t) = -\varrho\varepsilon_g(t)\,dt + \nu d\mathcal{B}_g(t) \tag{21}$$
$$d\varepsilon_i(t) = -\varrho\varepsilon_i(t)\,dt + \nu d\mathcal{B}_i(t) \tag{22}$$

where $\varrho > 0$ governs the speed of their mean-reversion while $\nu > 0$ their volatility.

**Definition 1.** An equilibrium of the model is a collection $\{\eta^x, \Lambda(t), Y(t), \pi_H(t), B(t), B_H(t), B_F(t), Z(t), \mathcal{U}(t), \mathcal{V}(t), \Delta(t), \varepsilon_g(t), \varepsilon_i(t)\}$ that satisfies (6), (7), (13) and the differential equations (12), (17), (18), (19), (20), (21), (22), (A.45), (A.46), (A.48), subject to (14), (15) and (16), given foreign variables $\{C^*(t), P^*(t), i^*(t)\}$, shocks $\{\mathcal{B}_g(t), \mathcal{B}_i(t), \mathcal{P}(t)\}$ and initial conditions $\{B_H(0), B_F(0), B_H^*(0), B_F^*(0), Z(0), \varepsilon_g(0), \varepsilon_i(0)\}$.

[11] The Brownian motions $\mathcal{B}^g$ and $\mathcal{B}^i$, and the Possion process $\mathcal{P}$ are defined on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}(t)\}, \mathbb{P})$. All stochastic processes are assumed to be adapted to $\{\mathcal{F}(t)\}_{t=0}^{\infty}$, the augmented filtration generated by $\{\mathcal{B}^g, \mathcal{B}^i, \mathcal{P}\}$. In what follows, we assume all regularity conditions which ensure that all processes introduced are well defined.

As common in the New Keynesian literature, we approximate the equilibrium dynamics of the model using a log-linear expansion around its deterministic steady state. We focus on a symmetric steady state in which the Home government is a net debtor and only part of its debt is held domestically.[12] We then use the log-linear approximation to study the response of the model to an unexpected increase in government spending and an unexpected tightening of the monetary policy rate. In what follows, a lowercase letter denotes the percentage deviation of the variable from its steady-state value.

Let $\xi \equiv \left(\bar{B}_H^* + \bar{\mathcal{S}}\bar{B}_F^*\right)/\bar{B}$ and $\iota \equiv \left(\bar{B}_F + \bar{B}_F^*\right)\bar{\mathcal{S}}/\bar{B}$ denote the steady-state share of external and foreign-currency debt, respectively. The equilibrium dynamics of the model around its deterministic symmetric steady state are described by the system of differential equations

$$\mathbb{E}\left[d\lambda\left(t\right)\right] = \left[\tilde{\eta}^x b\left(t\right)\right]dt \tag{23}$$

$$\mathbb{E}\left[dy\left(t\right)\right] = \left[\phi_\pi \pi_H\left(t\right) - \alpha\tilde{\eta}^x b\left(t\right) - \varrho_g \varepsilon_g\left(t\right) + \varepsilon_i\left(t\right)\right]dt \tag{24}$$

$$\mathbb{E}\left[d\pi_H\left(t\right)\right] = \left[\rho\pi_H\left(t\right) - \kappa\omega y\left(t\right) + \kappa\varepsilon_g\left(t\right)\right]dt \tag{25}$$

$$\mathbb{E}\left[db\left(t\right)\right] = \left[\left(\rho - \psi_b^x - \xi\tilde{\eta}^x\right)b\left(t\right) + \left(1 - \psi_\pi^x\right)\phi_\pi\pi_H\left(t\right) + \beta\varepsilon_g\left(t\right) + \varepsilon_i\left(t\right)\right]dt \tag{26}$$

$$\mathbb{E}\left[dz\left(t\right)\right] = \left[\rho z\left(t\right) + \alpha\lambda\left(t\right)\right]dt \tag{27}$$

and by the initial conditions $b\left(0\right) = \iota e\left(0\right)$ and $z\left(0\right) = 0$, where $\tilde{\eta}^x \equiv \left(\chi - \chi^*\right)\eta^x$, $\kappa \equiv \theta\left(\rho + \theta\right)$ and $\omega \equiv 1 + \varphi$, while $\beta^{-1} \equiv \bar{B}/\bar{Y}$ is the steady-state debt-to-GDP ratio. The exogenous variables evolve as

$$\mathbb{E}\left[d\varepsilon_g\left(t\right)\right] = -\varrho\varepsilon_g\left(t\right)dt \tag{28}$$

$$\mathbb{E}\left[d\varepsilon_i\left(t\right)\right] = -\varrho\varepsilon_i\left(t\right)dt \tag{29}$$

with $\varepsilon_g\left(0\right) > 0$ and $\varepsilon_i\left(0\right) > 0$. We focus on these two shocks since both increase the growth rate of government debt and tend to increase the probability of default, provided $\eta^x > 0$. This allows us to avoid the kink featured in equation (14) and approximate it with $\eta\left(t\right) = \eta^x b\left(t\right)$.[13]

# 4 Fiscal regimes and the exchange rate

In this section we prove the main theoretical results of the paper. We will proceed in steps. We first characterise the response of the exchange rate to fiscal and monetary shocks in the deterministic equilibria of the model, that is in the equilibria in which fiscal policy is always Ricardian or non-Ricardian. Then we characterize the behaviour of the exchange rate in a Markov-switching equilibrium in which fiscal policy shifts exogenously between the two regimes. Note that, while monetary and fiscal shocks are unexpected, the stochastic structure of the regime-shifting is preserved in the log-linearised model. Agents correctly anticipate that the policy regime might change in the future and form expectations accordingly. We close this section by showing that the different behaviour of the exchange rate in the non-Ricardian regime persists if we allow monetary policy to turn passive and inflate debt away as featured in the fiscal theory of the price level.

[12] By symmetric we mean a steady state in which the net foreign asset position of the Home country is zero

[13] Note that the shocks affect also the initial value of debt and their impact depend on the fraction of foreign-currency debt and on the response of the exchange rate. If, in response to a shock, the exchange rate appreciates and steady-state foreign-currency debt is positive ($\iota > 0$) then debt falls before rising. While this can occurr even in a non-Ricardian equilibrium, our approximation remains locally valid since $\bar{\eta} > 0$.

### Deterministic fiscal regimes

We start by characterizing the fiscal regimes of the model and the associated equilibria. We then study the response of the nominal exchange rate to fiscal and monetary shocks in each of them.

**Proposition 2.** *The model described by (23)-(29) has up to two equilibria:*

- *a* ***Ricardian equilibrium****, denoted with R, in which* $\psi_b^R > \rho$ *and* $\tilde{\eta}^R = 0$
- *a* ***non-Ricardian equilibrium****, denoted with N, in which* $\psi_b^N < \rho$ *and* $\tilde{\eta}^N = \frac{\rho - \psi_b^N}{\xi - \alpha(1-\psi_\pi^N)}$*, provided* $\xi > \frac{\alpha}{\rho}\left(1-\psi_\pi^N\right)\left(2\rho - \psi_b^N\right)$ *and* $\phi_\pi > \tilde{\eta}^N\left(1-\psi_\pi^N\right)\frac{\rho\alpha}{\kappa\omega}$.

The model has up to two equilibria, each associated with a different fiscal regime. If the fiscal authority commits to raising taxes commensurately to the servicing cost of the newly accumulated debt, then debt is sustainable. Hence, the probability of default is zero. This policy regime gives rise to the classical Ricardian equilibrium. In what follows we say that the fiscal regime is Ricardian if $\psi_b > \rho$. When the fiscal authority is unable or unwilling to commit to raising taxes sufficiently to ensure intertemporal solvency, then debt is unsustainable and default occurs with positive probability, that is $\eta^N(t) > 0$. In what follows we say that the fiscal regime is non-Ricardian if $\psi_b < \rho$.

To understand how the probability of default is determined and how it affects the dynamics of debt, solve (26) forward to obtain

$$\lim_{t\to\infty} \mathbb{E}\left[b(t)\right] = \lim_{t\to\infty}\left\{\int_0^t e^{\left(\rho-\psi_b^x-\xi\tilde{\eta}^x\right)(t-k)}\left[\left(1-\psi_\pi^x\right)\phi_\pi^x\pi_H(k)+\beta\varepsilon_g(k)+\varepsilon_i(k)\right]dk + e^{\left(\rho-\psi_b^x-\xi\tilde{\eta}^x\right)t}b(0)\right\} \quad (30)$$

In the non-Ricardian equilibrium, an increase in debt raises the probability of default, which in turn reduces the expected real interest rates paid by the government on its external liabilities. This reduces the expected growth rate of debt and ensures that its expected path is non-explosive.

The elasticity of default probability with respect to debt is decreasing in $\psi_b^N$. The more the government raises taxes in response to an increase in debt, the lower the default probability required to stabilize it. On the other hand, $\tilde{\eta}^N$ does not depend on the responsiveness of the monetary policy rate to inflation. However, while $\phi_\pi$ does not affect the debt elasticity of the default probability, it does determine the overall default probability through its impact on the path of debt. Similarly, while the currency composition of sovereign debt does not determine the debt elasticity of default, it does affect its Home-currency value and therefore the overall default risk. A depreciation of the domestic currency increases the real burden of Foreign-currency denominated liabilities and raise the value of debt. Hence, for a given $\tilde{\eta}^N$, $\eta(t)$ rises.

Proposition 2 shows that the non-Ricardian equilibrium can arise only under certain parameters restrictions. Specifically, a non-Ricardian equilibrium exists only if the share of externally-held debt ($\xi$) is sufficiently large and if the central bank is sufficiently aggressive in stabilising inflation. Both restrictions are due to our pricing assumption which features complete exchange-rate pass-through to imports and exports which introduces a feedback loop between the probability of default and domestic inflation. An increase in the default probability, as we shall see below, exerts a depreciating pressure on the exchange rate. This, in turn, makes domestic goods relatively cheaper than foreign goods, increasing their demand and raising domestic inflation. Since monetary policy is active, domestic inflation increases the real interest rate paid by the government and therefore the growth rate of debt. Hence, the probability of default must rise even further. If $\xi$ is small, the increase in the default probability required to stabilize debt is too large and this cycle does not converge. Similarly, an increase in the policy rate has a weaker effect on inflation since it increases the default probability which tends to depreciate the exchange rate. To avoid indeterminacy, the central bank must respond to inflation more forcefully than in a Ricardian equilibrium. This formalizes the argument put forward by Blanchard and Gali (2005) on the limit of inflation targeting in a regime of fiscal dominance. These restrictions can be weakened by adopting the, admittedly more realistic, assumption of imperfect exchange-rate pass-through. However, this would complicate the algebra unnecessarily. To keep the algebra tractable and to focus on the role of default risk alone, in what follows we sever the link between debt and inflation by assuming full indexation of taxes, that is, we set $\psi_\pi^N = 1$.[14] Under this assumption, a non-Ricardian equilibrium with default always exists and an active monetary policy is sufficient to guarantee its determinacy.

We are now ready to characterise the response of the nominal exchange rate to fiscal and monetary shocks in the two fiscal regimes. Since, $e(t) = s(t) - p^*(t) + p_H(t)$ and prices are pre-determined, the immediate response of the exchange rate to a time-zero shock is given by $e(0) = s(0)$. The next proposition proves our first main theoretical result.

**Proposition 3.** *The elasticities of the exchange rate to the unexpected time-zero shocks $\varepsilon_g(0) > 0$ and $\varepsilon_i(0) > 0$ in the two equilibria defined in Proposition 2 are:*

- *in the Ricardian equilibrium*

$$\frac{e^R(0)}{\varepsilon_g(0)} = -\frac{\kappa\varphi\phi_\pi}{\kappa\omega\phi_\pi + \varrho(\rho+\varrho)} \tag{31}$$

$$\frac{e^R(0)}{\varepsilon_i(0)} = -\frac{\rho+\varrho}{\kappa\omega\phi_\pi + \varrho(\rho+\varrho)} \tag{32}$$

- *in the non-Ricardian equilibrium (with $\psi_\pi^N = 1$)*

$$\frac{e^N(0)}{\varepsilon_g(0)} = \frac{-\frac{\kappa\varphi\phi_\pi}{\kappa\omega\phi_\pi+\varrho(\rho+\varrho)} + \beta\frac{\rho-\psi_b^N}{\rho\xi\kappa\omega\phi_\pi}\left[\kappa\omega\phi_\pi\frac{1-\alpha}{\rho+\varrho} + \alpha\rho\frac{\rho(\rho+\varrho)-\kappa\omega\phi_\pi}{\kappa\omega\phi_\pi+\varrho(\rho+\varrho)}\right]}{1-\iota\left(\rho-\psi_b^N\right)\frac{\kappa\omega\phi_\pi(1-\alpha)+\alpha\rho^2}{\rho\xi\kappa\omega\phi_\pi}} \tag{33}$$

$$\frac{e^N(0)}{\varepsilon_i(0)} = \frac{-\frac{\rho+\varrho}{\kappa\omega\phi_\pi+\varrho(\rho+\varrho)} + \frac{\rho-\psi_b^N}{\rho\xi\kappa\omega\phi_\pi}\left[\kappa\omega\phi_\pi\frac{1-\alpha}{\rho+\varrho} + \alpha\rho\frac{\rho(\rho+\varrho)-\kappa\omega\phi_\pi}{\kappa\omega\phi_\pi+\varrho(\rho+\varrho)}\right]}{1-\iota\left(\rho-\psi_b^N\right)\frac{\kappa\omega\phi_\pi(1-\alpha)+\alpha\rho^2}{\rho\xi\kappa\omega\phi_\pi}} \tag{34}$$

To understand the reaction of the exchange rate in the two regimes it is helpful to study its law of motion, given by equation (10). Linearise and solve it forward to obtain

$$e(0) = \int_0^\infty \left(\rho - i(t) + \pi_H(t) + \tilde{\eta}^x b(t)\right) dt \tag{35}$$

where we used the terminal condition $\lim_{t\to\infty} e(t) = \int_0^\infty (\pi^*(t) - \pi_H(t))\, dt$ and the fact that foreign variables are constant. The time-zero response of the exchange rate depends on three components: the interest rate, the path of domestic prices, and the probability of default. Thus, to understand the response of the exchange rate we need to understand how the shocks impact these components.

Consider first an expansionary fiscal shock. An unexpected increase in government expenditure raises output but also debt. In a Ricardian regime only the first effect matters for the exchange

[14] The solution for $\psi_\pi^N < 1$ can still be derived analytically, however its equations are very complex, even for an appendix. They are available upon request.

rate, since the probability of default is zero ($\tilde{\eta}^R = 0$). The increase in output puts upward pressure on prices and raises inflation. One the one hand, positive inflation depreciates the exchange rate. On the other hand, the increase in inflation prompts the central bank to hike the policy rate which has the opposite effect. In equilibrium, since $\phi_\pi > 0$ the central bank responds more than one-for-one to inflation and the second effect dominates ($\rho - i(t) + \pi_H(t) = -\phi_\pi \pi_H(t) < 0$). Hence, the exchange rate appreciates.We call this the monetary channel. This result is consistent with the empirical evidence presented by Corsetti, Meier, and Müller (2012) and Ilzetzki, Mendoza, and Végh (2013) which shows that, in countries with flexible exchange rate regimes, a positive spending shock tends to appreciate the real exchange rate on impact.

In a non-Ricardian regime the shock also affects the exchange rate through its impact on debt and the probability of default. An expansionary fiscal shock increases debt and the probability of default. This, in turn, tends to depreciate the exchange rate as foreign investors need to be compensated for the additional risk. This effect, which we call the debt channel, is captured by the second term at the numerator of equation (33).[15] The first term is identical to the response of the exchange rate in the Ricardian equilibrium and captures the monetary channel. The net effect of the shock on the exchange rate depends on the relative strength of these two channels. When the debt channel dominates, in the non-Ricardian regime the exchange rate depreciates in response to an expansionary fiscal shock. Whether this occurs or not in the data is an empirical question that we will tackle in Section 4. However, a quick back-of-the-envelope calculation suggests that this is the empirically relevant case. Using parameter values typically employed in the literature,[16] it is easy to show that (33) is positive, that is the debt channel dominates for values of the external public debt-to-GDP ratio ($\xi/\beta$) below 100%. For reference, the average of the per-country maximal external public debt-to-GDP ratio reported in the Institute of International Finance (IIF) database is 51%.

A similar logic applies when considering a monetary shock. In a Ricardian equilibrium, an increase in the policy rate appreciates the exchange rate, even though its response is partially muted by the fall in inflation caused by the contraction in output and the endogenous response of the monetary policy rule. In a non-Ricardian equilibrium this channel is compounded with the impact of the shock on debt. An increase in the policy rate raises the servicing cost of debt and increases its growth rate. Hence, the probability of default rises. When the debt channel dominates, the exchange rate depreciates, rather than appreciating in response to a contractionary monetary policy shock. As for the fiscal shock, a quick back-of-the-envelope calculation shows that (34) is positive for values of the external public debt-to-GDP ratio below 100%.

Note that the currency composition of government debt, captured by $\iota$, acts as a multiplier and affects the magnitude of the response of the exchange rate in the non-Ricardian regime. The larger the fraction of debt denominated in Foreign currency, the larger the depreciation. This is because the depreciation increases the value of Foreign-currency denominated liabilities, which in turn raises the default probability and causes the exchange rate to depreciate further.[17]

[15] The depreciation caused by the increase in the default probability raises domestic demand and therefore domestic inflation. This effect, captured by the negative term in the square brackets, offsets some of the depreciation but does not overturn it provided that $\alpha < \varrho/(\varrho + \rho)$. Since $\rho$, the household's time-discount factor, is small compared to $\varrho$, which determines the mean reversion of the shock, this condition is likely to be satisfied in any reasonable calibration of the model.

[16] We use $\rho = 0.04$, $\alpha = 0.4$, $\varphi = 3$, $\theta = 0.75$, $\beta = 0.6$, $\phi_\pi = 0.5$, $\psi_b = 0$, and $\varrho \in [0.1, 0.8]$.

