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Income polarization and stagnation in a stochastic model of growth: When the demand side matters

Thomas Gries

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Thomas Gries*

Abstract

This contribution is motivated by two stylized recent observations: the slowdown in growth and simultaneous income polarization in many advanced economies. We suggest an approach that departs from a typical endogenous growth model. As we want to look at polarization effects, we model two income groups, labor and financial wealth owners. The two groups have different intertemporal choices which lead to group-specific consumption and savings patterns. As a result, consumption and endogenous investments define an independent demand side. Prices are generally flexible. However, there is a mismatch which is related to stochastic aggregate demand and firm specific frictions. Adjustments to the mismatch are modeled through a reallocation of resources to a search and matching process. As an important result, firms and costumers find a market equilibrium growth path below potential growth. Thus, we obtain a hybrid neoclassical-keynesian model of growth. Even if potential growth can be generated only by the supply side, endogenous effective demand restricts the level and growth rate of the income path. In order to show that this path is a steady-state equilibrium path we suggest an unconventional equilibrium concept in a stochastic environment. We define an equilibrium as being stationary if all expected values are the effectively realized values (no systematic error in expectations). This equilibrium concept relates to the Nash idea of individual stationary behavior. Overall, our hybrid model bridges a gap between neoclassical (and endogenous growth) ideas and Keynesian ideas (demand restrictions) of economic growth.

JEL classifications: O40, E10, D33
Keywords: imperfect markets; market matching; product-variety model; demand-restricted growth; semi-endogenous growth

*) Thomas Gries, thomas.gries@notes.upb.de
Economics Department (C-I-E), www.C-I-E.org;
University of Paderborn, Germany

1 Introduction

While this contribution intends to rather generally bridge the gap between mainstream supply-side driven growth theory and a thinking that suggests that the demand side matters (even in long-run growth), it is motivated by two stylized observations: first, the slowdown in growth, and second, the simultaneously observed income polarization in many advanced economies.[1] Figure 1 graphically connects the two phenomena in a simple way. For the US economy this figure describes a negative relationship between the estimated long-term growth trends and the income share of the top income decile.[2]

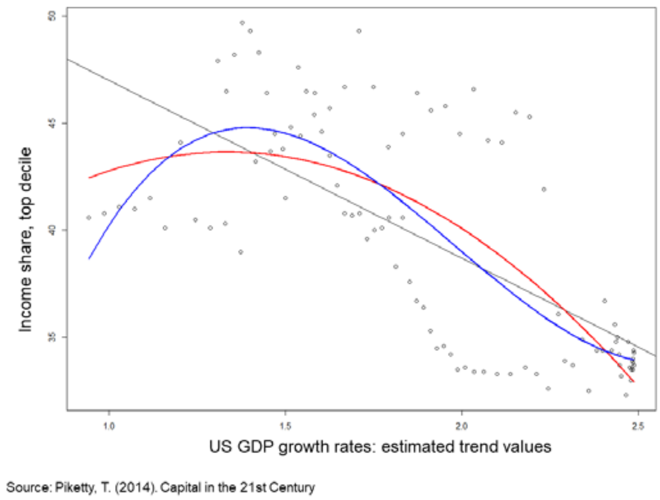

Figure 1: Relation between long term growth trend values and income share of top income decile

While polarization and low growth seem to coincide, it is not clear if they are economically connected and what the mechanism might be. A potentially important channel linking both phenomena may go through the demand side. However, the demand side has no place in mainstream growth theory. While

[1] By "long-term slowdown in growth" we mean the discussion related to Gordon (2012, 2015), Summers (2014), Plosser (2014), or Fritz et al. (2019). Examples of the identification of income polarization include contributions by Autor et al. (2003, 2006, 2008), Autor & Dorn (2013), Spitz-Oener (2006), Goos et al. (2014), or Van Reenen (2011).

[2] In figure 1 the long-term growth rate is estimated with a newly developed endogenous non-parametric trend estimation method introduced by Feng et al. (2020) and used in Fritz et al. (2019) or Gries et al. (2019). This method allows to identify a smoothly adjusting trend value for each point, with the interpretation that economic conditions may smoothly change the growth trend. Using these estimated trend data we run a simple linear, quadratic, and cubic regression and findd that the negative correlation is clearly significant. Even if this is only a simple regression and not a sophisticated econometric analysis, the result suggests that there is something we should not completely ignore.

we do not know yet, we should not rule out any channel in advance. Therefore, we cannot purely rely on standard supply side-driven growth theory, and thus use this occasion to suggest a general hybrid model of growth in which the demand side plays an important role. Further, we apply this general model to a discussion of the income polarization-growth nexus and obtain unconventional results.

In today's mainstream growth theory, there is no role for the demand side. Technical innovations and/or factors that can be accumulated determine potential production, and customers absorb whatever is potentially available in the economy. Economists often refer to this mechanism as Say's Law. As there is never a lack of demand, production capacity defines the limits of consumption, income, savings, and growth. With the automatic existence of sufficient demand the choice to save fully transforms to investments, and investments are spent on the factors that generate capacity for more production and growth. In this approach, the greatest economic problem is the intertemporal decision of how much of today's income we should save to be invested in tomorrow's consumption. The answer is given by the Cass-Koopmans-Ramsey[3] rule for the aggregate economy which completes the basic story of (constant return to scale) *neoclassical growth* mechanics.

This story still holds in the debate around *endogenous growth theory,* which is relevant for this paper. Endogenous growth theory started with the models by Romer (1986, 1987, 1990) or Lucas (1988) and has since become the dominant growth approach. The most prominent attribute of endogenous growth theory is the ability to generate a sustainable constant growth rate via various mechanics (often scale economies) and to select this sustainable growth rate as an optimal intertemporal choice. As excellent reviews and descriptions of these mechanisms are given in Aghion et al. (1998), Jones (1999), Aghion & Durlauf (2005), or Aghion & Howitt (2008), there is no need to repeat their comprehensive detailed discussion in this paper. Again, in all these models the demand side is irrelevant.

How can we be so sure that only the supply side determines growth? Why is it so difficult to think of another mechanism that allows the demand side to play a role? In the recent debate on the Great Recession and Secular Stagnation, some arguments go beyond a discussion of the supply side. E.g., Summers (2014) has concerns about a systematic savings-investment mismatch that may be a reason for secular stagnation. Gordon (2012, 2015) emphasizes six headwinds caused by an aging population, education, increasing inequality, and an increasing ratio of federal government debt to GDP, all of which impede growth. While most of these headwinds clearly address the supply side, increasing inequality may also relate to effects on the demand side, as we will show in section 3.6. Further, Chamley (2013) describes an adjustment process in a general equilibrium model in which aggregate supply creates the income that generates corresponding demand, which is exactly what Say's Law suggests. However, this mechanism may not work in a general equilibrium with decen-

[3] See Cass (1965) and Koopmans (1965).

tralized markets and savings in bonds or money, as convergence to that state is slow. Further, a small strand of literature considered the interactions between short-run macroeconomic dynamics under rigidities or time absorbing adjustments, and long run growth effects. Starting with Fatas (2000) and Comin and Gertler (2006) this literature continuously developed up to more recent papers such as Bianchi et al. (2014) or Anazoategui et al. (2019), who with respect to the Great Recession suggest that demand factors played a role in the slowdown in capacity growth. Similarly, Benigno & Fornaro (2018) provide a dynamic monetary macro model with Keynesian elements in which pessimistic expectations and a liquidity trap can lead to continuing slow growth. However, we do not pursue the idea that rigidities or slow adjustments in the short term dynamics of an otherwise mainstream growth approach cause longer-term growth effects. In this paper we suggest an alternative growth mechanism itself. We include a stochastic mismatch that relates to the demand side as a potentially constraining element even in steady state. In other words, while we build on a standard growth approach, we suggest a hybrid model that covers both elements – a modified (semi-)endogenous growth mechanism and a demand side that matters to the level and growth rate of the steady state path.

In detail, we depart from Romer's (1987, 1990) product-variety model and modify it in four ways. (i) Firms are in an uncertain environment with imperfect information. A mismatch in the final goods market is perceived as the result of stochastic frictions. Firms can counter this mismatch by shifting resources to a more successful sales process. This extra effort to sell describes the firm's major adjustment mechanism and leads to a reallocation of resources from pure production activites that counter this mismatch. Further, as this mismatch relates to the covariance of random aggregate demand and firm specific shocks, the independent demand side may restrict GDP and GDP growth. (ii) In order to obtain an independent demand side and to allow for income polarization, we move away from a representative household and introduce two separate income groups: labor and owners of financial assets. The two groups have different intertemporal choices which lead to group-specific consumption and savings patterns. Further, market entry conditions, specifically the match between potential new technologies and market opportunities, determine investments. Thus, consumption and endogenous investments which are not directly the result of the savings decision define the demand side. (iii) The growth rate of new technologies is driven by the market entry of these technologies. Market entry is determined not only by the given growth of ideas, but also by a match between endogenous market opportunities (demand) and new products offered, and thus again by demand-side elements. (iv) Why is the resulting market equilibrium a stationary equilibrium? Because we introduce an alternative stationary equilibrium concept which we refer to as the no-expectation-error equilibrium (n-e-ee). The basic idea of this concept was discussed in Gries (2020). With this concept we define equilibrium as rational stationary behavior (similar to Nash) which is possible at any level of income or production. With these modifications of an otherwise mainstream model, we obtain rather different economic mechanisms with very different policy implications for the resulting growth process.

2 The model

The model in this section is based on a product-variety approach similar to that of Romer (1987, 1990) as well as on a condensed, compact representation by Aghion & Howitt (2008, ch. 3) which we refer to as the standard model. Each of the following subsections differs slightly to this standard model in that the model mechanics may be rather different at the end. Subsection 2.1 describes the final-goods-producing firm. The most important difference to standard modeling is a market mismatch in the final goods market and an optimal counter activity from final-goods-producing firms. The innovative intermediate-goods-producing firms are described in subsection 2.2. Unlike the standard modeling, innovative intermediate goods do not automatically enter the economy. The market entry process of innovations is determined by another matching procedure, such that this successful market entry process determines innovation growth and investments. In section 2.3 and 2.4 we look at aggregate saving and consumption. As we wish to study effects of income polarization on growth, we look at two kinds of households and derive an aggregate consumption pattern. Labor households consume and wealth holders save. As consumers also may face non-matched demand, they also spend some income on collecting information and improving the matching process in final goods markets. We can hence determine the aggregate effective demand as the fourth innovative element of this growth model. In section 3 we solve the model and reveal that the mismatch is determined by a combination of aggregate and idiosyncratic conditions.

2.1 Final-goods-producing firms

In order to depart from a standard product variety model we use Aghion & Howitt (2008, ch. 3) as benchmark model. Except for the introduction of stochastic mismatch in equations (3) to (6), all other equations [(1) to (14)] introducing final-goods-producing and intermediate-goods-producing firms are identical to the model described by Aghion & Howitt (2008, ch. 3).

Production of final goods: A representative firm $i \in \mathcal{F}$ produces with labor used for production L_{Qi} and the number $N_i(t)$ of differentiated intermediate inputs $x_{ji}(t)$ offered by $N(t)$ small firms. Total *production* of firm i's good $Q_i(t)$ is

$$Q_i(t) = L_{Qi}^{1-\alpha}(t) N_i(t) x_i^{\alpha}(t), \tag{1}$$

with $x_i(t)$ being a representative variation of the intermediate goods $x_{ji}(t)$. Total production is the supply of final goods to the final goods market.

