AGENCY IN INTANGIBLES

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Abstract

I argue that intangible assets promote agency conflicts between outside investors and inside specialists. Their opacity and specialized nature provide a microfoundation for why highly intangible firms underinvest despite high valuations—a challenge for standard theories. This microfoundation and several other model predictions are supported in US and international data. I conclude that the rise of intangibles has likely aggravated agency frictions and account for about a third of the abnormal decline in investment.

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INTRODUCTION

Over the past 40 years or so, one marked shift in the economic landscape has been towards a dominant role played by intangible capital. Understanding its many impacts on the economy is thus vital. Yet how much can we truly understand? Intangibles are valued indirectly and hard to quantify. And their development is at the hands of specialists who may possess their own private interest. In this paper, I show that these essential features of intangible capital have likely exacerbated agency conflicts and plausibly account for puzzling patterns in the data.

To fix ideas, consider a firm's optimal choice for *physical* investment:

$$c'(i) = q. \tag{1}$$

Under an increasing and convex adjustment cost, $c(\cdot)$, the equation positively relates the investment rate, i, to the level of marginal q. The right side is unobserved and influenced by many factors, including the firm's intangibility.

Complementarity of physical and intangible capital in production implies that a high average Q, which is observed, should still predict substantial investment. The reasoning is straightforward and agrees with Tobin's original intuition: events that make firms valuable tend to make them good places to invest. This is a tenet of investment theory and an outcome of virtually all firm investment models in economics and finance.

I begin in Section I by documenting a set of stylized facts that show this tenet is violated: Average Q's rise with intangibility, yet physical investment rates remain flat or even fall. This pattern holds in panel data, non-parametric portfolio sorts, and across major economies and suggests that to better understand intangible capital we need to go beyond standard theory.

I develop a potential resolution to these facts in Section II by extending the standard firm investment model to distinguish physical capital, K, from intangible capital, N, by having the latter (i) not be directly observable to investors (principals) and (ii) be developed by specialists (agents) who have their own private interest. In the presence of agency conflicts (1) holds, crucially, from *investors' perspective*. Practically this means that marginal q now accounts for how physical investment improves or harms agents' incentives. If harmful enough, then investment will be optimally reduced, even in the presence of high average Q.

An appealing feature of the model is that investors can only infer what has happened to intangibles through other observable channels—and thus their understanding of the truth is limited. They write a contract based on observable variables to enforce the appropriate action because specialists could shirk unknowingly to investors for private benefits.

Common to the dynamic contracting literature, the optimal contract rewards agents following good historical performance, effectively granting them more ownership and lessening the agency conflict, and terminates them following serial bad performance. In contrast to this common result, however, greater profitability in my model is delivered via higher intangibility, $n = N/K$, so the conflict does not necessarily diminish following good histories. As a result, the agency model can replicate the stylized facts, reducing physical investment's marginal benefit as it potentially harms agents' effective ownership. Thus I provide a novel microfoundation for the great profitability, lofty valuations, yet puzzling small investment rates of highly intangible firms.

In order for the model to quantitatively capture the stylized facts, I need to overcome a technical challenge that has been largely sidestepped in the contracting literature, which commonly resorts to oversimplification at the cost of limited applicability. In Section III I show how to overcome this challenge and ultimately progress in this fruitful line of work. My second contribution is therefore to provide a numerical solution method that is general enough to be used to solve a wide class of problems, including those faced outside the contracting literature.

In Section IV I turn to validating several unique model predictions. First, intangible capital and agency conflicts are linked, and the optimal contract prescribes that compensation should grow in states in which the conflict is particularly severe. I show this prediction holds in the data and, interestingly, that greater rates of intangible development are responsible for shifting compensation of employees from current to deferred. Second, the model suggests that not only should compensation grow with intangibility, but also become more variable. I also find evidence of this does indeed show up in the second moment and is consistent with the model's form of the optimal contract.

After calibrating the model, I then analyze model predictions in Section V and compare them with what we see in the data. The standard model finds it challenging, if not impossible, to account for the stylized facts. On the other hand, I confirm that the agency model can even quantitatively replicate the key patterns of investment, valuations, and profitability.

I then provide evidence that thinking of intangibility as measuring the wedge between the interests of insiders and outsiders is a useful reduction. I corroborate this by documenting shifts to internal financing from external finance, reminiscent of patterns predicted by the Whited and Wu (2006) index of financial constraints. Finally, I conclude that many data patterns corroborate a simple economic mechanism where an agency conflict is embedded into the intangibility of a firm.

My paper contributes to topics in dynamic agency.[1] Like DeMarzo, Fishman, He and Wang (2012), this paper develops an optimal contracting framework for investment but distinguishes physical from intangible capital and analyzes how an agency friction embedded into the latter can account for features of data. A novelty here relative to that paper is that great historical profitability

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[1] A partial list is Albuquerque and Hopenhayn (2004), Quadrini (2004), Dow, Gorton and Krishnamurthy (2005), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007a), Biais, Mariotti, Plantin and Rochet (2007), Nikolov and Whited (2014), Morellec, Nikolov and Schürhoff (Forthcoming), and Tong and Ying (2019).

does not necessarily imply a small agency problem. Bolton, Wang and Yang (Forthcoming) study the impact of inalienability of human capital where a "key person's" potential departure is a risk to the firm. By paper instead focuses on the incentives to *develop* intangible capital holding fixed this departure risk, although this, too, could be analyzed in an extended version of the model. New to the literature, I overcome a technical challenge and embed the dynamic contracting environment into an stationary industry equilibrium (Hopenhayn (1992)). Industry analysis is an important consideration because implications of agency models often hinge on the likelihood of termination of these key agents and firm exit.

My paper also complements the large literature on financing, investment, and asset prices.[2] Rampini and Viswanathan (2013) show in a model of limited commitment that collateral is an primary determinant of capital structure. They provide a "user cost of (physical) capital" intuition where an additional term to the classic Jorgenson (1963) formula arises that captures the scarcity of internal funds. Collateral (tangibility) affects the marginal cost of investment in contrast to my paper where intangibility affects the marginal benefit. Ai and Li (2015) incorporate limited commitment of contracts into the neoclassical investment model and show that small firms have higher Tobin's Q and invest more. My paper also endogenizes contracting constraints yet connects them to intangibility. A unique prediction different from these papers is that highly intangible firms are typically smaller, possess high valuations and profitability, yet they invest less.

Albuquerque and Wang (2008) show that a priced investment-specific shock can raise investment volatility and risk, inducing outside shareholders to underinvest. Eisfeldt and Papanikolaou (2013) demonstrate that firms with a great deal of organizational capital are more likely to experience departure of "key talent", load more on a systematic risk factor, and possess higher discount rates. . In contrast to these two papers, I highlight a feature of intangibility driven by idiosyncratic risk.

Of course, there are other factors which matter to the development of intangible capital, like technological changes or market structure. For example, Lustig, Syverson and Van Nieuwerburgh (2011) document upward trends in managerial income inequality and pay-performance sensitivity and argue it reflects a shift towards general productivity growth from vintage-specific growth. Instead, I focus on the development of intangible capital and how changes in it affects agents' compensation and physical investment. The study of how incentive contracts shape the development of intangibles and vice versa is important in its own right as many models of intangible capital, as in this paper, are predicated on special agents who possess some control over it. My paper therefore connects technical change to the distribution of wages and executive compensation (Acemoglu (2002) and Violante (2002) and see Edmans, Gabaix and Jenter (2017) for a survey on

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[2] A short list is Gomes (2001), Krueger and Uhlig (2006), Hennessey and Whited (2007), Rampini and Viswanathan (2010), Ai, Croce and Li (2013), Glover and Levine (2017), and Sun and Xiaolan (Forthcoming).

executive compensation) and the decline in physical investment (Gutiérrez and Philippon (2017b) and Crouzet and Eberly (2018)).

In Section I I produce the stylized facts that are challenging for standard models. In Section II I set up the extended model predicated on an agency conflict. Section III describes the features of the solution to the agency model and also discusses in detail my novel solution method to solve this general class of problems. I validate some of the model's key predictions in Section IV before turning to the calibration. I analyze the model's predictions in Section V and compare them with the data. I then conclude.

I. MAIN DATA AND STYLIZED FACTS

I primarily use on public firms as they provide a useful lab to study agency conflicts. My US data begin in 1975, the year American accounting standards require the reporting of R&D, and end in 2017, although earlier data is used to construct intangible capital. Firm-level data comes from combining annual accounting data from Compustat with monthly equity and return data from CRSP. I additionally look at international accounting data spanning several major economies using Compustat Global's dataset. Details on data sources and variable definitions are provided in Appendix A.

A. INTANGIBLE CAPITAL AND PORTFOLIO SORTS

I quantify unmeasured intangibles with a perpetual inventory method used in the accounting and finance literature (Lev and Radhakrishnan (2005) and Eisfeldt and Papanikolaou (2013)). It is constructed for every firm i for each year t using selling, general, and administrative expenses:

$$N_{it} = (1 - 0.15) \times N_{it-1} + 0.3 \times \frac{SG\&A_{it}}{CPI_t}, \tag{2}$$

where CPI_t is the consumer price index. Following recent work, I attribute 30 percent of (real) $SG\&A$ expenditure to intangible development, use a 15 percent depreciation rate, and initialize a firm's N_{i0} at $SG\&A_{i0}/0.25$.

Compustat's record of $SG\&A$ includes expenditure on research and development and a large part of it consists of expenses related to training (human capital and business processes), skilled labor (programmers), information technology, as well as marketing (brand capital). Eisfeldt and Papanikolaou (2013) validate this measure with corroborating evidence on managerial quality, investment in information technology, and productivity. I measure a firm's relative opacity of total capital with its *intangibility*: the ratio of unmeasured intangible capital plus capitalized intangibles (goodwill and other intangible assets) to the value of (real) property, plant, and equipment.

I then form other common measures of investment, profitability, and value. I compute every firm's physical investment rate and, analogously, its intangible development rate according to (2). I calculate *physical* average Q (Peters and Taylor (2017)) to be the market value of the firm over the replacement cost of physical capital, as both numerator and denominator are relatively accurate. While previous work has shown it need not average near unity (Lindenberg and Ross (1981)), Tobin's basic intuition argues that higher values should correspond with more investment. Lastly, I define profitability as operating cash flows before depreciation and normalize by last year's physical capital.

After these key variables have been constructed, I create intangibility portfolios and look at their characteristics: every June, I rank firms by their previous fiscal year's intangibility and sort them into quintiles. Because accounting standards and production functions vary across industries, firms are ranked relative to their industry peers using two-digit NAICS codes. I then form five value-weighted portfolios based on these within-industry ranks and rebalance every year (ending in June 2018). Portfolio construction and rebalancing mitigates the effects of firm entry and exit, nonstationarity in firm-level data, and, importantly for intangibility, measurement error.

B. STYLIZED FACTS

I present the stylized facts in Table I by tabulating investment and development rates, (physical) average Q, and profitability for these portfolios of increasing intangibility. Common intuition suggests that a firm of great profitability and value would find it optimal to invest commensurately. Yet this is not what we see in the data: greater intangibility raises levels of average Q and profitability while rates of both physical investment and intangible development remain flat or even decline. This poses a challenge to standard models, which would predict a rise in physical investment rates across portfolios.[3]

Average returns, moreover, do not differ significantly across portfolios. Therefore, explanations based solely on differences in systematic risk, where higher discounting attributed to greater intangibility lowers marginal q and investment, are unlikely to resolve completely the stylized facts.[4] I believe that intangible firms are, in some sense, riskier, but there could be factors other than dis-

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[3] For completeness, I report in Appendix A univariate sorts on average Q and tangibility and show that in both sorts investment rates rise with average Q, suggesting it is necessary to capitalize unmeasured intangible investment.

[4] That average returns do not differ across portfolios is at odds with the findings in Eisfeldt and Papanikolaou (2013). My ranking metric differs from theirs in several ways: they divide unmeasured (organizational) capital by book assets, within rank relative to SIC codes, accumulate SG&A with a factor of 1 rather than 0.3, use a different sample period, only include firms with December-end fiscal years, among others. It should be noted, however, that both metrics produce an inverse relation between intangibility (organizational capital) and physical investment rates. Intangibility of course is difficult to measure, so it is expected that various constructions could produce different interpretations of the data. In Appendix B I extend the standard model to include a risk premium on intangibility. I find that even in the presence of this extension, it cannot fully explain the flatness of investment with respect to intangibility.

count rates at play, so I distinguish channels by conditionally sorting portfolios. That is, I first sort firms on characteristics known to correlate with discount rates and then, within each discount rate sort, sort on intangibility. The idea is to hold discount rates fixed while varying intangibility.

In practice, I use the well-known value (book-to-market) characteristic of Fama and French (1992) which explains much of the cross-section of average stock returns.[5] More specifically, I first sort firms by their book-to-market (BE/ME) ratio into terciles based on NYSE breakpoints. Within each book-to-market tercile, I then sort firms by their intangibility within each industry into terciles.

I report the results of these conditional sorts in Table II. Within a book-to-market sort and across intangibility terciles, I report portfolio spreads from the high tercile as the average time series difference and include robust standard errors. Looking within a sort on BE/ME we see the same pattern in the bottom two subpanels: average Q rises yet investment rates remain flat or even fall when moving from the medium to the high intangibility portfolio. These patterns are statistically distinguishable.

Average returns increase monotonically in book-to-market ratios, reproducing the value premium, with the exception when moving from medium to high BE/ME portfolio in the high intangibility tercile. Yet average returns across intangibility portfolios do not differ within a book-to-market grouping, so it is unlikely that differences in exposure to systematic risk are solely responsible for the results.

In conclusion, it is unlikely that discount rates completely explain the stylized patterns. A further advantage of sorting is that it ranks non-parametrically and does not impose a linear structure potentially inconsistent with (1). A disadvantage is that unobserved differences across firms within an industry could determine their intangibility choices and, in turn, drive the results. In the next section I go into finer detail to test the effects at the firm-level.

C. FIRM-LEVEL REGRESSIONS

In Table III I report the local average effects of classic investment panel regressions. As is the standard approach in the literature, all specifications use firm fixed effects to control for unobserved differences across firms and to let variation within a firm determine average responses. Because variation could come from aggregate or industry-level trends, I cluster standard errors at the firm-level.

The first specification in column (1) documents the well-known result that average Q (weakly) positively correlates with investment (Summers (1981)): an unit increase in it predicts just over a one percentage point rise in the investment rate. The second regression in (2) confirms the non-

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[5] The ability of the size (market equity) characteristic to explain variation in returns has been questioned in recent years (van Dijk (2011), Fama and French (2012)).

parametric results from the portfolio sorts: a growth in intangibility, holding Q fixed, lowers the physical investment rate of a firm.

I then study the interaction of these two regressors to emphasize the underlying channel. In column (3), the slope coefficient on the interaction term of Q and intangibility is negative and statistically significant. The predicted response is that an additional unit of Q's effect on the physical investment rate *decreases* as intangibility rises:

$$\frac{\partial \mathbb{E}[i|X]}{\partial Q} = 2.104 - 0.120 \times \text{Intangibility}, \tag{3}$$

where the conditional expectation of investment rate, $\mathbb{E}[i|X]$, is given the set of regressors, X. In this sense, greater intangibility widens the gap between the standard model's prediction of the positive relationship between Q and investment. One reason could be that intangibility measures the wedge between investors' and insiders' interests. A wider wedge would lower the expected marginal benefit of physical investment.

In classic models of dynamic agency, an agent's continuation payoff is tied to the entire history of profitability. Great historical profitability mitigates the agency conflict and enhances the marginal return of capital; thus cash flow positively predicts investment. Columns (4) and (5), which adds year fixed effects, show that this is not the case in the data: the coefficient on profitability and intangibility's interaction is negative. Even though profitability rises with intangibility, the response on physical investment is lessened. Thus a distinguishing feature of the data is that classic agency models may also find it challenging to generate the stylized facts.

Finally, in column (6) I add other known predictors of investment: book-to-market, cash-to-assets, and tangibility. Growth firms and those with large cash holdings invest more and tangibility, all else equal, enters with the wrong sign. These additional variables do not overturn any of the results and across all configurations the negative coefficients on the interaction terms remain stable.