[17] The denominator of equation (33) and (34) can, in principle, be negative and overturn the sign, causing the exchange rate to appreciate even when the debt channel dominates. However, this is a byproduct of the linearity of the model, rather than a meaningful prediction. The denominator is positive if $\xi$ is low and $\iota$ is high. That is, if most of the debt is held by domestic agents and denominated in Foreign currency. This implies that, when the exchange rate appreciates, the value of debt and the default probability fall rapidly. Since the default probability is bounded

### Stochastic fiscal regimes

In this section we characterize the equilibrium of the model and the behaviour of the exchange rate when fiscal policy shifts stochastically between the Ricardian and the non-Ricardian regimes. We assume that the probability of switching between the two regimes is constant over time and described by the transition matrix

$$\Sigma = \begin{bmatrix} -\sigma^N & \sigma^N \\ \sigma^R & -\sigma^R \end{bmatrix} \tag{36}$$

where $\sigma^N$ is the instantaneous probability of switching to the non-Ricardian regime and $\sigma^R$ is the instantaneous probability of switching to the Ricardian regime. As explained in the previous sections, solving the model involves computing the elasticity of default $\tilde{\eta}^N$ which ensures that the government intertemporal budget constraint holds. Being able to solve the model in closed-form greatly simplifies this step as it avoids the need to solve the fixed-point problem numerically. It turns out that the Markov-switching model can be solved analytically under very mild restrictions. In fact, assuming full indexation of taxes in the Ricardian regime ($\psi_\pi^R = 1$) is sufficient to obtain a closed-form solution. The next proposition derives the equilibrium elasticity of default in the Markov-switching model.

**Proposition 4.** *Assume that the model described by (23)-(29) switches stochastically between the Ricardian and the non-Ricardian regime according to the transition matrix (36), and $\psi_\pi^R = \psi_\pi^N = 1$. Then, the equilibrium of the model is mean-square stable if*

$$\tilde{\eta}^N = \frac{\rho - \psi_b^N}{\xi} - \frac{\sigma^R}{\xi} \frac{\psi_b^R - \rho}{2\left(\psi_b^R - \rho\right) + \sigma^N} \tag{37}$$

The elasticity of the default probability in the non-Ricardian regime is smaller the higher the probability of switching to the Ricardian one. This is because in forming expectations agents take into account the possibility of a regime switch and that taxes will increase to stabilize the dynamics of debt. This reduces the need for a default and therefore its probability. In fact, $\tilde{\eta}^N$ is smaller the more aggressively fiscal policy reduces debt in the Ricardian regime and the higher the persistence of the non-Ricardian regime, that is the lower $\sigma^N$. As for the deterministic case, the elasticity of the default probability does not depend on monetary policy parameter $\phi_\pi$.[18]

We are now ready to extend the results of Proposition 3 to the Markov-switching case. While the only restriction required to obtain a closed-form solution is $\psi_\pi^R = 1$, its equations are still very large. Here we focus on a specific calibration that yields a simpler solution.

**Proposition 5.** *Let $\psi_\pi^R = \psi_\pi^N = 1$, $\psi_b^N = 0$, $\psi_b^R \downarrow \rho$, and $\iota = 0$. Then the elasticities of the exchange rate to the unexpected time-zero shocks $\varepsilon_g(0) > 0$ and $\varepsilon_i(0) > 0$ in the Markov-switching*

below by zero, in this situation the debt channel should be pretty weak, if not completely absent. A model with a zero lower bound on the default probability would address this issue.

[18]However, in this case the result is due to our assumption of full indexation. In the proof of Proposition (4) reported in the appendix we derive the equation of $\tilde{\eta}^N$ in the more general case $\psi_\pi^N < 1$. When $\psi_\pi^N < 1$ monetary policy does affect the elasticity of default in the Markov-switching model. This has to do with the different behaviour of inflation in the two regimes. In the Ricardian regime inflation is more responsive to monetary policy than in the non-Ricardian one. In the latter, the depreciation induced by the increase in the default probability raises aggregate demand and sustains domestic prices. In the Ricardian regime, on the other hand, a higher $\phi_\pi$ leads to a lower inflation path. As a result, expected inflation falls even in the non-Ricardian regime and the real interest rate rises. This implies that the default probability must increase more to stabilize debt. Hence, $\tilde{\eta}^N$ is increasing in $\phi_\pi$.

*model are given by*

$$\frac{e^R(0)}{\varepsilon_g(0)} = \left.\frac{e^R(0)}{\varepsilon_g(0)}\right|_{\sigma^N=0} + \sigma^N\beta\Xi$$
$$\frac{e^R(0)}{\varepsilon_i(0)} = \left.\frac{e^R(0)}{\varepsilon_g(0)}\right|_{\sigma^N=0} + \sigma^N\Xi$$

*and*

$$\frac{e^N(0)}{\varepsilon_g(0)} = \left.\frac{e^N(0)}{\varepsilon_g(0)}\right|_{\sigma^R=0} - \sigma^R\beta\Xi$$
$$\frac{e^N(0)}{\varepsilon_i(0)} = \left.\frac{e^N(0)}{\varepsilon_i(0)}\right|_{\sigma^R=0} - \sigma^R\Xi$$

*where* $e^x(0)/\varepsilon_j(0)|_{\sigma^{-x}=0}$ *is the response of the exchange rate in regime* $x \in \{R, N\}$ *with respect to shock* $j \in \{g, i\}$ *in the deterministic model, and*

$$\Xi \equiv \frac{\alpha\rho}{\xi\varrho}\frac{\kappa\omega\phi_\pi - (\sigma+\rho+\varrho)(\rho+\varrho)}{[\kappa\omega\phi_\pi + (\sigma+\varrho)(\sigma+\rho+\varrho)][\kappa\omega\phi_\pi + \varrho(\rho+\varrho)]} + \frac{\alpha\rho}{\xi\varrho}\frac{\rho(\sigma+\rho) - \kappa\omega\phi_\pi}{\kappa\omega\phi_\pi + \sigma(\sigma+\rho)} + \frac{(1-\alpha)(\sigma+2\rho+\varrho)}{\xi(\rho+\varrho)(\sigma+\rho)(\sigma+\rho+\varrho)} \tag{38}$$

*where* $\sigma \equiv \sigma^R + \sigma^N$.

Under the assumed parametrization, the response of the exchange rate to fiscal and monetary shocks in the Markov-switching model takes a particularly simple form. The elasticity of the exchange rate in each regime is given by its elasticity in the associated deterministic equilibrium plus a component that is common across regimes and is proportional to the probability of switching. The sign of the common component $\Xi$ determines whether a stochastic fiscal policy reduces or increases the difference between the response of the exchange rate in the two regimes. If $\Xi > 0$ the exchange rate appreciates less in the Ricardian regime and it depreciates less in the non-Ricardian regime. Hence the difference narrows. On the other hand, if $\Xi < 0$ the exchange rate appreciates more in the Ricardian regime and it depreciates more in the non-Ricardian one. Hence, the difference increases. This case might seem counterintuitive. After all, the exchange rate is a forward-looking variable and the possibility of switching to other regime with opposite dynamics should counterbalance its response. However, the presence of the other regime affects the exchange rate not only directly, but also indirectly through its impact on the other equilibrium variables. Such impact might actually push the elasticity of the exchange rate in the two regimes further apart. The effect of the two regimes on inflation described above is a case in point.

As is clear from equation (38), the sign of $\Xi$ is ambiguous and depends on the parameters of the model. For most calibrations we expect $\Xi$ to be positive. Since the time discount factor $\rho$ is very small and the degree of openness $\alpha$ is typically below 0.5, the last term in equation (38) should dominate. However, if the degree of openness is sufficiently high then $\Xi$ could be be negative.

## The fiscal theory of the price level

When the fiscal authority is unable or unwilling to raise taxes, default is not the only solution to stabilize the dynamics of debt. The central bank can reduce the present discounted value of debt by allowing inflation to rise. Following the terminology set forth by Leeper (1991), we call this monetary policy regime, where $\phi_\pi < 0$, 'passive'. The combination of active fiscal policy and passive monetary policy is at the core of the FTPL and give rise to an equilibrium in which the price level is determined by the intertemporal budget constraint of the government, with no direct reference to monetary policy.

In this section we show how the differential response of the exchange rate in the Ricardian and non-Ricardian regime persists if we assume that in the latter monetary policy is passive. However, as the next proposition shows, when monetary policy is passive the debt elasticity of default is zero. This implies that the probability of default is constant across regimes. This is at odds with the empirical evidence presented in Section 5, suggesting that, at least for the case of Brazil, the model with active monetary policy and default is a better description of the data.

**Proposition 6.** *Assume $\phi_\pi \in \mathbb{R}$. Then the model described by (23)-(29) has one more **non-Ricardian equilibrium**, denoted with $N'$, in which $\phi_\pi^{N'} < 0$ and $\tilde{\eta}^{N'} = 0$, provided $\psi_\pi^{N'} < 1$.*

Note that even a Taylor coefficient arbitrarily close to but below one is sufficient to avert default. The mechanism through which inflation affects debt sustainability is similar to the case of default. A protracted period of high inflation implies low expected real interest rates which, in turn, raise the discount factor for government debt and offset the increased deficit. However, unlike default probability, inflation reduces the real interest rate paid on both domestic and external debt. This occurs if the fiscal rule does not offset the impact of inflation on the dynamics of debt, that is provided $\psi_\pi^{N'} < 1$. In what follows we assume $\psi_\pi^{N'} = 0$ as it is conventional in models with passive monetary policy.

The next proposition extends the result of Proposition 3 to the case of a non-Ricardian equilibrium in which monetary policy is passive.

**Proposition 7.** *The elasticity of the exchange rate to the unexpected time-zero shocks $\varepsilon_g(0) > 0$ and $\varepsilon_i(0) > 0$ in the non-Ricardian equilibrium $N'$ is given by*

$$\frac{e^{N'}(0)}{\varepsilon_g(0)} = \frac{\frac{\beta\omega(\mu+\rho+\varrho)\left(\mu+\rho-\psi_b^{N'}\right)-\varphi\mu(\mu+\rho)\left(\rho-\psi_b^{N'}\right)}{\omega\mu(\mu+\rho+\varrho)\left(\varrho+\rho-\psi_b^{N'}\right)}}{1-\iota\frac{\mu+\rho-\psi_b^{N'}}{\mu}} \tag{39}$$

$$\frac{e^{N'}(0)}{\varepsilon_i(0)} = \frac{\frac{\left(\rho-\psi_b^{N'}\right)(\rho+\varrho)}{\mu(\mu+\rho+\varrho)\left(\varrho+\rho-\psi_b^{N'}\right)}}{1-\iota\frac{\mu+\rho-\psi_b^{N'}}{\mu}} \tag{40}$$

*where* $\mu \equiv \left(\sqrt{\rho^2 - 4\kappa\omega\phi_\pi^{N'}} - \rho\right)/2 > 0$.

When monetary policy is passive, the response of the exchange rate to fiscal and monetary shocks depends on their impact on debt, since the latter determines the inflation path. Consider a fiscal shock first. On the one hand, the fiscal shock increases output and inflation in the short run. Since monetary policy is passive, the latter effect reduces the real interest rate and therefore the growth rate of debt. This implies that inflation will have to fall in the future to preserve its present discounted value. This channel, captured by the second term at the numerator of (39), tends to appreciate the exchange rate. On the other hand, the fiscal shock directly increases the growth rate of debt, which in turn implies that inflation will have to rise in the future to preserve its present discounted value. This channel, captured by the first term at the numerator of (39), tends to depreciate the exchange rate. The net effect depends on the relative strength of these two channels.

The response of the exchange rate to a monetary shock is unambiguous. An unexpected increase in the policy rate reduces output and inflation, and raises the servicing cost of debt. Both effects increase the growth rate of debt and inflation must rise to restore its present discounted value. Hence, the exchange rate depreciates. Note that, as in the equilibrium with default, the currency composition of debt does not affect the sign of the response of the exchange rate, but only its magnitude.

We close this section by studying the Markov-switching equilibrium when fiscal policy shifts between regimes $R$ and $N'$. Similarly to the case of default, we first derive the endogenous upper bound on $\phi_\pi^{N'}$ that guarantees that the equilibrium is stable and then we compute the elasticities of the exchange rate with respect to the two shocks. Unfortunately, the equations are even more complicated than before. While the stability condition is relatively compact, the equations of the elasticities are unmanageable. In the next proposition we focus on the particular case in which the Ricardian regime is absorbing, ie $\sigma^{N'} = 0$. That is, we assume that the system can jump from the non-Ricardian regime to the Ricardian one but not vice versa.

**Proposition 8.** *Assume that the model described by (23)-(29) switches stochastically from the non-Ricardian regime $N'$ to the Ricardian regime with transition probability $\sigma^R$. Let $\psi_\pi^R = 1$ and $\psi_\pi^N = 0$. Then the elasticities of the exchange rate to the unexpected time-zero shocks $\varepsilon_g(0) > 0$ and $\varepsilon_i(0) > 0$ in regime $N'$ are given by*

$$\frac{e^{N'}(0)}{\varepsilon_g(0)} = \left.\frac{e^{N'}(0)}{\varepsilon_g(0)}\right|_{\sigma^R=0} - \sigma^R \frac{\omega\beta + \frac{\varphi\mu\varrho}{\mu+\rho+\varrho+\sigma^R}\left[1 + \frac{\rho-\psi_b^{N'}}{\mu+\rho+\varrho} - \frac{\mu\left(\psi_b^{N'}+\varrho\right)+\sigma^R(\mu+\rho)+(\rho+\varrho)^2+\varrho\left(\rho-\psi_b^{N'}\right)}{\kappa\omega\phi_\pi^R+\varrho(\rho+\varrho)}\right]}{\mu\omega\left(\rho-\psi_b^{N'}+\varrho\right)} \tag{41}$$

$$\frac{e^{N'}(0)}{\varepsilon_i(0)} = \left.\frac{e^{N'}(0)}{\varepsilon_i(0)}\right|_{\sigma^R=0} - \sigma^R \frac{\sigma^R + \varrho + \frac{\mu\psi_b^{N'}+\rho(\rho+\varrho)}{\mu+\rho+\varrho} + \mu\frac{\mu\left(\psi_b^{N'}+\varrho\right)+\sigma^R(\mu+\rho)+(\rho+\varrho)^2+\varrho\left(\rho-\psi_b^{N'}\right)}{\kappa\omega\phi_\pi^R+\varrho(\rho+\varrho)}}{\mu\left(\varrho+\rho-\psi_b^{N'}\right)\left(\mu+\rho+\varrho+\sigma^R\right)} \tag{42}$$

*provided that $\mu > \sigma^R/2$, ie $\phi_\pi^{N'} < -\sigma^R\left(\sigma^R+2\rho\right)/4\kappa\omega$.*

Note that, while in the deterministic model stability requires $\phi_\pi^{N'} < 0$, in the Markov-switching this condition becomes $\phi_\pi^{N'} < -\sigma^R\left(\sigma^R+2\rho\right)/4\kappa\omega$. The higher the probability of switching to a Ricardian regime, the more monetary policy must be passive in the non-Ricardian one. The possibility of switching to a Ricardian regime with an active monetary policy decreases expected and therefore current inflation. Hence, in order to stabilize debt in the non-Ricardian regime monetary policy must be looser.

As in proposition 5 we can decompose the elasticity of the exchange rate to the shocks in two parts: the elasticity in the associated deterministic equilibrium and a component that is proportional to the probability of switching. In the case of a monetary shock, the possibility of switching to a Ricardian regime tends to reduce the depreciation of the exchange rate in the non-Ricardian one and to minimize the difference between the two. In the case of a fiscal shock, however, the presence of a Ricardian regime can affect the response of the exchange rate in both directions.[19]

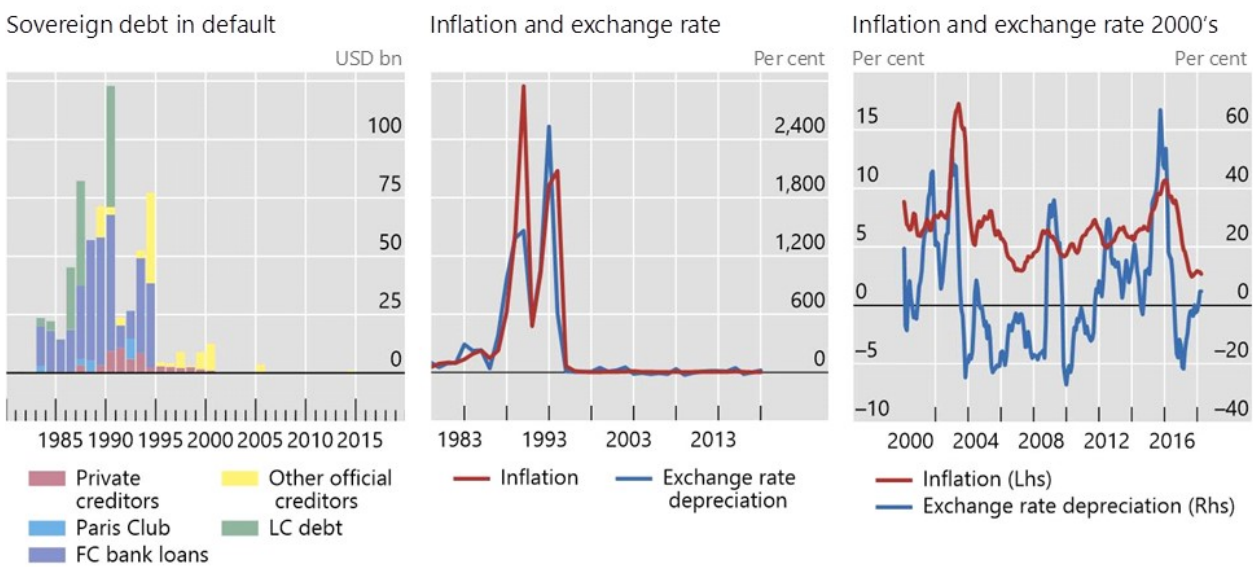
[19] Assume that the central bank sets $\phi_\pi^{N'}$ to the maximum value that ensures the stability of debt, that is consider the limit for $\mu \downarrow \sigma^R/2$. Then, if the probability of switching is low, the term multiplying $\sigma^R$ in equation 41 is positive and the exchange rate depreciates less than in the deterministic case. On the other hand, if $\sigma^R$ is sufficiently large then the term is negative and the exchange rate depreciates even more than in the deterministic case.