Market mismatch, countermeasures and expected sales: Firms are in a stochastic environment with random shocks, imperfect information, and frictions. As result of these imperfect information and frictions not all production directly and instantaneously finds a customer. When firms offer to the market, they observe that $\Phi_i(t)$ are their stochastic effective sales, such that randomly only fraction $\phi_i(t) = \frac{\Phi_i(t)}{Q_i(t)}$ of total output planned to sale is instantly sold. Thus, $\phi_i(t)$ is firm i's current *effective sales ratio.* Firm's subjective interpretation of $\phi_i(t) \leq 1$ is that this deviation is due to the firm's stochastic market mismatch $\delta_i(t)$, ,such that δ_i is the firm's non-matched individual fraction of supply

$$\phi_i(t) = 1 - \delta_i(t). \tag{2}$$

The market mismatch is perceived to be due to given stochastic market frictions $\delta_i'(t)$ which are perceived to be generated by costumers who are insufficiently informed about products, prices, qualities and general market conditions. There is no perfect instantaneous match between planned and produced output, and the respective demand. Further, firms experience that they can employ labor $L_{\phi i}$ to counter this friction and improve the chance of placing the not instantaneously sold goods in the market. With this match-improving experience $m_i(L_{\phi i})$ (with $\frac{\partial m_i}{\partial L_{\phi i}} > 0$) and perceived frictions $\delta_i'(t)$ the mismatch for an individual firm can be described as $\delta_i(t) = \delta_i'(t) - m_i(L_{\phi i})$. Then, from the perspective of this firm, the share of production that firm i can expect to sell in the market, the *expected effective sales ratio* $E\left[\phi_i(\delta_i', L_{\phi i})\right]$, monotonically increases in $L_{\phi i}$, and decreases[4] in $E\left[\delta_i'\right]$

$$E[\phi_i] = \phi_i\left(E\left[\delta_i'\right], L_{\phi i}\right), \quad 0 < E[\phi] \leqq 1, \quad \frac{\partial E[\phi_i]}{\partial E\left[\delta_i'\right]} < 0, \quad \frac{\partial E[\phi_i(t)]}{\partial L_{\phi i}(t)} > 0. \tag{3}$$

Note, that in the next subsection we show that profit-maximizing firms with individual perceptions and expectations about their observed mismatch will counter these frictions (mismatch). Further, in section 3.2 we reveal how this friction is not only an individual market problem of each firm, but also relates to the aggregate economy.

Demand for labor and wages: Wage $w'(t)$ is potentially paid for two kinds of labor: labor in production L_{Qi} and for labor countering the perceived market mismatch with sales related activities $L_{\phi i}$. Wages and intermediate goods prices are expressed in units of the final output good. With $p_x(t)$ denoting the prices of firm i's representative intermediate inputs $x_i(t)$, the final goods-producing firm's profit is

$$E[\Pi_{Qi}] = E[\phi_i] Q_i - N_i p_x x_i - w' L_{Qi} - w' L_{\phi i}. \tag{4}$$

For each firm the procedure to maximize profits has two components. Firms need to organize an efficient sales process, and firms have to determine optimal production.

First, in order to establish an *efficient sales process,* firm i allocates $L_{\phi i}$ to the matching process and improves it's effective sales. In order to get all production sold in the market the firm will increase $L_{\phi i}$ until all produced

[4] By definition $\phi_i(t) + \delta_i(t) = 1$, see the detatiled discussion of mismatch and sales in section 3.2.

and supplied goods are matched by market demand and can be expected to be absorbed by the market. Specifically, the firm's total expected revenues $E[\phi_i] Q_i$ are determined by the expected success rate of sales of already produced final goods $E[\phi_i]$, and the production of these goods Q_i. As each element depends on the respective labor input, we have to allocate labor to the two purposes, L_{Qi} and $L_{\phi i}$. Hence, the productivity of each use of labor is described in the following table:[5]

	$E[\phi_i] \leq 1$	otherwise
$\frac{d(E[\phi_i]Q_i)}{dL_{\phi i}} = \frac{\partial E[\phi_i]}{\partial L_{\phi i}} Q_i$	$\frac{\partial E[\phi_i]}{\partial L_{\phi i}} Q_i > 0$	0
$\frac{d(E[\phi_i]Q_i)}{dL_{Qi}} = E[\phi_i] \frac{\partial Q}{\partial L_{Qi}}$	$E[\phi_i](1-\alpha)\frac{Q_i}{L_{Qi}} > 0$	$(1-\alpha)\frac{Q_i}{L_{Qi}} > 0$

At this point we assume that it is more effective to place an already existing (but not yet demanded) output in the market than to produce a new unit of output. That is, as long as not all production immediately finds a customer, the marginal productivity of labor in the matching process is greater than marginal revenue of labor in production. Formally we suppose

$$\frac{\partial E[\phi_i]}{\partial L_{\phi i}} Q_i > E[\phi_i](1-\alpha)\frac{Q_i}{L_{Qi}} \quad for \quad E[\phi_i] \leq 1,$$

such that an efficient firm i, will increase L_{ϕ_i} until the expected sales ratio turns to

$$E[\phi_i] = 1, \tag{5}$$

and nothing is left in stock. Further, if $L^*_{\phi i}$ is the amount of labor that leads to $E[\phi_i] = 1$ it is the maximum amount of labor allocated to sales activities, since the mismatch has been solved and marginal productivity turns to $\frac{\partial E[\phi_i]}{\partial L_{\phi i}} = 0$. $L^*_{\phi i}$ is a stable corner solution. What is more, any time condition (5) holds, equation (3) implicitly defines a function for the allocation of sales-related labor. As result, the larger the firm's perceived mismatch $E[\delta_i]$, the more labor is allocated to sales activities[6]

$$L^*_{\phi i} = L_{\phi i}(E[\delta_i]), \quad \frac{\partial L_{\phi i}}{\partial E[\delta_i]} > 0. \tag{6}$$

Second, when the firm has solved it's sales and mismatch problem, it expects that it can sell what it produces such that $E[\phi] = 1$. Then, the firm's profit (4) can be described as

$$E[\Pi_{Qi}] = Q_i - N_i p_x x_i - w' L^*_{\phi i} - w' L_{Qi},$$

and we can derive the first order conditions. Defining $w'(t)$ as the wage potentially rewarded to producing labor, the first-order conditions for efficient use of labor in production gives

$$w'(t) = (1-\alpha)\frac{Q_i(t)}{L_{Qi}}. \tag{7}$$

[5] For a brief discussion see appendix A.1.
[6] For a brief discussion see appendix A.1.

Demand for intermediate goods is determined by marginal productivity. Prices are set by innovative firms as they offer a specific monopoly-like unique good. Using the f.o.c., we can derive the demand for each intermediate input,[7] namely

$$x_i(t) = \left(\frac{\alpha}{p_x(t)}\right)^{\frac{1}{1-\alpha}} L_{Qi}. \tag{8}$$

2.2 Intermediate-goods-producing firms

Market entry of new monopolistic firms: The intermediate goods firm is a monopoly because it sells a unique innovative product. c_x is the cost to produce one unit of x_i in units of the final output. Profits of an intermediate goods firm are $\pi_x = (p_x - c_x)\,x$. Being a monopoly, intermediate goods firms consider the demand function (8), and plugging in $p_x = \alpha L_Q^{1-\alpha} x^{-(1-\alpha)}$ we arrive at

$$\pi_x(t) = \alpha L_Q^{1-\alpha} x(t)^{-(1-\alpha)}\, x(t) - c_x x(t). \tag{9}$$

From the first-order conditions of the profit-maximizing intermediate goods firms, and using (8), we obtain the optimal price policy and determine[8] the production of $x(t)$,

$$p_x = \frac{c_x}{\alpha}, \tag{10}$$

$$x(t) = \left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} L_Q. \tag{11}$$

With (11) and (10), we eventually arrive at the maximum profit $\pi_x(t)$,

$$\pi_x(t) = \left(\frac{1}{\alpha} - 1\right)(c_x)^{\frac{-\alpha}{1-\alpha}}\, \alpha^{\frac{2}{1-\alpha}} L_Q^{*}. \tag{12}$$

As (10) gives us the flow of profits and the intermediate goods firm has an infinite lifetime, the present value of this profit flow - discounted with the interest rate $r(t)$ - is

$$V_x(t) = \frac{1}{r(t)}\pi_x(t) = \int_t^{\infty} \pi_x(t)\, e^{-r(v,t)(v-t)} dv. \tag{13}$$

Up to this point the model is close to the standard textbook versions of variety models as presented, e.g., by Aghion & Howitt (2008, ch. 3). Now we turn to the second major economic difference in this model. While $\frac{1}{r}\pi_x$ is the present value of profits per innovation, $\frac{1}{r}\pi_x \dot{N}$ is the total profit of the start-up when introducing $\dot{N}(t)$ new goods. Launching new goods on the market requires investment. Therefore, we assume that an investment ν is needed for the market entry of each new good. Thus, the total entry costs of the startup with innovation rate $\dot{N}$ is $\dot{N}\nu$, and these entry costs determine investments $I_x = \dot{N}\nu$. Due

[7] For calculations see appendix A.1.
[8] For calculations see A.2.

to competition, the net rent of a new firm turns to zero and the net present value of a new firm just about covers the total startup costs

$$\frac{1}{r(t)}\pi_x(t)\dot{N}(t) - I_x(t) = 0.$$

Further, plugging in $\dot{N}\nu = I_x$ we arrive at the well-known result that the return on an investment with infinite lifetime is profit flow over investment costs[9]

$$r(t) = \frac{\pi_x(t)}{\nu}. \tag{14}$$

Market entry of new technologies and goods: In this model, we do not intend to endogenize original technical change. There is a long-standing debate about this issue, A recent broad empirical finding in this respect is Bloom et al. (2017). Instead, we want to focus on *market entry* of new technologies. $\dot{A}(t) = \frac{dA(t)}{dt}$ is the exogenous number of new technologies invented at t which we refer to as innovative products. These innovative products are launched on the market by entrepreneurs and startups; however, they are not automatically successful in the market. According to the literature on startups and entrepreneurship, in general between 19 and 22 percent of firms exit the market in their first year of existence, and fail to bring an idea to the market. In this approach, we model the process of market entry as an *aggregate matching* process.[10] New, innovative goods $\dot{A}(t)$ are offered on the market and startup firms try to find buyers for their innovative products. The final goods sector provides opportunities for successful startups. Specifically, we assume that the number of new products successfully entering the market $\dot{N}$ is a function of two elements, (i) the exogenous number of new, innovative products $\dot{A}(t)$ potentially ready for market entry, and (ii) the opportunities for market entry that entrepreneurs discover. These opportunities are determined by the capacity of a market to absorb new intermediate products. This absorption capacity is indicated by the total effective demand for intermediate goods in the economy $X^D(t) = N(t)\,x(t)$. A large and booming economy with large effective demand offers more opportunities to what are sometimes referred to as "opportunity entrepreneurs" to launch new products and technologies.[11]

[9] Note: formally $\dot{N}\nu = I_x$ looks rather similar to the expression $\dot{N} = \frac{1}{\nu}I_x$ in the Romer-like reference model. However, the economic interpretation is reversed. Here, the number of new technologies in the economy is (semi-)exogenous and determined in a market matching process which we describe in the next section. Thus, causality runs from new goods entering the market, and this process leads to respective start-up investments ν. That is, when entering the market, the "market entry process leads to investments ($\dot{N}\nu \rightarrow I_x$)". In the Romer model, the interpretation of this relation is "investments generate innovation ($\dot{N} \leftarrow \frac{1}{\nu}I_x$)" and "$\frac{1}{\nu}$" is the linear spending efficiency of the R&D sector. However, both interpretations lead to the same result for returns on investment of the innovative firm, as given in (14).

[10] For a micro-foundation of this process, see Gries & Naudé (2011) and Gries et al. (2016).

[11] In a recent working paper, Fairlie & Fossen (2018) summarize these ideas and contrast the activities of "opportunity entrepreneurs" with those of "necessity entrepreneurs" who are forced to enter the market during economic downturns as a means of survival.

These two elements are combined in the aggregate matching function, $\dot{N} = f(\dot{A}, X^D)$. The number of surviving products equals the new technologies that match the requirements of the market. They add to the implemented products or technologies of the economy. For simplicity, we assume a matching technology with constant economies to scale and obtain for the number of new products in the market

$$\dot{N}(t) = \left(X^D(t)\right)^\gamma (\dot{A}(t))^{1-\gamma}, \tag{15}$$

where γ describes the contribution of market opportunities to this process. Although this assumption of a macro matching process is simplifying, it describes the main idea behind the mechanism.

Equation (15) makes clear that implementing technical change is a semi-endogenous process. The plain number of new technologies available $\dot{A}$ is exogenously given and not explained, but the number of new technologies that are indeed implemented in the economy $\dot{N}$ is endogenous. Thus, we refer to this as a semi-endogenous process of technical progress implemented in the economy.