D. INTERNATIONAL RESULTS

After completing this analysis in the US, I extend it to other major economies. The international sample is composed of British, German, French, and Japanese firms.[6] To ensure a large enough sample for the portfolio sorts that are constructed within industry, I limit coverage to manufacturing firms over a uniform period of 1994 to 2017 for all countries.

Table IV reproduces the main results of the empirical analysis. In Panel A I report intangibility-sorted portfolio statistics of average Q and investment by country. With the exception of France, the country with the least number of sampled firms, all countries display the eventual flat or declining

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[6] Canadian and Italian firms, completing the G7, are not sufficiently sampled in Compustat Global for all forms of the analysis.

investment profile with respect to greater intangibility.

In Panel B, I run the classic investment panel regressions on Q, intangibility, and their interaction for each country as well as a pooled estimate across these four economies. When the coefficients are estimated to be statistically significant, they are negative. Altogether, the evidence is in broad agreement with the US data, suggesting the phenomenon is global.

II. MODEL SETUP

The stylized facts presented in Section I present a challenge to standard models in generating the correct patterns among investment and value with respect to intangibility. In this section, I extend the standard model by introducing an agency conflict to better capture these facts.

In the model, investors hire agents who possess their own private interest to operate the firm and develop the firm's intangible capital, a process which is not directly observable. They write a contract that is incentive compatible with agents' interests to maximize firm value.

Investors have unlimited wealth and can therefore provide funds for the purchase of capital whenever suitable. They are risk-neutral and discount at rate r. Agents are also risk-neutral but discount at rate $\gamma > r$, a common assumption that reflects their assumed impatience or the presence of outside opportunities.[7] Agents have limited liability and no initial wealth.

A. PRODUCTION AND PHYSICAL CAPITAL

The firm uses capital and employs (unskilled) labor, L, at wage rate w_L to generate (instantaneous) cash flows

$$\Pi_t = \max_{L_t \geq 0} \left\{ \overline{A} F(K_t, N_t)^{\alpha} L_t^{1-\alpha} - i_t K_t - C_K(I_t, K_t) - g_t N_t - C_N(g_t N_t, N_t) - w_L L_t \right\}, \tag{4}$$

where $\overline{A} > 0$ is the level of productivity and α is the capital share of income. Capital comprises intangible capital, N, and physical capital, K, and is aggregated via the function $F(K, N)$. Being fully and instantaneously adjustable, optimal labor demand is proportional to capital, and substituting this optimum into (4) defines $A = \alpha \overline{A} \left(\frac{(1-\alpha)\overline{A}}{w_L} \right)^{\frac{1-\alpha}{\alpha}}$ and gives

$$\Pi_t = AF(K_t, N_t) - i_t K_t - C_K(i_t K_t, K_t) - g_t N_t - C_N(g_t N_t, N_t). \tag{5}$$

The physical investment rate is denoted by i and, following the vast literature on investment theory, it is subject to adjustment costs, $C_K(iK, K)$. The development rate of intangible capital,

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[7] While $\gamma = r$ may be a more neutral assumption, DeMarzo and Sannikov (2006) argue a contract can be made more robust by having investors assume that γ is higher than agents' true γ.

$g \geq 0$, is also chosen by principals to maximize firm value subject to adjustment costs, $C_N(gN, N)$. Finally, physical capital grows according to the standard accumulation equation subject to the depreciation rate $\delta_K \geq 0$:

$$dK_t = (i_t - \delta_K)K_t dt. \tag{6}$$

B. INTANGIBLE CAPITAL AND AGENCY FRICTION

Intangible capital is an agglomeration of technologies—information technology, research and development design, business processes, human capital, and brand capital—that complements traditional factors of production and is embodied by a firm. Its development is gradual and reflects the difficulty and effort in creating something new and verifying results. Agents' collective effort $e_t \in \{0, 1\}$ determines the growth of intangible capital that evolves as the persistent process

$$dN_t = (g_t e_t - \delta_N)N_t dt + \sigma N_t dZ_t. \tag{7}$$

The actual growth of intangible capital is hindered by obsolescence at rate $\delta_N \geq 0$ and fluctuates with the increments of a Brownian motion, dZ_t, with volatility $\sigma > 0$. Uncertainty here reflects the fundamental riskiness with developing a new technology or building a brand in the form of stochastic obsolescence or unproductive realized investment. Thus the degree of uncertainty associated with the technology, and the degree to which agents can hide their effort, scales with σ.

When agents do not exert effort ($e_t = 0$), they enjoy private benefits at rate $g_t \Lambda(K_t, N_t)dt$. Benefits being influenced by physical capital, $\Lambda(K, \cdot)$, is commonplace in the literature on empire building, and I naturally extend this function to include intangible capital, $\Lambda(\cdot, N)$, as it enhances revenue and profitability, power and prestige.[8] The potential for private benefits to rise with sales and wages is consistent with Edmans, Gabaix and Landier's (2008) treatment of private benefits as a normal good and the empirical evidence I provide in Section IV.

Effort only affects the drift of the process and a lack of it can be interpreted as shirking. In either interpretation, the function $\Lambda(\cdot) > 0$ measures the gap between outsiders' and insiders' interests and thus the severity of the agency friction.

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[8] It could also in part reflect an intellectual reward. Intangible capital is often developed by skilled labor who have typically chosen to attain advanced degrees. Cutting-edge research often goes hand-in-hand with traveling, attending conferences, and opining on technical issues that potentially influence policy. Moskowitz and Vissing-Jørgensen (2002) calculate the nonpecuniary benefits of entrepreneurship to be effectively 143 percent of total annual income; a number they find plausible given the compensation economists purportedly give up to remain in academia.

III. MODEL SOLUTION

I now provide details of the model described in Section II. I begin by highlighting the standard model's difficulty in reconciling the stylized facts. I then introduce the agency friction and discuss its solution.

To simplify the analysis, I assume that the capital function, $F(K,N)$, adjustment costs, $C_K(iK,K)$ and $C_N(gN,N)$, and private benefits, $\Lambda(K,N)$, are all homogeneous of degree one in their arguments. Specifically, $F(K,N)=Kf(n)$ where $f(\cdot)$ is increasing and concave; the cost of physical investment is $iK+C_K(iK,K)=Kc_K(i)$ and intangible development $gN+C_N(gN,N)=Nc_N(g)$; and $\Lambda(K,N)=K\lambda(n)$. The state variable $n=N/K$ is the intangibility of the firm and evolves, by Ito's lemma, as

$$dn_t = d\left(\frac{N_t}{K_t}\right) = ((g_t e_t-\delta_N)-(i_t-\delta_K))n_t dt+\sigma n_t dZ_t. \tag{8}$$

A. STANDARD MODEL

The standard model has no agency conflict. Without private benefits ($\lambda(\cdot)\equiv 0$), agents choose $e_t=1$ for all t. Homogeneity allows its solution to be represented as an ordinary differential equation[9]

$$rp(n)=\max_{i,g}\pi(n)+p(n)(i-\delta_K)+p'(n)(g-\delta_N-(i-\delta_K))n+\frac{1}{2}p''(n)n^2\sigma^2, \tag{9}$$

which equates the required return, $rp(n)$, to dividends, $\Pi/K=\pi(n)=Af(n)-c_K(i)-c_N(g)n$, plus expected capital gains: the gain, respectively, from an additional unit of physical capital and intangibility with an adjustment for variability.

The choice for the development rate intuitively sets $c_N'(g(n))=p'(n)$ and physical investment's choice follows from

$$\underbrace{c_K'(i(n))}_{\text{marginal cost}} = \underbrace{\underbrace{p(n)}_{\text{average } Q} - p'(n)n}_{\text{marginal } q} \tag{10}$$

where the right side is *physical* marginal q. Investment's marginal cost is equated to physical average Q, $p(n)$, net of the reduction in the firm's intangibility, $p'(n)n$, which is profitable in general. Optimal physical investment thus trades off the marginal values of physical and intangible

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[9] Firm value in (9) is before the agent has been compensated. If investors have promised to pay $W>0$ to agents, then it is optimal to pay out W in cash immediately because agents are relatively impatient. On a per physical capital basis, $w=W/K$, the value of the firm to investors is $p(n)-w$.

capital.

Under complementarity, a relative abundance of intangible capital implies that physical capital's marginal value is great and therefore a high investment rate desirable. Conversely, when intangibility is low, so are physical capital's marginal value and investment. Thus given an initial condition and absent uncertainty ($\sigma = 0$), a firm's physical investment choice would gravitate towards an interior point of intangibility. In the presence of uncertainty ($\sigma > 0$) and given the concavity of $p(n)$, the firm's intangibility would be on average below this steady state point, though these gravitational forces would remain.

Standard models therefore imply that investment, Q, and intangibility all positively relate, contradicting the facts of Section I. More practically, this implies that sorts on intangibility are equivalent to sorts on Q in regards to their predictions for investment, and controlling for intangibility is equivalent to controlling for Q in panel regressions.

B. AGENCY MODEL

I introduce the agency problem by setting $\lambda(\cdot) > 0$. I assume the firm's physical capital K_t and historical cash flow process $\{\Pi_s : 0 \leq s \leq t\}$ are observable and contractible. From (6), the investment rate i_t can therefore be contracted upon. Since physical capital and investment are contractible, cash flow realizations allows investors to write contracts on intangible capital N_t and its development rate g_t.

Principals therefore monitor other, better measured variables to assess and contract on agents' effort towards intangibles' development.[10] This feature captures the essence of intangible capital and agrees with approaches to valuing it—the Bureau of Economic Analysis (BEA), for example, indirectly measures intangible investment in software in part with programmers' wages.

The contract with agents can be terminated at any time. When it is, agents receive the value of their outside option that I normalize to zero. Investors recover a fraction $0 < l_N, l_K < 1$ of capital, altogether recovering $l_K K_t + l_N N_t$. Because not all is recovered, termination is inefficient ex post.

The contract offered by investors maximizes firm value and specifies agents' cumulative current compensation, U_t, investment and development policies, i_t and g_t, and a termination time, τ. All variables depend on the history of agents' performance, which is given by the evolution of intangibility. Limited liability requires incremental current compensation, dU_t, to be nondecreasing. I let $\mathcal{C} = (U, i, g, \tau)$ represent the contract.

Given the contract, agents choose their action process to maximize the present value of current

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[10] An example of what I have in mind is that when investors measure, say, a search engine's development of intangible capital, basically the efficacy of its algorithm, they measure the incremental amount of revenues obtained from ad-clicks or number of searches, controlling for other factors.

compensation and private benefits,

$$W(\mathcal{C}) = \max_{\{e_t \in \{0,1\}: 0 \leq t < \tau\}} \mathbb{E}^e \left[\int_0^\tau e^{-\gamma t} (dU_t + (1 - e_t) g_t \Lambda(K_t, N_t) dt) \right], \tag{11}$$

where the expectation, $\mathbb{E}^e[\cdot]$, is taken under the probability measure conditional on agents' effort process.

When the contract is written, the firm has K_0 units of physical capital and N_0 units of intangible capital. Investors write a contract that maximizes the value of the firm, the present value of cash flows less compensation paid to agents plus recovery in the event of termination,

$$P(K_0, N_0, W_0) = \max_{\mathcal{C}} \mathbb{E} \left[\int_0^\tau e^{-rt} (\Pi_t dt - dU_t) + e^{-r\tau} (l_K K_\tau + l_N N_\tau) \right]$$
$$\text{s.t. } \mathcal{C} \text{ is incentive compatible and } W(\mathcal{C}) = W_0. \tag{12}$$

Agents' initial wealth, W_0, is determined by the bargaining power of agents and investors when the contract is initiated. If, for example, investors possess all bargaining power, then $W_0 = \mathrm{argmax}_{W \geq 0} P(K_0, N_0, W)$; if agents possess all power, then $W_0 = \max\{W : P(K_0, N_0, W) \geq 0\}$. More generally, it could blend the two extremes and I implement this with a parameter $\varphi \in (0, 1)$ by having W_0 equal $\varphi \max\{W | P(K_0, N_0, W) > 0\} + (1 - \varphi)\, \mathrm{argmax}_{W \geq 0} P(K_0, N_0, W)$.

B.1. Incentive Compatible Contract

I focus on the case where the contract is incentive compatible and implements the efficient action, $e_t = 1$ for all t, and discuss its optimality as well as other model extensions in Appendix B. Given this contract and the history up until time t, agents' *continuation payoff* is given by

$$W_t(\mathcal{C}) = \mathbb{E}_t \left[\int_t^\tau e^{-\gamma(s-t)} dU_s \right]. \tag{13}$$

As is standard in dynamic contracting models, agents' incremental compensation at time t is composed of current compensation, dU_t, and the incremental change in the continuation payoff, dW_t (Spear and Srivastava (1987)). Promise keeping has these two sources compensate agents for their time preference on average:

$$\mathbb{E}_t[dW_t + dU_t] = \gamma W_t dt. \tag{14}$$

To maintain incentive compatibility, agents' compensation must remain sufficiently sensitive to the firm's cash flow process. When intangible capital can be contracted upon, I can formulate this sensitivity directly with the martingale representation theorem (see Sannikov (2008) for details):

$$dW_t + dU_t = \gamma W_t dt + \beta_t N_t \left(\frac{dN_t}{N_t} - (g_t - \delta_N)dt \right) = \gamma W_t dt + \beta_t N_t \sigma dZ_t. \tag{15}$$

In this environment, shocks to intangible capital and agents' continuation payoff, dN_t and dW_t, become perfectly correlated.

Agents who deviate reduce their compensation by $g_t \beta_t N_t dt$ and receive private benefits $g_t \Lambda(K_t, N_t)dt$. Incentive compatibility is then implemented with $\beta_t \geq \Lambda(K_t, N_t)/N_t$, for each t. Because liquidation is ex post inefficient and therefore costly to enforce, the optimal contract minimizes the likelihood of this event and sets

$$\beta_t = \frac{\Lambda(K_t, N_t)}{N_t} = \frac{\lambda(n_t)}{n_t}, \text{ for all } t. \tag{16}$$

This condition is natural and equalizes the incentive coefficient, β_t, to the stock of private benefits per unit of intangible capital. It is also potentially time-varying and could be more costly for investors to maintain agents' efficient action in highly intangible firms. Moreover, placing (16) into (15) suggests, intriguingly, that the functional form of private benefits, $\lambda(\cdot)$, can in principle be recovered from an analysis of how shocks to compensation depend on intangibility.[11]

This insight is more general than it first might appear. Optimal dynamic contracts endogenize incentive coefficients as functions of the underlying state space. While outside the scope of this paper, this observation should carry over to other potential distortions—a firm's cash holdings (Jensen (1986)) or financial intermediary's net worth (He and Krishnamurthy (2009)) for example. Treating private benefits as a function rather than a parameter encourages potentially fruitful empirical and structural work in how compensation sensitivities vary with conflicts of agency.

C. FORM OF SOLUTION AND BOUNDARY CONDITIONS

I now describe the form of the solution to the problem in (12). First, whatever the history of the firm up until date t, the only relevant state variables are the firm's capital stocks, K_t and N_t, and agents' continuation payoff, W_t. Therefore, investors' value function at time t, $P(K_t, N_t, W_t)$, can be solved using dynamic programming techniques. Second, homogeneity allows me to reduce the problem to two endogenous state variables—intangibility, $n = N/K$, and agents' (scaled) continuation payoff, $w = W/K$—and write $p(n, w) = P(K, N, W)/K$. The latter's dynamics

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[11] If agents are also risk averse, the incentive coefficient, more generally, is determined in part by the form of risk-sharing. Therefore $\lambda(\cdot)$ is therefore best thought of as a simple yet practical, reduced-form function more broadly capturing risk-sharing as well as private benefits.

follows directly from (6) and (15) under the incentive compatible contract and imply

$$dw_t = (\gamma - (i_t - \delta_K))w_t dt + \lambda(n_t)\sigma dZ_t. \tag{17}$$

Similar to the standard model, the solution to (12) has investors demanding required returns equal to expected returns through their choices of investment and development

$$rp(n,w)dt = \max_{i,g} \pi(n,w)dt + \mathbb{E}_t\left[d(Kp(n,w))/K\right]. \tag{18}$$

The solution here, however, is represented by a partial differential equation (PDE) subject to the boundary conditions described below. I defer its explicit form until Subsection D in which I describe my novel numerical method to solve it.