## 5 Empirical evidence: the case of Brazil

In this section we test the model using Brazil as a case study. The choice of Brazil is determined by its history of fiscal profligacy and recurrent fiscal crises, but also by the availability of daily survey data required by our empirical approach.

Like many other countries in Latin America, Brazil has a long track record of procyclical fiscal policies, as documented by by Alberola et al. (2016). In the 1980s and until the mid-1990s, fiscal indiscipline led to sovereign debt crises and bouts of hyperinflation. The Brazilian government defaulted on its domestic debt three times (1986, 1987 and 1990) and experienced three "technical" defaults on its foreign debt (1982, 1986 and 1990). At its peak, the value of defaulted debt reached USD 125bn (Figure 1, left-hand panel). In that period, inflation averaged 600% and the exchange rate depreciated on average 520% (Figure 1, centre panel).

Figure 1: **Sovereign debt, inflation and exchange rate depreciation** Nominal value (USD bn) of the different types of Brazilian sovereign government debt in default (left-hand panel). Yearly inflation and exchange rate depreciation (centre and right-hand panels). Source: Bloomberg, Beers, Jones, and Walsh (2020) and Reinhart and Rogoff (2009).




Since then, Brazil has improved its macro fundamentals. In 1999 the central bank of Brazil changed its exchange and monetary regime, abandoning a crawling peg in favor of a floating regime and an inflation targeting framework. This stabilized both inflation and the exchange rate (Figure 1, right-hand panel). However, not all problems have been solved. While in the past two decades fiscal discipline has improved, concerns about debt sustainability have periodically resurfaced. For example, from 2014 to 2017 gross government debt increased from 53.3% of GDP to 74% of GDP, amid low economic growth, declining tax revenues and political instability. In 2015 Brazil's sovereign debt lost its investment grade, raising the cost of issuing in international markets (Holland (2019)). These characteristics make Brazil an ideal case study for our model.

### The empirical model

To capture the regime differences in the reaction of the exchange rate to policy shocks we estimate the following Markov regime-switching model (see Hamilton (1994) for details):

$$\Delta e_d = \delta_{v,0} + \delta^x_{v,1}\xi_{v,d} + \delta_{v,Y} Y_d + \sigma^x_v \varepsilon_{v,d} \tag{43}$$

where $\varepsilon_{v,d} \sim \mathcal{N}(0,1)$ iid across time. The dependent variable, $\Delta e_{v,d}$ is the daily percentage change in the BRL/USD exchange rate. A positive value denotes a depreciation of the real vis-a-vis the dollar. Exchange rates are measured at 13:15 GMT. Therefore $d$ is the day of a policy announcement, when the latter took place before that time (almost all occurrences), while it is the day after, when the policy announcement took place after that time. The main independent variable in our regressions is the policy shock $\xi_{v,d}$. We estimate separate models for fiscal ($v = F$) and monetary ($v = M$) shocks. Our main focus is on the sign of $\delta^x_{v,1}$ in the two regimes. A positive (negative) sign means that the shock tends to depreciate (appreciate) the real vis-a-vis the dollar. We allow the error term variance to be regime-dependent. This allows the response of the exchange rate in one of the states to be more volatile than in the other state.

The daily BRL/USD exchange rate is obtained from the BIS foreign exchange statistics. The sample size depends on the number of fiscal and monetary policy announcements, as explained below. We estimate model (43) with and without control variables. We use the daily survey of the central bank of Brazil on current year's GDP growth to control for changes in expectations on economic growth. We proxy risk appetite toward emerging markets by calculating the first principal component of the JPMorgan global Emerging Market Bond Index (EMBI) spreads of Argentina, Chile, Colombia, Mexico, Peru, Venezuela, Asia and Eastern Europe. Finally, to control for other macro shocks, we consider the difference between realized and surveyed values around policy decisions of the 30-day unemployment rate, year-on-year growth of GDP, year-on-year growth of retail sales, year-on-year growth in economic activity, change in non-farm payrolls, quarter-on-quarter annualized growth in GDP in the US, year-on-year change in inflation in the US, and the FOMC rate in the US. All data is obtained from Bloomberg. If there are macro news from Brazil and the US on the same date, we choose the one from Brazil since it is likely to have a greater effect.[20]

We identify fiscal policy shocks as the difference between the announced primary deficit-to-GDP and the value expected the day before the announcement obtained from Bloomberg survey. The policy announcement is the official monthly release of the Brazilian primary deficit. The data spans from April 2003 to March 2018 and includes 180 fiscal policy announcements. Table 1 reports the shocks' summary statistics. In the sample we identify 95 expansionary shocks and 82 contractionary shocks. The two types of shock have similar means and standard deviations. Expansionary shocks have an average magnitude of 0.72 percentage points (pp) of GDP and a standard deviation of 0.74, while contractionary shocks have an average magnitude of 0.69 pp and a standard deviation of 0.76. Figure 2, left panel, plots the time series of the identified fiscal policy shocks in our sample. Note that most of the changes in the primary deficit are due to changes in expenditures, not taxes. This is consistent with our model which features shocks to government expenditures.

[20]For the monetary policy dates, US CPI inflation came out on the same day as retail sales and IBC-Br Index in Brazil on three occasions (17th November 2004, 19th April 2006 and 16th January 2013). Also, there were two instances when the FOMC Statement and US GDP were published on the same day (29th April 2009 and 29th April 2015). We chose the FOMC news. For the fiscal policy dates, there are overlaps of Brazilian and US macro news on 10 occasions, each involving the news about US GDP (29th May 2003, 27th February 2004, 31st January 2007, 28th February 2007, 30th January 2008, 30th April 2008, 30th January 2013, 30th April 2014 and 29th May 2015). All these choices have a negligible effect on our main results.

Table 1: **Shocks descriptive statistics**. The magnitude represent percentage points of GDP for the fiscal shocks and of the interest rate (Selic) for monetary shocks.

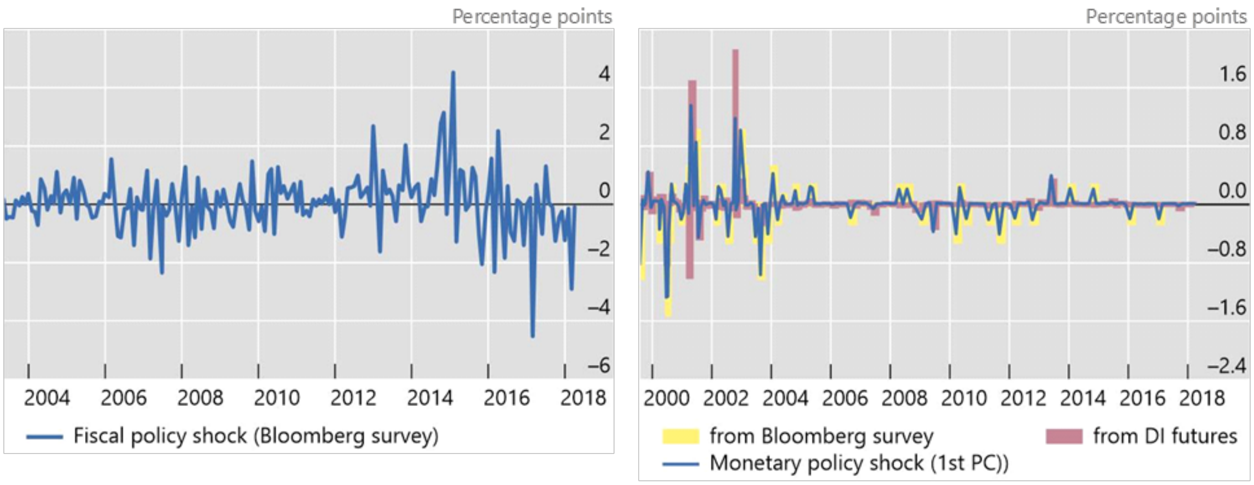
| | Fiscal policy shock | | | Monetary policy shock | | |
|---|---|---|---|---|---|---|
| | All | Negative | Positive | All | Negative | Positive |
| Count | 180 | 82 | 95 | 179 | 74 | 78 |
| Mean | 0.07 | -0.69 | 0.72 | 0.01 | -0.06 | 0.09 |
| S Dev | 1.02 | 0.76 | 0.74 | 0.23 | 0.16 | 0.3 |
| Min | -4.52 | -4.52 | 0.01 | -0.98 | -0.98 | 0 |
| 25% | -0.34 | -1.05 | 0.24 | -0.01 | -0.05 | 0.01 |
| 50% | 0.02 | -0.42 | 0.53 | 0.00 | -0.02 | 0.02 |
| 75% | 0.55 | -0.17 | 0.94 | 0.01 | -0.01 | 0.05 |
| Max | 4.52 | -0.02 | 4.52 | 2.09 | 0.00 | 2.09 |

We identify the monetary policy shock as the first principal component of two time-series. The first one is the daily change around policy announcements of *Deposito Interbancario* (DI) futures prices. The DI futures are written on an overnight interbank borrowing rate in Brazil, which are the most popular market-based forecasts for the Selic rate (see for example, Bernanke and Kuttner (2005), Cieslak and Schrimpf (2018), Cochrane and Piazzesi (2002), Kuttner (2001)). The second time-series is the difference between the announced Selic target rate and the its expected value the day before the announcement obtained from Bloomberg survey. We consider all the decisions made by the Monetary Policy Committee of the central bank of Brazil, including those where the Selic target rate was left unchanged. Our period of study starts with the implementation of inflation-targeting in July 1999 and ends in March 2018. In total, we consider 179 monetary policy decisions: 47 decisions to increase the Selic rate, 65 decisions to lower the rate and 67 where the rate was left unchanged. By combining these two series by its first principal component we capture the maximum variance in the unexpected component of the monetary policy shock. Table 1 reports the shocks' summary statistics. In the sample we identify 78 contractionary shocks and 74 expansionary shocks. The two shocks have similar means but different standard deviations. Expansionary shocks have an average magnitude of 6 basis points (bps) and a standard deviation of 0.16, while contractionary shocks have an average magnitude of 9 bps and a standard deviation of 0.3. Figure 2, right panel, plots the time series of the identified monetary policy shocks.

## Results

Conducting proper inference when identifying different regimes in the data can be challenging. The first step consists in testing for the presence of a second regime, which requires the use of a nonstandard test statistic and critical values that may differ across model specifications. Cho and White (2007) show that the appropriate measure for a test of multiple regimes in the Markov regime-switching framework is a quasi-likelihood-ratio statistic (QLR). The statistic tests the null hypothesis of one regime against the alternative of a two regimes by testing whether the estimated coefficients are significantly different across them. The QLR statistic for the fiscal policy model is 42.1 while the QLR statistic for the monetary policy model is 105.2. We compare these values with the simulated critical value for a 5 per cent QLR test. The number of simulations is standard (100,000) but we allow for the possibility of a wide separation between regimes (+/- 5 standard deviations between regime averages). We obtain a critical value of 7, which is significantly smaller

Figure 2: **Fiscal and Monetary policy shocks.** The figure shows differences between realized and expected primary deficit (left-hand panel) and differences between realized and expected change in the Selic rate (right-hand panel).




than the QLR statistics for both models. We therefore reject the null hypothesis of a single regime and can proceed to estimate both models with a two-regime specification.

We estimate (43) by maximizing the full log-likelihood function. Table 2 reports the outcome of the regression of the exchange rate on fiscal and monetary shocks. In both models the response of the exchange rate to the policy shock considered changes sign across regimes. Following the results of the theoretical model, we tentatively label the regime associated with an appreciation Ricardian (R) and the regime associated with a depreciation non-Ricardian (N).

For fiscal policy, an unexpected positive shock worth 1 percent of GDP appreciates the Brazilian real by around 0.1 pps in the Ricardian regime, while it depreciates it by around 0.5 pps in a non-Ricardian regime. The coefficients are identical when estimating the model with or without controls. The coefficient of the non-Ricardian regime is always significant at the 1% level, while the coefficient of the Ricardian regime is significant at the 10% level once controls are included in the regression. The estimated transition matrix suggests that the two regimes have similar persistence. Finally, the response of the exchange rate to policy shocks in the non-Ricardian regime is more volatile.

For monetary policy shocks, an unexpected 1% increase of the Selic rate appreciates the Brazilian real by 0.82 pps in the Ricardian regime, while it depreciates it by 0.84 pps in the non-Ricardian regime. Both coefficients are significantly different from zero, at the 5% and 10% level. The results including control variables are larger in magnitude and both coefficients are significantly different from zero at the 1% level. The estimated transition matrix suggests that the probability of switching from one regime to the other is higher compared to the estimates obtained using fiscal policy shocks, and that the non-Ricardian regime is less persistent than the Ricardian one. Finally, the response of the exchange rate to monetary policy shocks in the non-Ricardian regime is more volatile than in the Ricardian one.

Table 2: **Markov regime-switching.** The left panel reports the effects on the exchange rate of the fiscal policy shock (FP shock) and the right panel of the monetary policy shock (MP shock). A positive sign indicates a depreciation. Each panel shows the matrix of transition probabilities, estimated coefficients with standard errors (in parenthesis), $\sigma$ the estimated state-dependent standard deviation of the process and the number of observations. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and ***.

| | | Fiscal policy shock | | | | Monetary policy shock | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | No controls | | With controls | | No controls | | With controls | |
| | | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| | Regime | R | N | R | N | R | N | R | N |
| Trans. | R | 0.93 | 0.07 | 0.93 | 0.07 | 0.87 | 0.13 | 0.95 | 0.05 |
| Prob. | N | 0.17 | 0.83 | 0.16 | 0.84 | 0.40 | 0.60 | 0.70 | 0.30 |
| $\delta_1$ | | -0.11 | 0.52*** | -0.11* | 0.51*** | -0.82** | 0.84* | -0.91*** | 1.16*** |
| | | (0.07) | (0.16) | (0.06) | (0.16) | (0.32) | (0.37) | (0.29) | (0.00) |
| $\delta_0$ | | -0.17*** | | -0.16** | | -0.01 | | 0.104*** | |
| | | (0.05) | | (0.06) | | (0.06) | | (0.00) | |
| $\delta_{GDP}$ | | | | -0.05 | | | | -0.31*** | |
| | | | | (0.21) | | | | (0.01) | |
| $\delta_{EMBI}$ | | | | 0.94* | | | | 3.10*** | |
| | | | | (0.56) | | | | (0.03) | |
| $\delta_{Other}$ | | | | 0.12 | | | | 0.11*** | |
| | | | | (0.05) | | | | (0.01) | |
| $\sigma$ | | 0.57 | 1.54 | 0.57 | 1.52 | 0.70 | 1.22 | 0.77 | 1.68 |
| | | (0.05) | (0.20) | (0.08) | (0.22) | (0.09) | (0.22) | (0.08) | (0.10) |
| Observations | | 180 | | | | 179 | | | |

## Fiscal regimes

The empirical model provides strong support for the existence of two regimes in which the exchange rate reacts to policy shocks with opposite signs. While this result is consistent with the predictions of the theoretical model, it does not validate it, since the empirical estimation does not inform about the structural differences between the two regimes. In this section we argue that the difference between them is the expectation about future fiscal policies and market participants' assessment of the probability of a default. More specifically, we link the estimated probability of being in the non-Ricardian regime to various measures of the expected fiscal situation of Brazil and its probability of default.

Figure 3: **Probabilities of non-Ricardian regime.** Estimated probabilities of the non-Ricardian regime.

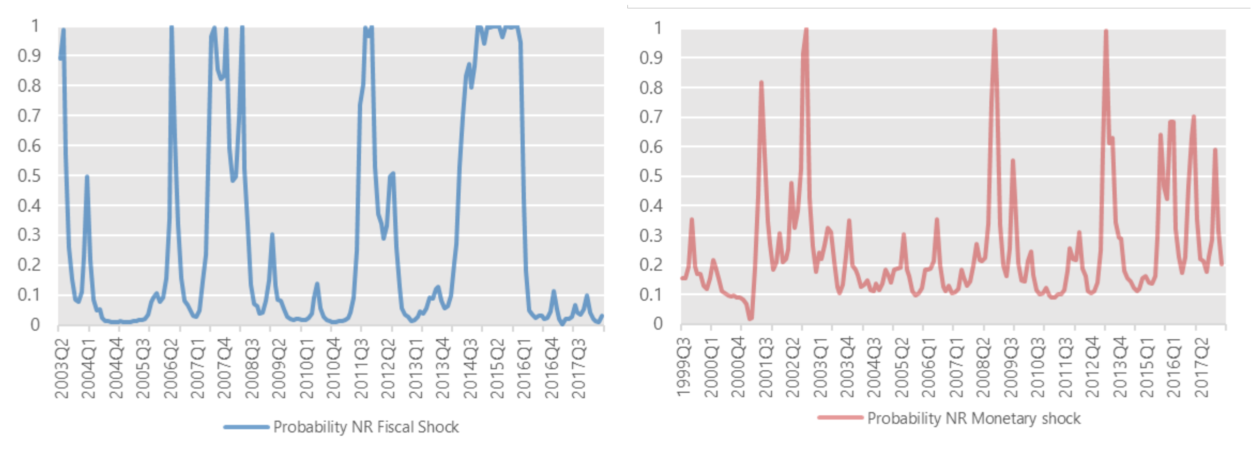



Figure 3 shows the time series of the probabilities of being in the non-Ricardian regime, estimated using fiscal (left panel) and monetary (right panel) shocks. Looking at the probabilities estimated using fiscal shocks, there are four major spikes: at the beginning of the sample in Q3 2003, from Q3 2007 to Q2 2008, in the second half of 2011, and from Q2 2014 to Q4 2015. Do these events correspond to periods of unsustainable fiscal policies in Brazil? The answer is yes. The first such event, was studied by Blanchard (2004) and labelled as a period of "fiscal dominance". In the running up to the 2002 elections, concerns about future fiscal policies led to an acute financial stress in Brazil. The interest rate on Brazilian government dollar-denominated debt increased sharply, reflecting an increase in the market's assessment of the probability of default. Simultaneously, a sharp depreciation of the real vis-a-vis the dollar led to a sharp increase in inflation. Despite its commitment to inflation targeting, the Brazilian central bank did not increase its policy rate. Blanchard (2004) argues that the level and the composition of Brazilian debt, together with the general level of risk aversion in international financial markets, were such as to imply that Brazil was in a regime of fiscal dominance and that an increase in interest rates would have had a perverse effect, leading to an increase in the probability of default, to further depreciation, and to a further surge in inflation.