2.3 Aggregate production, income and budget constraints

Labor income: In order to determine labor allocation in each representative firm and in the total economy, as well as wages, we need to include total labor market conditions. In this economy, total labor L is allocated to two occupations, production L_Q and sales promotion L_ϕ. As $L^*_{\phi i}$ is already determined for the representative firm by condition (5) and (6), all other labor is left for production, and obtain

$$L_Q = L - L_\phi. \tag{16}$$

From (7) we know that labor in production can be rewarded with wage rate $w'(t)$ according to it's marginal productivity. However, firms must not only pay labor used in physical production L_Q. Labor used in sales promotion L_ϕ also needs to be rewarded. As wages are paid in physical output goods at total amount $(1-\alpha)Q$, all labor has to be paid out of this physical labor productivity. Further, with homogeneous labor in a perfectly integrated labor market, only one wage rate $w(t)$ is paid to labor - no matter of its occupation. Further, as total wage income is $w(t)(L_Q + L_\phi)$, and this can be no more than the contribution of labor to the effective physical production $w'(t) L_Q$ (see 7) we obtain an average wage $w(t)$ paid to all workers no matter what they do, and total labor income is[12]

$$w(t) L = w' L_Q = (1-\alpha)Q. \tag{17}$$

[12] $w' L_Q = w\left(L_Q + L_\Phi\right) \Leftrightarrow w = w' \frac{L_Q}{(L_Q+L_\Phi)} = (1-\alpha)\frac{Q}{L_Q}\frac{L_Q}{(L_Q+L_\Phi)} = (1-\alpha)\frac{Q}{L}$. Note that Q depends on L_Q.

Wealth holder income: Further, $N(t)\nu$ is the value of all debt ever issued as all new products are always financed by newly issued financial assets, $\dot{N}(t)\nu = \dot{F}(t)$. Thus, with $N(t)\nu = F(t)$ we see that all profits are channeled to financial investors. Financial investors have bankrolled the process and now obtain a return on their investment

$$N(t)\pi_x(t) = r(t)F(t). \tag{18}$$

Production and income constraints: Effective output of the representative firm and of the total economy is generated by the two factors of production, thus the budget constraint for total production is

$$Q(t) = N(t)p_x(t)x(t) + w(t)L. \tag{19}$$

However, effective output is not GDP or income. As x is produced by using c_x units of final goods, net final output, and thus *GDP* or *income* is

$$Y(t) = Q(t) - N(t)x(t)c_x. \tag{20}$$

Further, from (19) and (20) we obtain $Q - Nxc_x = Np_xx - Nxc_x + wL$. With the definition of profits in the intermediate goods sector (9), the income constraint turns into

$$Y(t) = N(t)\pi_x(t) + w(t)L. \tag{21}$$

This constraint is already very familiar as it states that total income consists of profits and labor income. Further, equation (14) states that the return on an investment with an infinite lifetime is profit over investment costs $r = \frac{\pi_x}{\nu}$, and thus, $Y = rN\nu + wL$.

Using (18), we obtain the familiar income decomposition of the GDP

$$Y(t) = r(t)F(t) + w(t)L. \tag{22}$$

Income generated by innovative intermediate firms eventually generates a return for the financial asset owners. The growth process is implicitly a process of financial wealth accumulation through financing new products.

Income share of labor: As this approach explicitly refers to distributional aspects, we derive the income shares of two groups, labor income and income of financial investors. In appendix A.3 we show that the labor share of income can be derived by using (17) and (20) and is

$$\frac{w(t)L}{Y(t)} = \frac{1}{1+\alpha} < 1. \tag{23}$$

Income share of financial wealth: The profit share can also be explicitly determined using (20), (18) and (9). In the appendix A.3 we show how to arrive at

$$\frac{N_i(t)\pi_x(t)}{Y(t)} = \frac{\alpha}{1+\alpha} < 1. \tag{24}$$

2.4 Aggregate expenditure and income

The last aspect in which this model deviates from the standard endogenous growth approach is the assumption that we look at the consumption and savings behavior of two different groups of individuals. This is because we currently observe a polarization in income distribution in many countries[13] and want to examine the growth effects of this phenomenon. Therefore, a *representative* intertemporal choice with the help of a *representative* household's "Euler equation"[14] is not an adequate approach to this problem. It assumes away the effects of different intertemporal decisions of rich and poor households on aggregate consumption and savings. Thus the "*representative* household assumption" would not allow us to properly study effects of income polarization on growth. Thus, we suggest another approach. From income decomposition (22) we identify a group that earns wage income wL, and another group that earns income from financial assets rF. Each group exhibits its own expenditure pattern.

Consumption expenditure and labor income:
According to (23) the share of labor income is $\frac{wL}{Y} = \frac{1}{1+\alpha}$. If we define group specific intertemporal choice models and assume plausible group specific parameters for the choice problem, we can obtain that total wage income is fully consumed and that labor income turns into the only source of consumption expenditure in the economy. Total consumption in an economy with perfect matching in the final goods sector would be $C(t) = cY(t)$, with $c = \frac{1}{1+\alpha}$. While this is a traditional assumption in Keynesian growth models (inter alia by Dutt, 1984, Kaldor & Mirrlees, 1962, and Kalecki, 1968), we show in appendix A.4 that once we move away from the representative household approach, it is not difficult to motivate this assumption by group-specific optimal intertemporal choices. The important assumption is that groups are different and have different expenditure behavior.

Further, in an economy with non-perfect matching, consumers also devote income to the search and matching process whenever their desired consumption cannot find an output that could match consumption demand. However, searching for the desired goods leads to the experience that using fraction θ_j of their income for the search and matching procedure reduces the mismatch.[15] There-

[13] Examples include Autor et al. (2003, 2006, 2008) or Autor & Dorn (2013); for Germany, Spitz-Oener (2006), Dustmann et al. (2009); for the UK, Goos et al. (2014) and Van Reenen (2011) for various advanced countries.

[14] $\frac{\dot{C}}{C} = \frac{r_D - \rho}{\eta_U}$ with ρ denoting the representative agent's time preference rate and η_U the intertemporal elasticity of substitution. In appendix A.4 we give examples for specific intertemporal choices at individual or group level. Further, if group preferences are diverse, they may lead to diverse consumption and savings behavior which is consistent with the suggestions in our model. Thus, we do not intend to assume away the idea of rational intertemporal choices. However, what we do assume away is the idea of a simple aggregation rule like a *representative* household.

[15] In section 3.2 when we introduce the aggregate *match-improvement function* (35) we will see, how θ affects the matching process.

fore, we can aggregate to describe the effective aggregate consumption

$$C(t) = c(1-\theta)Y(t) + \varepsilon, \quad E[\varepsilon] = 0, \tag{25}$$

with $c = \frac{1}{1+\alpha}$ as being the economy's marginal and average rate of consumption. ε denotes a randomness in consumption demand with an expected value $E[\varepsilon] = 0$.

Investments and market entry of new goods:
In this model, investments are determined by innovations and an endogenous market entry of new technologies. Bringing a new technology to the market requires investments. We assume that these investments are identical for each innovation and thus total start-up investments $I(t)$ are

$$I(t) = \nu \dot{N}(t). \tag{26}$$

To simplify, we do not assume a matching problem for investors. Further, note that the idea that technological change relates to investments also links up with the Keynesian traditions and started with Kaldor (1957).

Effective demand for GDP (Keynesian income-expenditure equilibrium)
By the definition of national accounting, effective income Y can be used for effective consumption C and investment I. Thus, effective demand is defined by $Y \equiv C + I$. While the consumption rate (determined by 25) is a constant fraction of total effective income, investments are driven only by the endogenous market entry of new goods $\dot{N}$. With a given consumption rate the Keynesian income-expenditure mechanism can be applied to determine effective spending and thus effective total demand Y^D. Therefore, in income-expenditure equilibrium, aggregate effective demand equals effective income

$$Y(t) \overset{!}{=} Y^D(t) \equiv C(t) + I(t), \tag{27}$$

and we obtain the well-known Keynesian income-expenditure multiplier for the effective expected demand in aggregate goods market

$$Y^D(t) = \frac{I(t) + \varepsilon(t)}{1 - c(1-\theta)}. \tag{28}$$

Effective demand for total production Q
The demand for total output Q is the demand for GDP plus the demand in terms of input goods taken from final goods sector $N(t)x(t)c_x$. From the Keynesian income-expenditure mechanism (expected 28) we know that effective aggregate demand for GDP is $\frac{\nu\dot{N}+\varepsilon}{1-c(1-\theta)}$. Adding $N(t)x(t)c_x$ gives the aggregate demand for total production Q, namely

$$Q^D = \frac{\nu\dot{N}(t) + \varepsilon(t)}{1 - c(1-\theta)} + N(t)x(t)c_x.$$

As a result, demand is an endogenous value in which investment expenditures are independent of savings decisions of households. Further, to determine the expected demand ratio under current demand conditions, we need to divide by $Q(t)$. As a result, the aggregate effective demand ratio $\lambda(t)$ describes the ratio of effective aggregate demand to current output

$$\lambda(t) = \frac{Q^D(t)}{Q(t)}, \tag{29}$$

and in expected values we obtain the *ratio of expected effective aggregate demand* [16]

$$E\left[\lambda(t)\right] = \frac{\nu}{1 - c\left(1-\theta\right)} \frac{1}{L_Q^* \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}} g_N + \alpha^2. \tag{30}$$

3 Solving the model

In this section we move from the perspective of individual firms and consumers to that of an omniscient observer of the mechanisms. We will see that the perceived sales problem is driven by an aggregate market mismatch, and that the mismatch provokes a reallocation of resources. This reallocation can neutralize the mismatch. However, it also reduces resources in production and decreases output. As the mismatch is closely related to the demand side, the demand side plays a major role and restricts the current output and growth rate. The growth path is demand-restricted. Further, we also show that this growth path is stationary in the sense that there is no individual incentive to change behavior, such that the economy will not automatically return to the path of potential growth.

3.1 Solving for technology growth

As the first step towards solving the model, we determine the semi-endogenous growth rate of new products successfully entering and staying in the market $g_N = \dot{N}/N$. This growth rate is the result of market opportunities. Equation (15) describes the aggregate matching process for new technologies successfully entering the market. Thus, the growth rate of implemented technologies depends on effective demand for intermediate goods and thus depends on labor in effective production L_Q, and is[17]

$$g_N = \frac{\dot{N}(t)}{N(t)} = \left(\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} L_Q\right)^{\gamma} (g_A)^{1-\gamma}. \tag{31}$$

We call this process semi-endogenous, as the rate g_N is basically driven by the exogenous g_A. However, to what extent the exogenous innovative process g_A becomes usable and implemented in the economy is endogenous.

[16] $\frac{E[Q^D]}{Q} = \frac{\nu}{1-c(1-\theta)} \frac{\dot{N}(t)}{Q(t)} + \alpha^2$ and using (1) and (11) we obtain (30).

[17] For calculations see appendix A.5.

3.2 From perceived individual frictions to aggregate market mismatch

To discuss individual frictions in the final goods market we need to refer back to the discussion of section 2.1. In section 2.1 we looked at activities from the perspective of single firms i. Firms face an individual market mismatch δ_i which they perceive as their individual market problem. They use labor $L_{\phi i}$ for the placement of goods and for finding customers and mitigate their individual sales problems accordingly. However, when consumers j search for their desired consumption goods they also observe a market mismatch and use a fraction $\theta_j(t)$ of their potential expenditure to find the appropriate goods.
In this section, we suggest that firms' sales and customers' purchase problems are not purely due to individual (idiosyncratic) problems in the markets. They are in fact also a result of aggregate market conditions, even if individual decision-makers are not aware of it.
However, being the omniscient observer of this economy we can now ask: What is the source of the sales and matching problem? Effective sales are impeded by a mismatch $\delta_i(t)$ [see (2), $\phi_i(t) = 1 - \delta_i(t)$] which is caused by stochastic market friction $\delta_i'(t)$. So, what exactly determines the mismatch, and what is behind the perceived stochastic market frictions $\delta_i'(t)$?

Friction: We assume that the stochastic friction $\delta_i'(t)$ has two components, (i) aggregate market conditions and (ii) an idiosyncratic component for each individual firm.
(i) The first component, the aggregate component, is a *current shortage of stochastic aggregate demand* $\delta^D(t)$. It is the difference between total supply and effective aggregate demand $Q^D(t)$

$$\delta^D(t) = \frac{Q(t) - Q^D(t)}{Q(t)} = 1 - \lambda(t). \tag{32}$$

(ii) Second, and in addition, while $\delta^D(t)$ indicates the aggregate market component without idiosyncratic elements, we need to identify and account for this individual component. These sales problems are indeed firm i -specific or idiosyncratic obstacles in each firm's market and are denoted by the random variable ε_{Fi}, with $1 > E[\varepsilon_{Fi}] > 0$. For given aggregate market conditions $\delta^D(t)$, ε_{Fi} is the component of total mismatch that is caused by individual firm conditions. Therefore, the total individual friction perceived by each firm i combines the market and idiosyncratic component and can be described as

$$\delta_i'(t) = \delta^D(t)\,\varepsilon_{Fi}. \tag{33}$$

However, this insight into the decomposition of the mismatch is not available either to individual firms or to customers. An individual firm only perceives a sales ratio $E[\phi_i(t)] = 1 - E[\delta_i(t)]$, interpreting it as being due to an exogenous friction $\delta_i'(t)$ which can be countered by allocating more labor towards the matching process $[\frac{\partial E[\phi_i(t)]}{\partial E[\delta_i']} < 0, \quad \frac{\partial E[\phi_i(t)]}{\partial L_{\phi i}(t)} > 0$ see (3) section 2.1].