Also similar to the standard model, the development rate satisfies $c'_N(g(n,w)) = p_n(n,w)$. The investment rate, by contrast, takes the form

$$c'_K(i(n,w)) = p(n,w) - p_n(n,w)n - p_w(n,w)w \tag{19}$$

and newly accounts for shifts in agents' continuation payoff w. Investors now internalize the effect of investment on the agency conflict.

Now consider a growth in intangibility. Given complementarity, the rate of cash flows will generally increase and make it efficient to reduce the likelihood of termination. It will therefore be optimal to raise agents' continuation payoff and better align incentives in these states in which the incentive problem is more costly. More generally, the optimal contract will dynamically *smooth* this cost.

The desire for smoothing, in turn, makes large rates of physical investment unappealing, especially when intangibility is high and agency conflicts severe. Specifically, if firm value becomes extremely sensitive to shifts in w, then investors may optimally choose to invest very little, even if the average gain on a unit of physical capital, $p(n,w)$, is large.

C.1. Boundary Conditions

The boundary conditions that define the state space over which (18) is solved make the problem challenging and I now discuss them in turn. First, the agent will be terminated immediately once their continuation payoff reaches the value of their outside option (normalized to zero), because otherwise they would immediately consume private benefits and therefore

$$p(n,0) = l_K + l_N \times n, \text{ for each } n. \tag{20}$$

In effect, consequent to a sequence of adverse shocks ($dZ_t < 0$), the firm's cash flow rate slows, leading principals to believe agents have not been sufficiently developing intangible capital and to terminate the contract and fire the agents.

Second, because agents can always be paid current compensation ($du_t = dU_t/K_t > 0$), it will cost investors at most one dollar to increase w by one dollar, implying a lower bound of $p_w(n,w) \geq -1$, for all n. As w grows, the likelihood of termination and costly liquidation falls. It is thus optimal to grow w as quickly as possible at low levels by setting du_t to zero. Since agents are impatient relative to investors, however, current compensation will be required at some threshold of w to discourage agents from consuming private benefits. At this threshold, investors will be indifferent between reducing their value by one dollar to pay agents one dollar in current compensation,

$$p_w(n, \overline{w}(n)) = -1, \text{ for each } n. \tag{21}$$

And finally, the super contact condition is required to ensure that this threshold is optimal on behalf of investors,

$$p_{ww}(n, \overline{w}(n)) = 0, \text{ for each } n. \tag{22}$$

Note that these conditions hold for each n, so I highlight the dependence of the payment boundary on the firm's intangibility, $\overline{w}(n)$. That the payment boundary is jointly determined with the solution creates a challenge to solving (18).

D. NUMERICAL SOLUTION METHOD

Much of the existing work in the dynamic contracting literature has been largely restricted to analyses involving problems of single-state ordinary differential equations.[12] This technical constraint limits the proper assessment of these models and ultimately the progress in this line of work. In this section I describe how to overcome this limitation and, because the contribution is material, document it in detail here.[13]

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[12] Contemporary contributions are DeMarzo and Sannikov (2006) and Sannikov (2008) in continuous-time mathematics and DeMarzo and Fishman (2007a) and DeMarzo and Fishman (2007b) in discrete-time. Phelan and Townsend (1991) provided an early solution to the recursive formulation of the contracting problem. More recently, Piskorski and Tchistyi (2010) and DeMarzo et al. (2012) solve nested ODEs over two exogenous states. I elaborate on the technical constraint in Appendix B.

[13] Two papers in the economics literature that solve a similar numerical problem are Murto and Terviö (2014) who employ a discrete-time approximation to a continuous model and Décamps and Villeneuve (2015) who use a quasi-explicit solution. Different from these papers, my problem is of a general class that does not permit a quasi-explicit solution and cannot use a discrete-time approximation as they require randomization of the termination decision (see DeMarzo and Sannikov (2006)).

Written out fully the equation in (18) is

$$\begin{aligned} rp(n,w) = \max_{i,g} & Af(n) - c_K(i) - c_N(g)n + p(n,w)(i-\delta) \\ & + p_n(n,w)(g - \delta_N - (i-\delta))n + p_w(n,w)(\gamma - (i-\delta))w \\ & + \frac{1}{2}p_{nn}(n,w)(\sigma n)^2 + \frac{1}{2}p_{ww}(n,w)(\lambda(n)\sigma)^2 + p_{nw}(n,w)\sigma^2 n\lambda(n). \end{aligned} \tag{23}$$

I solve it with a finite difference method that approximates the value function $p(n,w)$ on a non-rectangular grid: $n \in \{n_i\}_{i=1}^{I}$ and $w \in \{w_j(n_i)\}_{j=1}^{J^i}$. The set of grid points along a given j, $w_j(n_i)$, depend on the value of n_i because of each payment boundary point, $w_{J^i}(n_i) = \overline{w}(n_i)$.

I approximate first derivatives of p using both backward and forward differences and second derivatives with central differences. All differences of n and w are calculated respectively over the fixed increments Δ_n and Δ_w. I use an implicit difference method, following Candler (1999) and Achdou, Han, Lasry, Lions and Moll (2017) who discuss the "verify" part of the solution, which solves the vector

$p^{b+1} = (p_{1,1}^{b+1}, \ldots, p_{1,J^1}^{b+1}, p_{2,1}^{b+1}, \ldots, p_{2,J^2}^{b+1}, \ldots, p_{I,J^I}^{b+1})'$ using notation $p_{i,j} = p(n_i, w_j)$.[14] It begins with a guess $b = 1$ and proceeds to iterate until convergence ($\max(|p^{b+1} - p^b|) < 10^{-9}$) on the equation

$$p^{b+1}\left[\left(\frac{1}{\Delta} + r - (i - \delta_K)\right) - \mathbf{Q}\right] = Af(n) - c_K(i) - c_N(g)n + D + p^b/\Delta, \tag{24}$$

where i solves (19) and g solves $c_N'(g(n,w)) = p_n(n,w)$ on each iteration, $\Delta > 0$ is the step size of the iteration, and the $\sum_{i=1}^{I} J^i \times \sum_{i=1}^{I} J^i$-matrix $\mathbf{Q}$ is the transition matrix defined by the

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[14] Barles and Souganidis (1991) show that if the solution method satisfies monotonicity, stability, and consistency, then as Δ_n and Δ_w get small the solution converges locally uniformly to the unique viscosity solution. Here, monotonicity is ensured by the upwind scheme; stability, by the implicit method (on a uniformly bounded value function that is independent of Δ_n and Δ_w); and consistency, by the backwards time-step of the iterative method.

diffusion processes of the states n and w,

$$\mathbf{Q} = \begin{bmatrix} q^{ss}_{1,1} & q^{su}_{1,1} & 0 & \cdots & 0 & q^{us}_{1,1} & q^{uu}_{1,1} & 0 & \cdots & 0 & \cdots & 0 \\ q^{sd}_{1,2} & q^{ss}_{1,2} & q^{su}_{1,2} & \ddots & \vdots & q^{ud}_{1,2} & q^{us}_{1,2} & q^{uu}_{1,2} & \ddots & \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & q^{sd}_{1,J^1} & q^{ss}_{1,J^1} & 0 & \cdots & \cdots & q^{ud}_{1,J^1} & q^{us}_{1,J^1} & \ddots & \vdots \\ q^{ds}_{2,1} & q^{du}_{2,1} & 0 & \cdots & 0 & q^{ss}_{2,1} & q^{su}_{2,1} & 0 & \cdots & 0 & \ddots & \vdots \\ q^{dd}_{2,2} & q^{ds}_{2,2} & q^{du}_{2,2} & \ddots & \vdots & q^{sd}_{2,2} & q^{ss}_{2,2} & q^{su}_{2,2} & \ddots & \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & q^{dd}_{2,J^2} & q^{ds}_{2,J^2} & 0 & \cdots & \cdots & q^{sd}_{2,J^2} & q^{ss}_{2,J^2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & q^{sd}_{I,J^I} & q^{ss}_{I,J^I} \end{bmatrix}. \tag{25}$$

The matrix $\mathbf{Q}$ is the discretized infinitesimal generator of (dn, dw): $\mathcal{A}\varphi(n, w)$ for some arbitrary function $\varphi(\cdot)$. Its elements are derived from an upwind scheme and defined as

- $q^{ss}_{i,j} = -\max(\mathbb{E}_t[dw], 0)/\Delta_w + \min(\mathbb{E}_t[dw], 0)/\Delta_w - \max(\mathbb{E}_t[dn], 0)/\Delta_n + \min(\mathbb{E}_t[dn], 0)/\Delta_n - \mathbb{E}_t[dw^2]/\Delta_w^2 - \mathbb{E}_t[dn^2]/\Delta_n^2$
- $q^{su}_{i,j} = \max(\mathbb{E}_t[dw], 0)/\Delta_w + \mathbb{E}_t[dw^2]/(2\Delta_w^2)$
- $q^{sd}_{i,j} = -\min(\mathbb{E}_t[dw], 0)/\Delta_w + \mathbb{E}_t[dw^2]/(2\Delta_w^2)$
- $q^{us}_{i,j} = \max(\mathbb{E}_t[dn], 0)/\Delta_n + \mathbb{E}_t[dn^2]/(2\Delta_n^2)$
- $q^{ds}_{i,j} = -\min(\mathbb{E}_t[dn], 0)/\Delta_n + \mathbb{E}_t[dn^2]/(2\Delta_n^2)$
- $q^{uu}_{i,j} = q^{du}_{ij} = q^{ud}_{ij} = q^{dd}_{ij} = \mathbb{E}_t[dwdn]/(4\Delta_n\Delta_w)$,

where the conditional moments of state variables are $\mathbb{E}_t[dw] = (\gamma - (i - \delta_K))w$, $\mathbb{E}_t[dn] = (g - \delta_N - (i - \delta_K))n$, $\mathbb{E}_t[dw^2] = (\lambda(n)\sigma)^2$, $\mathbb{E}_t[dn^2] = (\sigma n)^2$, and $\mathbb{E}_t[dwdn] = \sigma^2 n\lambda(n)$.

D.1. Boundaries

The main technical innovation to Achdou et al. (2017) is that (some, but could be all) boundaries are endogenous. I impose the usual reflecting boundaries for n: $p(n_0, w_j) = p(n_1, w_j)$ and $p(n_{I+1}, w_j) = p(n_I, w_j)$ for all j. The boundary conditions for w imply that at the termination point $p(n, 0) = l_K + l_N n \Rightarrow p(n_i, w_0) \approx l_K + l_N n_i$ and at the payment boundary point

$p_w(n, \overline{w}(n)) = -1 \Rightarrow p(n_i, w_{J^i+1}) \approx p(n_i, w_{J^i}) - \Delta_w$ under a forward difference, where both conditions hold for all i. These conditions require further adjusting $\mathbf{Q}$ for the non-rectangular grid as it is an approximation to the boundary curve:

1. For all i and each $j = J^i$, I add $q^{su}_{i,j}$ to each $q^{ss}_{i,j}$
2. For all $i > 1$ and each j satisfying $j = J^i \geq J^{i-1}$, I adjust transition probabilities so that a downward move from n_i to n_{i-1} appropriately shifts the value function to $p(n_{i-1}, w_{J^{i-1}})$
3. For all $i < I$ and each j satisfying $j = J^i \geq J^{i+1}$, I similarly adjust transition probabilities so that an upward move from n_i to n_{i+1} appropriately shifts the value function to $p(n_{i+1}, w_{J^{i+1}})$

These adjustments ensure that the non-termination-boundary rows of the transition matrix $\mathbf{Q}$ sum to zero—which is also a useful check. The termination-boundary rows do not sum to zero as they measure the (absorbing) exiting mass of firms.

Lastly, the boundaries require a stacked $\sum_{i=1}^{I} J^i$-vector D to possess the following properties:

- For all i and $j = 1$, D is $(q^{dd}_{i,j} + q^{ud}_{i,j} + q^{sd}_{i,j}) \times (l_K + l_N n_i)$
- For all i and each $j = J^i$, D equals $-q^{su}_{i,J^i} \times \Delta_w$
- For all $i > 1$ and each j satisfying $j = J^i \geq J^{i-1}$, I subtract $\Delta_w \times (J^i - J^{i-1} + 1)$ from D for each transition probability that appropriately shifts the value function to $p(n_{i-1}, w_{J^{i-1}})$
- For all $i < I$ and each j satisfying $j = J^i \geq J^{i+1}$, I similarly subtract $\Delta_w \times (J^i - J^{i+1} + 1)$ from D for each transition probability that appropriately shifts the value function to $p(n_{i+1}, w_{J^{i+1}})$
- D is zero otherwise

Looking at the right side of (24), D intuitively adjusts the flow to principals for the liquidation value of the firm at the termination boundary and for paying agents in current (cash) compensation at the payment boundary. Given this method, the numerical approximation in (24) is entirely consistent with the PDE of (23).

E. AGGREGATION AND STATIONARY DISTRIBUTION

Because the upper boundary $\overline{w}(n)$ is never breached, a given contract will be terminated with probability one. Therefore contract termination, equivalently exit, is a salient force that shapes agents' incentives and intangible capital accumulation. With the description of individual firm behavior complete, I now characterize the distribution of firms in the economy.

Because each firm is described by its current state (n, w), the density of firms is defined over this state space. The non-stationary distribution at time t, $h(n, w, t)$, satisfies the Kolmogorov forward equation

$$\frac{\partial h(n,w,t)}{\partial t} = \mathcal{A}^* h(n,w,t) + \psi(n,w)m(t), \tag{26}$$

where $\mathcal{A}^* h(n, w, t)$ is the adjoint of the infinitesimal generator of the bivariate diffusion process (dn, dw).[15] By construction this generator contains the rates of exit that occur along the termination boundary, $w = 0$, and so to ensure a stationary mass of firms, I add a product of an entry rate, $m(t)$, and an entry mass, $\psi(n, w)$, which integrates to one.

I pin down the entry rate with the requirement that the total mass of firms is constant, which I normalize to one: $\int_0^\infty \int_0^{\overline{w}(n)} h(n, w, t)dwdn = 1$ for all t. Twice integrating (26) then implies that the time derivative of this constant distribution on the left side is zero and therefore that the right side is time invariant. Thus after integration the equation can be rearranged for the stationary entry rate, which of course equals the exiting mass of firms:

$$m = -\int_0^\infty \int_0^{\overline{w}(n)} \mathcal{A}^* h(n,w)dwdn. \tag{27}$$

When a firm's contract is terminated I assume that a new, replacing firm's intangibility is drawn from a distribution with positive support. The entrant, however, also starts with a new continuation payoff, w_0, determined by agents' and investors' bargaining powers. It should be greater than zero because it prevents the economy from exhibiting a "replacement frenzy" (DeMarzo et al. (2012)) over a short period of time and mimics an economy where there is some cost to investors of starting a new firm.

E.1. Computation

Given the transition matrix, $\mathbf{Q}$, the computation of the stationary distribution follows almost immediately. The stationary distribution vector of length $\sum_i J^i$, $h(n, w)$, is calculated by solving, $h(n, w) = -(\mathbf{Q}^T)^{-1}\psi$, where ψ is the similarly-sized entry vector. The rows of ψ that are non-zero are determined by the assumptions on the shape of the intangibility entry distribution that isolates n and in turn on how agents' initial continuation utility w is determined through bargaining. The normalization of $h(n, w)$ to one implies that the entry rate equals $m = -\sum_i \mathbf{Q}^T h(n, w)\Delta_w \Delta_n$.[16]

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[15] Specifically, $\mathcal{A}^* h(n,w,t) = -\mathbb{E}_t[dn_t]h_n(n,w,t) - \mathbb{E}_t[dw_t]h_w(n,w,t) + \frac{1}{2}\mathbb{E}_t[(dn_t)^2]h_{nn}(n,w,t) + \frac{1}{2}\mathbb{E}_t[(dw_t)^2]h_{ww}(n,w,t) + \mathbb{E}_t[(dn_t dw_t)]h_{nw}(n,w,t)$.

[16] In the standard model there is no exit and I compute the stationary distribution $h(n)$ by solving an eigenvalue problem of the adjoint of the $I \times I$-sized transition matrix $\mathbf{Q}$: $\mathbf{Q}^T h(n) = \mathbf{0}$.