Similarly, the periods between 2014 and 2015 was characterized by a distressed fiscal and political situation that could increase the perception of a non-Ricardian policy. Between 2013 and 2015 the persistent decline in the primary deficit, coupled with a long sequence of interest rate hikes, led the Brazilian government debt-servicing costs to double. Over the course of 2015 the real depreciated significantly against the dollar, despite further tightening by the central bank. These observations led Bolle (2015) to label this period as a period of fiscal dominance and to argue in favor of a temporary return to a crawling exchange rate band regime.

Finally, the other two spikes in the probability of a non-Ricardian regime in Figure 3 occur around 2007/2008 and 2011/2012, two periods characterised by strained international financial conditions due to the global financial crisis and the European debt crisis, respectively. This highlights the fact that debt sustainability is not only a function of domestic policies, but also of global financial conditions. A country can move into a non-Ricardian regime not only due to changes in the domestic fiscal policy, but also as a consequence of shifts in the price of risk (absent adequate adjustment of the former).

Overall, the model estimated using the exchange rate response to fiscal shocks correctly identifies periods of heightened concerns about the sustainability of Brazilian debt. To corroborate the narrative evidence, in our last exercise we relate the estimated probabilities of the non-Ricardian regime to various measures of expected future fiscal policies. We proxy expectations about fiscal policies with changes in the expected evolution of Brazil's fiscal fundamentals. We use changes in expectations, rather than levels, as they give a better sense of the perceived medium-term dynamics of the fiscal situation in the country. We use the current-year expectations about the primary deficit, interest payments and net debt obtained from the central bank of Brazil's Market Expectations System, a framework that collects daily projections from professional forecasters about the main macroeconomic variables in Brazil. Since policy shocks occur only at announcement dates, for each fiscal variable we use the change in expectations between policy announcements. That is, the change in expectations between the day after the previous announcement and the day before the current one.[21] Finally, as an overall proxy for expected future fiscal policies we use the change in the probability of default between policy announcements. Following Hull (2006), we model the default probability using the hazard rate $\lambda$ obtained from CDS spreads using $\lambda(t) = S(t)/\left(1-R\right)$ where $S(t)$ is the Brazil 5 year flat CDS spread flat CDS spread and $R$ is the recovery rate which, following Andritzky and Singh (2006), we assume equal to 25%.

Table 3 reports the correlation coefficients estimated from individual regressions of the estimated probability of being in the non-Ricardian regime on the expected fiscal variables and the probability of default. For fiscal policy shocks, upward revisions in the current year expected net debt, primary balance and interest payments are positively and significantly correlated with the probability of a non-Ricardian regime. A higher probability of default is also associated with a higher probability of being in the non-Ricardian regime. All variables are significant when simultaneously included in the regression.[22] These results suggest that the structural difference between the two regimes identified in the data is related to the expected fiscal situation of the country and its expected fiscal policies. In line with the mechanism of the theoretical model developed in Section 2, an unconventional response of the exchange rate to a fiscal policy shock is more likely to occur when economic agents expect weaker fiscal policies and their concern about debt sustainability rises.

Turning to monetary policy shocks, the probabilities of being in the non-Ricardian regime estimated using them paint a less clear picture. While they still spike around the periods identified using fiscal policy shocks, they are more volatile and much less persistent. This is probably not

[21] These measures might be correlated with the sign of the policy shock, for example if market participants are more or less conservative in their forecasts. To rule out the possibility of any mechanical link between them and the estimated probabilities, we test whether the latter are correlated with the sign of the policy shock. For both models the correlation is insignificant. That is, the unconventional response of the exchange rate is as likely to occur in response to a positive shock as it is in response to a negative shock.

[22] We run a robustness check using one-year, two-year-ahead and long-term expectations. The results including all the variables in each regression are reported in the appendix (Table 4).

Table 3: **Non-Ricardian regime probabilities and fiscal fundamentals.** The table reports coefficients estimated from individual regressions of the non-Ricardian regime probabilities on fiscal variables. Robust standard errors in parenthesis. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and *** .

| | FP | MP | FP | MP | FP | MP | FP | MP | FP | MP |
|---|---|---|---|---|---|---|---|---|---|---|
| $\Delta E_t$[Net debt] | 0.04* | 0.03* | | | | | | | 0.07*** | 0.02 |
| | (0.02) | (0.01) | | | | | | | (0.02) | (0.01) |
| $\Delta E_t$[Primary deficit] | | | 0.61*** | 0.08 | | | | | 0.66*** | 0.09 |
| | | | (0.23) | (0.06) | | | | | (0.20) | (0.06) |
| $\Delta E_t$[Int Payments] | | | | | 0.27* | -0.01 | | | 0.39*** | -0.01 |
| | | | | | (0.15) | (0.05) | | | (0.14) | (0.05) |
| Prob default | | | | | | | 20.06** | 3.76** | 16.67** | 3.50** |
| | | | | | | | (8.13) | (1.71) | (7.65) | (1.70) |
| Constant | 0.27*** | 0.25*** | 0.25*** | 0.25*** | 0.27*** | 0.26*** | 0.28*** | 0.26*** | 0.25*** | 0.26*** |
| | (0.026) | (0.01) | (0.02) | (0.01) | (0.02) | (0.01) | (0.02) | (0.01) | (0.02) | (0.01) |
| Observations | 179 | 172 | 179 | 172 | 179 | 148 | 179 | 148 | 179 | 148 |
| R-squared | 0.00 | 0.03 | 0.04 | 0.00 | 0.01 | 0.00 | 0.03 | 0.05 | 0.11 | 0.08 |

surprising given the difficulty in properly identifying monetary policy shocks. While we only look at unexpected changes in the Selic rate, monetary policy announcements convey much more information, in particular about the economic outlook as well as the future path of monetary policy. Both information are relevant for the determination of the exchange rate. In order to properly estimate the impact of the Selic rate on the exchange rate one would need to control for such omitted variables. Unfortunately, many of the approaches used in the literature use a form of sign restriction that would interact with our identification of the regimes. See for example Nakamura and Steinnson (2018) and Jarocinski and Karadi (2020).

Given these differences, one might question the validity of the coefficients reported in Table 2. To test their robustness, we estimate the response of the exchange rate to monetary policy shocks in the Ricardian and non-Ricardian regimes identified using the fiscal policy shock. Formally we estimate the following equation:

$$\Delta e_d = \delta_{M,0} + (\delta^{NR}_{M,1}\beta^{NR}_{F,d} + \delta^{R}_{M,1}(1-\beta^{NR}_{F,d}))\xi_{M,d} + \delta_{M,Y}Y_d + \sigma_M\varepsilon_{M,d} \tag{44}$$

where $\beta^{NR}_{F,d}$ is a dummy variable that takes a value of 1 if on the date of the episode, the probability of the non-Ricardian regime identified by the fiscal policy shock is greater than 0.5.[23] Since the dates of the monetary policy announcements and the fiscal policy announcements do not coincide, we match them at the monthly frequency. We report the results in the appendix table 5. As expected, the estimate coefficients are closer together but they still have significantly different signs. We corroborate that during episodes identified as Ricardian by the fiscal shock an unexpected rise in the Selic rate tends to appreciate the exchange. By contrast, in episodes identified as non-Ricardian, an unexpected increase in the policy rate tends to depreciate the exchange rate.

[23]In order to maintain comparability with the results presented in Table 2, we estimate the regression on the full sample of monetary policy shocks. At the beginning of the sample, from July 1999 to March 2003, when fiscal data is not available we use the probabilities estimated with the monetary policy shock to create $\beta^{NR}_{F,d}$. This involves 48 out of 179 observations.

# 6 Conclusion

Given the critical importance of the exchange rate for internal and external stability, understanding its response to domestic policies is fundamental for policymakers.

In this paper we have argued that the effect of monetary and fiscal policies on the domestic currency depends crucially on the fiscal regime. A contractionary monetary (expansionary fiscal) shock can lead to a depreciation, rather than an appreciation, of the exchange rate if debt is not fully backed by future surpluses.

We have shown how this differential behaviour can arise in a model characterised by stochastic fiscal regimes and asymmetric recovery rates between domestic and foreign investors. In a non-Ricardian regime policy shocks affect the probability of default and the currency risk premium, reversing the response of the exchange rate.

We have tested these theoretical predictions and the underlying mechanism using Brazil as a case study. By looking at daily movements of the BRL/USD exchange rate around monetary and fiscal policy announcements we have found strong support for the existence of two regimes with opposite signs. Consistent with our model, we have found that the unconventional response of the exchange rate is more likely to arise when economic agents expect weaker fiscal policies and their concern about debt sustainability rises.

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# Appendix A Tables

Table 4: **Non-Ricardian regime probabilities and long term fiscal fundamentals.** The table reports coefficients estimated from individual regressions of the non-Ricardian regime probabilities on fiscal variables. Robust standard errors in parenthesis. Statistical significance at the 10%, 5% and 1% levels is denoted by *, **, and *** .

| | t+1 | | t+2 | | LT | |
|---|---|---|---|---|---|---|
| | FP | MP | FP | MP | FP | MP |
| $\Delta E_t$[Net debt] | -0.05* | 0.04** | -0.02 | 0.02* | -0.00 | 0.03** |
| | (0.03) | (0.016) | (0.02) | (0.01) | (0.00) | (0.01) |
| $\Delta E_t$[Public deficit] | 0.51*** | 0.15** | 0.42* | 0.04 | 0.38*** | 0.08 |
| | (0.18) | (0.07) | (0.23) | (0.11) | (0.14) | (0.11) |
| $\Delta E_t$[Interest payments] | 0.47*** | -0.04 | 0.43*** | -0.01 | 0.40*** | -0.01 |
| | (0.15) | (0.04) | (0.14) | (0.05) | (0.13) | (0.04) |
| Prob of default | 17.37** | 3.32* | 18.54** | 3.53** | 18.45** | 3.56** |
| | (7.67) | (1.68) | (7.69) | (1.73) | (7.67) | (1.73) |
| Constant | 0.26*** | 0.25*** | 0.26*** | 0.26*** | 0.26*** | 0.21*** |
| | (0.02) | (0.01) | (0.02) | (0.01) | (0.02) | (0.01) |
| Observations | 179 | 148 | 179 | 148 | 179 | 148 |
| R-squared | 0.10 | 0.12 | 0.09 | 0.08 | 0.09 | 0.10 |

Table 5: **Monetary policy shock and non-Ricardian regime episodes** The table reports the response of the exchange rate to monetary policy shocks in the Ricardian and non-Ricardian regimes identified using the fiscal policy shock.

| | $\Delta e_d$ | |
|---|---|---|
| | No controls | With controls |
| $\delta^{R}_{M,1}$ | -0.56* | -0.77*** |
| | (0.28) | (0.22) |
| $\delta^{NR}_{M,1}$ | 0.66 | 0.91** |
| | (0.47) | (0.40) |
| $\delta_{GDP}$ | | -1.37* |
| | | (0.75) |
| $\delta_{EMBI}$ | | 3.62*** |
| | | (0.75) |
| $\delta_{other}$ | | 0.11 |
| | | (0.07) |
| $\delta_0$ | 0.052 | 0.056 |
| | (0.06) | (0.06) |
| Observations | 179 | 179 |
| R-squared | 0.026 | 0.199 |

# Appendix B Model equations

To simplify the algebra in this appendix we omit time arguments except where strictly necessary. In what follows we will make repeatedly use of Ito's formula for jump-diffusion processes so it is worth it to report it here. Let $\mathbf{S}$ be a generic vector of variables of size $J$ with law of motion

$$d\mathbf{S} = \left(\mathbf{S}\boldsymbol{\mu}^{\mathbf{S}} + \eta\mathbf{S}\boldsymbol{\delta}^{\mathbf{S}}\right) dt + \mathbf{S}\boldsymbol{\sigma}_g^{\mathbf{S}} d\mathcal{B}_g + \mathbf{S}\boldsymbol{\sigma}_i^{\mathbf{S}} d\mathcal{B}_i + \mathbf{S}\boldsymbol{\delta}^{\mathbf{S}} d\tilde{\mathcal{P}}$$

where $\tilde{\mathcal{P}}$ is a compensated Poisson process which stasifies $d\tilde{\mathcal{P}} = d\mathcal{P} - \eta dt$. Then, the law of motion of $X = F(\mathbf{S})$ is

$$dX = X\left(\mu^X + \eta\delta^X\right) dt + X\sigma_g^X d\mathcal{B}_g + X\sigma_i^X d\mathcal{B}_i + X\delta^X d\tilde{\mathcal{P}}$$

where

$$\begin{aligned}
\mu^X &= \sum_j \frac{F_{S_j}}{F} S_j \mu^{S_j} + \frac{1}{2}\sum_{j,k} \frac{F_{S_j S_k}}{F} S_j S_k \left(\sigma_g^{S_j}\sigma_g^{S_k} + \sigma_i^{S_j}\sigma_i^{S_k}\right) \\
\sigma_g^X &= \sum_j \frac{F_{S_j}}{F} S_j \sigma_g^{S_j} \\
\sigma_i^X &= \sum_j \frac{F_{S_j}}{F} S_j \sigma_i^{S_j} \\
\delta^X &= \frac{F\left(\mathbf{S} + \mathbf{S}\boldsymbol{\delta}^{\mathbf{S}}\right) - F(\mathbf{S})}{F(\mathbf{S})}
\end{aligned}$$

## B.1 The households' problem

The representative Home household maximizes

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\ln C - \frac{L^{1+\varphi}}{1+\varphi}\right) dt\right]$$

subject to

$$\begin{aligned}
dA = {} & \left[(i - \pi_H) A + WL + \Upsilon - C\mathcal{S}^\alpha - T\right] dt \\
& + B_H \left[(i_H - i) dt - (1-\chi) d\mathcal{P}\right] \\
& + \bar{B}_F \mathcal{S}\left[d\mathcal{E}/\mathcal{E} + (i^* - i) dt\right]
\end{aligned}$$

where

$$d\mathcal{E} = \mathcal{E}\mu^{\mathcal{E}} dt + \mathcal{E}\sigma_g^{\mathcal{E}} d\mathcal{B}_g + \mathcal{E}\sigma_i^{\mathcal{E}} d\mathcal{B}_i + \mathcal{E}\delta^{\mathcal{E}} d\mathcal{P}$$

Her intertemporal problem can be formulated as follows:

$$V(A, \mathbf{S}) = \max_{C,L} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\ln C - \frac{L^{1+\varphi}}{1+\varphi}\right) dt\right]$$

subject to

$$\begin{aligned}
dA/A &= \left(\mu^A + \eta\delta^A\right) dt + \sigma_g^A d\mathcal{B}_g + \sigma_i^A d\mathcal{B}_i + \delta^A d\tilde{\mathcal{P}} \\
d\mathbf{S}/\mathbf{S} &= \left(\boldsymbol{\mu}^{\mathbf{S}} + \eta\boldsymbol{\delta}^{\mathbf{S}}\right) dt + \boldsymbol{\sigma}_g^{\mathbf{S}} d\mathcal{B}_g + \boldsymbol{\sigma}_i^{\mathbf{S}} d\mathcal{B}_i + \boldsymbol{\delta}^{\mathbf{S}} d\tilde{\mathcal{P}}
\end{aligned}$$

with

$$
\begin{aligned}
A\mu^A &= (i-\pi_H)\,A + WL + \Upsilon - C\mathcal{S}^\alpha - T + B_H\,(i_H - i) + \bar{B}_F\mathcal{S}\left(i^* - i + \mu^{\mathcal{E}}\right) \\
A\sigma_g^A &= \bar{B}_F\mathcal{S}\sigma_g^{\mathcal{E}} \\
A\sigma_i^A &= \bar{B}_F\mathcal{S}\sigma_i^{\mathcal{E}} \\
A\delta^A &= \bar{B}_F\mathcal{S}\delta^{\mathcal{E}} - B_H\,(1-\chi)
\end{aligned}
$$

where $\mathbf{S}$ is a generic vector of states of size $J$ and $\boldsymbol{\mu}^{\mathbf{S}}$, $\boldsymbol{\sigma}^{\mathbf{S}}$ and $\boldsymbol{\delta}^{\mathbf{S}}$ are a function of the states only. The HJB for this problem is

$$
\rho V\,(A,\mathbf{S}) = sup_{C,L,B_H}\left\{\ln C - \frac{L^{1+\varphi}}{1+\varphi} + \mathbb{E}\,[dV\,(A,\mathbf{S})]\right\}
$$

where

$$
\begin{aligned}
\mathbb{E}\,[dV] \;=\; & V_A A\mu^A + \sum_j V_{S_j} S_j \mu^{S_j} + \eta\left[V\left(A + A\delta^A, \mathbf{S} + \mathbf{S}\boldsymbol{\delta}^{\mathbf{S}}\right) - V\,(A,\mathbf{S})\right] \\
& + \frac{1}{2}\left[V_{AA}\left(A\sigma_g^A\right)^2 + 2\sum_j V_{AS_j} A S_j \sigma_g^A \sigma_g^{S_j} + \sum_j\sum_k V_{S_jS_k} S_j S_k \sigma_g^{S_j}\sigma_g^{S_k}\right] \\
& + \frac{1}{2}\left[V_{AA}\left(A\sigma_i^A\right)^2 + 2\sum_j V_{AS_j} A S_j \sigma_i^A \sigma_i^{S_j} + \sum_j\sum_k V_{S_jS_k} S_j S_k \sigma_i^{S_j}\sigma_i^{S_k}\right]
\end{aligned}
$$