Next, we have to aggregate to link up these individual activities with total and current market conditions to determine aggregate market equilibrium. If we now assume that ε_{Fi} are i.i.d. for $i \in \mathcal{I}$, we can aggregate $(\varepsilon_{Fi} = \varepsilon_F,)$ and obtain as representative friction $\delta'(t)$ which in expected values is

$$E\left[\delta'(t)\right] = (1 - E[\lambda])\,E[\varepsilon_F] - cov(\varepsilon, \varepsilon_F), \quad \text{with} \quad cov(\varepsilon, \varepsilon_F) < 0. \tag{34}$$

We assume that $cov(\varepsilon, \varepsilon_F)$ is negative because a random increase of ε describes a random increase in aggregate demand that reduces aggregate market tightness. This increase in aggregate demand (and reduction of market tightness) is accompanied by a reduction in the firm's idiosyncratic difficulty of finding a customer ε_F. Further, we assume that $-cov(\varepsilon, \varepsilon_F)$ is sufficiently large, such that $E\left[\delta'(t)\right]$ is allways positive.

Aggregate mismatch and matching equilibrium: Now we know about the individual mismatch and its decomposition. We have revealed the full mechanism that leads to the mismatch. Yet, we have not introduced how counter measures by firms and customers affect the mismatch. To this end we define the aggregate *match-improvement function* $m(t)$ for the aggregate market. We assume that matching of the two market sides is determined by the firms' allocation of labor to combat mismatch L_ϕ and - according to the discussion in section 2.4 - fraction θ of consumers' income to find the desired consumption good

$$m = L_\phi (1 - \theta)^{-1}, \quad \text{with } \frac{dm}{dL_\phi} > 0, \quad \frac{dm}{d\theta} > 0. \tag{35}$$

Thus, the *rate of expected effective aggregate mismatch* - after implementing counter-measures - is

$$E[\delta(t)] = E\left[\delta'(t)\right] - m. \tag{36}$$

When the mismatch is completely eliminated, such that the aggregate expected mismatch becomes zero, we obtain a *perfect match*

$$E[\delta(t)] = 0. \tag{37}$$

Further, and most importantly, equation (37) implies that firms are in sales equlibrium, and the aggregate market is in equilibrium too. Specifically, (i) firms are in sales equilibrium as the e*xpected effective sales ratio* turns to one,

$$E[\phi(t)] = 1 - E[\delta(t)] = 1. \tag{38}$$

(ii) for the aggregate market we know that effective sales require the respective effective demand, $E[\Phi] = E\left[Q^D\right]$. As the effecive sales ratio is $1 - E[\delta(t)]$, we obtain that the mismatch $E[\delta(t)]$ also describes the gap betweeen effective demand and production $1 - E[\delta(t)] = E[\lambda(t)] = \frac{E\left[Q^D(t)\right]}{Q(t)}$. Thus, whenever

(37) holds and the mismatch is eliminated, the aggregate goods market is also in equlibrium

$$E[\lambda(t)] = 1. \quad (39)$$

3.3 The aggregate model in two equations

After the steps taken in section 3.1 and section 3.2 we can now summarize the model and reduce the system to two simultaneous equations, namely (38a) and (39a).

Firms' sales and matching equilibrium: From the discussion in section 2.1 and equation (5) we know that a firm will allocate labor to the market placement process until all output is sold. Hence, when all mismatch is eliminated (37) firm's sales and matching equilibrium (38) is reached. Using the labor market constraint (16) we obtain

$$L_Q = cov(\varepsilon, \varepsilon_F)(1-\theta) + L. \quad (38a)$$

$$\begin{aligned} L_\phi (1-\theta)^{-1} &= -cov(\varepsilon, \varepsilon_F) \\ -L_Q &= -cov(\varepsilon, \varepsilon_F)(1-\theta) - L \end{aligned}$$

Note, that in this equilibrium, the match improving activities always and only neutralize the "natural friction" $-cov(\varepsilon, \varepsilon_F) = m$ which is generated by the (negative) covariance of aggregate market and idiosyncratic elements.

Aggregate market equilibrium: In section 2.4 we determined aggregate demand and the aggregate effective demand ratio $\lambda(t)$ (see 29). Perfect matching (37) also implies aggregate goods market equilibrium, as expected demand equals production, and thus, the expected effective demand ratio turns to one. Further, plugging (31) in (30) gives [18]

$$L_Q = \left(\frac{\nu}{(1-\alpha^2)(1-c(1-\theta))}\right)^{\frac{1}{(1-\gamma)}} \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{(1-\alpha)(1-\gamma)}} g_A. \quad (39a)$$

Thus, we are left with two equations (38a) and (39a) to solve for the two endogenous variables, namely labor used in production L_Q and consumers' spending on search and matching θ.

[18] For details see appendix A.5..

3.4 Current market equilibrium

We can now solve for equilibrium. Plugging in[19] we are left with only equation (40) and one variable, L_Q

$$0 = F = L_Q^{(1-\gamma)} - \frac{\nu}{(1-\alpha^2)\left(1 - c\frac{L-L_Q}{-cov(\varepsilon,\varepsilon_F)}\right)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma}. \tag{40}$$

As we cannot explicitly solve for L_Q we apply the *Implicit Function Theorem* to determine the equilibrium $\tilde{L}_Q$, and other interesting variables.

Proposition 1 ***Current market equilibrium:*** *Equation (40) implicitly defines a function for*
(i) the equilibrium value of $\tilde{L}_Q$

$$\tilde{L}_Q = L_Q\left(\nu, \alpha, g_A, c_x, cov\left(\varepsilon, \varepsilon_F\right)\right). \tag{41}$$

Further, using equation (41) leads to current market equilibrium values for
(ii) consumers' fraction of income used for improving the matching process

$$\tilde{\theta} = 1 - \frac{L - \tilde{L}_Q}{-cov\left(\varepsilon, \varepsilon_F\right)}, \quad \text{with} \quad cov\left(\varepsilon, \varepsilon_F\right) < 0 \tag{42}$$

(iii) total production of the final good

$$\tilde{Q}(t) = N(t)\,\tilde{L}_Q\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}, \tag{43}$$

(iv) total income and hence the level of the growth path

$$\tilde{Y}(t) = N(t)\left(1-\alpha^2\right)\tilde{L}_Q\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}, \tag{44}$$

(v) the growth rate of income (GDP) gives

$$\tilde{g}_Y = \frac{\dot{Y}(t)}{Y(t)} = \tilde{g}_N = \frac{\dot{N}(t)}{N(t)} = \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma}{1-\alpha}}\left(\tilde{L}_Q\right)^{\gamma}(g_A)^{1-\gamma}, \tag{45}$$

and (vi) the real rate of return on financial investment

$$\tilde{r} = \tilde{g}_Y. \tag{46}$$

For a proof, see appendix A.6.

[19] For details see appendix A.5

With $\tilde{L}_Q$ and $\tilde{L}_\phi$ we have determined a current market equilibrium at a level below potential output $\tilde{L}_Q < L$. Further, as $\tilde{L}_Q$ depends on demand side parameters, e.g. ν, the level of the income path is restricted by the demand side. This result, a demand-restricted stationary growth path below the level of the potential growth is the major difference to mainstream neoclassical or endogenous growth mechanics. Under the perfect market conditions of the neoclassical or endogenous growth world, this could not happen. So why and how can this happen here? Broadly speaking, there are two reasons. First, firms observe excess supply and customers observe excess demand which they perceive as market mismatch (due to market frictions) and which incentivizes firms and customers to act. In response to this mismatch between supply and demand, both can allocate resources to reduce perceived frictions and improve the match between demand and supply. In consequence, labor potentially available for production is allocated to improve the matching process, and expenditure potential usable for consumption demand is spent on search. This reallocation of resources leaves the economy below the potential production level.

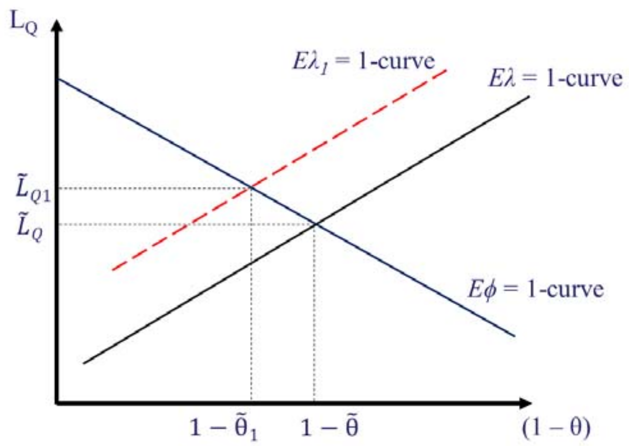

Figure 2: Equilibrium allocation of matching relevant resources

Further, it is interesting to note that in this kind of economy the return on investment is equal to the growth rate. While (46) is a result that we also find in other mainstream modeling, the interpretation of causality is different. In this model g_N is the driver of r. As more products enter the market, profits improve and return on investments increase. In endogenous growth theory, r is the result of an intertemporal choice and drives the growth rate as well as the savings rate.

An even simpler way to determine the equilibrium of this growth model and its determinants is illustrated in figure 2. The two equilibrium conditions (38a) and

(39a) are drawn in figure 2. Both conditions can be described by an equilibrium relation of L_Q and $1-\theta$. The $E\phi = 1$-*curve* is downward sloping and the $E\lambda = 1$-*curve* is upward sloping with an intersection at the equilibrium point $[\tilde{L}_Q, 1-\tilde{\theta}]$[20].

3.5 Stationarity of Equilibrium

While section 3.4 describes market equilibrium for each period, two important questions remain. 1) How can the equilibrium output steadily remain below potential output and represent a long-term stationary equilibrium? 2) How can aggregate demand become central, and determine the stationary level as well as the speed of the growth path?
While the answer to these two questions is given in the next two subsections, we first need to discuss the following question. Why do we need to ask these two questions? Mainstream dynamic macroeconomics is based on the idea that the path of potential output is the only relevant long-term growth path. This means, that after some temporary deviation from this path the economy normally returns to it and continues to grow in the manner that the supply side driven growth model describes. No permanent deviation is considered. Therefore, unlike this standard notion we suggest that the demand-restricted equilibrium we just derived can be maintained and become a permanent, stationary process. The economy does not necessarily return to the path of potential growth.

Concept of stationary no-expectation-error equilibrium

Related to stochastic modeling, we suggest a different concept for a stationary equilibrium. We take the perspective of individual decision-makers and describe stationary behavior. According to this concept a change in behavior is the result of inconsistent experiences. If an individual plans and organizes a specific outcome - according to their subjective expectations in a stochastic environment -, and their plan and outcome do not coincide with observed expected values, we refer to this difference as "expectation-error". As consequence the individual tries to learn from this error, meaning they the individual change their behavior and adjust their plans. Individual behavior becomes stationary if the planned and realized outcome is indeed the expected outcome. This condition defines a behavioral equilibrium such that it implies no (need) for a change in behavior. Thus, we refer to this condition as the "no-expectation-error equilibrium" (n-e-ee).
In this approach, the general concept of a no-expectation-error equilibrium can be illutrated by looking at the matching procedure. The mismatch $E[\delta]$ defines the gap between planned production $Q_i(t)$ and the mean of effective sales $E[\Phi_i(t)] = (1 - E[\delta(t)]) Q_i(t)$. Thus, as long as firms and customers do not allocate sufficient resources to counter the mismatch they cannot expect the mismatch to disappear, and $E[\delta] = E[\delta'] - m(L_\phi, \theta) > 0$. Thus, individuals face an expectation error as their actions do not coincide with the expected

[20] For a brief discussion see appendix A.6.

values. In other words, there is an error in their planning as their subjective expectations are false. Thus, they continue to adjust their plans until they correctly expect the mismatch and plan their counter activities, such that the expected mismatch is on average fully eliminated $0 = E[\delta] = E\left[\delta'\right] - m(L_\phi, \theta)$. As result, there is no expectation error with respect to the final goods matching mechanism. As a consequence, firms are in sales equilibrium ($E[\phi] = 1$), and we also obtain equilibrium in the aggregate goods market $E[\lambda(t)] = 1$.