IV. MODEL VALIDATION AND CALIBRATION

The main innovation to the standard model proposed in Section II is the introduction of an agency conflict. In this section, I explore some of the agency model's distinguishing predictions and validate certain choices made in the calibration.

To start, I supplement my primary US dataset by merging it with annual compensation data from Execucomp, which I restrict to begin in 1993 when its coverage becomes virtually complete. My preferred measure is total compensation—salary, bonus, long-term incentive plans, and option and stock awards—summed across all managers within each firm-year and scaled by the firm's physical capital, although it can be decomposed into current (salary and bonus) and deferred (total less current).[17]

The model interprets any form of labor not obtained in competitive markets (L_t) as specialists; that is, these could be executives, division managers, and technicians. Unfortunately Execucomp only reports data on upper management, usually the top five compensated employees, so these data understate compensation earned by all specialists. The sample is also biased towards large firms and significantly backfilled, so I follow Gillan, Hartzell, Koch and Starks (2018) and exclude observations where $salary$ is available yet the item $tdc1$ is missing.

Unfortunately, only a few studies have looked at the distribution of compensation within a firm beyond Execucomp.[18] Based on these studies I infer specialists' compensation by estimating it as a constant multiple of the upper management's compensation.[19] I relegate my calculations in Appendix A that lead me to a multiple of two; that is, all employees responsible for the creation of intangible capital, which include upper management, earn twice what just upper management does.

In addition, a rule change in accounting standards (SFAS No. 123R) in 2004 mandated the expensing of stock-based bonuses and deferred compensation on the income statement, although some firms previously did so voluntarily. When it is available in Compustat, I use it as an alternative measure of deferred compensation for all employees.

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[17] In the language of Clementi and Cooley (2010), I use the classical definition of compensation as a flow that is transparent, rather than the model's notion of the stock of total compensation (see (13)) as it would require the difficult and unavoidably subjective calculation of the expected discounted value of future salaries and bonuses. I instead bridge model and data by converting the model's stock of compensation into a flow.

[18] Important examples are Aggarwal and Samwick (2003), which was one of the first to use Execucomp over a period of study that spanned only 1993 to 1997. Wulf (2007) secured a longer panel derived from a compensation firm's survey that had more detailed data on lower-level employees. Recently, Song, Price, Guvenen, Bloom and von Wachter (2019) have analyzed the US Social Security Administration's records of W-2 earnings records within an employer identification numbers (EIN).

[19] Largely concentrated among firms of more than 10,000 employees, Song et al. (2019) document a result that within-firm income inequality has risen whereby the top 10 percent of earners has grown away from the bottom 90 percent. Within this top decile, however, the trend in inequality is much less pronounced and nearly static, which supports the assumption of a constant multiple.

A. MODEL VALIDATION

My model implies that intangibility, agency conflicts, and compensation are intimately linked. In this section, I consider several ways of validating their link.

A.1. Private Benefits and Contract Form

The analysis in Section III suggested that the functional form of $\lambda(n)$ can be in part recovered from an analysis of how shocks to compensation depend on intangibility. As motivation, Figure 1 shows the coefficient estimates and standard errors of quantile regressions of total compensation on lagged intangibility in conjunction with firm and year fixed effects. The coefficients rise with intangibility, supporting that $\lambda(n)$ should be increasing, and the distribution of estimates fans out as intangibility grows, corroborating the heteroskedastic form of (17).[20]

This motivating example, however, is potentially inconsistent with the explicit contract form, so I produce a model-consistent estimate of $\lambda(n)$ in the following steps. First, obtain shocks to wages by running a panel regression of total compensation, w_{it}, on lagged total compensation, w_{it-1}, and year, f_t, and firm, f_i, fixed effects. The regression has the coefficient estimate (with standard errors clustered at the firm-level in parentheses) of

$$w_{it} = f_i + f_t + \underset{(0.015)}{0.399} \times w_{it-1} + \epsilon_{it}, \tag{28}$$

and an overall R-squared of nearly 70 percent. I estimate the residuals, which average to zero across all firm-years, and standardize them.

Next, I place the residuals into five portfolios that are annually rebalanced and formed on the quintiles of some underlying firm characteristic. Within a particular quantile-year I calculate the mean and standard deviation of the residuals. I then take the time series average of these statistics for each portfolio. Table V reports the results for four portfolios formed on intangibility, profitability, book-to-market, asset productivity, and log assets.

The mean and standard deviation of residuals rises across portfolios of greater intangibility. Consistent with the model, the pattern reflects the history-dependence of the contract: a sequence of positive shocks to intangibility raises the growth of compensation and widens its dispersion. Comparing the patterns across other characteristics shows that something is indeed special about intangibility: profitability, book-to-market, asset productivity, and log assets all produce non-monotone relationships among characteristics and their means and variances. Thus the conditional

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[20] Kogan, Papanikolaou, Schmidt and Song (2017) empirically study workers' earning growth within a firm in response to changes in the firm's market value as a result of patent innovation. A key finding is that the gains and losses among the upper tail of the firm's income distribution are particularly dispersed. This is also consistent with the private benefits being increasing and compensation being heteroskedastic in intangibility.

distribution of residuals moves with intangibility in the right way and validates the model.

In the third and final step I regress the compensation residuals on a polynomial expansion of intangibility to ascertain the functional form of private benefits. I consider three measures of compensation: total and deferred for specialists and deferred for all employees. Panel B of Table V analyzes the predictions for the form of $\lambda(n)$.

The first-order expansion produce point estimates ranging from 0.092 to 0.164 and capture how a unit change in intangibility impacts the stock of compensation. Nearly 20 percent of the variation in the residual of total compensation is explained by intangibility alone. It is common in the literature, however, to understand how an additional dollar of value-added attributed to innovation changes workers' earnings. For example, Kline, Petkova, Williams and Zidar (2019) study how patent-induced shocks causally affect worker compensation. They estimate that for every dollar of new firm surplus, workers' earnings rise by nearly 30 cents and find that this effect is particularly concentrated among the firm's inventors.

To compare to these estimates, I regress average Q in the data on intangibility, with firm and year fixed effects, to obtain a slope coefficient of 0.55. Thus increasing intangibility by $1/0.55 = 1.81$ units increases (scaled) firm value by a dollar. Focusing on total compensation, a rise in 1.81 units of intangibility would then imply a dollar rise in compensation, dw, of 0.164×1.81 = 30 cents, which is consistent with estimates in the literature.

Altogether, I specify the function of private benefits to be

$$\sigma\lambda(n) = 0.15n. \tag{29}$$

A.2. Intangible Investment and High-Powered Incentives

The previous subsection provides evidence that private benefits are increasing in intangible capital. One would expect, then, that the incentive constraint would be tightened post-intangible development and, in turn, worsen the agency conflict and make termination more likely. Recall from Section III that in response, principals prefer to smooth this tightening by raising agents' compensation to mitigate this conflict. Thus the model predicts that increased rates of development, all else equal, will correlate with compensation growth.

I test this prediction with panel regressions. Table VI tabulates the estimates of the model that regresses compensation on its own lag, development rates, and their interaction, in the presence of important controls—lagged sales, sales growth, and lagged intangibility—and firm and year fixed effects. Columns (1) and (2) use my preferred measure of total compensation among specialists. Consistent with the model prediction, the development rate positively predicts compensation. The sign of the interaction coefficient, moreover, agrees with the intuition above.

Columns (3) through (6) decompose compensation into current and deferred forms. Interestingly, development enters negatively on current compensation yet positively on deferred compensation, suggesting that compensation shifts from current to deferred as development rises. This is consistent with development primarily affecting the long-term incentives of agents. In particular, raising the development rate by one standard deviation (7.72 percent) increases deferred compensation by $0.037 \times 7.72 = 0.286$ percent. And because the development rate is positively autocorrelated (coefficient 0.72), this effect is expected to persist and is therefore economically significant.

The final two columns look at a broader measure of deferred compensation expensed to all employees, not just upper management. Here there is also a statistically significant effect but this time the coefficient estimate on the development rate is over three times larger (0.129/0.037).

B. CALIBRATION

I tabulate the calibration targeted towards only US moments in Table VII. I begin by setting the real interest rate, r, to 1.1 percent and $\delta_K = 0.1$. While physical capital's depreciation rate is largely agreed upon, intangible capital's rate is less so. The BEA uses 15 percent to R&D and Corrado, Hulten and Sichel (2009) provide informed assumptions ranging from 20 percent (R&D) to 60 percent (brand equity). I set $\delta_N = 0.19$.

B.1. Production Function

Krusell, Ohanian, Ríos-Rull and Violante (2000) and Eisfeldt, Falato and Xiaolan (2018) argue for complementarity between physical capital and skilled labor and physical capital and human capital, respectively. While intangible capital is neither skilled labor nor human capital, they are plausibly all correlated. Following these papers, I specify a constant elasticity of substitution (CES) production function between the two types of capital:

$$F(K_t, N_t) = (\phi K_t^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) N_t^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} = K_t \left((1-\phi) + \phi n_t^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} = K_t f(n_t), \qquad (30)$$

where $\phi \in (0,1)$ is intangible capital's share of total capital and $\epsilon \in (0,\infty)$ is the elasticity between capital types. Krusell et al. (2000) estimate an elasticity of 0.67 and Eisfeldt et al. (2018) 0.87 in their benchmark models. I choose $\epsilon = 0.80$.

I then calibrate A and ϕ jointly to match the average values of intangibility and cash flows per unit of physical capital in the data. In my data sample I estimate average intangibility and average cash flow across all firm-years to be $n^{ss} = 2.2$ and 35 percent, respectively. These estimates require $A = 0.321$ and $\phi = 0.810$. In recent work, Bhandari and McGrattan (2019) find the ratio of intangible-to-total asset value has a median of 64 percent among private firms and Ewens, Peters and Wang (2019) report an average of 60 percent among public firms.

B.2. Stationary Distribution

Because I focus on industry equilibrium, parameters that govern the shape of steady state distribution and entry and exit are important. First is the variability of the growth rate of intangibility, σ, which influences the severity of the agency friction and the likelihood of contract termination. To calibrate this, I examine within-firm volatility. Specifically, for every firm in my sample, I construct intangibility and then compute the volatility of its growth rate. This measure is fixed at the firm-level. I then take the median and winsorized mean across all firms to get 18.6 and 30 percent, respectively. I settle on $\sigma = 0.2$.

I decondition the entry mass to a conditional and marginal distribution, $\psi(n_0, w_0) = \psi_w(w_0|n_0)\psi_n(n_0)$. I assume initial intangibility, n_0, draws from a log-normal distribution with mean n^{ss} and standard deviation σ. Given the intangibility draw, n_0, the distribution of w_0 is degenerate and the value of initial w_0 comes from an assumption of investors' and agents' bargaining power. Because agents are integral to the success of the firm, I pick $\varphi = 0.8$. I later conduct a sensitivity analysis on the choice of this parameter.

I specify smooth adjustment cost technologies as I am interested in the model's long-run properties. I assume that both intangible and physical capital have the same adjustment cost function that takes a quadratic form of the growth rate

$$c_j(x) = x + \frac{\kappa}{2}(x - \delta_j - z)^2, \text{ for } j = N, K \tag{31}$$

where κ measures the magnitude of the adjustment cost and z is an exogenous growth rate that locates the function. Following Hall (2001), I interpret the parameter as a doubling time. The BEA reports data on quantity indices for the net stock of fixed assets that I use to calculate the compound annual growth rate of nonresidential equipment to be 3.6 percent per year from 1975 to 2017, implying a doubling time of 19 years. Hall (2001) uses either 2 or 8 years for his upward adjustment cost and 20 or 80 years for his downward adjustment cost. I set $\kappa = 20$ and $z = 0.01$.

Specialists' time rate of preference is $\gamma > r$. Its value influences the length of the interval $[0, \overline{w}(n)]$, as more impatient agents (higher γ) require relatively sooner current payments out of continuation utility which lowers $\overline{w}(n)$. I convert the agent's stock of compensation w to a flow by multiplying it by γ. Altogether, I parameterize γ by calibrating to average compensation in the data and correspondingly choose $\gamma = 0.028$.

The threat and implementation of termination in the optimal contract is used to ensure agents' proper incentives. The liquidation values of capital in the model are best thought of as direct bankruptcy costs. Estimates vary in the literature with the earlier studies of Warner (1977), Altman

(1984), Weiss (1990) finding values in the range of 3 to 6 percent of pre-bankruptcy firm value.[21] I impose a cost on both capital types on the order of 7 percent, putting $l_K = l_N = 0.93$.

V. MODEL ANALYSIS

Having solved the calibrated model, I first describe the properties of its solution before comparing its predictions with the data. In what follows, I look at policy and value functions across four breakpoints based on the marginal distribution of intangibility: the 30th, 50th, 70th, and 90th percentiles.

A. INVESTORS' VALUE FUNCTION AND STATIONARY DISTRIBUTION

In Panel A of Figure 2 I plot the first derivative of investors' value function with respect to agents' scaled continuation payoff, w. The first derivative measures the marginal cost of compensation: the marginal cost to investors of promising an additional dollar in continuation utility to agents. Optimality of the solution requires that all first derivatives equal -1 at the payment boundary. We can see that as intangibility rises, the marginal cost of compensation increases for every w in general. This response is paramount in understanding the model.

Recall that in designing the optimal contract, investors smooth the marginal cost of compensation to minimize agency costs. There are several effects that determine the exact response of the marginal cost of compensation to a growth in intangibility:

1. Firm Value—As intangibility rises, cash flows and firm value do too, thus raising the benefit of avoiding termination. This decreases the marginal cost of improving agents' payoff, that is p_w rises, and therefore investors will optimally increase w to reduce agency costs.

2. Liquidation Value—The firm's liquidation value rises with the quantity of intangible capital. This reduces the benefit of avoiding termination and counteracts, and could completely overturn, the first effect of a higher firm value.

3. Likelihood of Termination—Equation (17) shows that this likelihood grows with intangibility for a given w because the upper payment boundary is never breached and the lower termination boundary absorbing. Therefore for high n it is more likely an adverse shock will result in contract termination and liquidation. This amplifies the second effect.

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[21] More recent literature has estimated the costs of financial, not economic, distress to be much higher; for example, Andrade and Kaplan (1998), Davydenko, Strebulaev and Zhao (2012), and Glover (2016) put these costs in the range of 10 to 45 percent.

The values of the second derivative at the payment boundary (along $\overline{w}(n)$) are plotted in Panel B. The super contact condition requires that these should all be zero to be optimal for investors. The norm (error) per grid point of intangibility is less than 1 percent. The error is not uniformly zero, but numerical sensitivity analysis for different, local boundary curves suggest that the results which follow are not acutely affected by this small inaccuracy.

The boundary curve is a two-part, piece-wise quadratic function that is evident in the shape of the value function depicted in Panel C. The general strategy to attain super contact is to raise the maximum of $\overline{w}(n)$ if $p_{ww}(\cdot)$ is negative and vice versa. The end points are set to flatten the profile of the curve. The concavity of the graph of $\overline{w}(n)$ is a consequence of decreasing returns to scale and the relative strengths of 1., which dominates at low levels of n, and 2. and 3. above.

Investors' scaled value function $p(n, w)$ is concave in w. Concavity arises from investors' aversion to fluctuations in agents' payoff and is generated by two opposing effects. First, agency conflicts are reduced as w grows away from the termination boundary as investors' and agents' incentives align. Agents' fear of termination dissipates and agrees with investors' desire to weaken this threat; $p(n, w)$ therefore grows at low w. Second, as w rises, agents extract a larger share of firm value, reducing investors' share and decreasing $p(n, w)$. The function in general declines with w. The value function increases in intangibility as profitability rises and because investors recover more in liquidation.

The stationary distribution of firms is depicted in Panel D. There are two parts to the distribution. The first part tracks out a path from $(n, w) = (1, 0)$ to the boundary curve near $(n^{ss}, \overline{w}(n^{ss}))$; this is the model's saddle path. Its particular gradient arises from the commonality in the drifts of states $(\mathbb{E}[dn], \mathbb{E}[dw])$. The path of investment is chosen so that both n and w grow together. But since the optimal contract minimizes agency costs that grow with n, investors desire w to increase relatively faster.

The second part has to do with firms that have reached the payment boundary. Incentives are structured so that firms accumulate near the payment boundary as this is when agency frictions are minimized. Because of the model's stochastic property, the mass of firms are spread along the boundary.