The first order conditions with respect to $C$ is

$$
\frac{1}{C} = \mathcal{S}^\alpha V_A
$$

Apply Ito's lemma to $V_A\,(A,\mathbf{S})$ to obtain its law of motion

$$
dV_A = \left(V_A\mu^{V_A} + \eta V_A\delta^{V_A}\right)dt + V_A\sigma_g^{V_A}d\mathcal{B}_g + V_A\sigma_i^{V_A}d\mathcal{B}_i + V_A\delta^{V_A}d\tilde{\mathcal{P}}
$$

where

$$
\begin{aligned}
V_A\mu^{V_A} &= V_{AA}A\mu^A + \sum_j V_{AS_j}S_j\mu^{S_j} \\
&\quad + \frac{1}{2}\left[V_{AAA}\left(A\sigma_g^A\right)^2 + 2\sum_j V_{AAS_j}AS_j\sigma_g^A\sigma_g^{S_j} + \sum_{j,k} V_{AS_jS_k}S_jS_k\sigma_g^{S_j}\sigma_g^{S_k}\right] \\
&\quad + \frac{1}{2}\left[V_{AAA}\left(A\sigma_i^A\right)^2 + 2\sum_j V_{AAS_j}AS_j\sigma_i^A\sigma_i^{S_j} + \sum_{j,k} V_{AS_jS_k}S_jS_k\sigma_i^{S_j}\sigma_i^{S_k}\right] \\
V_A\delta^{V_A} &= V_A\left(A + A\delta^A, \mathbf{S} + \mathbf{S}\boldsymbol{\delta}^{\mathbf{S}}\right) - V_A\,(A,\mathbf{S}) \\
V_A\sigma_g^{V_A} &= V_{AA}A\sigma_g^A + \sum_j V_{AS_j}S_j\sigma_g^{S_j} \\
V_A\sigma_i^{V_A} &= V_{AA}A\sigma_i^A + \sum_j V_{AS_j}S_j\sigma_i^{S_j}
\end{aligned}
$$

Now derive both sides of the HJB with respect to $A$ to obtain

$$\begin{aligned}\rho V_A &= V_A\left(i-\pi_H\right)+V_{AA}A\mu^A+\sum_j V_{AS_j}S_j\mu^{S_j}+\eta\left[V_A\left(A+A\delta^A,\mathbf{S}+\mathbf{S}\boldsymbol{\delta}^{\mathbf{S}}\right)-V_A\left(A,\mathbf{S}\right)\right]\\ &+\frac{1}{2}\left[V_{AAA}\left(A\sigma_g^A\right)^2+2\sum_j V_{AAS_j}AS_j\sigma_g^A\sigma_g^{S_j}+\sum_j\sum_k V_{AS_jS_k}S_jS_k\sigma_g^{S_j}\sigma_g^{S_k}\right]\\ &+\frac{1}{2}\left[V_{AAA}\left(A\sigma_i^A\right)^2+2\sum_j V_{AAS_j}AS_j\sigma_i^A\sigma_i^{S_j}+\sum_j\sum_k V_{AS_jS_k}S_jS_k\sigma_i^{S_j}\sigma_i^{S_k}\right]\end{aligned}$$

Therefore

$$\mu^{V_A}=\rho-i+\pi_H-\eta\delta^{V_A}$$

Finally, apply Ito's lemma to the first order condition to obtain

$$\begin{aligned}dC/C=&\left\{-\left(\rho-i+\pi\right)+\sigma_g^{V_A}\sigma_g^{V_A}+\sigma_i^{V_A}\sigma_i^{V_A}+\alpha\frac{1+\alpha}{2}\left(\sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}}+\sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}\right)+\alpha\left(\sigma_g^{V_A}\sigma_g^{\mathcal{S}}+\sigma_i^{V_A}\sigma_i^{\mathcal{S}}\right)\right\}dt\\ &+\left\{\eta\left[\frac{\left[\left(1+\delta^S\right)^\alpha-1\right]^2}{\left(1+\delta^S\right)^\alpha\left(1+\delta^{V_A}\right)}+\delta^{V_A}\frac{\left(1+\delta^S\right)^\alpha-1+\delta^{V_A}}{1+\delta^{V_A}}\right]\right\}dt\\ &-\left(\alpha\sigma_g^{\mathcal{S}}+\sigma_g^{V_A}\right)d\mathcal{B}_g-\left(\alpha\sigma_i^{\mathcal{S}}+\sigma_i^{V_A}\right)d\mathcal{B}_i+\left[\frac{1}{\left(1+\delta^S\right)^\alpha\left(1+\delta^{V_A}\right)}-1\right]d\tilde{\mathcal{P}}\end{aligned}$$

where we used

$$d\mathcal{S}/\mathcal{S}=\left(\mu^S+\eta\delta^S\right)dt+\sigma_g^{\mathcal{S}}d\mathcal{B}_g+\sigma_i^{\mathcal{S}}d\mathcal{B}_i+\delta^{\mathcal{S}}d\tilde{\mathcal{P}}$$

and $\pi\equiv\pi_H+\alpha\mu^S+\eta\left[\left(1+\delta^S\right)^\alpha-1\right]$. Finally, the FOC with respect to $L$ is $W=L^\varphi C\mathcal{S}^\alpha$.

## B.2 The firms' problem

A measure one of monopolistic firms (indexed by $j\in[0,1]$) engage in infrequent price setting a la Calvo. Each firm re-optimizes its price $P_{H,j}(t)$ only at discrete dates determined by a Poisson process with intensity $\theta$. A firm that is allowed to re-optimize its price at time $t$ maximizes the present discounted value of future profits[24]

$$max_{\bar{P}_{H,j}(t)}\mathbb{E}\left[\int_t^\infty\frac{P(t)C(t)}{P(u)C(u)}e^{-(\rho+\theta)(u-t)}\left\{\bar{P}_{H,j}(t)Y_j(u|t)-\mathcal{C}_H\left(Y_j(u|t)\right)\right\}du\right]$$

subject to the demand schedule

$$Y_j(u|t)=\left[\frac{\bar{P}_{H,j}(t)}{P_H(u)}\right]^{-\epsilon}Y(u)$$

where $\mathcal{C}(\cdot)$ is the firms nominal cost function. The first-order condition associated with the problem is

$$\mathbb{E}\left[\int_t^\infty\frac{P(t)C(t)}{P(u)C(u)}e^{-(\rho+\theta)(u-t)}Y_j(u|t)\left\{\bar{P}_{H,j}(t)-\mathcal{M}MC\left(Y_j(u|t)\right)\right\}du\right]=0$$

[24]We assume that firms commit to supply whatever quantity demanded at the posted price, even if that implies negative profits

where $MC$ is the nominal marginal cost function and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$. Note that in the limiting case of no price rigidities ($\theta \to \infty$), this condition collapses to the familiar optimal price-setting condition under flexible prices $P_{H,j}(t) = \mathcal{M}MC(Y_j(t))$.

The firm's cost function is $\mathcal{C}(Y_j(u|t)) = Y_j(u|t)(1-\tau)W(u)P_H(u)$, therefore the nominal marginal cost is

$$MC(Y_j(u|t)) = (1-\tau)W(u)P_H(u) \equiv MC(u)$$

The FOC can be rewritten as

$$\bar{P}_{H,j}(t) = P_H(t)\frac{\mathcal{U}(t)}{\mathcal{V}(t)}$$
$$1 = \mathcal{M}(1-\tau)W(u)$$

where

$$\mathcal{U}(t) \equiv \mathbb{E}\left[\int_t^\infty \left\{e^{-\int_t^u[\rho+\theta-\epsilon\pi_H(s)]ds}\frac{\mathcal{M}MC(u)Y(u)}{P(u)C(u)}\right\}du\right]$$
$$\mathcal{V}(t) \equiv \mathbb{E}\left[\int_t^\infty \left\{e^{-\int_t^u[\rho+\theta-(\epsilon-1)\pi_H(s)]ds}\frac{P_H(u)Y(u)}{P(u)C(u)}\right\}du\right]$$

and we used the result that the dynamics of the price level is locally deterministic (see below). Now assume $\mathcal{U}(t) = \mathcal{U}(\mathbf{S})$ and $\mathcal{V}(t) = \mathcal{V}(\mathbf{S})$. We can apply the Feynman-Kac representation formula to obtain

$$\mathbb{E}[d\mathcal{U}/\mathcal{U}] = \rho + \theta - \epsilon\pi_H - \mathcal{M}\frac{YMC}{\mathcal{U}PC}$$
$$\mathbb{E}[d\mathcal{V}/\mathcal{V}] = \rho + \theta - (\epsilon-1)\pi_H - \frac{YP_H}{\mathcal{V}PC}$$

Therefore, their laws of motion are of the form

$$d\mathcal{U}/\mathcal{U} = \left(\rho + \theta - \epsilon\pi_H - \mathcal{M}\frac{YMC}{\mathcal{U}PC}\right)dt + \sigma_g^{\mathcal{U}}dZ_g + \sigma_i^{\mathcal{U}}dZ_i + \delta^{\mathcal{U}}d\tilde{\mathcal{P}} \tag{A.45}$$
$$d\mathcal{V}/\mathcal{V} = \left(\rho + \theta - (\epsilon-1)\pi_H - \frac{YP_H}{\mathcal{V}PC}\right)dt + \sigma_g^{\mathcal{V}}dZ_g + \sigma_i^{\mathcal{V}}dZ_i + \delta^{\mathcal{V}}d\tilde{\mathcal{P}} \tag{A.46}$$

Since all firms resetting prices choose an identical price $\bar{P}_H$ the law of motion of producer price index is PPI is $dP_H/P_H = \pi_H dt$ where PPI inflation is given by (see Cavallino (2019) for a complete proof)

$$dP_H/P_H = \pi_H dt = \frac{\theta}{\epsilon-1}\left[1-\left(\frac{\bar{P}_H}{P_H}\right)^{1-\epsilon}\right]dt$$

By applying Ito's lemma and using the laws of motion for $\mathcal{U}$ and $\mathcal{V}$ we obtain

$$\begin{aligned}\mathbb{E}[d\pi_H] = {} & [(\epsilon-1)\pi_H-\theta]\left\{\pi_H + \mathcal{M}\frac{YMC}{\mathcal{U}PC} - \frac{YP_H}{\mathcal{V}PC} + \frac{\epsilon}{2}\left(\sigma_g^{\mathcal{U}}\sigma_g^{\mathcal{U}} + \sigma_i^{\mathcal{U}}\sigma_i^{\mathcal{U}}\right) - (\epsilon-1)\left(\sigma_g^{\mathcal{U}}\sigma_g^{\mathcal{V}} + \sigma_i^{\mathcal{U}}\sigma_i^{\mathcal{V}}\right)\right\}dt \\ & + [(\epsilon-1)\pi_H-\theta]\left\{-\frac{2-\epsilon}{2}\left(\sigma_g^{\mathcal{V}}\sigma_g^{\mathcal{V}} + \sigma_i^{\mathcal{V}}\sigma_i^{\mathcal{V}}\right) + \eta\left[\frac{\left(\frac{1+\delta^{\mathcal{U}}}{1+\delta^{\mathcal{V}}}\right)^{1-\epsilon}-1}{\epsilon-1} + \delta^{\mathcal{U}} - \delta^{\mathcal{V}}\right]\right\}dt\end{aligned} \tag{A.47}$$

Finally, let $\Delta \equiv \int_0^1\left[\frac{P_{H,j}}{P_H}\right]^{-\epsilon}dj$ denote the aggregate loss of efficiency induced by price dispersion among firms. Then, its law of motion is (see Cavallino (2019) for a complete proof)

$$d\Delta = \left[\theta\left(1-\frac{\epsilon-1}{\theta}\pi_H\right)^{\frac{\epsilon}{\epsilon-1}} + \Delta(\epsilon\pi_H-\theta)\right]dt \tag{A.48}$$

## B.3 Foreign investors and no-arbitrage conditions

The Home stochastic discount factor for payoffs in units of the domestic good is $\mathcal{D} = e^{-\rho t} V_A$ while the return of the Home-currency bond for Home households is

$$dB_H/B_H = \left[i_H - \pi_H - \eta\left(1-\chi\right)\right]dt - \left(1-\chi\right)d\tilde{\mathcal{P}}$$

where $\zeta$ is our linearization parameter, such that when $\zeta = 0$ the model is deterministic. The portfolio optimality condition is $\mathbb{E}\left[d\left(\mathcal{D}B_H\right)\right] = 0$ and gives rise to the following no-arbitrage equation

$$i_H - i = \eta\left(1-\chi\right)\left(1+\delta^{V_A}\right)$$

Similarly, let $\mathcal{D}^* = e^{-\rho^* t} V_A^*$ be the Foreign stochastic discount factor for payoffs in units of the foreign good and

$$dB_H^*/B_H^* = \left[i_H - \pi_H - \eta\left(1-\chi^*\right)\right]dt - \left(1-\chi^*\right)d\tilde{\mathcal{P}}$$
$$dB_F^*/B_F^* = \left[i_F - \pi^* - \eta\left(1-\chi^*\right)\right]dt - \left(1-\chi^*\right)d\tilde{\mathcal{P}}$$

Then the foreign investors portoflio optimality conditions, $\mathbb{E}\left[d\left(\mathcal{D}^* B_H^*/\mathcal{S}\right)\right] = 0$ and $\mathbb{E}\left[d\left(\mathcal{D}^* B_F^*\right)\right] = 0$, yield

$$i_H - i^* = \mu^{\mathcal{S}} + \pi_H - \pi^* + \eta\left(1+\delta^{V_A^*}\right)\frac{\delta^{\mathcal{S}} + 1 - \chi^*}{1+\delta^{\mathcal{S}}} + \left(\sigma_g^{V_A^*} - \sigma_g^{\mathcal{S}}\right)\sigma_g^{\mathcal{S}} + \left(\sigma_i^{V_A^*} - \sigma_i^{\mathcal{S}}\right)\sigma_i^{\mathcal{S}}$$
$$i_F - i^* = \eta\left(1+\delta^{V_A^*}\right)\left(1-\chi^*\right)$$

Now, since the Home economy is small, ie has a negligible size, $\sigma_g^{V_A^*} = \sigma_i^{V_A^*} = \delta^{V_A^*} = 0$. Hence

$$i_H - i^* = \mu^{\mathcal{S}} + \pi_H - \pi^* + \eta\frac{1 - \chi^* + \delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} - \sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} - \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}$$
$$i_F - i^* = \eta\left(1-\chi^*\right)$$

and

$$\mu^{\mathcal{S}} = i - i^* - \pi_H + \pi^* - \eta\left[\delta^{\mathcal{S}} + \left(\chi - \chi^*\right) - \left(1-\chi\right)\delta^{V_A} - \delta^{\mathcal{S}}\frac{\left(1-\chi^*\right) + \delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}}\right] + \sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}$$

Now use $\mathcal{E} = \mathcal{S}P_H/P^*$ to obtain

$$\mathbb{E}\left[d\mathcal{E}/\mathcal{E}\right] = i - i^* - \eta\left[\chi - \chi^* - \left(1-\chi\right)\delta^{V_A} - \delta^{\mathcal{S}}\frac{\delta^{\mathcal{S}} + \left(1-\chi^*\right)}{1+\delta^{\mathcal{S}}}\right] + \sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}$$

## B.4 Public sector

Government debt evolves according to

$$dB = \left(G - T\right)dt + dB_H + dB_H^* + d\left(\mathcal{S}B_F^*\right)$$

Using the equations derived above we can rewrite it as

$$\begin{aligned} dB = &\left\{\left[i - \pi_H + \eta\left(1-\chi\right)\delta^{V_A} - \eta\left(\chi - \chi^*\right)\frac{B_H^* + \mathcal{S}B_F^*}{B}\right]B + G - T + \eta\chi^*\frac{\delta^{\mathcal{S}}\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}}\mathcal{S}B_F^*\right\}dt \\ &+ \left(\sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}\right)\mathcal{S}B_F^* dt + \mathcal{S}B_F^*\sigma_g^{\mathcal{S}}d\mathcal{B}_g + \mathcal{S}B_F^*\sigma_i^{\mathcal{S}}d\mathcal{B}_i \\ &- \left\{\mathcal{S}B_F^*\left[\zeta\left(1-\chi\right) - \delta^{\mathcal{S}} + \zeta\left(1-\chi^*\right)\delta^{\mathcal{S}}\right] + B_H\zeta\left(1-\chi\right) + B_H^*\zeta\left(1-\chi^*\right)\right\}d\tilde{\mathcal{P}} \end{aligned}$$

### B.5 Equilibrium

Let $\Lambda = \frac{C}{\mathcal{Q}C^*}$ and use $\mathcal{Q} = \mathcal{S}^{1-\alpha}$ to derive its law of motion

$$
\begin{aligned}
d\Lambda/\Lambda = &\left\{\rho^* - \rho + \eta\left[\chi - \chi^* - (1-\chi)\,\delta^{V_A} - (1-\chi^*)\,\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \frac{\delta^{V_A}}{1+\delta^{V_A}}\left(\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \delta^{V_A}\right)\right]\right\} dt \\
&+ \left(\sigma_g^{V_A}\sigma_g^{V_A} + \sigma_i^{V_A}\sigma_i^{V_A} + \sigma_g^{V_A}\sigma_g^{\mathcal{S}} + \sigma_i^{V_A}\sigma_i^{\mathcal{S}}\right) dt - \left(\sigma_g^{V_A} + \sigma_g^{\mathcal{S}}\right) d\mathcal{B}_g - \left(\sigma_i^{V_A} + \sigma_i^{\mathcal{S}}\right) d\mathcal{B}_i \\
&+ \left[\frac{1}{\left(1+\delta^{S}\right)\left(1+\delta^{V_A}\right)} - 1\right] d\tilde{\mathcal{P}}
\end{aligned}
$$