Definition 2 ***No-expectation-error equilibrium:*** *(i) Firms and customers are in "no-expectation-error equilibrium" (n-e-ee) if the expected mismatch is correctly predicted, such that match-impvoring measures fully eliminate the expected mismatch*

$$E\left[\delta'\right] = m(L_\phi, \theta). \tag{47}$$

(ii) As expectations, plans and realizations coincide, firms and customers behave in a stationary manner (no change in behavior), such that firms remain in sales equilibrium, and the aggregate market equilibrium sustains,

$$\begin{aligned} E[\phi] &= 1, \text{ and} \\ E[\lambda] &= 1. \end{aligned}$$

Using definition 2 we see that the equilibrium which is determined in proposition 1 is indeed a stationary equilibrium. Thus, we can state the following proposition.

Proposition 3 ***Steady state equilibrium:*** *The current market equilibrium derived in proposition 1 is a no-expectation-error equilibrium, and thus, a stationary equilibrium.*[21]

In equilibrium all actions are adjusted towards the expected values. All planning is on average consistent with effective real market conditions. If all firms obtain what they expect, there is no need for further action and the economy remains stationary in this position. What is the difference to a rational expectation equilibrium? Conceptually, the ideas are indeed close. However, a rational expectation equilibrium is normally associated with well-functioning markets and a unique market equilibrium at the level of potential output. This is different in this approach. No-expectation-error equilibrium is a behavioral rule. When I obtain what I expect and optimally plan with individually available information, I do not change my behavior, no matter if the market clears at the level of potential output or any other level below. In fact the level of potential out put is even unknown for individuals. Thus, under this interpretation the no-expectation-error equilibrium can exist anywhere below potential market output.

[21] Proof: As (41) and (42) in proposition 1 satisfies condition (37), resepctively (38) and (39), conditions of definition 2 are satsfied. Thus, firms and cutomers are in a no-expectation-error equilibrium and exhibit stationary behavior.

Interpretation of individual firms' adjustment and steady state behavior: For each firm, perceived stochastic market mismatch δ_i reduces the firm's *sales ratio* $\phi_i(t)$ in the market (see 3). Thus, each firm reallocates labor towards sales activities ($L^*_{\phi i}$) to counter the mismatch and increase the expected share of sales $E[\phi_i]$. This reallocation from final goods supply to reducing market mismatch continues to improve the expected effective sales ratio as long as the firm still expects an oversupply of its final goods in the final goods market. At the same time – due to these activities – the expectation error declines, so that in no-expectation-error equilibrium firm i's effective supply of final goods equals the expected effective sales, and thus ratio $E[\phi_i] = 1$ (Definition 2ii). Since δ_i is the firm's perceived mismatch, $L^*_{\phi i}$ is the best individual response to this stochastic mismatch.

Interpretation of aggregate markets and coordinated market behavior: While – from the perspective of each individual firm i – frictions δ_i' are regarded exogenous and random. They lead to the mismatch δ_i. Equation (34) reveals that individually perceived frictions are driven by idiosyncratic and aggregate market sources. First, we assume symmetrical firms with the same idiosyncratic stochastic variations (ε_{Fi} are i.i.d., $E[\varepsilon_{Fi}] = E[\varepsilon_F]$). Further, customers also perceive the market mismatch. Consequently customers (on the demand side) try to counter the mismatch and allocate resources to improve the aggregate market match. Reallocating some income originally planned for consumption towards improving the search and matching process reduces not only effective demand, but also the mismatch. With this process both, $E[\phi(t)]$ and $E[\lambda(t)]$, move towards one. So in aggregate equilibrium not only does $E[\phi(t)]$ turn to one, the aggregate market also turns to equilibrium. The planning of both, firms and consumers becomes consistent. They do not consider changing their activities further. Thus, we regard this as behavioral equilibrium because plans and expected outcome do not generate an incentive to change behavior.

3.6 Role of the demand side and polarization

The simplest way to identify how the demand side affects this equilibrium is to look at the determinants of the equilibrium $\tilde{L}_Q$. Equation (41) suggests that ν is an exogenous determinant. Further, ν is the start-up expenditure which a new firm has to undertake to develop the new technology and enter the market. So far, this is interpreted as an investment expenditure to enter the market. However, it is not an investment in the traditional sense. In this model ν is not accumulating to a capital stock. Thus, the value of ν does not affect the supply side (production factors) but only the demand side. It could be the demand for innovation and market entry related services. Having identified ν as a demand-side element we can look at comparative static results for an increase in this demand-side element. Proposition 4 states the derivatives for the most interesting variables, namely labor in production ($\tilde{L}_Q$ see 41), GDP ($Y(t)$ see 44) and the growth rate (g_Y see 45).

Proposition 4 ***Demand side matters:*** *An increase in demand elements like investments expenditure ν generates an increase in both, the level and the growth rate of the income path. In particular (i) labor in production $\frac{d\tilde{L}_Q}{d\nu} > 0$, (ii) the level of the income path $\frac{d\tilde{Y}}{d\nu} > 0$, (iii) the growth rate of income $\frac{d\tilde{g}_Y}{d\nu} > 0$, and (iv) returns $\frac{d\tilde{r}}{d\nu} > 0$.*

For a proof see appendix A.6.

The demand side and income level From proposition 4 we know that an increase in ν as an element of demand expenditure changes the factor allocation and the steady state level of the income path.

What happened? In the original no-expectation-error equilibrium, all firms and consumers have no reason to change their behavior. On average, they expect what they obtain under the current aggregate market conditions. Neither would activities by individual firms or consumers change the expected values. Only coordinated action could do that. However, an exogenous increase in ν would be a change that affects aggregate market demand. Thus, the mean of aggregate demand moves up and firms and consumers need to adjust their expectations towards a higher average demand level. Since $\tilde{L}_Q < L$ and expansion of labor in production is possible, the effective production and income will increase. Graphically, this can be also seen in figure 2. As result of an increase in ν the $E\lambda_1$-*curve* shifts upwards and a new equilibrium is established at a higher level of $\tilde{L}_Q$ and θ. Consequently the demand side plays a major role. Showing that the demand side determines the current and stationary market equilibrium was the objective of this exercise.

The demand side and growth rate: While this mechanism describes the equilibrium level, there is also an effect on the growth rate. According to (45), GDP growth g_Y is (semi-)endogenously determined. First, there are new ideas or technologies. In this model the growth rate of these new ideas is exogenous. Second, the market entry of these ideas is endogenous. The market entry process is determined by an aggregate entrepreneurial market matching process. The better the market conditions, the easier it is for a new product to survive the first phase of market entry and the higher the rate of survival of new technologies. Not only the growth rate of inventions of potential technologies is crucial, market opportunities which are represented by aggregate demand are, too. As a result, in this hybrid long-term growth approach the demand side matters. Thus, the model neglects neither the demand side nor the supply side. Indeed, the supply side is the most important of the two and remains the growth-generating engine. However, even if growth is driven by the supply side and cannot be generated by the demand side, it can be sustainably restricted by the demand side. Further, it is also notable that interest rates are determined by the market entry rate of new innovative products. It is not the savings decision that determines the investments and the rate of return; rather, the rate of return is determined by innovations and investment opportunities. Savings will adjust. In this interpretation, a low innovation rate is responsible for the low interest rate and the low level of investments.

Income polarization For comparative static analysis, we consider a change in the elasticity of production α. It is well known that due to the properties of the production function, α affects income distribution and the labor share of income $\frac{d(wL/Y)}{d\alpha}$. In other words, income polarization can be considered as result of a shift in technology. What is new is that we look at both supply- and demand-side channels. The change in income distribution affects the allocation of labor $\frac{d\tilde{L}_\phi}{d\alpha}$, the level of the income path $\frac{d\tilde{Y}}{d\alpha}$, and the semi-endogenous growth rate $\frac{d\tilde{g}_N}{d\alpha}$. Specifically, with two distinct income groups and distinct consumption behavior, we can analyze the role of income polarization on the demand side and the entire growth process. With all labor income being consumed,[22] the aggregate consumption rate is $c = \frac{1}{1+\alpha}$. Thus, changes in technology that affect income distribution also affect the consumption rate and may affect growth. Further, the supply side is affected as there is a shift in each factor's contribution in production.
Why is this interesting? Turning to the discussion of "secular stagnation", as mentioned in the motivation of this paper and looking at Gordon's (2012) headwinds, we can easily analyze the effects of increasing inequality that Gordon mentions. According to recent studies, e.g., Karabarbounis & Neiman (2014) or Lawrence (2015), there has been a decline in the share of labor income over recent decades. In proposition 5, we state the implications of this stylized fact on the growth path, as an increase in α leads to a reduction of the income share of labor.

Proposition 5 ***Polarization matters**: Assuming $\frac{(1-\tilde{\theta})}{2} > \alpha > \gamma$, and c_x being sufficiently small, changes in technology ($d\alpha$) leading to a change of income distribution in favor of financial wealth holders*
(i) decreases labor used in production $\frac{d\tilde{L}_Q}{d\alpha} < 0$,
(ii) increases the level of the steady state income path $\frac{d\tilde{Y}}{d\alpha} > 0$, *and*
(iii) decreases the growth rate of the economy $\frac{d\tilde{g}_Y}{d\alpha} < 0$.

For a proof see appendix A.6.

Figure 3 illustrates the effects graphically. In figure 3 an increase of α, and thus a lower income share of labor affect the dynamics of the growth path via the demand and the supply side.
As all labor income is consumed, the consumption rate $c = \frac{1}{1+\alpha}$ is directly affected by the change of α. With $\tilde{Y}(0)$ the economy would start at level $\ln \tilde{Y}(0)$ and grow at rate $\tilde{g}_Y$. At $t = 1$ we consider a shock on α that affects the demand side by a decreasing share of labor income, leading to a decrease in consumption rate c. As result, the equilibrium share of labor in production declines. However,

[22] See the discussion in section 2.4 and in appendix A.4.

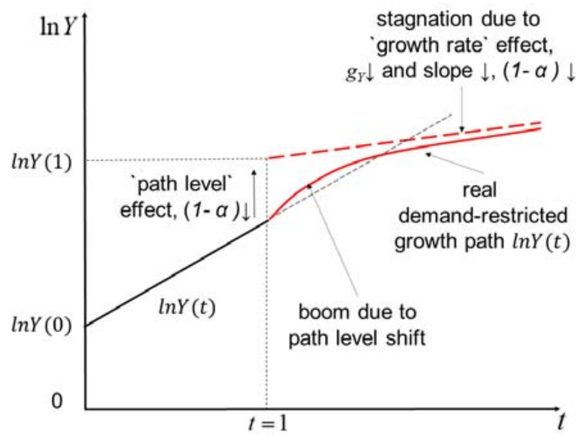

Figure 3: Path of an economy with a demand-restricted growth process, and a declining income share of labor.

even if labor in production declines, there is a second effect on potential output. The increase in α improves the supply side through intermediate inputs and leads to an increase in potential supply ($\frac{d\tilde{Y}}{d\alpha} > 0$) in proposition 5). The net effect is positive, as the supply-side effect dominates. This positive total effect shifts the income path upwards in figure 3.

Further, as result of a lower expected effective supply ratio the equilibrium semi-endogenous growth rate $\tilde{g}_Y$ declines ($\frac{d\tilde{g}_Y}{d\alpha} < 0$ in proposition 5). With $\tilde{g}_Y'$ the economy realizes a lower speed of growth. The systematic decline of opportunities in the market for new ideas and technologies reduces the number of newly implemented technologies.

The combination of both effects gives an interesting interpretation that is quite different to the standard narrative. The supply-side effect of an increase in production elasticity of intermediate goods may allow for higher production. However, the negative demand-side effect due to a decreasing share of consuming labor income affects the market entry of new goods negatively and thus reduces the growth rate. Therefore, we may obtain a supply side-induced boom at the beginning of the process and a demand side-driven depression of the growth rate which dominates the growth path in the long term and reduces growth opportunities.

Further, this decline in investment opportunities also reduces the profits of innovative firms and leads - along with the decline in growth - to a decline in real returns on investments (46). Growth, profits, real returns, and investments decline. These results are consistent with current observations.