Tian (2018) estimates average firm birth and death rates of 11 and 9 percent, respectively, from the Business Dynamic Statistics database over the period 1979 to 2013. Gutiérrez and Philippon (2017a) classify entry as the first year in which a firm appears in Compustat. With their definition, I estimate an average entry rate of 8 percent in my sample. The calibrated model produces an entry/exit rate of 11 percent that is in line with these estimates.

B. INCENTIVES, INTANGIBILITY, AND INVESTMENT

In this section, I discuss how agents' incentives and intangibility interact to alter the predictions of standard investment theory. Before doing so, I first define some variables in the model.

Total firm value including agents' claim is $P(K, N, W) + W$. Physical average Q, the ratio of total firm value to the physical capital stock, is given by

$$Q(n, w) = \frac{P(K, N, W) + W}{K} = p(n, w) + w. \tag{32}$$

Using this definition I rewrite (physical) marginal q of the Euler equation (19):

$$c'_K(i(n, w)) = q(n, w) = Q(n, w) - p_n(n, w)n - (1 + p_w(n, w))w. \tag{33}$$

Marginal q quantifies the incremental increase in firm value of a unit of physical capital whereas average Q is used in empirical studies due to the simplicity of its construction as a proxy for q. As can be seen from the equation, the two right-most terms can potentially alter the relationship between q and Q. Moreover, whereas (19) reflects only investors' concerns, the rewritten equation (33) reflects both investors' and agents' concerns. I decompose these effects below.

Panel A of Figure 3 displays the value of physical Q as a function of w indexed by the four breakpoints. As intangibility rises, the firm generates more cash flows and total firm value split by both parties raises for any w.

Panel B depicts the marginal cost of intangibility. Recall that physical investment reduces intangibility, which is profitable, and so it subtracts from the benefit of investment. The effects of a reduction in cash flows are quantitatively large, especially for firms of great intangibility. This effect is crucial to understanding why investment rates are flat across portfolios of increasing intangibility.

Why is profitability so essential? The reason relates to Panel C that depicts the net benefit to both parties of shifting ownership in response to investment, $-(1 + p_w(n, w))w$. Physical investment reduces agents' effective claim on the firm and hence induces a more severe agency problem. Hence, this net benefit must be weakly negative (a cost). When w is near zero, shifts in ownership do not matter as contract termination is likely and liquidation becomes more certain. As w rises towards $\overline{w}(n)$, agents and investors incentives are aligned and again shifting ownership becomes insensitive to changes in intangibility.

Recall that the optimal contract in this dynamic environment smooths the marginal cost of compensation, increasing agents' rents and aligning incentives in states in which the incentive problem is more costly. This feature manifests itself in this panel whereby for greater levels of intangibility in the interior of $[0, \overline{w}(n)]$, the cost of shifting ownership is smaller.

The desire for smoothing makes large rates of investment unappealing, especially when intangibility is large and agency conflicts severe. Even in response to a greater level of intangibility that in the standard model would prescribe a high rate of investment, the smoothing motive ensures that investment is largely unresponsive. This can be seen in the patterns of investment rates in Panel D. Holding w fixed and raising intangibility results in a smaller and smaller rates of investment. Moreover, this smoothing motive can be so pressing that the investment response can actually begin to fall at percentiles above the 90th. Investment naturally grows with w holding n fixed.

We can now reconnect to profitability. It increasing with intangibility follows endogenously from the smoothing of the marginal cost of compensation in response to a greater agency conflict. This reduction in variability is chosen to minimize the agency problem, both to the benefit of investors and agents. Alternatively put, great levels of profitability make the contract that commits to a high level of smooth compensation more credible, aligning incentives. Consequently high, stable profits become extremely valuable and this explains the quantitatively large effects of reducing intangibility in Panel B.

This paper therefore provides a microfoundation for the high cash flows, high valuation, and low investment of highly intangible firms through the stability of the marginal cost of compensation sought in designing the optimal contract. This is new and complements the existing literature. Michelacci and Quadrini (2009) and Guiso, Pistaferri and Schivardi (2013) show financially constrained firms defer wage payments as a source of internal financing. Sun and Xiaolan (Forthcoming) demonstrate this deferment is concentrated within highly intangible firms and further show it substitutes for debt capital.

C. MODEL PREDICTIONS

In this section, I compare the model's predictions with those in the data. I first look at the direct and indirect predictions in portfolio sorts before turning to panel regressions.

C.1. Portfolio Results

In Table VIII I tabulate statistics across the intangibility-sorted portfolios in the standard and agency models using the stationary distribution. I first discuss the direct predictions before turning to secondary, indirect predictions.

First, the standard model is wholly unable to generate the stylized patterns presented in Section I. As discussed in Section V, greater value or intangibility unwaveringly predicts more investment. The agency model, however, can generate higher profitability and value along with a flat investment response. It is even able to capture an eventual decline in investment rates as we sample into the right tail of the intangibility distribution.

Altogether, the expected return on capital has fallen subject to these agency frictions. Comparing the average investment rate of the agency model (11.9 percent) to the standard model (18.4 percent) implies that agency frictions account for around a third (11.9/18.4-1) of the decline in abnormal investment, as documented recently by Alexander and Eberly (2018) and Gutiérrez and Philippon (2017b).

Patterns of compensation, moreover, in the data and model quantitatively agree. In the data, deferred compensation of all employees rises up to 2.4 percent, slightly larger than the model's 2.0 percent of the top intangibility quintile. Principals optimally shift ownership to the agents to offset the deleterious effects of greater intangibility.

C.2. Indirect Predictions

The agency friction microfounds and endogenizes an external cost of finance. It predicts that these firms would be constrained, in spite of high valuations of physical capital and profitability. Whereas economists distinguish internal from external finance, the model here makes no distinction among cash holdings, lines of credit, debt, and equity.[22] But one would naturally expect that firms possessing a greater agency conflict to find external finance less attractive than internal finance. The ability to finance internally, say by retaining holding cash, would therefore be in the spirit of the model but only an indirect prediction of it. Moreover, the frequency of external finance should also be lower for firms with greater agency frictions.

I examine this empirically in the bottom two rows of Table VIII. Shifts in both cash holdings and the rate of external finance agree with intuition.[23] Altogether, these relationships are accordant with a shift from external to internal finance and could reflect a growing divergence between insiders' and outsiders' interests.[24]

C.3. US Listing Gap

Doidge, Karolyi and Stulz (2017) analyze US stock market listings and describe a "gap" between the post-1996 experience and what historical norms would have dictated. They hypothesize that part of the explanation has to do with intangible capital. I reconduct their exercise in spirit by looking across US industries of changes in intangibility and comparing them with changes in variables

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[22] While much of the dynamic contracting literature focuses on decentralizing the optimal contract with simple securities, my setup is not conducive to this analysis, for reasons similar to those discussed in He (2009).

[23] I construct an indicator for external financing that takes the value one if the firm issues common or preferred equity greater than 3 percent of its market value of equity in a given year (McKeon (2015)) or if long-term debt issuance is positive.

[24] The patterns are reminiscent of the Whited and Wu (2006) measure who find that constrained firms are small, invest little in spite of apparently good investment opportunities, and hold a lot of cash. One difference is that in their classification constrained firms are unprofitable.

predicted to be relevant by the model. As a summary measure, I also compute the correlations across these industry changes.

Figure 4 depicts the result. It shows that in the industries which experienced the biggest changes in intangibility correspond to those that display the largest increases in cash holdings and mean and dispersion of compensation along with the greatest decreases in issuance and IPO rates. The effects are particularly concentrated in the industries of manufacturing, information, professional, and education and health. Therefore, the model's predictions are evident in the changes that occur at the industry level over time.

C.4. Panel Regressions

To further highlight predictions of the agency in intangibles model, I replicate the investment panel regressions of Section I in the model. I do this by simulating the model at a monthly frequency and keeping track of the entry and exit of firms according to optimal policy. Monthly investment is summed over the year and, as in the data, regressed on a firm fixed effect and last year's controls of value, intangibility, and profitability.

I tabulate these regression coefficients for various specifications across the standard and agency models. The standard model confounds the distinction between Q and intangibility: holding Q fixed while increasing intangibility raises investment, and this is counterfactual to the data.

On the other hand, the agency model can distinctively recreate the empirical patterns of the negative coefficients on the interaction of intangibility with Q, capturing its interpretation as the wedge in interests between insiders and outsiders.[25] The negative coefficient on the interaction on profitability, moreover, is consistent with the data and further distinguishes the model described here and classic agency models.

CONCLUSION

I embed an agency friction into a firm's intangibility on the idea that intangible capital is opaque and largely developed by specialists who may possess their own private interest. The model provides a microfoundation for the high profitability, high valuation, yet puzzling low investment rates of highly intangible firms. Many of the model's predictions are confirmed in the data, both in the US and internationally, including features about the form of the optimal contract.

I overcome a technical challenge typically sidestepped in the dynamic contracting literature that allows progress in this fruitful line of work. The numerical solution method, moreover, is

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[25] The magnitudes in the model are greater than those in the data. They could be brought closer by increasing the value of κ but adjustment costs of this magnitude are not plausible (Tobin (1981)).

general enough to be used to solve a wide class of problems, including those faced outside the contracting literature.

I assumed no frictions in unskilled labor markets or general equilibrium effects of industry competition or systematic risk. In reality, these factors could affect the investment opportunities of intangible firms. The partial equilibrium responses developed in this paper would still be present, however, even in a more fleshed out model. Less debatable is that standard models, which are widely used in the literature, will find it challenging to explain several economic forces surrounding intangible capital.

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A. DATA APPENDIX

A. DATA CONSTRUCTION

I use all industrial, standard format, consolidated accounts of firms in Compustat and Compustat Global. For US firms I only include firms with common shares (shrcd = 10 and 11) that trade on the NYSE, AMEX, and NASDAQ (exchcd = 1, 2, and 3). I exclude firms without a NAICS code and in the utilities (22), financial (52-53), other (91), and public (92) industries. To ensure a sufficient number of firms for each portfolio, I group agriculture (11) and mining (21), education (61) and health (62), and arts (71) and accommodation (72), leaving 10 industries. For non-US firms I use the security level file and restrict to initial issues (iid = 01W) and manufacturing firms. I use a consumer price index compiled by the OECD for all countries and convert all non-US countries' balance sheet variables to US dollars at the time that corresponds to the *CPI*'s base year. I remove firms with missing or non-positive book assets (at) or sales ($sale$) and those with net property, plant, and equipment ($ppent$) of less than 5 million US dollars as of the base year, as is standard in the literature. I replace missing values of $SG\&A$ with zeros.

Asset Productivity = sales (sale(t)) / assets (at(t-1))
Average Q (physical) = (market equity + long-term debt (dltt) + current debt (dlc) $-$ inventory (invt) $-$ cash (che)) / physical capital (ppegt)
Book Equity = stockholders' equity (seq) - preferred stock + deferred taxes and investment tax credits (txditc)
Cash-to-Assets = cash (che) / assets (at)
Compensation, Specialists = 2$\times$ salary + bonus + LTIP + equity rewards (tdc2) (see Section C) / physical capital (ppegt(t-1))
Debt Issuance = 1 if long-term debt issuance (dltis) $>$ 0; 0 otherwise
Deferred Comp., All = stock compensation expense (stkco) / physical capital (ppegt(t-1))
Development Rate = 0.3 $\times$ real SG&A (xsga(t)/cpi(t)) / organizational capital (N(t-1), see (2))
Entry = 1 if first $permno$ in Compustat
Equity Issuance = 1 if sale of common and preferred stock (sstk)/market equity $>$ 0.03; 0 otherwise
Equity Rewards = stock awards (stock_awards) + option awards (option_awards_blk_value)
Intangibility = unmeasured intangible capital (N, see (2)) + recorded intangibles (intan) / real physical capital (ppegt/cpi)
Investment Rate = physical investment (capx(t)) / physical capital (ppegt(t-1))
Issuance = 1 if either equity issuance or debt issuance equal 1
Market Equity = price per share $\times$ shares outstanding (December values of abs(prc) x shrout from CRSP and abs(prccd) x cshoc from Compustat Global)
Tangibility = net physical capital (ppent) / assets (at)
Preferred Stock = Use the redemption value (pstkrv), liquidation value (pstkl), book value (pstk), or zero, in decreasing order of preference
Profitability = EBITDA (ebitda(t)) / physical capital (ppegt(t-1))

B. SORTS ON Q AND TANGIBILITY

I report univariate sorts on Q and Tangibility in Tables A-I and A-II. Both sorts display patterns consistent with the standard model: as average Q rises, so too does investment. In contrast to intangibility, tangibility crucially ignores the capitalized costs of intangible development, so the estimation of these costs is necessary.

C. SPECIALISTS' COMPENSATION

Compensation data is limited and comes from either Execucomp, which reports usually only for the top five earners, or from Compustat, which reports only a stock-based compensation, ignoring cash bonuses and long-term incentive plan contributions as well as only becoming effectively complete after 2004. Few papers in the literature have looked at how compensation is distributed within a firm and based on these few, I estimate the total compensation earmarked for the firm's specialists, which include top management but could also include division managers and technicians. In what follows because the distribution of the upper tail of earnings is likely skewed, I use medians when available to evaluate central tendency.

Aggarwal and Samwick's (2003) Table II reports the median values of total flow compensation across the CEO, other executives and key chairpersons, division managers, and other employees (production- or infrastructure-related or other senior managers) in Execucomp data for the years from 1993 until 1997. I sum the median compensation paid to division managers and other employees across all firms and divide by the sum of median compensation to CEOs across all firms to get a ratio of 1.02. Next, Wulf's (2007) Table II similarly provides the median values of total compensation across division managers and CEOs in her longer data sample spanning 1986 to 1999. I do similar steps to division managers and CEOs as above to get a ratio of 0.90.

Finally, Fatih Guvenen of Song et al. (2019) provides on his website annual data that spans 1978 to 2013 of statistics of the earnings distribution for firms of various employment sizes. Within firms of more than 10,000 employees, I divide firms' 90th percentile of earnings by 0.1 and further divide this quotient by the average earnings of the top 5 earners to get an annual time series of this ratio. While it has slightly trended downwards over time, its temporal mean is 0.98.

The average of all three of these ratios (0.90, 0.98, 1.02) is near one and I therefore conclude that specialists earn roughly double what top management earns.

B. TECHNICAL APPENDIX

A. RISK PREMIUM EXTENSION ON STANDARD MODEL

In this appendix I show that including a risk premium as a function of intangibility does not change the standard theory's qualitative implication that investment monotonically rises with intangibility.

To study the effects of a discount rate correlated with intangibility I specify an exogenous process for a stochastic discount factor, M, as $dM_t = -rM_t dt - \Gamma M_t dB_t$, where Γ is the price of risk and dB_t is an aggregate Brownian shock that has correlation ρ with dZ_t. I also expose intangible capital to both shocks and have it follow the process

$$dN_t = (g_t - \delta_N)N_t dt + \sigma N_t \left(\rho dB_t + \sqrt{1-\rho^2} dZ_t\right). \tag{B1}$$

Girsanov's transformation ($dB_t^{\mathbb{Q}} = dB_t + \Gamma dt$) allows me to solve for the value of the firm under the risk-neutral distribution, $dB_t^{\mathbb{Q}}$, implying that the firm's intangibility evolves as $dn_t = (g_t - \delta_N - \rho\Gamma\sigma - (i_t - \delta_K))n_t dt + \sigma n_t \left(\rho dB_t^{Q} + \sqrt{1-\rho^2} dZ_t\right)$. Given this transformation, the analysis in Section III's standard case goes through and results in a risk premium of

$$\mathbb{E}_t[dR_t] - rdt = -\text{cov}_t\left(\frac{dM_t}{M_t}, \frac{dp(n_t)}{p(n_t)}\right) = \rho\Gamma p'(n_t) n_t \sigma^2 dt \tag{B2}$$

that captures the idea that investors will demand compensation for exposure to intangibility.

Panel A of Table A-III summarizes the calibration targets of this exercise. The top two rows take the reported average values of organizational capital (their O/K ratio) and risk premia (net of market risk equal to 4.4 percent) across portfolio quintiles from Eisfeldt and Papanikolaou (2013), denoted EP. Following their paper, I set $\Gamma = 0.53$. I then calibrate the potential for risk premia to explain the investment slope by first reporting my extension's risk premia at their values of the O/K ratio under the calibration in Table VII but with a linear production function, similar to EP. I set $\rho = 0.7$ to match these patterns in premia.