Use the market clearing condition $Y = C_H + C_H^* + G = [\alpha + (1-\alpha)\,\Lambda]\,\mathcal{S}C^* + G$ to compute the law of motion of $Y$

$$
\begin{aligned}
dY = &(Y-G)\left\{i - \pi_H - \rho + \eta\frac{\delta^{V_A}\delta^{V_A}}{1+\delta^{V_A}} + (1-\alpha)\,\Lambda\frac{\sigma_g^{V_A}\sigma_g^{V_A} + \sigma_i^{V_A}\sigma_i^{V_A}}{\alpha + (1-\alpha)\,\Lambda} + (Y-G)\,\alpha\frac{\sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}}{\alpha + (1-\alpha)\,\Lambda} dt\right\} dt \\
&+ (Y-G)\,\alpha\frac{\rho - \rho^* - \eta\left[\chi - \chi^* - (1-\chi)\,\delta^{V_A} - (1-\chi^*)\,\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \frac{\delta^{V_A}\delta^{V_A}}{1+\delta^{V_A}} - \frac{\delta^{\mathcal{S}}\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}}\right]}{\alpha + (1-\alpha)\,\Lambda} dt \\
&+ dG + (Y-G)\,\frac{\alpha\sigma_g^{\mathcal{S}} - (1-\alpha)\,\Lambda\sigma_g^{V_A}}{\alpha + (1-\alpha)\,\Lambda} d\mathcal{B}_g + (Y-G)\,\frac{\alpha\sigma_i^{\mathcal{S}} - (1-\alpha)\,\Lambda\sigma_i^{V_A}}{\alpha + (1-\alpha)\,\Lambda} d\mathcal{B}_i \\
&+ (Y-G)\,\frac{\alpha\delta^{\mathcal{S}} - \frac{\delta^{V_A}}{1+\delta^{V_A}}\,(1-\alpha)\,\Lambda}{\alpha + (1-\alpha)\,\Lambda} d\tilde{\mathcal{P}}
\end{aligned}
$$

Finally, let $Z = \frac{B-A}{\mathcal{S}C^*}$ and derive its law of motion

$$
dZ = [Z\rho^* + \alpha\,(\Lambda - 1)]\,dt - \frac{B_H^*}{\mathcal{S}C^*}\sigma_g^{\mathcal{S}} d\mathcal{B}_g - \frac{B_H^*}{\mathcal{S}C^*} d\mathcal{B}_i - \frac{(1-\chi^*)\left(\mathcal{S}B_F^* + B_H^*\right) + \delta^{\mathcal{S}}\,\frac{1-(1-\chi^*)}{1+\delta^{\mathcal{S}}}B_H^*}{\mathcal{S}C^*} d\tilde{\mathcal{P}}
$$

### B.6 Log-linearization

Using the policy rules described in the main text, the equilibrium of the model can be reduced to the following system of equations

$$\mathbb{E}\left[d\Lambda\right] = \Lambda\left\{\rho^* - \rho + \eta\left[\left(\chi - \chi^*\right) - \left(1-\chi\right)\delta^{V_A} - \left(1-\chi^*\right)\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \frac{\delta^{V_A}}{1+\delta^{V_A}}\left(\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \delta^{V_A}\right)\right]\right\} dt$$
$$+ \Lambda\left(\sigma_g^{V_A}\sigma_g^{V_A} + \sigma_i^{V_A}\sigma_i^{V_A} + \sigma_g^{V_A}\sigma_g^{\mathcal{S}} + \sigma_i^{V_A}\sigma_i^{\mathcal{S}}\right) dt$$

$$\mathbb{E}\left[dY\right] = \left(Y-G\right)\left[\phi_\pi^x \pi_H + \varepsilon_i + \eta\frac{\delta^{V_A}\delta^{V_A}}{1+\delta^{V_A}} + \left(1-\alpha\right)\Lambda\frac{\sigma_g^{V_A}\sigma_g^{V_A} + \sigma_i^{V_A}\sigma_i^{V_A}}{\alpha + \left(1-\alpha\right)\Lambda} + \alpha\frac{\sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}}{\alpha + \left(1-\alpha\right)\Lambda}\right] dt$$
$$+ \left(Y-G\right)\alpha\frac{\rho - \rho^* - \eta\left[\left(\chi-\chi^*\right) - \left(1-\chi\right)\delta^{V_A} - \left(1-\chi^*\right)\frac{\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \frac{\delta^{V_A}\delta^{V_A}}{1+\delta^{V_A}} - \frac{\delta^{\mathcal{S}}\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}}\right]}{\alpha + \left(1-\alpha\right)\Lambda} dt + \mathbb{E}\left[dG\right]$$

$$\mathbb{E}\left[dB\right] = B\left[\rho + \phi_\pi^x\pi_H + \varepsilon_i + \eta\left(1-\chi\right)\delta^{V_A} - \eta\left(\chi-\chi^*\right)\frac{Z + \frac{\bar{B}_F}{C^*}\frac{Y-G}{\alpha+(1-\alpha)\Lambda}}{B}\right] dt$$
$$+ \frac{Y-G}{\alpha + \left(1-\alpha\right)\Lambda}\frac{B_F^*}{C^*}\left[\eta\left(1 - \left(1-\chi^*\right)\right)\frac{\delta^{\mathcal{S}}\delta^{\mathcal{S}}}{1+\delta^{\mathcal{S}}} + \sigma_g^{\mathcal{S}}\sigma_g^{\mathcal{S}} + \sigma_i^{\mathcal{S}}\sigma_i^{\mathcal{S}}\right] dt$$
$$+ \left[G - \bar{T} - \psi_b^x\left(B - \bar{B}\right) - \psi_\pi^x\phi_\pi^x\pi_H\bar{B}\right] dt$$

$$\mathbb{E}\left[dZ\right] = \left[Z\rho^* + \alpha\left(\Lambda - 1\right)\right] dt$$

$$\mathbb{E}\left[d\pi_H\right] = \left[\left(\epsilon-1\right)\pi_H - \theta\right]\left\{\pi_H + \left(\frac{\alpha}{\Lambda} + 1 - \alpha\right)\left[\frac{\Delta^\varphi Y^{1+\varphi}}{\left(1-\alpha\right)\mathcal{U}} - \frac{Y/\mathcal{V}}{Y-G}\right] + \frac{\epsilon}{2}\left(\sigma_g^{\mathcal{U}}\sigma_g^{\mathcal{U}} + \sigma_i^{\mathcal{U}}\sigma_i^{\mathcal{U}}\right)\right\} dt$$
$$+ \left[\left(\epsilon-1\right)\pi_H - \theta\right]\left[\left(1-\epsilon\right)\left(\sigma_g^{\mathcal{U}}\sigma_g^{\mathcal{V}} + \sigma_i^{\mathcal{U}}\sigma_i^{\mathcal{V}}\right) - \frac{2-\epsilon}{2}\left(\sigma_g^{\mathcal{V}}\sigma_g^{\mathcal{V}} + \sigma_i^{\mathcal{V}}\sigma_i^{\mathcal{V}}\right)\right] dt$$
$$+ \left[\left(\epsilon-1\right)\pi_H - \theta\right]\eta\left[\frac{\left(\frac{1+\delta^{\mathcal{U}}}{1+\delta^{\mathcal{V}}}\right)^{1-\epsilon} - 1}{\epsilon - 1} + \delta^{\mathcal{U}} - \delta^{\mathcal{V}}\right] dt$$

$$\mathbb{E}\left[d\mathcal{U}\right] = \mathcal{U}\left[\rho + \theta - \epsilon\pi_H - \left(\frac{\alpha}{\Lambda} + 1 - \alpha\right)\frac{\Delta^\varphi Y^{1+\varphi}}{\left(1-\alpha\right)\mathcal{U}}\right] dt$$

$$\mathbb{E}\left[d\mathcal{V}\right] = \mathcal{V}\left[\rho + \theta - \left(\epsilon-1\right)\pi_H - \left(\frac{\alpha}{\Lambda} + 1 - \alpha\right)\frac{Y}{Y-G}\frac{1}{\mathcal{V}}\right] dt$$

$$\mathbb{E}\left[d\Delta\right] = \left[\theta\left(1 - \frac{\epsilon-1}{\theta}\pi_H\right)^{\frac{\epsilon}{\epsilon-1}} + \Delta\left(\epsilon\pi_H - \theta\right)\right] dt$$

with $G = \varepsilon_g$ and $\eta = \max\left\{0, \bar{\eta} + \eta^x\frac{B-\bar{B}}{\bar{B}}\right\}$, subject to

$$d\varepsilon_g = -\varrho_g\varepsilon_g dt + \sigma_g d\mathcal{B}_g$$
$$d\varepsilon_i = -\varrho_i\varepsilon_i dt + \sigma_i d\mathcal{B}_i$$

To linearize the model, premultiply $\bar{\eta}$, $\sigma_g$, and $\sigma_i$ by the perturbation parameter $\zeta$, such that when $\zeta = 0$ the model is deterministic. Then, in a deterministic steady state we have

$$
\begin{aligned}
\bar{\pi}_H &= 0 \\
\bar{\Delta} &= 1 \\
\bar{Z} &= \frac{\alpha}{\rho^*}\left(1-\bar{\Lambda}\right) \\
\bar{\mathcal{U}} &= \frac{\frac{\alpha}{\bar{\Lambda}}+1-\alpha}{\rho+\theta}\frac{\bar{Y}^{1+\varphi}}{1-\alpha} \\
\bar{\mathcal{V}} &= \frac{\frac{\alpha}{\bar{\Lambda}}+1-\alpha}{\rho+\theta}\frac{\bar{Y}}{\bar{Y}-\bar{G}}
\end{aligned}
$$

We chose a symmetric steady state with $\Lambda = 1$, and we normalize $\bar{Y} = 1$ and $\bar{G} = 0$. Then, the log-linear dynamics around this points are described by the system of differential equations

$$
\begin{aligned}
\mathbb{E}\left[d\lambda\right] &= \tilde{\eta}^x b dt \\
\mathbb{E}\left[dy\right] &= \left(\phi_\pi^x \pi_H - \alpha\tilde{\eta}^x b - \varrho_g\varepsilon_g + \varepsilon_i\right) dt \\
\mathbb{E}\left[db\right] &= \left[\left(\rho - \psi_b^x - \xi\tilde{\eta}^x\right) b + \left(1-\psi_\pi^x\right)\phi_\pi^x\pi_H + \beta\varepsilon_g + \varepsilon_i\right] dt \\
\mathbb{E}\left[dz\right] &= \left(\rho z + \alpha\lambda\right) dt \\
\mathbb{E}\left[d\pi_H\right] &= \left(\rho\pi_H - \kappa\omega y + \kappa\varepsilon_g\right) dt \\
\mathbb{E}\left[d\varepsilon_g\right] &= -\varrho_g\varepsilon_g dt \\
\mathbb{E}\left[d\varepsilon_i\right] &= -\varrho_i\varepsilon_i dt
\end{aligned}
$$

where we used

$$
\begin{aligned}
\rho &= \rho^* + \bar{\eta}\left(\chi-\chi^*\right) \\
\bar{T} &= \left[\rho - \bar{\eta}\left(\chi-\chi^*\right)\xi\right]\bar{B}
\end{aligned}
$$

and $\tilde{\eta}^x = \left(\chi-\chi^*\right)\eta^x$, $\beta = \bar{Y}/\bar{B}$, $\kappa = \theta\left(\rho+\theta\right)$, $\omega = 1+\varphi$. We can omit the linearized laws of motion of $\mathcal{U}$,$\mathcal{V}$,and $\Delta$ since these variables do not enter the linearized dynamic. Note that, like in most small open economy models, the net foreign asset position of the country $z$, and therefore $\lambda$, are not mean-reverting variables. This well-known problem can be solved by assuming that the cost of borrowing abroad is increasing in net foreign debt. This would introduce a term $\varsigma z$ with $\varsigma > 0$ in the equation for $d\lambda$ and make both variable mean-reverting. Since this is true for any $\varsigma > 0$, to simplify the algebra we directly take the limit for $\varsigma \downarrow 0$ .

# Appendix C Proofs

**Solution method** We solve the model using the method of undetermined coefficients. The system of linear differential equation derived above can be written in matrix form

$$\begin{bmatrix} \mathbb{E}\left[d\mathbf{m}\left(t\right)|x\right] \\ \mathbb{E}\left[d\mathbf{n}\left(t\right)|x\right] \end{bmatrix} = \begin{bmatrix} \mathbf{A}^x & \mathbf{B}^x \\ \mathbf{C}^x & \mathbf{D}^x \end{bmatrix} \begin{bmatrix} \mathbf{m}\left(t\right) \\ \mathbf{n}\left(t\right) \end{bmatrix} dt \tag{A.49}$$

where $\mathbf{m}(t) \equiv \begin{bmatrix} \lambda(t) & y(t) & \pi_H(t) \end{bmatrix}^\top$, $\mathbf{n}(t) \equiv \begin{bmatrix} z(t) & b(t) & \varepsilon_g(t) & \varepsilon_i(t) \end{bmatrix}^\top$ and

$$\mathbf{A}^x \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \phi_\pi^x \\ 0 & -\kappa\omega & \rho \end{bmatrix} \qquad \mathbf{B}^x \equiv \begin{bmatrix} 0 & \tilde{\eta}^x & 0 & 0 \\ 0 & -\alpha\tilde{\eta}^x & -\varrho_g & 1 \\ 0 & 0 & \kappa & 0 \end{bmatrix}$$

$$\mathbf{C}^x \equiv \begin{bmatrix} \alpha & 0 & 0 \\ 0 & 0 & \phi_\pi^x\left(1-\psi_\pi^x\right) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{D}^x \equiv \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & \rho-\psi_b^x-\xi\tilde{\eta}^x & \beta & 1 \\ 0 & 0 & -\varrho_g & 0 \\ 0 & 0 & 0 & -\varrho_i \end{bmatrix}$$

for $x \in \{R, N\}$. The transition matrix between the two states is given by

$$\mathbf{\Sigma} = \begin{bmatrix} -\sigma^N & \sigma^N \\ \sigma^R & -\sigma^R \end{bmatrix}$$

We guess a solution of the form $\mathbf{m}(t) = \mathbf{\Gamma}^x\mathbf{n}(t) + \boldsymbol{\zeta}^x(t)$ where

$$\mathbf{\Gamma}^x \equiv \begin{bmatrix} \gamma_{\lambda z}^x & \gamma_{\lambda b}^x & \gamma_{\lambda g}^x & \gamma_{\lambda i}^x \\ \gamma_{yz}^x & \gamma_{yb}^x & \gamma_{yg}^x & \gamma_{yi}^x \\ \gamma_{\pi z}^x & \gamma_{\pi b}^x & \gamma_{\pi g}^x & \gamma_{\pi i}^x \end{bmatrix} \quad \boldsymbol{\zeta}^x(t) \equiv \begin{bmatrix} \zeta_\lambda^x(t) \\ \zeta_y^x(t) \\ \zeta_\pi^x(t) \end{bmatrix}$$

and apply Ito's lemma to obtain

$$\mathbb{E}\left[d\mathbf{m}(t)|x\right] = \mathbf{\Gamma}^x\mathbb{E}\left[d\mathbf{n}(t)|x\right] + \sigma^{-x}\left(\mathbf{\Gamma}^{-x}-\mathbf{\Gamma}^x\right)\mathbf{n}(t) + \sigma^{-x}\left(\boldsymbol{\zeta}^{-x}(t)-\boldsymbol{\zeta}^x(t)\right) + d\boldsymbol{\zeta}^x(t)$$

By replacing our guess into the equilibrium system of differential equations and matching coefficients we obtain the system of polynomials

$$0 = \mathbf{\Gamma}^R\left(\mathbf{C}^R\mathbf{\Gamma}^R + \mathbf{D}^R\right) + \sigma^N\left(\mathbf{\Gamma}^N - \mathbf{\Gamma}^R\right) - \mathbf{A}^R\mathbf{\Gamma}^R - \mathbf{B}^R \tag{A.50}$$

$$0 = \mathbf{\Gamma}^N\left(\mathbf{C}^N\mathbf{\Gamma}^N + \mathbf{D}^N\right) + \sigma^R\left(\mathbf{\Gamma}^R - \mathbf{\Gamma}^N\right) - \mathbf{A}^N\mathbf{\Gamma}^N - \mathbf{B}^N \tag{A.51}$$

and the system of differential equations

$$\begin{bmatrix} d\boldsymbol{\zeta}^R(t) \\ d\boldsymbol{\zeta}^N(t) \end{bmatrix} = \mathbf{U}\begin{bmatrix} \boldsymbol{\zeta}^R(t) \\ \boldsymbol{\zeta}^N(t) \end{bmatrix} \tag{A.52}$$

where

$$\mathbf{U} \equiv \begin{bmatrix} \mathbf{A}^R - \mathbf{\Gamma}^R\mathbf{C}^R + \sigma^N\mathbf{I}_{|\mathbf{m}|} & -\sigma^N\mathbf{I}_{|\mathbf{m}|} \\ -\sigma^R\mathbf{I}_{|\mathbf{m}|} & \mathbf{A}^N - \mathbf{\Gamma}^N\mathbf{C}^N + \sigma^R\mathbf{I}_{|\mathbf{m}|} \end{bmatrix} \tag{A.53}$$

and $\mathbf{I}_{|\mathbf{m}|}$ is the identity matrix of size $|\mathbf{m}| \times |\mathbf{m}|$. The pair of matrices $\left\{\mathbf{\Gamma}^R, \mathbf{\Gamma}^N\right\}$ is a solution to A.49 if it satisfies (A.50) and (A.51). The solution is determined (or unique) if $\boldsymbol{\zeta}^R(t) = \boldsymbol{\zeta}^N(t) = 0$

is the only stable solution to (A.52). That is, iff all eigenvalues of $\mathbf{U}$ are strictly positive. Finally, define

$$\begin{aligned}\mathbf{F}^R &\equiv \mathbf{C}^R\mathbf{\Gamma}^R + \mathbf{D}^R \\ \mathbf{F}^N &\equiv \mathbf{C}^N\mathbf{\Gamma}^N + \mathbf{D}^N \\ \mathbf{S} &\equiv \mathbf{\Sigma}^\top \otimes \mathbf{I}_{|\mathbf{m}|^2} + \text{diag}\left(\mathbf{F}^x \otimes \mathbf{I}_{|\mathbf{n}|} + \mathbf{I}_{|\mathbf{n}|} \otimes \mathbf{F}^x\right)\end{aligned} \tag{A.54}$$

The solution is mean-square stable iff all eigenvalues of $\mathbf{S}$ are strictly negative. For a proof of this result, see Theorem 3.15 in Costa et al. (2013). The endogenous elasticity of default with respect to debt, $\eta^x$, is obtained by setting the maximum eigenvalue of A.54 equal to zero.