4 Summary and conclusions

Mainstream growth theory is dominated by variations of neoclassical and endogenous growth models. In these approaches, growth is explained fully by elements of the supply side. In light of a long-term slowdown in growth – with the latest manifestation being the 2008 crisis and the following Great Recession – and a simultaneous income polarization starting in the 1990s, it is fair to ask why are we so sure that only the supply side determines growth. Instead of following a fully supply-side-driven neoclassical or endogenous growth approach, we suggest a hybrid approach that allows for growth restrictions induced by demand-side elements. We depart from an endogenous growth product-variety approach and modify the model. The four most important modifications are the following. (i) Firms face stochastic a mismatch in the market of their final goods, so that their effective sales ratio is below the potential. (ii) The second modification is the notion of "no-expectation-error equilibrium" as a concept for stationary equilibrium behavior. (iii) The third modification is an independent demand side that is not driven by inner solutions of representative intertemporal choices (Euler Equation) and Say's Law. Instead, the demand side is determined by investments in the market entry of new products and technologies. (iv) Finally, the growth rate is semi-endogenous, that is, technology growth is exogenous, but the implementation of resulting new products in the economy is determined by endogenous market opportunities.

As a result of these variations, changes in demand affect the expected effective income as well as profits and returns in the final and intermediate goods sector. The economy is not pinned down to a unique equilibrium path. When independent demand increases, excess demand in the markets for final products drives total production and final goods firms' input of intermediate goods. Further, intermediate goods profits increase and the return on financial investments in intermediate-good startups rises. The economy remains at this new higher level of activity as a new no-expectation-error equilibrium will be established. The new stationary equilibrium can be established, because firms and households have no reason to engage in further adjustments. Their expectations, behavior, and experiences are consistent. Furthermore, in this new equilibrium the growth rate increases because higher demand leads to more opportunities for market entry of new products, so the success rate in the market entry matching process improves. With more market entries the growth rate of implemented technologies increases. Potential growth is driven by technology growth and thus by the supply side, but the realized effective income and income growth is limited if demand is insufficient. Thus, the model neglects neither the demand side nor the supply side. Indeed, the supply side is the most important of the two and remains the growth-generating engine. However, even if growth is driven by the supply side and cannot be generated by the demand side, it can be sustainably restricted by the demand side.

To address income polarization, we consider a technology shock account of the labor share of income and in favor of financial wealth holders. Even if the resulting effects are not completely unambiguous, we may obtain an upward shift in the path level of income and a reduction in the long-term growth rate. In other words, we obtain a short-term supply side-driven boom and a long-term demand side-induced stagnation in growth.

The policy implications of such a growth process are clearly different than those of standard mainstream growth theory. According to standard theory, policies that encourage an increase in savings would make more resources available for growth (including R&D expenditure etc.). In our approach, the economy is bounded by investment opportunities. Policies that encourage higher demand instead of higher savings would improve these investment opportunities and thus lower the restrictions on the growth process. Thus, this hybrid model allows for both mechanisms and may bridge the gap between two hitherto distinct views: the view of mainstream growth focusing purely on the supply side, and the Keynesian view that is oriented purely towards the demand side.

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A Appendix:

A.1 Final-goods-producing firm

Allocation of labor to sales effort: As firms cannot sell more than their current output we restrict the function $E[\phi_i]$ to take values only in the interval $[0,1]$. Further, we assume for $0 < E[\phi_i] \leq 1$ that the marginal contribution of a unit $L_{\phi i}$ is always more than the marginal contribution of L_{Qi}, $\frac{\partial E[\phi_i]}{\partial L_{\phi i}} Q_i > (1-\alpha) E[\phi_i] \frac{Q_i}{L_{Qi}}$, such that $\frac{\partial E[\phi_i]}{\partial L_{\phi i}} \frac{L_{\phi i}}{E[\phi_i]} > (1-\alpha) \frac{L_{\phi i}}{L_{Qi}}$. Consequently, as long as $E[\phi_i] \leq 1$ employing labor to improve the sales ratio is always a higher priority than increasing employment in the production process L_{Qi}. This procedure implies that the firm will expand $L_{\phi i}$ up to $E[\phi_i] = 1$ is reached. Thus, for the employment $L_{\phi i}$ there is no inner solution, but we reach a corner solution at $E[\phi_i] = 1$.

Implicit Function Theorem for optimal $L_{\phi i}$ **:** Conditons for applying the Implicit function theorem hold: $0 = F = E\left[\phi_i\left(\delta_i, L_{\phi i}\right)\right] - 1$, and $\frac{dF}{dL_{\phi i}} = \frac{\partial E[\phi_i(t)]}{\partial L_{\phi i}(t)} > 0$. For the effect of $E\left[\delta_i\right]$ we use $\frac{dF}{dE[\delta_i]} = \frac{\partial E[\phi_i(t)]}{\partial E[\delta_i]} < 0$.

F.O.C.: F.O.C with respect to x_i is $\frac{d\Pi_i}{dx_i} = \alpha x_i^{\alpha-1} L_{Qi}^{*(1-\alpha)} - p_x = 0$, thus $p_x = \alpha L_{Qi}^{*(1-\alpha)} x_i^{-(1-\alpha)} \Leftrightarrow x_i^{1-\alpha} = \alpha \frac{1}{p_x} L_{Qi}^{*(1-\alpha)}$.

A.2 Intermediate-goods-producing firm

F.O.C. is $\frac{\partial \pi_x}{\partial x} = \alpha^2 L_Q^{*(1-\alpha)} x^{\alpha-1} - c_x = 0$, thus $c_x = \alpha^2 L_Q^{*(1-\alpha)} x^{\alpha-1} \Leftrightarrow x^{1-\alpha} = \left(c_x\right)^{-1} \alpha^2 L_Q^{*(1-\alpha)}$, and we obtain (11) .

A.3 Determining the labor and profit share of income

Income Y (GDP) and total production Q : Before we determine the income shares, we determine the simple relation between income Y (GDP) and total production Q. According to (20)

$$Y(t) = Q(t) - N(t)x(t)c_x = \left(1 - \frac{Nxc_x}{Q}\right) Q(t).$$

Applying (11) ($x\left(t\right) = \left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} L_Q^*$) gives

$$\frac{Nxc_x}{Q} = \frac{Nxc_x}{NL_Q^{1-\alpha} x^{\alpha}} = \frac{x^{1-\alpha} c_x}{L_Q^{1-\alpha}} = \frac{\left(\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} L_Q^*\right)^{1-\alpha} c_x}{L_Q^{1-\alpha}} = \left(\frac{\alpha^2}{c_x}\right)^{\frac{1-\alpha}{1-\alpha}} c_x = \alpha^2$$

and for $Y(t)$ we obtain

$$Y(t) = \left(1 - \alpha^2\right) Q(t) = \left(1 - \alpha^2\right) N(t) \alpha^{\frac{2\alpha}{1-\alpha}} \left(c_x\right)^{-\frac{\alpha}{1-\alpha}} L_Q. \tag{48}$$

Labor share of income:

Labor share of income is defined as $\frac{wL}{Y} = \frac{(1-\alpha)}{(1-\alpha^2)}$. The fact that $\left(1 - \alpha^2\right) = \left(1 - \alpha\right)\left(1 + \alpha\right)$ and applying (11) gives $\frac{wL}{Y} = \frac{1}{1+\alpha}$.

Profit share of income: Profit share of income is defined as $\frac{N_i \pi_x}{Y} = \frac{N\alpha(1-\theta_i)L^{1-\alpha}x^{\alpha} - Nc_x x.}{NL^{1-\alpha}x^{\alpha}(1-\theta_i) - Nxc_x} = \frac{\alpha - \frac{Nc_x x}{N(1-\theta_i)L^{1-\alpha}x^{\alpha}}}{1 - \frac{Nxc_x}{NL^{1-\alpha}x^{\alpha}(1-\theta_i)}} = \frac{\alpha - \alpha^2}{1-\alpha^2} = \frac{\alpha}{1+\alpha}$.

A.4 Intertemporal choices for labor and capital owners

In standard models, aggregate consumption expenditure and savings are determined by a representative household conducting an optimal intertemporal choice according to the Euler equation

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\eta_U}.$$

However, this assumption of a *representative household* is *rather restrictive.* Further, the implication of the aggregate Ramsey rule is a postive correlation (if not even causality) between interest rates and growth rates. Figure 4 indicates that there is no obvious emprical evidence for the existence of such correlation at least in the data for the US economy from 1961 to 2018.

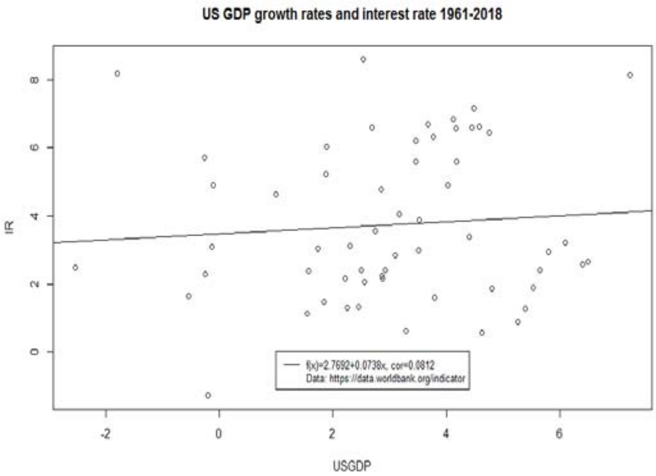

Figure 4: Correlation between interest rates and GDP growth rates for the US

Moreover, for modeling a polarized economy it makes sense to substitute this assumptions by proposing two groups of households differing with respect to their consumption and savings behavior. We also argue that our assumptions are not in contrast to individual optimal intertemporal choices. In fact, in this appendix we provide a simple example that indicates that individual intertemporal choices can be integrated in this kind of modeling. However, since the main focus of the paper is to describe mechanisms of demand-restricted growth in a polarized economy, we reduce complexity by skipping over an explicit modeling of intertemporal choice mechanisms in the main text. The following example, however, indicates how our simplifying assumptions can be related to intertemporal choice modeling.

This is the example: (i) We assume that workers with wage income represent the "low per capita income" group. The second group, the owners of financial assets F, represent the "high per capita income" group. For these households returns r are the only source of income. (ii) Households in each group make their own intertemporal choices. We generally suggest that both ρ and η_U vary across low and high income households. a) Low-income wage-earning households: If we assume that the time preference rate of low-income households is high and if we do not allow for household debt, then wage-earning households would consume their full income. The result of individual intertemporal choice – even when applying the Euler equation $\frac{\dot{C}_L}{C_L} = \frac{r-\rho_L}{\eta_{U_L}}$ – is that these households do not intend to shift intertemporal consumption and simply consume what they earn from wages. The consumption rate is $c_L = 1$. b) High-income households obtain their total income from returns on financial assets $F(t)\,r$. Thus, the budget constraint of high-income financial asset owners is $S_F(t) \leqq F(t)\,r - C_F(t)$. As savings are used to purchase newly issued financial assets $\dot{F}(t)$ and these assets finance investments (26), we obtain

$$\begin{aligned} \dot{N}(t)\,\nu &= I(t) = \dot{F}(t) = F(t)\,r - C_F(t) \\ \frac{\dot{N}(t)}{N(t)}\nu &= \frac{F(t)}{N(t)}r - \frac{C_F(t)}{N(t)}. \end{aligned} \tag{A1}$$

From the perspective of the individual investor the interest rate r is given. A stationary interest rate together with the Euler equation for financial investors would lead to a constant consumption growth for this group, $g_{C_F} = \frac{\dot{C}_F}{C_F} = \frac{r-\rho_F}{\eta_{U_F}}$. If parameters of the Euler equation of financial wealth holders determine consumption growth as $g_{C_F} < g_N$, $\frac{C_F(t)}{N(t)}$ turns to zero in the long term, $\underset{\lim t\to\infty}{\frac{C_F(t)}{N(t)}} = \frac{C_F(0)e^{g_{C_F}t}}{N(0)e^{g_N t}} = \frac{C_F(0)}{N(0)}e^{\left(g_{C_F}-g_N\right)t} = 0$. Even if there is a positive consumption ratio during adjustment, the consumption ratio will turn to zero when taking the long term limit $\lim_{t\to\infty} c_F = \frac{C_F(t)}{Y(t)} = \frac{C_F(t)}{N(t)\left(\left(1-\tilde{\theta}\right)L^{1-\alpha}x^{\alpha}-xc_x\right)} = \frac{C_F(t)}{N(t)}\left(\left(1-\tilde{\theta}\right)L^{1-\alpha}\tilde{x}^{\alpha}-\tilde{x}c_x\right)^{-1} = 0$ for steady state values $\tilde{\theta}$ and $\tilde{x}$. As consequence $g_N\nu = \frac{F(0)e^{g_F t}}{N(0)e^{g_N t}}r$. This savings and investment process is consistent with a long term steady state process. Thus, we assume that 1) the start value of this steady state process is $\frac{F(0)}{N(0)} = \nu$, and 2) in steady state the ratio $\frac{F(t)}{N(t)}$ turns constant and thus $g_F = g_N$. Then, from $g_N\nu = \frac{F(0)e^{g_F t}}{N(0)e^{g_N t}}r$ we obtain $g_N = r$. In other words, in the choice model financial investors would start with both consumption and savings. However, their savings rate converges to one, such that in steady state, the consumption rate of financial investors is $c_F = 0$. As $c_L = 1$ and $c_F = 0$ are the simplifying assumptions in our model, we find in appendix A.7 that $\frac{F(0)}{N(0)} = \nu$ is the start value, and that $g_F = g_N$ are the growth rates which are required for dynamic consistency of the full model mechanism.