Panel B tabulates investment rate statistics from intangibility portfolios and compares the data, the standard model, and the risk premia model, now with CES production, that was calibrated according to the steps in Panel A. The investment rate at a high level of intangibility falls when accounting for risk exposure, reducing the range from 10.0 percent to 5.8 percent. Thus attaching discounting to intangibility altogether reduces the slope by approximately 43 percent. I take this as evidence that discount rates cannot explain fully the cross-sectional investment patterns in these intangibility portfolios.

B. AN EFFORT CONDITION

Define the gain process $\{G\}$ under any incentive-compatible contract $\mathcal{C} = (U, i, g, \tau)$ for any $t \leq \tau$ as

$$G_t(\mathcal{C}) = \int_0^t e^{-rs}(\Pi_s ds - dU_s) + e^{-rt} P(K_t, N_t, W_t), \tag{B3}$$

where W_t evolves as in (15). Homogeneity and Ito's lemma imply

$$e^{rt}G_t(\mathcal{C}) = K_t \left\{ \begin{array}{c} \left[\begin{array}{c} -rp + Af(n_t) - c_K(i_t) - c_N(g_t)n_t + (i_t - \delta_K)(p - p_n n_t - p_w w_t) \\ +p_n(g_t - \delta_N)n_t + p_w \gamma w_t + \frac{\sigma^2 n_t^2}{2} p_{nn} + \frac{\beta_t^2 \sigma^2 n_t^2}{2} p_{ww} + \beta_t \sigma^2 n_t^2 p_{nw} \end{array} \right] dt \\ -(1 + p_w)du_t + (p_n + \beta_t p_w)\sigma n_t dZ_t \end{array} \right\}, \tag{B4}$$

where $p(\cdot)$'s dependence on states (n_t, w_t) has been omitted for brevity.

Under the optimal investment i_t^*, development g_t^*, and incentive policies $\beta_t^* = \lambda(n_t)/n_t$ the top two lines in the square brackets are the optimized PDE in (23) and therefore zero. For models in which the only state variable is agents' continuation utility this nonpositivity condition follows from the concavity of $p(w)$ (and $\beta_t \geq \beta_t^*$). In this more general case, I show numerically that for any other incentive-compatible policy the sum of the second-order terms, $\frac{\sigma^2 n_t^2}{2} p_{nn} + \frac{\beta_t^2 \sigma^2 n_t^2}{2} p_{ww} + \beta_t \sigma^2 n_t^2 p_{nw}$ are nonpositive. Panel A of Figure A-1 depicts the sum of these terms under $\beta_t = \beta_t^*$. In all cases these terms are negative under the calibration.

Next, the term capturing the optimality of the cash payment policy, $-(1 + p_w)du_t$, is nonpositive since $p_w \geq -1$ but equals zero under the optimal contract. Therefore, under any incentive-compatible contract $e^{rt}G_t(\mathcal{C})$ has nonpositive drift and zero drift for the optimal contract. This implies that $\{G\}$ is a supermartingale.

Now consider a policy that has agents shirk. When they do $(e_t = 0)$ they enjoy private benefits at rate $\widehat{g}_t \Lambda(K_t, N_t)dt$. Intangible capital and agents' continuation payoff now evolve as

$$dN_t = -\delta_N N_t dt + \sigma N_t dZ_t \text{ and } dW_t = (\gamma W_t - \widehat{g}_t \Lambda(K_t, N_t))dt + \widehat{\beta}_t N_t \sigma dZ_t, \tag{B5}$$

where $\widehat{\beta}_t \leq \Lambda(K_t, N_t)/N_t$. For this not to be the case and for effort $(e_t = 1)$ to remain optimal, it must be that investors' payoff rate from allowing agents to shirk be lower than under the optimal contract and equivalently that investors' optimal gain process remain a supermartingale with respect to this shirking policy:

$$\begin{aligned} rp \geq{}& Af(n_t) - c_K(i_t^*) - c_N(\widehat{g}_t)n_t + (i_t^* - \delta_K)(p - p_n n_t - p_w w_t) \\ &- p_n \delta_N n_t + p_w(\gamma w_t - \widehat{g}\lambda(n_t)) + \frac{\sigma^2 n_t^2}{2} p_{nn} + \frac{\widehat{\beta}_t^2 \sigma^2 n_t^2}{2} p_{ww} + \widehat{\beta}_t \sigma^2 n_t^2 p_{nw} \end{aligned} \tag{B6}$$

for some t.

Since p is concave in w, it is optimal to set the incentive coefficient to zero: $\beta = 0$. The first-order conditions with respect to investment remains as in (19). Lastly, the necessary optimality condition respect to $\widehat{g}$ becomes

$$c_N'(\widehat{g}) = -p_w \frac{\lambda(n)}{n} \tag{B7}$$

which gives an interesting observation: that the marginal benefit of intangible capital is limited by the magnitude of the agency friction under the shirking policy.

Enforcing this choice and rearranging (B6) gives the inequality required to ensure that never shirking is the optimal incentive policy:

$$\begin{aligned}(r+\delta_K)p + c_K(i^*) + c_N(\widehat{g})n - p_w((\gamma+\delta_K)w - \widehat{g}\lambda(n)) + p_n(\delta_K+\delta_N)n \\ - Af(n) - i^*q^* - \frac{\sigma^2 n^2}{2}p_{nn} \geq 0, \text{ for all } n \text{ and } w.\end{aligned} \tag{B8}$$

Panel B of Figure A-1 plots this inequality and shows that it is positive and above zero for all w at the four percentile thresholds of intangibility.

Why this condition holds even in the presence of increasing $\lambda(n)$ is worth commenting on. When agents shirk, intangible capital is expected to shrink by a rapid $-\delta_N + \sigma_N^2/2 = 17$ percent per year. Intuitively, agents' effort is crucial to the success of the firm; this can be seen by δ_N increasing the left side of (B8). In contrast to the setup of DeMarzo et al. (2012) where shirking only has a transient effect, agents' effort here has a long-lasting effect and is therefore very valuable to the firm.

In more general situations where the inequality binds, then a more complicated contract than the one described in this paper would need to be considered. Zhu (2013) considers this extended contracting environment in the context of the DeMarzo and Sannikov (2006) model.

C. POTENTIAL MODEL EXTENSIONS

The model could be extended in several ways, including additional shocks, endogenous recovery, private benefits of control, or the lack of commitment of human capital. While not within the scope of the paper, all of these extensions are presumably of interest when modeling intangible capital.

C.1. Shock to Physical Capital Shock

The accumulation of physical capital could be extended to vary stochastically, similarly to Cox, Ingersoll and Ross (1985),

$$dK_t = (I_t - \delta_K K_t)dt + \sigma_K dZ_{Kt}, \tag{B9}$$

where σ_K is the volatility of the capital depreciation shock. Intangibility would then evolve as

$$dn_t = ((g_t e_t - \delta_N) - (i_t - \delta_K) + (\sigma_K^2 - \sigma\sigma_K\rho))n_t dt + \sigma n_t dZ_t - \sigma_K n_t dZ_{Kt}, \tag{B10}$$

where ρ specifies the correlation between Brownian shocks. Adding another shock effectively translates intangible capital's net growth rate by $\sigma_K^2 - \sigma\sigma_K\rho$. One could also define a new normal shock, without loss of generality, with standard deviation $n_t\sqrt{\sigma^2 + \sigma_K^2 - 2\sigma\sigma_K\rho}$.

C.2. Endogenous Recovery

DeMarzo et al. (2012) show the recovery fraction could be made endogenous. If the contract is terminated when the firm has intangibility n_t, then investors could be allowed to hire new specialists at payoff w_0

$$l_K + l_N n_t = \max_{w_0}(1-\xi)p(n_t, w_0), \tag{B11}$$

where $\xi \in (0,1)$ is the cost replacing the specialists. The entry rate discussed above effectively assumes that the cost ξ becomes arbitrarily small.

C.3. Private Benefits of Control

DeMarzo and Sannikov (2006) discuss the extension where agents receive private benefits of control from running the firm. If prior to termination agents earn additional utility at rate $\gamma\omega$, then with this private benefit agents' continuation payoff would evolve as

$$dW_t = \gamma(W_t - \omega)dt - dU_t + \lambda(N_t)\sigma dZ_t. \tag{B12}$$

Intuitively, the threat of liquidation and therefore the risk of losing private benefits improves agents' incentives.

C.4. Lack of Commitment of Human Capital

Bolton et al. (Forthcoming) analyze a model where an entrepreneur's human capital is inalienable and can leave a firm to pursue an outside option. My model could accommodate this lack of commitment by redefining the default boundary in (20) to

$$p(n, \underline{w}) = p(\nu n, w_0), \text{ for each } n. \tag{B13}$$

where $\nu \in (0,1)$ measures agents' collective talent that would determine the new firm's intangibility and $\underline{w}$ is their endogenous quitting boundary, which could also depend on n.

D. DISCUSSION OF ADVANTAGE OF SOLUTION METHOD

In DeMarzo et al. (2012), building on the prior solution technique of Piskorski and Tchistyi (2010), solve a system of two ODEs, one for a low (L) and high (H) productivity state. The optimal compensation policy requires a state-dependent compensation adjustment ψ that is defined by $p'_L(w_L) = p'_H(w_L + \psi_{LH}(w_L)) = p'_H(w_H) = p'_L(w_H + \psi_{HL}(w_H))$, which measures how agents' compensation changes in response to a change in state (from L to H and vice versa). This is an adjustment that needs to be added to the *domain* of the numerical derivative of the principal's value function $p'(w)$. This adjustment is solved for numerically in conjunction with an iterative procedure that alternates on solving the ODEs and determining the free boundaries $\{\overline{w}_L, \overline{w}_H\}$.

The main advantage of my approach replaces the compensation adjustment with a second-order partial derivative p_{wn}, which is analogous to limit of the compensation adjustment as the jump between productivity states goes to zero. This obviates the need to numerically solve for $\psi(\cdot)$ within each step of the solution. The advantage is particularly obvious when considering more than two states, as the number of compensation adjustments requiring a solution equals $S(S-1)/2$, where S is the number of states. These $S(S-1)/2$ adjustments need to be solved in addition to the S ODEs and S free boundaries. I conjecture this becomes infeasible once S becomes large.

I overcome this dimensionality problem by solving the PDE on a non-rectangular grid. I guess a payment boundary curve, the collection $\{\overline{w}(n)\}$, and for each guess solve the PDE, which solves very quickly with the implicit method. It is then computationally cheap to guess boundary curves until the boundary conditions and solution to the PDE numerically satisfy some criterion. The cost of the approach is having to program the matrix $\mathbf{Q}$, which requires care. Overall, I think the most important advantage is that the solution is feasible.

TABLE I: STYLIZED FACTS

This table lists characteristics of portfolios sorted on intangibility. Firms are sorted into quintiles by intangibility within their two-digit NAICS code and are rebalanced every June. Portfolios are value-weighted by a firm's market capitalization based on these within-industry ranks. I report time series averages of median portfolio characteristics for all variables except for returns which is an average and annualized. Variable definitions are in Appendix A. The sample period covers 1975 until 2017.

	Intangibility Quintile				
	1	2	3	4	5
Intangibility	0.3	0.8	1.5	2.5	4.7
Average Q	1.1	2.9	4.0	5.4	6.1
Investment Rate (%)	10.4	13.2	13.7	13.4	12.5
Development Rate (%)	17.8	19.2	20.0	19.3	19.0
Profitability (%)	19.8	32.0	43.2	56.3	74.3
Average Return (%)	11.6	12.6	12.1	13.1	12.4

TABLE II: DOUBLE SORTS ON BOOK-TO-MARKET AND INTANGIBILITY

This table lists characteristics of portfolios sorted on book-to-market (BE/ME) ratios and intangibility. Firms are first sorted into portfolios on book-to-market ratios based on NYSE breakpoints and then within each book-to-market portfolio are sorted by intangibility within their two-digit NAICS code. Portfolios are rebalanced every June and are value-weighted by a firm's market capitalization based on these book-to-market/within-industry ranks. Terciles are grouped into Low (0-33), Medium (34-66), and High (67-100). I report time series averages of median portfolio characteristics for all variables except returns and report average monthly returns, which I annualize. The columns under Portfolio Spread report the average time-series difference between a particular portfolio and the high (H) portfolio within a book-to-market grouping. Variable definitions are in Appendix A. The sample period is from 1975 until 2017. Standard errors are in parentheses and computed using a Newey-West estimator with a one month lag.

	Intangibility			Intangibility		
	Portfolio Spread		Portfolio	Portfolio Spread		Portfolio
BE/ME	H-L	H-M	H	H-L	H-M	H
	Intangibility			Average Return (%)		
L	2.8	1.7	3.4	1.0	-0.2	12.4
				(1.2)	(1.1)	
M	3.4	2.3	3.8	2.3	1.9	14.7
				(1.5)	(1.2)	
H	4.0	2.9	4.2	-0.6	-1.2	13.0
				(1.6)	(1.5)	
	Average Q			Investment Rate (%)		
L	4.4	1.7	7.5	-1.6	-0.6	13.7
	(0.5)	(0.3)		(0.6)	(0.4)	
M	2.5	1.6	3.5	2.0	0.1	12.0
	(0.4)	(0.3)		(0.5)	(0.4)	
H	1.4	1.0	2.1	2.0	0.6	11.5
	(0.3)	(0.3)		(0.5)	(0.5)	

TABLE III: INVESTMENT PANEL REGRESSIONS

This table reports empirical estimates of coefficients of the panel regression

$$i_{it} = f_i + f_t + \beta_i' X_{it} + \epsilon_{it},$$

where i_{it} is the investment rate indexed by firm (i) and year (t), f_i is a firm fixed effect, f_t a year fixed effect, and X_{it-1} are lagged firm-level regressors and controls. Variable definitions are in Appendix A. The Compustat sample is from 1975 until 2017. All variables are winsorized at the 5-95 percent level across all firm-year observations. Standard errors in parentheses are clustered at the firm-level.

	Investment Rate (%)					
	(1)	(2)	(3)	(4)	(5)	(6)
Average Q	1.644	1.664	2.104	1.200	1.425	1.123
	(0.037)	(0.036)	(0.052)	(0.055)	(0.049)	(0.049)
Intangibility		-0.144	0.621	0.791	1.303	1.150
		(0.069)	(0.086)	(0.086)	(0.069)	(0.076)
Q × Intangibility			-0.120	-0.042	-0.073	-0.058
			(0.009)	(0.010)	(0.008)	(0.008)
Profitability (%)				0.173	0.127	0.108
				(0.005)	(0.004)	(0.004)
Profitability × Intangibility				-0.014	-0.010	-0.008
				(0.001)	(0.001)	(0.001)
BE/ME						-3.192
						(0.130)
Cash-to-Assets (%)						0.084
						(0.008)
Tangibility (%)						-0.046
						(0.009)
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	No	No	No	No	Yes	Yes
Observations	86,266	86,266	86,266	86,169	86,169	83,687
Overall $\bar{R}^2$	0.126	0.126	0.134	0.210	0.290	0.310

TABLE IV: INTERNATIONAL EVIDENCE

Panel A lists investment rates and values of average Q of portfolios sorted on intangibility. Within each country, firms are sorted into quintiles by intangibility within the manufacturing two-digit NAICS code (31-33) and are rebalanced every June. Portfolios are value-weighted by a firm's market capitalization based on these within-industry ranks. I report time series averages of median portfolio characteristics. Panel B reports empirical estimates of coefficients of the panel regression

$$i_{it} = f_i + f_t + \beta_i' X_{it-1} + \epsilon_{it},$$

where i_{it} is the investment rate indexed by firm (i) and year (t), f_i is a firm fixed effect, f_t a year fixed effect, and X_{it-1} are lagged firm-level regressors and controls. Variable definitions are in Appendix A. The sample period is from 1994 until 2017. All variables are winsorized at the 5-95 percent level across all firm-year observations. Standard errors clustered at the firm-level are reported in parentheses.