Given a solution $\left\{\mathbf{\Gamma}^R, \mathbf{\Gamma}^N\right\}$of the model, the response of the nominal exchange rate in regime $x \in \{R, N\}$ to fiscal and monetary shocks can be calculated using $e(0) = y(0) - (1-\alpha)\lambda(0) - \varepsilon_g(0)$, which yields

$$e(0) = \frac{\gamma^x_{yg} - (1-\alpha)\gamma^x_{\lambda g} - 1}{1 - \iota\left[\gamma^x_{yb} + (1-\alpha)\gamma^x_{\lambda b}\right]}\varepsilon_g(0) + \frac{\gamma^x_{yi} - (1-\alpha)\gamma^x_{\lambda i}}{1 - \iota\left[\gamma^x_{yb} + (1-\alpha)\gamma^x_{\lambda b}\right]}\varepsilon_i(0)$$

where we used the initial conditions $b(0) = \iota e(0)$ and $z(0) = 0$.

**Proof of Propositions 2-3 and 6-7** To adapt the solution method described above to the deterministic case, set $\sigma^R = \sigma^N = 0$ and let $\tilde{\eta}^R = \tilde{\eta}^N = \tilde{\eta}^x$, $\psi^R_\pi = \psi^N_\pi = \psi^x_\pi$, $\psi^R_b = \psi^N_b = \psi^x_b$, and $\phi^R_\pi = \phi^N_\pi = \phi^x_\pi > 0$. First of all, note that output and inflation do not depend directly on $z$, but only indirectly through $\lambda$ which captures the consumption absorption share of the Home economy. Therefore we must have $\gamma^x_{yz} = 0$ and $\gamma^x_{\pi z} = 0$, which implies $\gamma^x_{\lambda z} = -\rho/\alpha$.[25] Now solve for the eigenvalues of A.54. The sign of the largest eigenvalues is determined by

$$\text{Sign}\left[\rho - \psi^x_b - \xi\eta^x + \gamma^x_{\pi b}\left(1 - \psi^x_\pi\right)\phi^x_\pi\right] \tag{A.55}$$

Now guess $\tilde{\eta}^x = 0$. Solve (A.50)-(A.51) for $\gamma^x_{yb}$and $\gamma^x_{\pi b}$ (they can be solved independently of the other coefficients). There are 3 solutions. The first one has $\gamma^x_{yb} = \gamma^x_{\pi b} = 0$ which implies that the system is stable iff $\psi^x_b > \rho$. Solving for the eigenvalues of A.53 it's easy to show that they are all positive iff $\phi^x_\pi > 0$. This is the Ricardian equilibrium and its solution is $\gamma^x_{\lambda b} = \gamma^x_{\lambda g} = \gamma^x_{\lambda m} = 0$ and

$$\begin{aligned}\gamma^x_{yg} &= \frac{\kappa\phi^x_\pi + \varrho(\rho+\varrho)}{\kappa\omega\phi^x_\pi + \varrho(\rho+\varrho)} \\ \gamma^x_{\pi g} &= \frac{\kappa(\omega-1)\varrho}{\kappa\omega\phi^x_\pi + \varrho(\rho+\varrho)} \\ \gamma^x_{ym} &= -\frac{\rho+\varrho}{\kappa\omega\phi^x_\pi + \varrho(\rho+\varrho)} \\ \gamma^x_{\pi m} &= -\frac{\kappa\omega}{\kappa\omega\phi^x_\pi + \varrho(\rho+\varrho)}\end{aligned}$$

The second solution has

$$\gamma^x_{\pi b} = -\frac{\rho - 2\psi^x_b - \sqrt{\rho^2 - 4\kappa\omega\phi^x_\pi}}{2\left(1 - \psi^x_\pi\right)\phi^x_\pi}$$

[25] Note that the model contains a unit root for $\lambda$ and $z$ which can be eliminated by using a debt-elastic foreign interest rate rule.

which implies that implies that it is unstable since A.55 simplifies to Sign $\left[\frac{\rho+\sqrt{\rho^2-4\kappa\omega\phi_\pi^x}}{2}\right]$ whose real part is always positive. The third solution has

$$\gamma_{\pi b}^x = -\frac{\rho - 2\psi_b^x + \sqrt{\rho^2 - 4\kappa\omega\phi_\pi^x}}{2\left(1-\psi_\pi^x\right)\phi_\pi^x}$$

which implies that A.55 simplifies to Sign $\left[\frac{\rho-\sqrt{\rho^2-4\kappa\omega\phi_\pi^x}}{2}\right]$. Thus, the solution is stable iff $\phi_\pi^x < 0$. Finally, the minimum eigenvalue of A.53 is $\rho - \psi_b^x$, which means that the solution is determinate iff $\psi_b^x < \rho$. This is the non-Ricardian inflationary equilibrium and its solution is $\gamma_{\lambda b}^x = \gamma_{\lambda g}^x = \gamma_{\lambda m}^x = 0$ and

$$\begin{aligned}
\gamma_{yb}^x &= \frac{-\psi_b + \mu + \rho}{\mu - \mu\psi_\pi} \\
\gamma_{\pi b}^x &= -\frac{\kappa\omega\left(-\psi_b + \mu + \rho\right)}{\mu\left(\psi_\pi - 1\right)\left(\mu + \rho\right)} \\
\gamma_{yg}^x &= \frac{\frac{-\psi_b(\mu+\rho+\omega\varrho)+\omega\varrho(\mu+2\rho+\varrho)+\rho(\mu+\rho)}{\omega(\mu+\rho+\varrho)} - \frac{\beta(-\psi_b+\mu+\rho)}{\mu(\psi_\pi-1)}}{-\psi_b + \rho + \varrho} \\
\gamma_{\pi g}^x &= \frac{\kappa\left(\frac{(\omega-1)\varrho}{\mu+\rho+\varrho} - \frac{\beta\omega(-\psi_b+\mu+\rho)}{\mu(\psi_\pi-1)(\mu+\rho)}\right)}{-\psi_b + \rho + \varrho} \\
\gamma_{ym}^x &= \frac{\psi_b\left(\mu\psi_\pi + \rho + \varrho\right) - \mu\psi_\pi(\mu + 2\rho + \varrho) - \rho(\rho + \varrho)}{\mu\left(\psi_\pi - 1\right)\left(\mu + \rho + \varrho\right)\left(-\psi_b + \rho + \varrho\right)} \\
\gamma_{\pi m}^x &= -\frac{\kappa\omega\left(\left(\mu + \rho\right)\left(\mu\psi_\pi + \rho + \varrho\right) - \psi_b(\mu + \rho + \varrho)\right)}{\mu\left(\psi_\pi - 1\right)\left(\mu + \rho\right)(\mu + \rho + \varrho)\left(-\psi_b + \rho + \varrho\right)}
\end{aligned}$$

where we defined $\mu \equiv \left(\sqrt{\rho^2 - 4\kappa\omega\phi_\pi^N} - \rho\right)/2$.

Now assume $\tilde{\eta}^x > 0$ and set it $\tilde{\eta}^x = \frac{\rho-\psi_b^x+\gamma_{\pi b}^x(1-\psi_\pi^x)\phi_\pi^x}{\xi}$, such that A.55 is equal to zero. Solve (A.50)-(A.51) for $\gamma_{yb}^x$and $\gamma_{\pi b}^x$ (they can be solved independently of the other coefficients) to obtain

$$\begin{aligned}
\gamma_{yb}^x &= \frac{\alpha\rho}{\kappa\omega\phi_\pi^x}\frac{\rho - \psi_b^x}{\xi - \alpha\left(1 - \psi_\pi^x\right)} \\
\gamma_{\pi b}^x &= \frac{\alpha}{\phi_\pi^x}\frac{\rho - \psi_b^x}{\xi - \alpha\left(1 - \psi_\pi^x\right)}
\end{aligned}$$

This implies that the elasticity of the default probability becomes

$$\tilde{\eta}^x = \frac{\rho - \psi_b^x}{\xi - \alpha\left(1 - \psi_\pi^x\right)}$$

which, assuming $\xi > \alpha\left(1 - \psi_\pi^x\right)$, is positive iff $\psi_b^x < \rho$. Finally, under these assumption, the minimum eigenvalue of A.53 is

$$\frac{\xi\rho + \alpha\left(\psi_\pi^x - 1\right)\left(2\rho - \psi_b^x\right) - \sqrt{\left(\alpha\psi_b^x\psi_\pi^x - \alpha\psi_b^x + \xi\rho\right)^2 - 4\kappa\omega\phi_\pi^x\left(\alpha\psi_\pi^x - \alpha + \xi\right)^2}}{2\left(\alpha\psi_\pi^x - \alpha + \xi\right)}$$

which is strictly positive iff

$$\phi_\pi > \frac{\alpha\rho\left(1 - \psi_\pi^x\right)\left(\rho - \psi_b^x\right)}{\kappa\omega\left(\xi - \alpha + \alpha\psi_\pi^x\right)}$$

This is the non-Ricardian default equilibrium and its solution is

$$
\begin{aligned}
\gamma_{yg}^{x} &= \frac{1}{\omega}\frac{-\alpha\left(\rho\omega(\beta+2\varrho)+\rho^{2}+\omega\varrho^{2}\right)+\alpha\psi_{\pi}\left[\rho^{2}+\omega\varrho\left(2\rho+\varrho\right)\right]}{\kappa\omega\phi_{\pi}\left(\alpha\psi_{\pi}-\alpha+\xi\right)+\left(\rho+\varrho\right)\left\{-\alpha\left(\rho+\varrho\right)+\alpha\left[\psi_{\pi}\left(-\psi_{b}+\rho+\varrho\right)+\psi_{b}\right]+\xi\varrho\right\}} \\
&+\frac{1}{\omega}\frac{\alpha\psi_{b}\left[\omega\left(\beta+\varrho\right)-\psi_{\pi}\left(\rho+\omega\varrho\right)+\rho\right]+\xi\omega\varrho\left(\rho+\varrho\right)}{\kappa\omega\phi_{\pi}\left(\alpha\psi_{\pi}-\alpha+\xi\right)+\left(\rho+\varrho\right)\left\{-\alpha\left(\rho+\varrho\right)+\alpha\left[\psi_{\pi}\left(-\psi_{b}+\rho+\varrho\right)+\psi_{b}\right]+\xi\varrho\right\}} \\
&+\frac{\kappa^{2}\omega\phi_{\pi}^{2}\left(\alpha\psi_{\pi}-\alpha+\xi\right)+\alpha\beta\rho(\rho+\varrho)\left(\rho-\psi_{b}\right)}{\kappa\omega\phi_{\pi}\left(\kappa\omega\phi_{\pi}\left(\alpha\psi_{\pi}-\alpha+\xi\right)+\left(\rho+\varrho\right)\left(-\alpha(\rho+\varrho)+\alpha\left(\psi_{\pi}\left(-\psi_{b}+\rho+\varrho\right)+\psi_{b}\right)+\xi\varrho\right)\right)} \\
\gamma_{\pi g}^{x} &= \frac{\kappa(\omega-1)\varrho\phi_{\pi}\left(\alpha\psi_{\pi}-\alpha+\xi\right)+\alpha\beta(\rho+\varrho)\left(\rho-\psi_{b}\right)}{\phi_{\pi}\left(\kappa\omega\phi_{\pi}\left(\alpha\psi_{\pi}-\alpha+\xi\right)+\left(\rho+\varrho\right)\left(-\alpha(\rho+\varrho)+\alpha\left(\psi_{\pi}\left(-\psi_{b}+\rho+\varrho\right)+\psi_{b}\right)+\xi\varrho\right)\right)} \\
\gamma_{ym}^{x} &= \frac{\kappa\omega\phi_{\pi}\left((\alpha-\xi)(\rho+\varrho)+\alpha\psi_{\pi}\left(\psi_{b}-2\rho-\varrho\right)\right)+\alpha\rho(\rho+\varrho)\left(\rho-\psi_{b}\right)}{\kappa\omega\phi_{\pi}\left(\kappa\omega\phi_{\pi}\left(\alpha\psi_{\pi}-\alpha+\xi\right)+\left(\rho+\varrho\right)\left(-\alpha(\rho+\varrho)+\alpha\left(\psi_{\pi}\left(-\psi_{b}+\rho+\varrho\right)+\psi_{b}\right)+\xi\varrho\right)\right)} \\
\gamma_{\pi m}^{x} &= \frac{\alpha(\rho+\varrho)\left(\rho-\psi_{b}\right)-\kappa\omega\phi_{\pi}\left(\alpha\psi_{\pi}-\alpha+\xi\right)}{\phi_{\pi}\left(\kappa\omega\phi_{\pi}\left(\alpha\psi_{\pi}-\alpha+\xi\right)+\left(\rho+\varrho\right)\left(-\alpha(\rho+\varrho)+\alpha\left(\psi_{\pi}\left(-\psi_{b}+\rho+\varrho\right)+\psi_{b}\right)+\xi\varrho\right)\right)}
\end{aligned}
$$

For each equilibrium, the response of the nominal exchange rate to fiscal and monetary shocks is then calculated using $e\left(0\right)=y\left(0\right)-\left(1-\alpha\right)\lambda\left(0\right)-\varepsilon_{g}\left(0\right)$, which yields

$$
e\left(0\right)=\frac{\gamma_{yg}^{x}-\left(1-\alpha\right)\gamma_{\lambda g}^{x}-1}{1-\iota\left[\gamma_{yb}^{x}+\left(1-\alpha\right)\gamma_{\lambda b}^{x}\right]}\varepsilon_{g}\left(0\right)+\frac{\gamma_{yi}^{x}-\left(1-\alpha\right)\gamma_{\lambda i}^{x}}{1-\iota\left[\gamma_{yb}^{x}+\left(1-\alpha\right)\gamma_{\lambda b}^{x}\right]}\varepsilon_{i}\left(0\right)
$$

where we used the initial conditions $b\left(0\right)=\iota e\left(0\right)$ and $z\left(0\right)=0$.

**Proof of Propositions 4-5** Assume $\phi_{\pi}^{R}=\phi_{\pi}^{N}=\phi_{\pi}>0$, $\psi_{b}^{R}>\rho$, and $\psi_{b}^{N}<\rho$, such that $\tilde{\eta}^{R}=0$. Note that output and inflation do not depend directly on $z$, but only indirectly through $\lambda$ which captures the consumption absorption share of the Home economy. Therefore we must have $\gamma_{yz}^{R}=\gamma_{yz}^{N}=0$ and $\gamma_{\pi z}^{R}=\gamma_{\pi z}^{N}=0$, which implies $\gamma_{\lambda z}^{R}=\gamma_{\lambda z}^{N}=-\rho/\alpha$. Now solve for the eigenvalues of A.54 and show that the maximum one is zero iff

$$
\tilde{\eta}^{N}=\frac{1}{2\xi}\left[2\left(\rho-\psi_{b}^{N}\right)+2\gamma_{\pi b}^{N}\left(1-\psi_{\pi}^{N}\right)\phi_{\pi\mathrm{N}}-\sigma^{R}-\frac{\sigma^{N}\sigma^{R}}{2\left(\rho-\psi_{b}^{R}\right)-\sigma_{N}+\frac{2\alpha\rho\sigma^{N}(1-\psi_{\pi}^{R})(\rho+\sigma^{N}+\sigma^{R})}{\xi\kappa\omega\phi_{\pi}+\xi(\sigma^{N}+\sigma^{R})(\rho+\sigma^{N}+\sigma^{R})}}\right] \tag{A.56}
$$