A.5 Calculations to solve the model (section 3)

Determine the growth rate g_N: From (15) and (11) we obtain $X^{eD} = Nx = N\alpha^{\frac{2}{1-\alpha}} (c_x)^{-\frac{1}{1-\alpha}} L_Q$. and $\dot{N} = \left(\alpha^{\frac{2}{1-\alpha}} (c_x)^{-\frac{1}{1-\alpha}} L_Q\right)^{\gamma} N^{\gamma}(\dot{A}))^{1-\gamma}$. Rearranging gives $\frac{\dot{N}(t)}{N(t)} = \left(\alpha^{\frac{2}{1-\alpha}} (c_x)^{-\frac{1}{1-\alpha}} L_Q\right)^{\gamma} (\frac{\dot{A}(t)}{A(t)})^{1-\gamma}$ for $N(t) = A(t)$.

Frim's sales equilibrium: Combing (2) for the aggregate economy $\phi(t) = 1 - \delta(t)$ with $\delta(t) = \delta'(t) - m$ (see 36) leads to $\phi(t) = 1 - \delta'(t) + m$, and with the definition of the firms sales frction friction $\delta'(t) = (1-\lambda)\varepsilon_F$, and taking expectations gives

$$\begin{aligned} E[\phi(t)] &= 1 - E\left[\delta'(t)\right] + m \\ &= 1 - E[\varepsilon_F] + E[\lambda] E[\varepsilon_F] + cov(\varepsilon, \varepsilon_F) + L_\phi (1-\theta)^{-1} \end{aligned}$$

Thus, in sales equilibrium described by condition (38) $E[\phi(t)] \overset{!}{=} 1$, we can determine the relation between L_ϕ and $(1-\theta)$

$$\begin{aligned} 1 &= E[\phi(t)] = 1 - E[\varepsilon_F] + E[\lambda] E[\varepsilon_F] + cov(\varepsilon, \varepsilon_F) + L_\phi (1-\theta)^{-1} \\ L_\phi (1-\theta)^{-1} &= E[\varepsilon_F] - E[\lambda] E[\varepsilon_F] - cov(\varepsilon, \varepsilon_F) \end{aligned}$$

After using (16) and market equlibrium condition (39) we obtain condition (38a)

$$\begin{aligned} L_\phi &= [(1 - E[\lambda]) E[\varepsilon_F] - cov(\varepsilon, \varepsilon_F)] (1-\theta) \\ L_Q &= cov(\varepsilon, \varepsilon_F)(1-\theta) + L, \quad \text{with} \quad cov(\varepsilon, \varepsilon_F) < 0, \end{aligned}$$

and respectively

$$(1-\theta) = \frac{L - L_Q}{-cov(\varepsilon, \varepsilon_F)}.$$

Determine current aggregate market equilibrium: According to (39) market equlibrium requires $1 = E[\lambda(t)]$. With (30)

$$1 = E[\lambda(t)] = \frac{\nu}{1 - c(1-\theta)} \frac{1}{L_Q \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}} g_N + \alpha^2,$$

and using (31) leads to

$$1 = \frac{\nu}{1 - c(1-\theta)} \frac{1}{L_Q \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}} \left(\left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} L_Q\right)^{\gamma} (g_A)^{1-\gamma} + \alpha^2.$$

When rearranging for L_Q we arrive at (39a)

A.6 Proof of proposition 1:

Proof of the Implicit Function Theorem: Using (39a) and plugging in (38a, respectively $(1-\theta)=\frac{L-L_Q}{-cov(\varepsilon,\varepsilon_F)}$) we obtain (40)

$$\begin{aligned}
F &= 0 = L_Q^{(1-\gamma)} - \frac{\nu}{(1-\alpha^2)\left(1-c\frac{L-L_Q}{-cov(\varepsilon,\varepsilon_F)}\right)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma}. \\
&= \left(1-c\frac{L-L_Q}{-cov\left(\varepsilon,\varepsilon_F\right)}\right)L_Q^{(1-\gamma)} - \frac{\nu}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma}. \\
&= \left[-cov\left(\varepsilon,\varepsilon_F\right)-c\left(L-L_Q\right)\right]L_Q^{(1-\gamma)} - \frac{-cov\left(\varepsilon,\varepsilon_F\right)\nu}{(1-\alpha^2)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma}.
\end{aligned}$$

$$\begin{aligned}
\frac{dF}{dL_Q} &= -\left(1-\gamma\right)cov\left(\varepsilon,\varepsilon_F\right)L_Q^{-\gamma} - c\left(1-\gamma\right)LL_Q^{-\gamma} + c\left(2-\gamma\right)L_Q^{(1-\gamma)} \\
&= L_Q^{-\gamma}\left[-\left(1-\gamma\right)cov\left(\varepsilon,\varepsilon_F\right) - c\left(1-\gamma\right)L + c\left(2-\gamma\right)L_Q\right]
\end{aligned}$$

$$\frac{dF}{dL_Q} = L_Q^{-\gamma}\left(1-\gamma\right)\left[-cov\left(\varepsilon,\varepsilon_F\right) - cL_\phi + \frac{c}{(1-\gamma)}L_Q\right] > 0$$

for $L_\phi < L_Q$ which we assume such that the condition for the implicit function theorem $\frac{dF}{dL_Q} \neq 0$ is satisfied, and the Implicit Function Theorem can be applied. Further derivatives of F are

$$\begin{aligned}
\frac{dF}{dg_A} &= -\frac{(1-\gamma)\,\nu}{(1-\alpha^2)\left(1-c\frac{(L-L_Q)}{-cov(\varepsilon,\varepsilon_F)}\right)}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{-\gamma} < 0 \\
\frac{dF}{d\nu} &= -\frac{\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma}}{(1-\alpha^2)\left(1-c\frac{(L-L_Q)}{-cov(\varepsilon,\varepsilon_F)}\right)} < 0
\end{aligned}$$

The derivative with respect to α, $\frac{dF}{d\alpha}$ requires more effort to calculate: $\frac{dF}{d\alpha} > 0$, for $\frac{L-L_Q}{2[-cov(\varepsilon,\varepsilon_F)]} > \alpha > \gamma$, and $\frac{\alpha^2}{c_x} > 1$

$$(1-\theta) = \frac{L-L_Q}{-cov\left(\varepsilon,\varepsilon_F\right)} > 2$$

$$\begin{aligned}
F &= L_Q^{1-\gamma} - \frac{\nu}{(1-\alpha^2)-(1-\alpha)\frac{L-L_Q}{-cov(\varepsilon,\varepsilon_F)}}G \\
G &= \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}} = e^{\frac{\gamma-\alpha}{1-\alpha}\left(\ln\alpha^2-\ln c_x\right)}
\end{aligned}$$

$$
\begin{aligned}
\frac{dG}{d\alpha} &= e^{\frac{\gamma-\alpha}{1-\alpha}\left(\ln\alpha^2-\ln c_x\right)}\left(\frac{-(1-\alpha)+(\gamma-\alpha)}{(1-\alpha)^2}(\ln\alpha^2-\ln c_x)+\frac{\gamma-\alpha}{1-\alpha}\frac{2}{\alpha}\right)\\
&= \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}\left(-\frac{1}{1-\alpha}\ln\left(\frac{\alpha^2}{c_x}\right)+\frac{(\gamma-\alpha)}{(1-\alpha)^2}\ln\left(\frac{\alpha^2}{c_x}\right)+\frac{\gamma-\alpha}{1-\alpha}\frac{2}{\alpha}\right)\\
&= \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}\left(-\frac{1}{1-\alpha}\ln\left(\frac{\alpha^2}{c_x}\right)+\frac{(\gamma-\alpha)}{(1-\alpha)}\left(\frac{1}{1-\alpha}\ln\left(\frac{\alpha^2}{c_x}\right)+\frac{2}{\alpha}\right)\right)\\
&< 0 \text{ if } \frac{\alpha^2}{c_x} > 1 \text{ and } \gamma<\alpha
\end{aligned}
$$

$$
\begin{aligned}
\frac{dF}{d\alpha} &= \frac{\nu}{\left((1-\alpha^2)-(1-\alpha)\frac{L-L_Q}{-cov(\varepsilon,\varepsilon_F)}\right)^2}\left(-2\alpha+\frac{L-L_Q}{-cov\left(\varepsilon,\varepsilon_F\right)}\right)G\\
&\quad -\frac{\nu}{(1-\alpha^2)-(1-\alpha)\frac{L-L_Q}{-cov(\varepsilon,\varepsilon_F)}}\frac{dG}{d\alpha} > 0
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\nu}{\left((1-\alpha^2)-(1-\alpha)\frac{L-L_Q}{-cov(\varepsilon,\varepsilon_F)}\right)^2}\left(-2\alpha+\frac{L-L_Q}{-cov\left(\varepsilon,\varepsilon_F\right)}\right)\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}\\
&\quad -\frac{\nu}{(1-\alpha^2)-(1-\alpha)\frac{L-L_Q}{-cov(\varepsilon,\varepsilon_F)}}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}\left(\begin{array}{c}-\frac{1}{1-\alpha}\ln\left(\frac{\alpha^2}{c_x}\right)\\ +\frac{(\gamma-\alpha)}{(1-\alpha)}\left(\frac{1}{1-\alpha}\ln\left(\frac{\alpha^2}{c_x}\right)+\frac{2}{\alpha}\right)\end{array}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\nu}{\left((1-\alpha^2)-(1-\alpha)\frac{L-L_Q}{-cov(\varepsilon,\varepsilon_F)}\right)^2}\left(-2\alpha+\frac{L-L_Q}{-cov\left(\varepsilon,\varepsilon_F\right)}\right)\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}\\
&\quad +\frac{\nu}{(1-\alpha^2)-(1-\alpha)\frac{L-L_Q}{-cov(\varepsilon,\varepsilon_F)}}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}\left(\begin{array}{c}\frac{1}{1-\alpha}\ln\left(\frac{\alpha^2}{c_x}\right)\\ +\frac{(\alpha-\gamma)}{(1-\alpha)}\left(\frac{1}{1-\alpha}\ln\left(\frac{\alpha^2}{c_x}\right)+\frac{2}{\alpha}\right)\end{array}\right) > 0\\
&: \text{if, } \frac{L-L_Q}{2\left[-cov\left(\varepsilon,\varepsilon_F\right)\right]} = \frac{\left(1-\tilde{\theta}\right)}{2} > \alpha > \gamma, \quad \text{and } \frac{\alpha^2}{c_x} > 1.
\end{aligned}
$$

This leads to comparative static result for interesting exogenous variables at equilibrium point $\tilde{L}_Q$ such as

$$
\begin{aligned}
0 &= \frac{\partial F}{\partial L_Q^*}dL_Q^* + \frac{\partial F}{\partial g_A}dg_A\\
\frac{dL_Q^*}{dg_A} &= -\frac{\overset{(-)}{\frac{\partial F}{\partial g_A}}}{\underset{(+)}{\frac{\partial F}{\partial L_Q^*}}} > 0, \qquad \frac{dL_Q^*}{d\nu} = -\frac{\frac{\partial F}{\partial \nu}}{\frac{\partial F}{\partial L_Q^*}} > 0, \qquad \frac{dL_Q^*}{d\alpha} = -\frac{\frac{dF}{d\alpha}}{\frac{\partial F}{\partial L_Q^*}} < 0.
\end{aligned}
$$

Show $r = g_N$:
$r = g_N$ can also be easily derived. According to (26) investments are $I_x(t) = \dot{N}(t)\nu$ and from (14) we know $r(t) = \frac{\pi_x(t)}{\nu}$. With all profits being saved we obtain $\dot{N}(t)\nu = I_x(t) = S(t) = N(t)\pi_x(t:)$ and plugging in (14) shows that in this kind of model returns on investments equalize to the growth rate of new products in the market

$$r = \frac{\pi_x(t)}{\nu} = \frac{\dot{N}(t)}{N(t)} = g_N.$$

Equilibrium $\tilde{\theta}$: Equation (43) gives equlibrium labor input in production. Plugging this in (38a) and rearranging gives (42).