Panel A: Portfolio Statistics by Country

	Intangibility Quintile				
	1	2	3	4	5
Germany					
Average Q	0.7	0.9	0.8	1.4	2.7
Investment Rate (%)	8.7	10.3	10.5	10.8	8.8
France					
Average Q	0.8	1.0	1.6	1.5	6.2
Investment Rate (%)	9.1	7.0	7.3	9.5	15.7
Great Britain					
Average Q	0.7	1.7	2.3	5.8	6.2
Investment Rate (%)	8.5	9.0	10.5	10.6	10.2
Japan					
Average Q	0.7	1.1	0.8	1.2	2.2
Investment Rate (%)	6.7	9.0	7.3	7.7	6.6

Panel B: Investment Regressions

	Country's Firm-Level investment Rate (%)				
	Germany	France	G. Britain	Japan	Pooled
Average Q	2.810	1.857	1.560	2.089	1.987
	(0.435)	(0.387)	(0.123)	(0.123)	(0.089)
Intangibility	-0.417	-0.648	-0.295	1.713	0.088
	(0.381)	(0.311)	(0.137)	(0.248)	(0.144)
$Q \times$ Intangibility	-0.082	-0.067	-0.058	-0.621	-0.248
	(0.171)	(0.083)	(0.024)	(0.126)	(0.048)
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes
Observations	3,250	1,717	4,805	22,904	32,676
Overall $\bar{R}^2$	0.106	0.098	0.141	0.062	0.080

TABLE V: ESTIMATION OF PRIVATE BENEFITS FUNCTION

Panel A reports the time series average values of the mean and standard deviation (in italics) of the standardized residuals within a portfolio obtained from the regression

$$w_{it} = f_i + f_t + \beta_w \times w_{it-1} + \beta_{iw} \times i_{it} \times w_{it-1} + \epsilon_{it},$$

where i indexes the firm and t the year, w_{it} is compensation scaled by lagged physical capital, i_{it} is the firm's investment rate, and year, f_t, and firm, f_i, are fixed effects. The period is from 1993 to 2017. The residuals are grouped into portfolios defined by quintiles of a characteristic and are rebalanced every year. The four characteristics are intangibility, cash flow, book-to-market ratio, asset productivity, and log assets. Variable definitions are in Appendix A. Panel B reports results from regressing standardized residuals, obtained from a regression above, on a polynomial expansion of intangibility across two orders:

$$\epsilon_{it} = f_i + f_t + \beta_1 n_{it} + \beta_2 n_{it}^2 + u_{it}.$$

The residuals are obtained for three measures of compensation. Data for specialists cover the period 1993 to 2017. Data for all employees deferred compensation expense ends in 2017 and dates back to 2000 but becomes widely available after 2004. Standard errors clustered at the firm-level are in parentheses.

Panel A: Conditional Distribution of Residuals

Quintile		Intangibility	Cash Flow	BE/ME	Productivity	Log(Assets)
1	Mean	-0.37	0.11	0.40	0.05	1.00
	Stdev	*0.34*	*1.11*	*1.21*	*1.14*	*1.16*
2	Mean	-0.23	-0.25	0.17	0.21	0.77
	Stdev	*0.49*	*0.51*	*0.95*	*1.03*	*1.31*
3	Mean	0.01	-0.19	0.13	0.28	0.28
	Stdev	*0.72*	*0.48*	*0.89*	*1.07*	*1.00*
4	Mean	0.29	0.10	0.07	0.18	0.01
	Stdev	*0.97*	*0.84*	*0.89*	*0.99*	*0.78*
5	Mean	0.98	0.81	0.04	0.07	-0.29
	Stdev	*1.37*	*1.30*	*0.94*	*0.74*	*0.50*

Panel B: Functional Form by Compensation of Employees, Type

	Specialists, Total		Specialists, Deferred		All, Deferred	
n	0.164	0.185	0.159	0.165	0.092	-0.012
	(0.0056)	(0.0141)	(0.0056)	(0.0140)	(0.0047)	(0.0141)
n^2		-0.002		-0.001		0.011
		(0.0016)		(0.0016)		(0.0015)
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	27,491	27,491	27,349	27,349	25,095	25,095
$\bar{R}^2$	0.207	0.207	0.196	0.196	0.084	0.092

TABLE VI: COMPENSATION AND DEVELOPMENT

This table reports empirical estimates of coefficients of the panel regression

$$w_{it} = f_i + f_t + \beta_i' X_{it-1} + \epsilon_{it},$$

where i indexes the firm and t the year, w_{it} is compensation scaled by lagged physical capital, i_{it} is the firm's investment rate, and year, f_t, and firm, f_i, are fixed effects. Coefficients of interest in X_{it-1} are lagged compensation, w_{it-1}, and the (real) development rate, $0.3 \times SG\&A_{it}/N_{it-1}$, and their interaction. Controls are sales growth, lagged sales, and lagged intangibility. Variable definitions are in Appendix A. The sample period is from 1994 until 2017. All variables are winsorized at the 5-95 percent level across all firm-year observations. Standard errors clustered at the firm-level are reported in parentheses.

	Compensation by Employees, Type							
	Specialists, Total		Specialists, Current		Specialists, Deferred		All, Deferred	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Lag Compensation	0.336	0.241	0.577	0.603	0.227	0.093	0.081	-0.077
	(0.015)	(0.028)	(0.012)	(0.020)	(0.016)	(0.032)	(0.018)	(0.033)
Development Rate (%)	0.023	-0.006	-0.010	-0.007	0.037	0.010	0.129	0.080
	(0.008)	(0.008)	(0.002)	(0.002)	(0.007)	(0.007)	(0.018)	(0.017)
Lag Comp. × Dev. Rate		0.004		-0.001		0.006		0.008
		(0.001)		(0.001)		(0.001)		(0.002)
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	26,526	26,526	26,402	26,402	26,402	26,402	24,091	24,091
Overall $\bar{R}^2$	0.259	0.262	0.589	0.589	0.135	0.139	0.062	0.069

TABLE VII: CALIBRATION (ANNUAL)

This table summarizes the chosen values of the calibration discussed in Section IV.

Parameter	Value	Description/Target
Preferences		
(r, γ)	(0.011, 0.028)	Interest rate/Average compensation
$\sigma\lambda(n)$	$0.15n$	See Section B and Table V
Technology		
(A, ϕ)	(0.321, 0.810)	Average intangibility (n^{ss}) and profitability
ϵ	0.8	Capital complementarity
(δ_K, δ_N)	(0.1, 0.19)	Depreciation rates
(κ_K, κ_N, z)	(20, 20, 0.01)	Adjustment cost parameters/Doubling time of capital
φ	0.8	Relative bargaining power of agents
σ	0.2	Variability of intangibility growth
(l_K, l_N)	(0.93, 0.93)	Recovery rates

TABLE VIII: PORTFOLIO STATISTICS

This table lists characteristics of portfolios sorted on intangibility. Firms are sorted into quintiles by intangibility within their two-digit NAICS code and are rebalanced every June. Portfolios are value-weighted by a firm's market capitalization based on these within-industry ranks. I report time series averages of median portfolio characteristics for all variables except for returns and issuance frequency which are averages and annualized. Variable definitions are in Appendix A. The sample period is from 1975 until 2017. In the model, intangibility is n, average Q is $p(n)$ or $p(n, w) + w$, investment rate is $i = I/K$, profitability is $Af(n) - c_K(i) - c_N(g)n$, agents' flow share of the firm (compensation) is $(1 - \alpha)\gamma w$, and the agency friction, $\lambda(n)$, takes the form in (29).

		Intangibility Quintile				
		1	2	3	4	5
Standard	Intangibility	1.0	1.4	1.8	2.2	3.1
	Average Q	3.4	4.4	5.1	5.9	7.4
	Investment Rate (%)	13.9	16.4	18.1	18.8	23.9
	Development Rate (%)	32.9	30.8	29.7	27.5	27.3
	Profitability (%)	32.2	43.0	50.8	60.1	78.0
Agency	Intangibility	1.1	1.5	1.9	2.3	3.2
	Average Q	2.1	2.6	2.9	3.3	4.0
	Investment Rate (%)	10.5	11.2	12.1	12.9	12.7
	Development Rate (%)	20.2	19.7	19.3	18.9	19.0
	Profitability (%)	35.7	45.3	53.2	62.0	79.0
	Compensation (%)	0.7	1.2	1.6	1.9	2.0
	Interquartile Range	0.8	1.1	1.3	1.5	1.5
	Agency, $\sigma\lambda(n)$ (%)	17.2	23.1	28.3	34.4	47.3
Data	Compensation (%)					
	Specialists, Total	0.2	0.4	1.0	1.2	2.0
	Interquartile Range	0.4	1.4	3.0	3.2	6.3
	Specialists, Deferred	0.1	0.3	0.7	0.9	1.4
	All, Deferred	0.3	2.0	2.0	2.1	2.4
	Cash-to-Assets (%)	4.5	7.5	11.3	11.3	9.6
	Issuance Frequency (%)	87.5	78.0	70.8	72.9	69.3

TABLE IX: MODEL PANEL REGRESSIONS

I run annual panel regressions in the model using simulated monthly data and including firm fixed effects. Monthly investment is summed over the year and, as in the data, is regressed on last year's lagged average Q $(p(n, w) + w)$ and intangibility (n).

	Investment Rate (%)							
	Standard Model				Agency Model			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Average Q	2.289	0.011	0.894	-0.416	0.584	4.626	3.156	-13.641
Intangibility		4.132	1.288	-21.022		-3.626	-0.076	70.983
Q × Intangibility			0.125	0.368			-0.382	8.051
Profitability (%)				0.877				-1.838
Profitability × Intangibility				0.032				-0.563
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

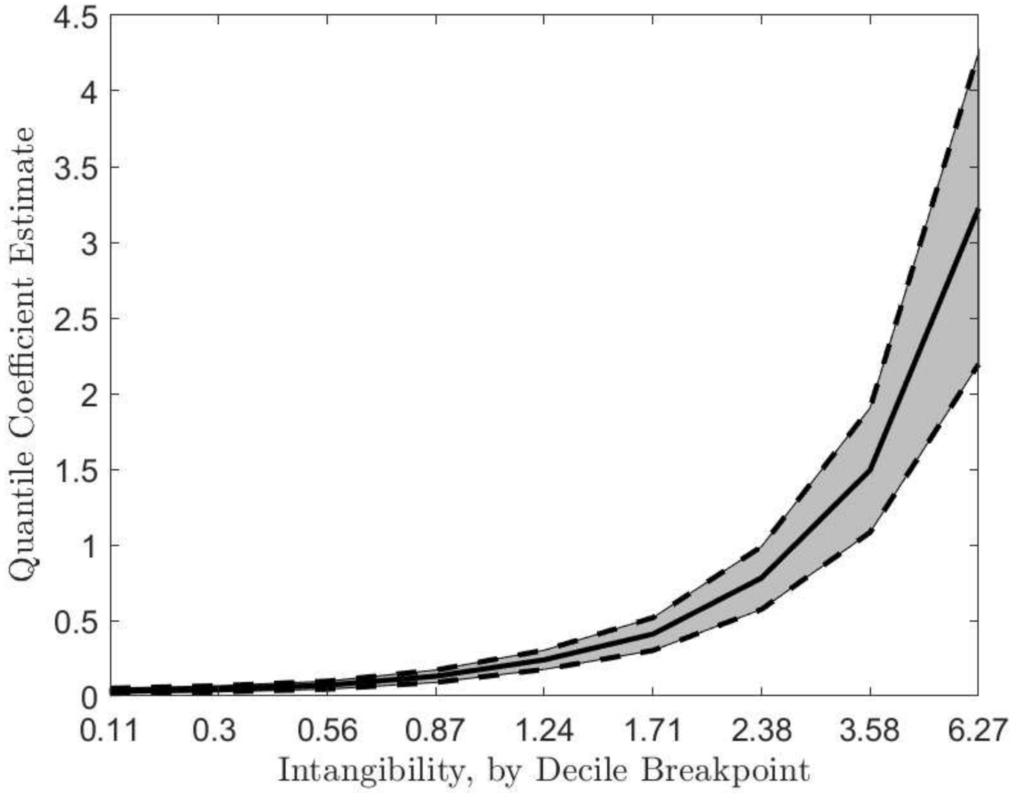

FIGURE 1: QUANTILE REGRESSIONS: COMPENSATION ON INTANGIBILITY

This figure displays the point estimates and two times the standard errors of coefficients obtained from quantile regressions of compensation on lagged intangibility controlling for firm and year fixed effects. Nine breakpoints separating deciles were estimated on annual firm-level data over the period 1975 to 2017. Intangibility levels at each breakpoint are recorded on the horizontal axis. Standard errors are clustered at the firm-level.

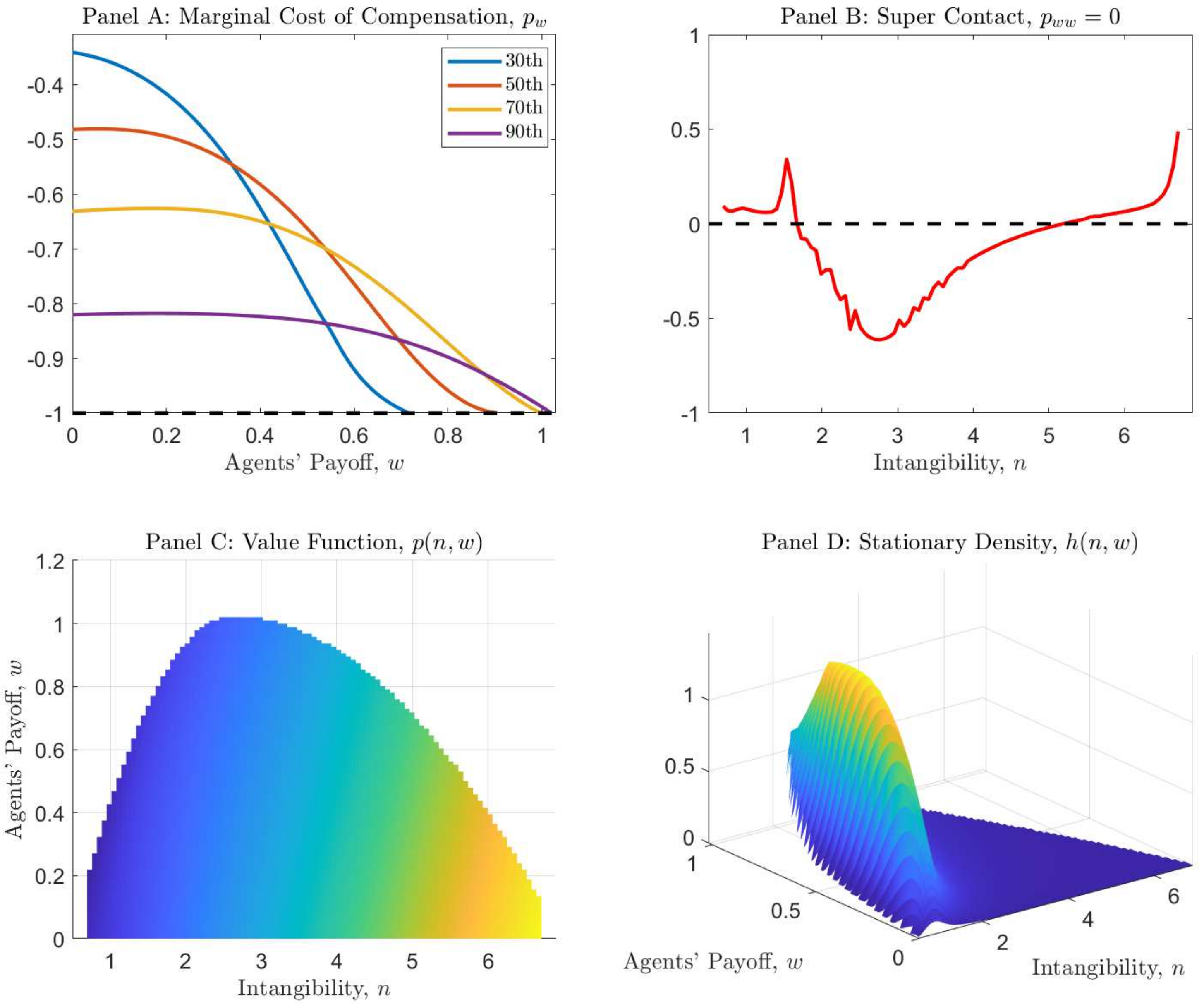

FIGURE 2: PROPERTIES OF MODEL SOLUTION

Panel A plots the first derivatives of investors' scaled value function with respect to w. I denote the 30th, 50th, 70th, and 90th percentiles of the marginal distribution of intangibility. At the payout boundary, smooth pasting holds: $p_w(n, \overline{w}(n)) = -1$ for all n. Panel B plots the second derivative of $p(n, w)$ with respect to w for each value of intangibility; this is the super contact condition on the payout boundary, $p_{ww}(n, \overline{w}(n))$. Panel C plots the investor's scaled value function, $P(K, N, W)/K$, as a function of intangibility $n = N/K$ and agents' scaled continuation payoff $w = W/K$. The domain of the solution is non-rectangular. Panel D plots the stationary density of the model. In Panels C and D a brighter color represents a higher value.