Using (A.56) and assuming $\psi_{\pi}^{R}=1$, (A.50)-(A.51) can be solved analytically. Unfortunately the equations are too large to be reported here (they are available upon request). To simplify them, assume also $\psi_{\pi}^{N}=1$, $\psi_{b}^{R}=\rho$, and $\psi_{b}^{N}=0$. Then (A.56) simplyfies to $\tilde{\eta}^{N}=\rho/\xi$ and the solution of (A.50)-(A.51) is

$$
\begin{aligned}
\gamma_{yb}^{R} &= \frac{\alpha\rho\sigma^{N}\left(\rho\left(\sigma^{N}+\rho+\sigma^{R}\right)-\kappa\omega\phi_{\pi}\right)}{\kappa\xi\omega\phi_{\pi}\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}\right)\left(\sigma^{N}+\rho+\sigma^{R}\right)\right)} \\
\gamma_{\pi b}^{R} &= \frac{\alpha\rho\sigma^{N}\left(\sigma^{N}+\rho+\sigma^{R}\right)}{\xi\phi_{\pi}\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}\right)\left(\sigma^{N}+\rho+\sigma^{R}\right)\right)} \\
\gamma_{\lambda b}^{R} &= -\frac{\sigma^{N}}{\xi\left(\sigma^{N}+\rho+\sigma^{R}\right)}
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{yg}^{R} &= \sigma^{N}\frac{\phi_{\pi}\left((\rho+2\varrho)\left(\kappa-\beta\gamma_{\pi b}^{N}\right)+\sigma^{R}\left(2\kappa-\beta\left(\gamma_{\pi b}^{N}+\gamma_{\pi b}^{R}\right)\right)\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&+\sigma^{N}\frac{\beta\gamma_{yb}^{N}\left((\rho+\varrho)\left(\rho+\sigma^{R}+\varrho\right)-\kappa\omega\phi_{\pi}\right)+\varrho(\rho+\varrho)\left(\rho+2\sigma^{R}+2\varrho\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&+\frac{\sigma^{N}\left(\phi_{\pi}\left(\kappa-\beta\gamma_{\pi b}^{N}\right)+(\rho+\varrho)\left(\beta\gamma_{yb}^{N}+\varrho\right)\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&+\frac{\beta\gamma_{yb}^{R}\left(\kappa\omega\phi_{\pi}\left(\sigma^{N}+\rho+\varrho\right)+(\rho+\varrho)\left(\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&+\frac{\left(\phi_{\pi}\left(\kappa-\beta\gamma_{\pi b}^{R}\right)+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_{\pi}+\left(\sigma^{R}+\varrho\right)\left(\rho+\sigma^{R}+\varrho\right)\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
\gamma_{\pi g}^{R} &= \frac{\sigma^{N}\left(\beta\gamma_{\pi b}^{N}\varrho+\beta\kappa\omega\gamma_{yb}^{N}+\kappa(\omega-1)\varrho\right)+\sigma^{R}\left(\beta\gamma_{\pi b}^{R}\varrho+\beta\kappa\omega\gamma_{yb}^{R}+\kappa(\omega-1)\varrho\right)}{\left(\sigma^{N}+\sigma^{R}\right)(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))}\\
&+\frac{\beta\sigma^{N}\left(\kappa\omega\left(\gamma_{yb}^{R}-\gamma_{yb}^{N}\right)-\left(\gamma_{\pi b}^{N}-\gamma_{\pi b}^{R}\right)\left(\sigma^{N}+\sigma^{R}+\varrho\right)\right)}{\left(\sigma^{N}+\sigma^{R}\right)\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
\gamma_{\lambda g}^{R} &= -\frac{\beta\sigma^{N}\left(\sigma^{N}+2\rho+\sigma^{R}+\varrho\right)}{\xi(\rho+\varrho)\left(\sigma^{N}+\rho+\sigma^{R}\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)}
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{ym}^{R} &= -\phi_{\pi}\frac{\sigma^{N}\left(\gamma_{\pi b}^{N}\left(\sigma^{N}+\rho+\sigma^{R}+2\varrho\right)+\kappa\omega\gamma_{yb}^{N}\right)-\kappa\omega\gamma_{yb}^{R}\left(\sigma^{N}+\rho+\varrho\right)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&-\phi_{\pi}\frac{\gamma_{\pi b}^{R}\left(\sigma^{R}\left(\sigma^{N}+\rho+\sigma^{R}+2\varrho\right)+\varrho(\rho+\varrho)\right)+\kappa\omega(\rho+\varrho)}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
&+\frac{(\rho+\varrho)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\left(\left(\gamma_{yb}^{N}-1\right)\sigma^{N}+\gamma_{yb}^{R}\left(\sigma^{R}+\varrho\right)-\sigma^{R}-\varrho\right)+\gamma_{\pi b}^{R}(-\kappa)\omega\phi_{\pi}^{2}}{(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
\gamma_{\pi m}^{R} &= \frac{\sigma^{N}\left(\kappa\omega\left(\gamma_{yb}^{N}-1\right)+\gamma_{\pi b}^{N}\varrho\right)+\sigma^{R}\left(\kappa\omega\left(\gamma_{yb}^{R}-1\right)+\gamma_{\pi b}^{R}\varrho\right)}{\left(\sigma^{N}+\sigma^{R}\right)(\kappa\omega\phi_{\pi}+\varrho(\rho+\varrho))}\\
&+\frac{\sigma^{N}\left(\kappa\omega\left(\gamma_{yb}^{R}-\gamma_{yb}^{N}\right)-\left(\gamma_{\pi b}^{N}-\gamma_{\pi b}^{R}\right)\left(\sigma^{N}+\sigma^{R}+\varrho\right)\right)}{\left(\sigma^{N}+\sigma^{R}\right)\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}+\varrho\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)\right)}\\
\gamma_{\lambda m}^{R} &= -\frac{\sigma^{N}\left(\sigma^{N}+2\rho+\sigma^{R}+\varrho\right)}{\xi(\rho+\varrho)\left(\sigma^{N}+\rho+\sigma^{R}\right)\left(\sigma^{N}+\rho+\sigma^{R}+\varrho\right)}
\end{aligned}
$$

and

$$
\begin{aligned}
\gamma_{yb}^{N} &= \frac{\alpha\rho\left(\rho\sigma^{N}\left(\sigma^{N}+\rho+\sigma^{R}\right)+\kappa\omega\phi_{\pi}\left(\rho+\sigma^{R}\right)\right)}{\kappa\xi\omega\phi_{\pi}\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}\right)\left(\sigma^{N}+\rho+\sigma^{R}\right)\right)}\\
\gamma_{\pi b}^{N} &= \frac{\alpha\rho\left(\kappa\omega\phi_{\pi}+\sigma^{N}\left(\sigma^{N}+\rho+\sigma^{R}\right)\right)}{\xi\phi_{\pi}\left(\kappa\omega\phi_{\pi}+\left(\sigma^{N}+\sigma^{R}\right)\left(\sigma^{N}+\rho+\sigma^{R}\right)\right)}\\
\gamma_{\lambda b}^{N} &= -\frac{\sigma^{N}+\rho}{\xi\left(\sigma^{N}+\rho+\sigma^{R}\right)}
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{yg}^{N} &= \phi_\pi \frac{\beta\gamma_{\pi b}^{N}\left(-\left(\sigma^N+\varrho\right)\left(\sigma^N+\rho+\varrho\right)-\sigma^N\sigma^R\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&+\phi_\pi \frac{\sigma^R\left(\kappa\left(-\beta\omega\gamma_{yb}^{R}+2\sigma^N+\rho+2\varrho\right)-\beta\gamma_{\pi b}^{R}\left(\sigma^N+\rho+2\varrho\right)\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&+\phi_\pi \frac{\sigma^R\left(\kappa-\beta\gamma_{\pi b}^{R}\right)+\kappa(\omega+1)\varrho(\rho+\varrho)+\kappa\sigma^N\left(\sigma^N+\rho+2\varrho\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&+\frac{\kappa\omega\phi_\pi^2\left(\kappa-\beta\gamma_{\pi b}^{N}\right)+\beta\gamma_{yb}^{N}\left(\left(\rho+\varrho\right)\left(\sigma^N+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)+\kappa\omega\phi_\pi\left(\rho+\sigma^R+\varrho\right)\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&+\frac{\left(\rho+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\left(\sigma^R\left(\beta\gamma_{yb}^{R}+\varrho\right)+\varrho\left(\sigma^N+\varrho\right)\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
\gamma_{\pi g}^{N} &= \frac{\sigma^N\left(\beta\gamma_{\pi b}^{N}\varrho+\beta\kappa\omega\gamma_{yb}^{N}+\kappa(\omega-1)\varrho\right)+\sigma^R\left(\beta\gamma_{\pi b}^{R}\varrho+\beta\kappa\omega\gamma_{yb}^{R}+\kappa(\omega-1)\varrho\right)}{\left(\sigma^N+\sigma^R\right)\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)} \\
&+\frac{\beta\sigma^R\left(\left(\gamma_{\pi b}^{N}-\gamma_{\pi b}^{R}\right)\left(\sigma^N+\sigma^R+\varrho\right)+\kappa\omega\left(\gamma_{yb}^{N}-\gamma_{yb}^{R}\right)\right)}{\left(\sigma^N+\sigma^R\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
\gamma_{\lambda g}^{N} &= -\frac{\beta\left(\left(\sigma^N+\rho\right)\left(\sigma^N+\rho+\varrho\right)+\sigma^N\sigma^R\right)}{\xi(\rho+\varrho)\left(\sigma^N+\rho+\sigma^R\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)}
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{ym}^{N} &= -\phi_\pi \frac{\gamma_{\pi b}^{N}\left(\left(\sigma^N+\varrho\right)\left(\sigma^N+\rho+\varrho\right)+\sigma^N\sigma^R\right)+\gamma_{\pi b}^{R}\sigma^R\left(\sigma^N+\rho+\sigma^R+2\varrho\right)+\kappa\omega\left(\gamma_{yb}^{R}\sigma^R+\rho+\varrho\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&+\frac{\gamma_{\pi b}^{N}\kappa\omega\phi_\pi^2+\gamma_{yb}^{N}\left(\left(\rho+\varrho\right)\left(\sigma^N+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)+\kappa\omega\phi_\pi\left(\rho+\sigma^R+\varrho\right)\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
&-\frac{\left(\rho+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\left(-\left(\gamma_{yb}^{R}-1\right)\sigma^R+\sigma^N+\varrho\right)}{\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
\gamma_{\pi m}^{N} &= \frac{\sigma^N\left(\kappa\omega\left(\gamma_{yb}^{N}-1\right)+\gamma_{\pi b}^{N}\varrho\right)+\sigma^R\left(\kappa\omega\left(\gamma_{yb}^{R}-1\right)+\gamma_{\pi b}^{R}\varrho\right)}{\left(\sigma^N+\sigma^R\right)\left(\kappa\omega\phi_\pi+\varrho(\rho+\varrho)\right)} \\
&+\frac{\sigma^R\left(\left(\gamma_{\pi b}^{N}-\gamma_{\pi b}^{R}\right)\left(\sigma^N+\sigma^R+\varrho\right)+\kappa\omega\left(\gamma_{yb}^{N}-\gamma_{yb}^{R}\right)\right)}{\left(\sigma^N+\sigma^R\right)\left(\kappa\omega\phi_\pi+\left(\sigma^N+\sigma^R+\varrho\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)\right)} \\
\gamma_{\lambda m}^{N} &= -\frac{\left(\sigma^N+\rho\right)\left(\sigma^N+\rho+\varrho\right)+\sigma^N\sigma^R}{\xi(\rho+\varrho)\left(\sigma^N+\rho+\sigma^R\right)\left(\sigma^N+\rho+\sigma^R+\varrho\right)}
\end{aligned}
$$

Finally, compute the eigenvalues of (A.53) and check that they are all positive iff $\phi_\pi > 0$. The response of the nominal exchange rate to fiscal and monetary shocks is then calculated using $e(0) = y(0) - (1-\alpha)\lambda(0) - \varepsilon_g(0)$, which yields

$$
e^x(0) = \frac{\gamma_{yg}^{x}-(1-\alpha)\gamma_{\lambda g}^{x}-1}{1-\iota\left[\gamma_{yb}^{x}+(1-\alpha)\gamma_{\lambda b}^{x}\right]}\varepsilon_g(0)+\frac{\gamma_{yi}^{x}-(1-\alpha)\gamma_{\lambda i}^{x}}{1-\iota\left[\gamma_{yb}^{x}+(1-\alpha)\gamma_{\lambda b}^{x}\right]}\varepsilon_i(0)
$$

for $x \in \{R, N\}$, where we used the initial conditions $b(0) = \iota e(0)$ and $z(0) = 0$.

**Proof of Proposition 8** Assume $\tilde{\eta}^R = \tilde{\eta}^N = 0$, $\psi_\pi^R = 1$ $\psi_\pi^N = 0$, $\psi_b^R = \rho$, $\psi_b^N = 0$ and $\sigma^N = 0$. Note that output and inflation do not depend directly on $z$, but only indirectly through $\lambda$ which captures the consumption absorption share of the Home economy. Therefore we must have $\gamma_{yz}^R = \gamma_{yz}^N = 0$ and $\gamma_{\pi z}^R = \gamma_{\pi z}^N = 0$, which implies $\gamma_{\lambda z}^R = \gamma_{\lambda z}^N = -\rho/\alpha$. Define $\mu \equiv \left(\sqrt{\rho^2 - 4\kappa\omega\phi_\pi^N} - \rho\right)/2$. Then the solution of (A.50)-(A.51) is

$$\gamma_{yb}^R = 0$$
$$\gamma_{\pi b}^R = 0$$
$$\gamma_{\lambda b}^R = 0$$

$$\begin{aligned}
\gamma_{yg}^R &= \frac{\kappa\phi_\pi^R + \varrho(\rho+\varrho)}{\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)} \\
\gamma_{\pi g}^R &= \frac{\kappa(\omega-1)\varrho}{\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)} \\
\gamma_{\lambda g}^R &= 0
\end{aligned}$$

$$\begin{aligned}
\gamma_{ym}^R &= -\frac{\rho+\varrho}{\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)} \\
\gamma_{\pi m}^R &= -\frac{\kappa\omega}{\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)} \\
\gamma_{\lambda m}^R &= 0
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{yb}^N &= \frac{\mu+\rho-\sigma^R}{\mu} \\
\gamma_{\pi b}^N &= \frac{\kappa\omega\left(\mu+\rho-\sigma^R\right)}{\mu(\mu+\rho)} \\
\lambda_{\mathrm{bN}} &= 0
\end{aligned}$$

$$\begin{aligned}
\gamma_{yg}^N = {} & \frac{\left(\mu^2(\omega(\beta+\varrho)+\rho) + \mu\left(2\rho\omega(\beta+\varrho) + \omega\varrho(\beta+\varrho) + \rho^2\right) + \beta\rho\omega(\rho+\varrho)\right)\left(\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)\right)}{\mu\omega(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)\left(\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)\right)} \\
& + \sigma^R \frac{\kappa\omega\phi_\pi^R(\mu(\rho+\varrho) - \beta\omega\varrho) + \varrho\left(-\beta\omega\varrho(\rho+\varrho) + \mu^2(\omega-1)\varrho + \mu\left(\rho^2\omega + \rho(3\omega-1)\varrho + \omega\varrho^2\right)\right)}{\mu\omega(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)\left(\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)\right)} \\
& + \sigma^R \frac{-\beta\kappa\omega^2\phi_\pi^R - \varrho\left(\beta\omega(\rho+\varrho) - \mu^2(\omega-1) + \mu(\rho-\rho\omega)\right)}{\mu\omega(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)\left(\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)\right)} \\
\gamma_{\pi g}^N = {} & \frac{\kappa\left((\mu+\rho)(\mu(\beta\omega + (\omega-1)\varrho) + \beta\omega(\rho+\varrho)) - \sigma^R\left(\beta\omega\varrho + \mu^2 + \mu\rho\right) - \beta\omega\sigma^R\right)}{\mu(\mu+\rho)(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)} \\
& - \frac{\sigma^R\left(\kappa\phi_\pi^R + \varrho(\rho+\varrho)\right)\left(-\kappa\omega\phi_\pi^R + \varrho\sigma^R + \varrho^2\right)}{\phi_\pi^R\left(\kappa\omega\phi_\pi^R + \varrho(\rho+\varrho)\right)(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)} + \frac{\varrho\sigma^R\left(\sigma^R+\varrho\right)}{\phi_\pi^R(\rho+\varrho)\left(\mu+\rho+\sigma^R+\varrho\right)} \\
\gamma_{\lambda g}^N = {} & 0
\end{aligned}$$

$$\gamma_{ym}^{N} = \frac{\rho(\rho+\varrho)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)-\sigma^{R}\left(\kappa\omega\phi_{\pi}^{R}+\mu^{2}+\mu\rho+\varrho(\rho+\varrho)\right)}{\mu(\rho+\varrho)\left(\mu+\rho+\sigma^{R}+\varrho\right)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)}$$

$$-\sigma^{R}\frac{\kappa\omega\varrho\phi_{\pi}^{R}+\mu^{2}\varrho+\mu\left(\rho^{2}+3\rho\varrho+\varrho^{2}\right)+\varrho^{2}(\rho+\varrho)}{\mu(\rho+\varrho)\left(\mu+\rho+\sigma^{R}+\varrho\right)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)}$$

$$\gamma_{\pi m}^{N} = \kappa\omega\frac{(\mu+\rho)(\rho+\varrho)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)-\sigma^{R}\left(\kappa\omega\phi_{\pi}^{R}+\mu^{2}+\mu\rho+\varrho(\rho+\varrho)\right)}{\mu(\mu+\rho)(\rho+\varrho)\left(\mu+\rho+\sigma^{R}+\varrho\right)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)}$$

$$-\sigma^{R}\kappa\omega\frac{\kappa\omega\varrho\phi_{\pi}^{R}+\mu^{2}(\rho+2\varrho)+\mu\rho(\rho+2\varrho)+\varrho^{2}(\rho+\varrho)}{\mu(\mu+\rho)(\rho+\varrho)\left(\mu+\rho+\sigma^{R}+\varrho\right)\left(\kappa\omega\phi_{\pi}^{R}+\varrho(\rho+\varrho)\right)}$$

$$\gamma_{\lambda m}^{N} = 0$$

Compute the eigenvalues of (A.53) and check that they are all positive iff $\phi_{\pi}^{R} > 0$ and $\mu > 0$, which implies $\phi_{\pi}^{N} < 0$. Finally, compute the eigenvalues of (A.54) and show that the maximum eigenvalue is nonpositive iff $\mu \geq \sigma^{R}/2$. The response of the nominal exchange rate to fiscal and monetary shocks is then calculated using $e(0) = y(0) - (1-\alpha)\lambda(0) - \varepsilon_{g}(0)$, which yields

$$e^{x}(0) = \frac{\gamma_{yg}^{x} - (1-\alpha)\gamma_{\lambda g}^{x} - 1}{1-\iota\left[\gamma_{yb}^{x} + (1-\alpha)\gamma_{\lambda b}^{x}\right]}\varepsilon_{g}(0) + \frac{\gamma_{yi}^{x} - (1-\alpha)\gamma_{\lambda i}^{x}}{1-\iota\left[\gamma_{yb}^{x} + (1-\alpha)\gamma_{\lambda b}^{x}\right]}\varepsilon_{i}(0)$$

for $x \in \{R, N\}$, where we used the initial conditions $b(0) = \iota e(0)$ and $z(0) = 0$.