Equilibrium $\tilde{Q}(t)$: Using the production function (1), the optimal intermediate goods input (11) ($x(t) = \left(\frac{\alpha^2}{c_x}\right)^{\frac{1}{1-\alpha}} L_Q$) we obtain (43).

Equilibrium level of income path $\tilde{Y}(t)$: From appendix A.3 and in particular (48) we obtain the Y. Combining with (43) gives (48).

Equilibrium level of income path $g_Y(t)$: Taking the time derivative of (44) in equilibrium we obtain $\dot{Y}(t) = \dot{N}(t)\left(1-\alpha^2\right) L_Q \left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}$ and thus $g_Y = \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{N}(t)}{N(t)}$, and using (31) we arrive at (45).

Derivatives of curves, figure 2: The derivative of (39a) gives the slope of the $E\lambda = 1$-*curve* as $\left.\frac{dL_Q}{d(1-\theta)}\right|_{E\lambda=1} = \frac{g_A c}{(1-\gamma)(1-c(1-\theta))}\left(\frac{\nu}{(1-\alpha^2)(1-c(1-\theta))}\right)^{\frac{1}{(1-\gamma)}}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{(1-\alpha)(1-\gamma)}} > 0$.
The derivative of (38a) gives the slope of the $E\phi = 1$-*curve* as $\left.\frac{dL_Q}{d(1-\theta)}\right|_{E\phi=1} = -E[\varepsilon_F](1-\theta)\frac{dE[\lambda(t)]}{d(1-\theta)} - E[\varepsilon_F](E[\lambda(t)]-1) - [-cov(\varepsilon,\varepsilon_F)] < 0$, with $E[\lambda(t)] = \frac{\nu}{1-c(1-\theta)}\frac{1}{L_Q^{(1-\gamma)}}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma} + \alpha^2$, and $\frac{dE[\lambda(t)]}{d(1-\theta)} = \frac{1}{L_Q^{(1-\gamma)}}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma-\alpha}{1-\alpha}}(g_A)^{1-\gamma}\frac{\nu c}{(1-c(1-\theta))^2} > 0$.

A.7 Dynamic consistency:

- Consistent motion of the demand and supply side in n-e-ee:
From (29) and (1), we obtain

$$\frac{\overbrace{\frac{\nu \dot{N}(t)+E\left[\varepsilon\left(t\right)\right]}{1-c}+N(t)x(t)c_{x}}^{\text{effective total demand for product } Q^{D}}}{\underbrace{N(t)L_{Q}^{1-\alpha}\left(x(t)\right)^{\alpha}}_{\text{output } Q}} = \lambda = 1, \quad \text{with} \quad E\left[\varepsilon\left(t\right)\right]=0,$$

$$\begin{aligned} \text{with } \tilde{x}_{j} &= \alpha^{\frac{2}{1-\alpha}}L_{Q}\left(c_{x}\right)^{-\frac{1}{1-\alpha}}, \\ \text{and } Q\left(t\right) &= N(t)L_{Q}\alpha^{\frac{2\alpha}{1-\alpha}}\left(c_{x}\right)^{-\frac{\alpha}{1-\alpha}} \end{aligned}$$

and in equilibrium

$$\frac{\nu}{1-c}\dot{N}(t)=N(t)L_{Q}\alpha^{\frac{2\alpha}{1-\alpha}}\left(c_{x}\right)^{-\frac{\alpha}{1-\alpha}}\left(1-\alpha^{2}\right).$$

A change over time is described by

$$\begin{aligned} \frac{1}{1-c}\nu\ddot{N}(t) &= \dot{N}(t)L_{Q}\alpha^{\frac{2\alpha}{1-\alpha}}\left(c_{x}\right)^{-\frac{\alpha}{1-\alpha}}\left(1-\alpha^{2}\right) \\ \frac{1}{1-c}\nu\frac{\ddot{N}(t)}{\dot{N}(t)} &= L_{Q}\alpha^{\frac{2\alpha}{1-\alpha}}\left(c_{x}\right)^{-\frac{\alpha}{1-\alpha}}\left(1-\alpha^{2}\right). \end{aligned}$$

Second, the growth rate of innovation related investments is $\frac{\ddot{N}}{\dot{N}}=g_{N}$ for exponential growth ($N(t)=e^{g_{N}t}$)) and

$$\frac{1}{1-c}\nu g_{N}=L_{Q}\alpha^{\frac{2\alpha}{1-\alpha}}\left(c_{x}\right)^{-\frac{\alpha}{1-\alpha}}\left(1-\alpha^{2}\right).$$

q.e.d.

- Consistent start values of Financial and Technology stocks:
Finally, we can show consistency by deriving the savings, and demonstrate that these savings indeed can finance the process from the start. Financial wealth income is $rF\left(t\right)$. According to our discussion in section 2.3 financial asset owners only save, and as these savings finance the investments for newly introduced goods, we obtain

$$rF\left(t\right)=S\left(t\right)=\dot{F}\left(t\right)=\dot{N}\left(t\right)\nu.$$

For this debt and technology growth mechanism, we need to show that savings in deposits and financing investments are consistent in their stock and flow mechanism, and we can derive a relation for the start period $F(0)/N(0)$ that leads to this consistent growth process [23]

$$\begin{aligned} r\frac{F(t)}{N(t)} &= \nu\frac{\dot{N}(t)}{N(t)} \Longleftrightarrow \\ rF(0)\,e^{(g_N)t} &= \nu N(0)\,e^{g_N t} g_N \Longleftrightarrow \\ rF(0) &= \nu N(0)\,g_N \end{aligned}$$

$$\frac{F(0)}{N(0)} = \frac{\nu}{r} g_N = \nu.$$

q.e.d.

A.8 Proof of proposition 5

Equilibrium reaction $\frac{d\tilde{L}_Q}{d\alpha} < 0$ **:** See appendix A.6.

Equilibrium reaction $\frac{d\tilde{g}_Y}{d\alpha} < 0$ **:**

$$g_Y = \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma}{1-\alpha}} L_Q^{\gamma} g_A^{1-\gamma} = e^{\frac{\gamma}{1-\alpha}(\ln\alpha^2 - \ln c_x)} L_Q^{\gamma} g_A^{1-\gamma}$$

$$\begin{aligned} \frac{dg_Y}{d\alpha} &= e^{\frac{\gamma}{1-\alpha}(\ln\alpha^2 - \ln c_x)} \left(\frac{\gamma}{1-\alpha}\frac{2}{\alpha} - \frac{\gamma}{(1-\alpha)^2}(\ln\alpha^2 - \ln c_x)\right) L_Q^{\gamma} g_A^{1-\gamma} \\ &\quad + e^{\frac{\gamma}{1-\alpha}(\ln\alpha^2 - \ln c_x)} \gamma L_Q^{\gamma-1} \frac{dL_Q}{d\alpha} g_A^{1-\gamma} \\ &= \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma}{1-\alpha}} \left(\frac{\gamma}{1-\alpha}\frac{2}{\alpha} - \frac{\gamma}{(1-\alpha)^2}(\ln\alpha^2 - \ln c_x)\right) L_Q^{\gamma} g_A^{1-\gamma} + \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma}{1-\alpha}} \gamma L_Q^{\gamma-1} \frac{dL_Q}{d\alpha} g_A^{1-\gamma} \\ &= \left(\frac{\alpha^2}{c_x}\right)^{\frac{\gamma}{1-\alpha}} \frac{L_Q^{\gamma}\gamma}{(1-\alpha)^2} g_A^{1-\gamma} \left[\overbrace{(1-\alpha)^2 L_Q^{-1}\frac{dL_Q}{d\alpha}}^{(i)} + \overbrace{\frac{2}{\alpha} - 2 - (\ln\alpha^2 - \ln c_x)}^{(ii)}\right] < 0 \end{aligned}$$

(i) $(1-\alpha)^2 L_Q^{-1}\frac{dL_Q}{d\alpha} < 0$ as we assume in appendix A.6 that $\frac{(1-\tilde{\theta})}{2} > \alpha > \gamma$, and $\frac{\alpha^2}{c_x} > 1$, leading to $\frac{dF}{d\alpha} > 0$ and $\frac{dL_Q}{d\alpha} < 0$.

[23] $r\frac{F(t)}{N(t)} = \nu\frac{\dot{N}(t)}{N(t)} \Longleftrightarrow rF(0)\,e^{(g_N)t} = \nu N(0)\,e^{g_N t} g_N \Longleftrightarrow rF(0) = \nu N(0)\,g_N$

(ii) $\frac{2}{\alpha} - 2 - (\ln \alpha^2 - \ln c_x) < 0$ holds if c_x is sufficiently small relatively to α :

$$\begin{aligned} \frac{2}{\alpha} - 2 - (\ln \alpha^2 - \ln c_x) &< 0 \\ \frac{2}{\alpha} - 2 &< (\ln \alpha^2 - \ln c_x) \\ e^{2\left(\frac{1}{\alpha}-1\right)} &< \alpha^2 c_x^{-1} \\ c_x &< \left(\frac{\alpha}{e^{\left(\frac{1}{\alpha}-1\right)}}\right)^2 \end{aligned}$$

q.e.d.

Equilibrium reaction $\frac{d\tilde{Y}(t)}{d\alpha} > 0$ **:**

$$Y = N(1-\alpha^2)L_Q\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}} = N(1-\alpha^2)L_Q e^{\frac{\alpha}{1-\alpha}(\ln \alpha^2 - \ln c_x)}$$

$$\begin{aligned} \frac{dY}{d\alpha} &= N(1-\alpha^2)\frac{dL_Q}{d\alpha}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}} + NL_Q(1-2\alpha)e^{\frac{\alpha}{1-\alpha}(\ln \alpha^2 - \ln c_x)} \\ &\quad + NL_Q(1-\alpha^2)e^{\frac{\alpha}{1-\alpha}(\ln \alpha^2 - \ln c_x)}\left(\frac{\alpha}{1-\alpha}\frac{2}{\alpha} + \frac{1-\alpha+\alpha}{(1-\alpha)^2}(\ln \alpha^2 - \ln c_x)\right) \\ &= N(1-\alpha^2)\frac{dL_Q}{d\alpha}\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}} + NL_Q(1-2\alpha)\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}} \\ &\quad + NL_Q(1-\alpha^2)\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{1-\alpha}\frac{2}{\alpha} + \frac{1}{(1-\alpha)^2}(\ln \alpha^2 - \ln c_x)\right) \\ &= N\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}} L_Q\left[(1-\alpha^2)\frac{dL_Q}{d\alpha}\frac{1}{L_Q} + 1 - 2\alpha + (1-\alpha^2)\left(\frac{2}{1-\alpha} + \frac{1}{(1-\alpha)^2}(\ln \alpha^2 - \ln c_x)\right)\right] \\ &= N\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}} L_Q\left[(1-\alpha^2)\frac{dL_Q}{d\alpha}\frac{1}{L_Q} + 1 - 2\alpha + \frac{2(1-\alpha^2)}{1-\alpha} + \frac{(1-\alpha^2)}{(1-\alpha)^2}(\ln \alpha^2 - \ln c_x)\right] \end{aligned}$$

using $\left(1-\alpha^2\right) = (1-\alpha)(1+\alpha)$

$$\begin{aligned} &= N\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}} L_Q\left[\begin{array}{c} (1-\alpha^2)\frac{dL_Q}{d\alpha}\frac{1}{L_Q} + 1 - 2\alpha + \frac{2(1-\alpha)(1+\alpha)}{1-\alpha} \\ +\frac{(1-\alpha)(1+\alpha)}{(1-\alpha)^2}(\ln \alpha^2 - \ln c_x) \end{array}\right] \\ &= N\left(\frac{\alpha^2}{c_x}\right)^{\frac{\alpha}{1-\alpha}} L_Q\left[(1-\alpha^2)\frac{dL_Q}{d\alpha}\frac{1}{L_Q} + 3 + \frac{(1+\alpha)}{(1-\alpha)}(\ln \alpha^2 - \ln c_x)\right] \end{aligned}$$

if $(\ln \alpha^2 - \ln c_x) > 0$

As we have already assumed that $\frac{\alpha^2}{c_x} > 1$, $(\ln \alpha^2 - \ln c_x)$ is clearly positive leading to the interesting case we discuss it in proposition 5.

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