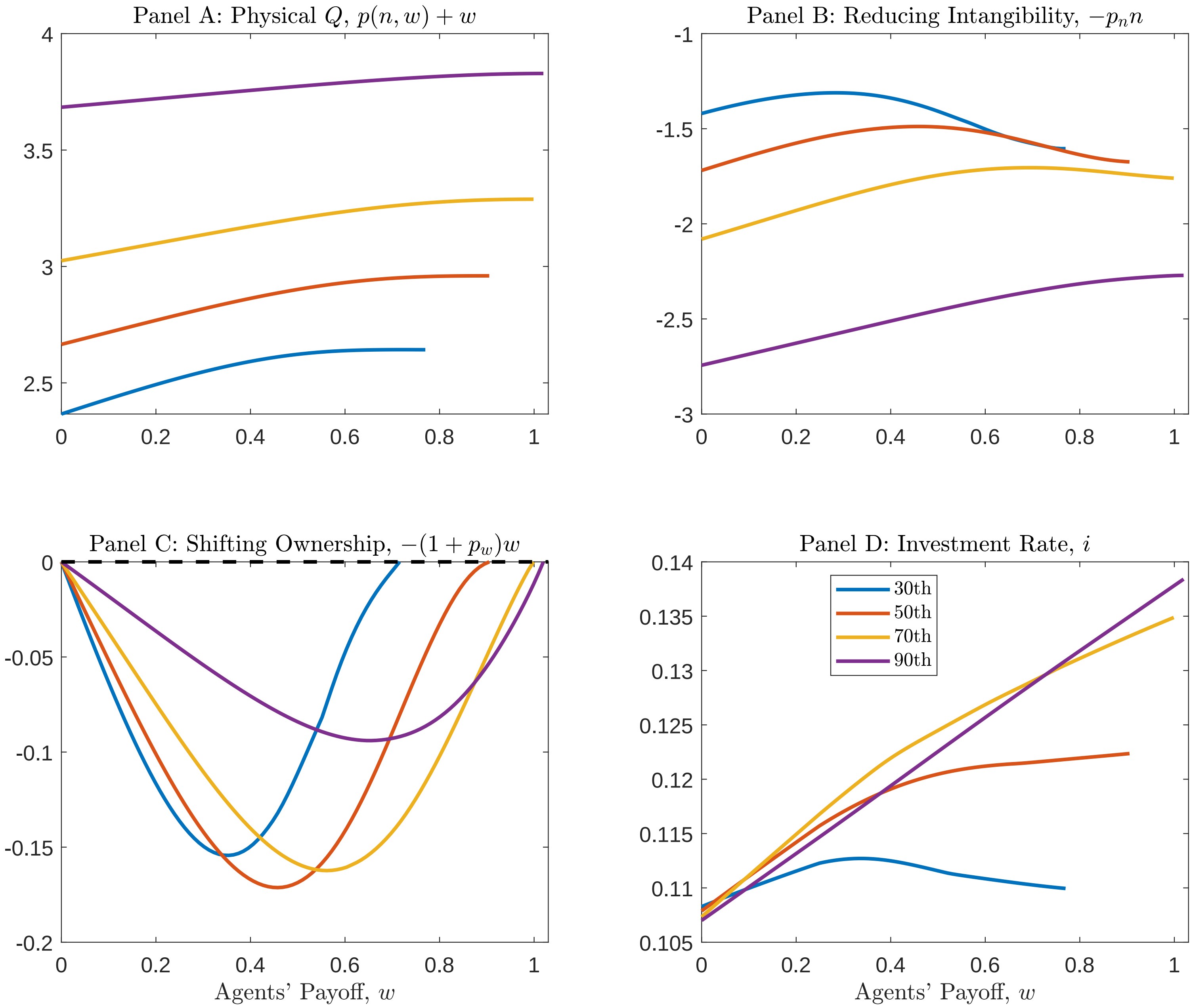

Figure 3: The Investment Decision

This figure decomposes the investment policy function: $c'(i) = p(n,w) + w - p_n(n,w)n - (1 + p_w(n,w)w$. I denote the 30th, 50th, 70th, and 90th percentiles of the marginal distribution of intangibility. Panel A plots average Q, $p(n,w) + w$. Panels B and C plot the cost of reducing intangibility, $-p_n n$, and shifting ownership, $-(1 + p_w)w$, respectively. Panel D plots the investment rate which, since $c(i)$ is quadratic, is linear in the sum of panels $A + B + C$.

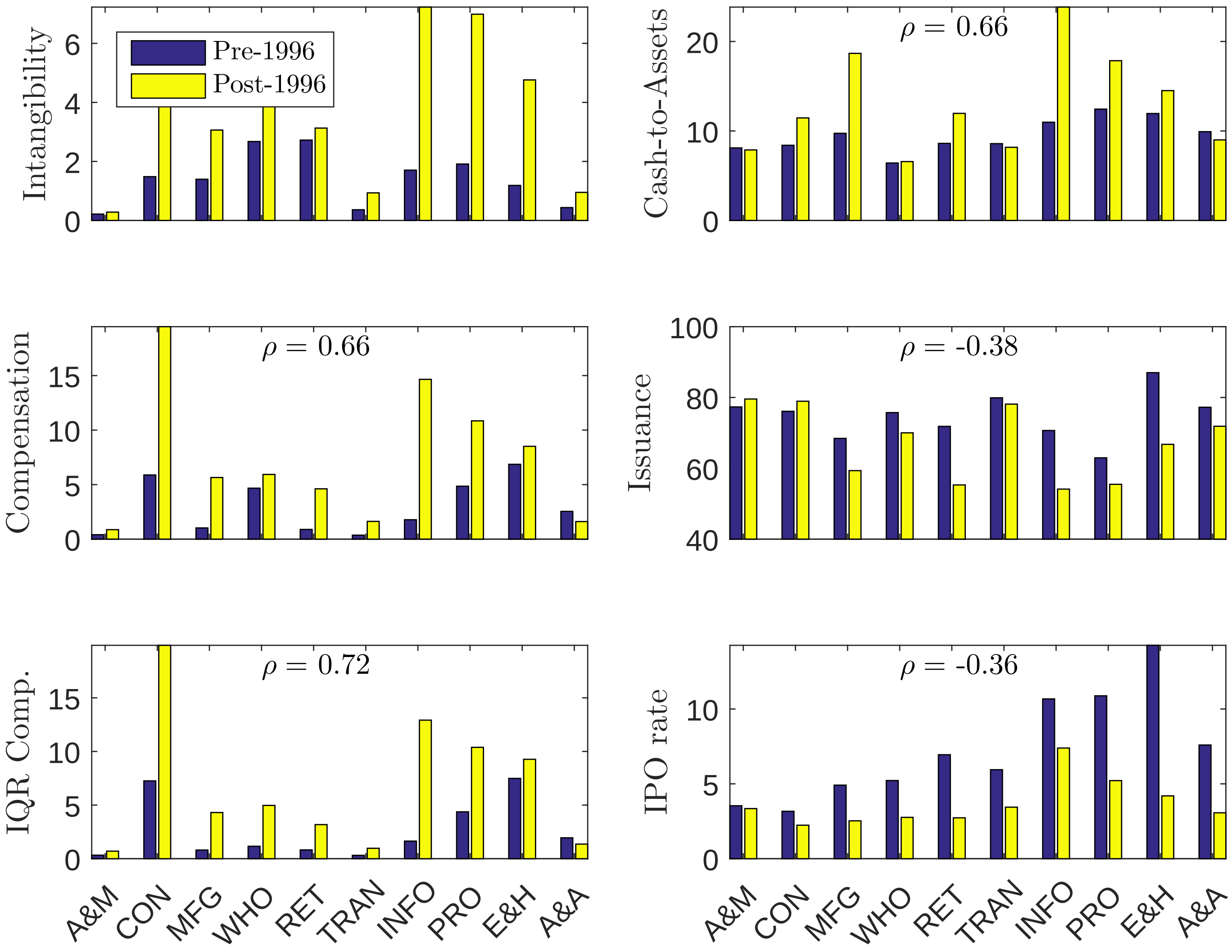

FIGURE 4: RISE IN INTANGIBILITY AND SHIFT IN FINANCING BY INDUSTRY

This figure shows changes for several average and interquartile (IQR) characteristics of firms and rate of initial public offerings (IPOs) by industry. Data are from 1975 until 2017. Reported coefficients, ρ, measure the correlation across industries of the change in the particular panel's characteristic with the change in intangibility in the top-left panel. Characteristic variables are defined in Appendix A. IPO data are from Jay Ritter's website. The IPO rate is the number of IPOs divided by the number of last year's listed firms. Industry groups are based on 2-digit NAICS codes: 11–21, Agriculture and Mining (A&M); 23, Construction (CON); 31–33, Manufacturing (MFG); 42, Wholesale Trade (WHO); 44-45, Retail Trade (RET); 48–49, Transportation (TRAN); 51, Information (INFO); 54–56, Professional (PRO); 61–62, Education & Health (E&H); 71–72, Arts & Accommodation (A&A).

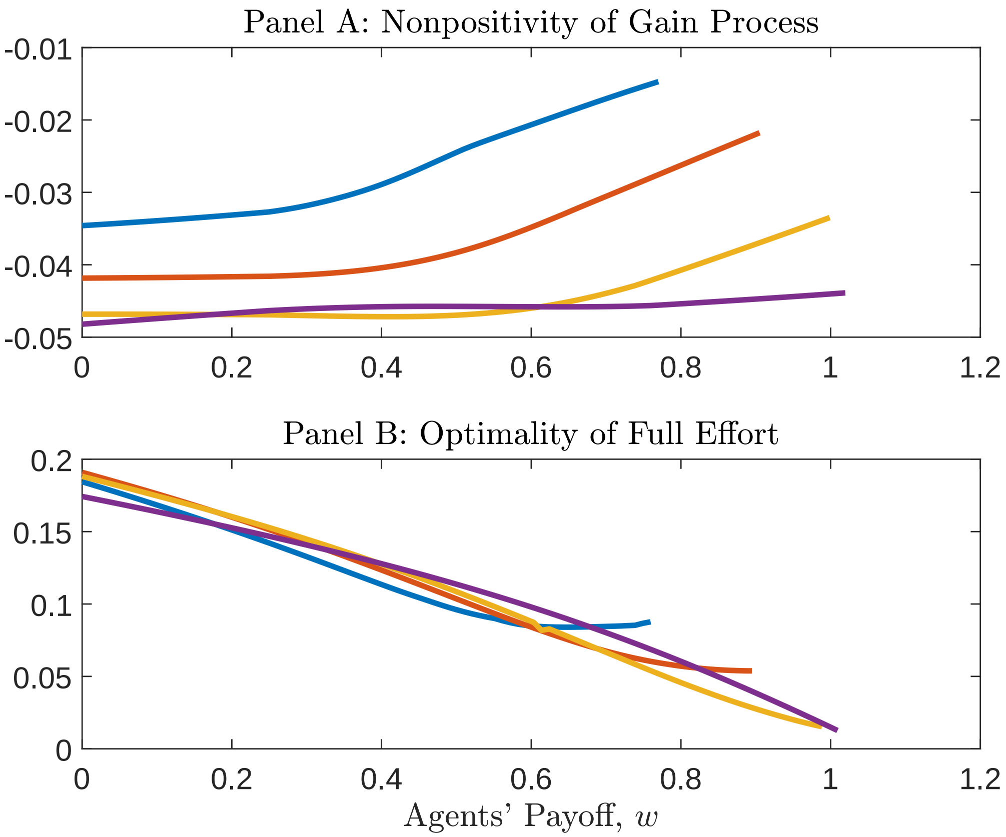

FIGURE A-1: FULL EFFORT CONDITION

This figure displays conditions required for full effort $(e_t = 1)$ to be the optimal incentive strategy as I discuss in Appendix B. Panel A shows the sum of all second-order terms in (B4): $\frac{\sigma^2 n^2}{2} p_{nn} + \frac{\beta^2 \sigma^2 n^2}{2} p_{ww} + \beta \sigma^2 n^2 p_{nw}$. I denote the 30th, 50th, 70th, and 90th percentiles of the marginal distribution of intangibility. Solid lines are terms evaluated at $\beta = \beta^* = \lambda(n)/n$. The nonpositivity of these lines are necessary for the supermartingale property of the gain process $\{G\}$. Panel B plots the value of the inequality that is required to ensure that full effort is preferred to a policy in which agents shirk $(e_t = 0)$.

TABLE A-I: PORTFOLIOS SORTS ON AVERAGE Q

This table lists characteristics of portfolios sorted on physical average Q. Firms are sorted into portfolios by Q and rebalanced every June. Portfolios are value-weighted by a firm's market capitalization. I report time series averages of median portfolio characteristics. The sample period is from 1975 until 2017.

	Q Quintile				
	1	2	3	4	5
Intangibility	0.3	0.3	0.7	1.8	2.5
Average Q	0.5	0.9	1.6	3.3	8.9
Investment Rate (%)	8.9	9.3	11.0	12.9	17.1

TABLE A-II: PORTFOLIOS SORTS ON TANGIBILITY

This table lists characteristics of portfolios sorted on tangibility. Firms are sorted into quintiles and rebalanced every June. Portfolios are value-weighted by a firm's market capitalization. I report time series averages of median portfolio characteristics. The sample period is from 1975 until 2017.

	Tangibility Quintile				
	1	2	3	4	5
Intangibility	4.1	2.4	1.7	0.8	0.2
Tangibility (%)	10.5	19.4	27.5	40.4	65.7
Average Q	11.3	4.6	3.6	2.0	1.0
Investment Rate (%)	15.2	12.8	13.1	11.9	9.9

TABLE A-III: RISK PREMIA EXTENSION TO STANDARD MODEL

Panel A lists risk premia (net of market risk) by organizational capital (O/K) quintile from Eisfeldt and Papanikolaou (2013), henceforth EP. The top two rows, O/K and Risk Premia, are directly from EP. The third row calibrates risk premia to EP in my risk premia model with a linear production function $(\epsilon, \phi) \to (\infty, 1)$ and price of risk $\Gamma = 0.53$, following EP, and a correlation $\rho = 0.7$ that is otherwise under the calibration in Section IV. Panel B lists investment rates, in percent, of portfolios sorted on intangibility across data, the standard model, and the risk premia model, now with CES production, that otherwise matches the calibration in Panel A's final row. In the data, firms are sorted into quintiles and rebalanced every June. I report the time series average of the portfolio's median investment rate. The sample period is from 1975 until 2017. The column (5-1) reports the range: the difference between a portfolio 1 and portfolio 5. The right-most column reports the relative difference to the standard model.

Panel A: Risk Premia by O/K Quintile

	O/K Quintile				
Variable	1	2	3	4	5
O/K (EP)	0.19	0.42	0.66	1	1.65
Risk Premia (%, EP, Net of Market Risk)	0.05	1.07	1.86	2.79	4.29
Risk Premia Model (%, Linear Production)	0.48	1.03	1.58	2.32	3.65

Panel B: Investment Rates by Intangibility Quintile

	Intangibility Quintile						
Investment Rates (%)	1	2	3	4	5	(5-1)	Relative
Data	10.4	13.2	13.7	13.4	12.5		
Standard Model	13.9	16.4	18.1	18.8	23.9	10.0	
Risk Premia Model (CES Production)	16.4	17.8	18.8	19.6	22.1	5.8	-43%