This Time is Different: Disentangling the Heterogeneity of Business Cycle Fluctuations

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Abstract

We apply the hierarchical Markov normal mixture model (HMNM) with time varying transition probabilities to analyze the effect of credit on business cycle phases using a worldwide dataset. HMNM enables us to examine the effect of credit on mild recessions and depressions as well as recoveries and secular growth phases of the business cycle in a hierarchical pattern endogenously determined by the data. We show that (1) past credit accumulation increases the probability of entering into recessions and depressions, (2) this effect is stronger for countries with high credit/GDP ratios, and (3) credit impulse (change in credit growth) has a second order importance after credit growth. It is only by disentangling the recessions further into depressions and mild recessions that the effect of credit can be seen clearly. Our results are consistent with financial accelerator models and imply that ignoring the different dynamics of tail events and ”normal times” both costs precision and can lead to wrong inference.

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1 Introduction

The link between economic and financial fluctuations has been area of active research at least since the Great depression of 1930's following the crash in financial markets. The discussion has been further intensified since the Great recession, as the global economic downturn coupled with a financial crisis hit many countries much severely compared to earlier mild economic downturns towards the end of the 20th century. Not surprisingly, empirical evidence indicates that economic downturns tend to be more severe when they are coupled with a financial crisis, see for example Kaminsky and Reinhart (1999); Eichengreen and Arteta (2001) among others. Therefore, the predictive content of financial variables, most notably, credit related variables is likely to be amplified for more severe recessions compared to economic downturns with milder course.

Empirical models of cyclical variation in economic conditions often employ nonlinear models with alternating parameters during economic downturns and expansions following the seminal paper of Hamilton (1989). These regime switching models have the attractive feature of incorporating the notion of economic fluctuations by endogenously estimating the cyclical phases in econometric inference. The predictive content of financial variables could then be efficiently tested by examining whether those variables have an impact on the probability of cyclical phases, leading to a trough examination of causal links between the financial variables and cyclical phases of business cycle, see Gadea Rivas and Perez-Quiros (2015) for a discussion and a recent analysis.

While the quest of the business cycle naturally implies economic fluctuations classified as recessions and expansions, nevertheless it does not necessarily implies homogeneous cyclical phases. Still, empirical analysis relies on the two phases of the cycle as recessions and expansions with the implicit assumption of homogeneity among recessions as well as among expansions. While it is clear that business cycle fluctuations are comprised by upward and downward swings it is not certain whether all recessions as well as expansions share common features and dynamics. Especially, as the economic downturns tend to be more severe when they are coupled with financial distress the predictive ability of financial variables for signaling recessions might be tempered due to the failure to predict mild recessions that are more frequent observed. Therefore, empirical analysis of business cycle fluctuations require to explicitly account for heterogeneity among recessions as well as expansions.

Departing from this point we propose the hierarchical Markov regime switching model with time varying regime probabilities for efficient analysis of business cycle fluctuations and the link with the financial variables. Two-state MNM models (MNM-2) typically assume a Gaussian distribution for the distribution of data in regime of recession and expansion. The problem with this approach is that the distribution of growth rates in recession (expansion) phases are not normal: they are left (right) skewed. If credit effects the tail events and secular phases differently, then commonly used MNM-2 model would provide imprecise estimates and can lead to wrong inference. Alternatively, a four-state MNM model (MNM-4) model can capture depression, mild recession, secular growth and recovery phases separately. However MNM-4 model with time varying transition probabilities requires estimation of excessive number of parameters that often leads to intractable specifications. Identifying the effects of macroeconomic and financial variables on normal recessions and depressions separately requires a model that identifies recovery, expansion, recession and depression states separately, and does not overparametrize the transition probabilities as the business cycles are long and data is simply not vast enough to examine various channels. For this purpose, we use an extended version of Hierarchical Normal Mixture (HMNM) Model of Geweke and Amisano (2011) to allow for time varying transition probabilities.

Second, severe recessions or economic recoveries are rare events, focusing on a single country, US for example, would reduce the significance of econometric inference. Using this rich dataset facilitates econometric analysis as it enables use to gather many observations of these rare events.

Dynamics of business cycle → credit channel Neither view provided a complete explanation of what was observed in the data, which led others to investigate the credit channel (Mishkin, 1978; Bernanke, 1983; Gertler and Mark, 1988). In the "credit view", quantities and mechanisms of credit mattered, not as an independent source but as a propagator of shocks. This interest in the dynamics of business cycles has slowly faded as the developed economies, especially US, entered into a long and steady expansion period following the great moderation. Crises seemed to affect only the emerging markets and not the developed nations.

Focus on the interaction of financial variables, in particular, credit, financial development → heterogeneous recessions. We observe that if economic crisis are coupled with financial crisis, it tends to be more severe compared to recessions with only an underlying economic crisis. Following the 2008 crisis, the interest in the business cycles and its relationship with financial variables has reawakened. An important body of literature has analyzed historical records, documented the timing of financial crises and detected their similarities and differences. Two important contributions in this field are Reinhart and Rogoff (2009) and Laeven and Valencia (2012). While Reinhart and Rogoff (2009) construct an enormous database that spans the entire world for eight centuries, Laeven and Valencia (2012) provide statistical information on the dates and characteristics of banking crisis during the period 1970-2009.

These documentations led a group of studies to explore the dynamics of financial and macroeconomic variables alongside the periods of financial unrest. Following the seminal work of Kaminsky and Reinhart (1999), these papers focus on how certain key variables, especially credit, behave during the crisis periods and how and if these variables can be used to predict recessions. Schularick and Taylor (2012) construct a dataset of 14 countries over the years 1870-2008 and find that credit growth is a powerful predictor of financial crises. Gourinchas and Obstfeld (2012) use different databases to classify financial crises into different types and finds that domestic credit expansion (high Domestic Credit/GDP ratio) and real currency appreciation are robust predictors of financial crises. Mendoza and Terrones (2008, 2012) identify credit booms and document how certain macroeconomic and financial variables are affected around their peak. Jorda et al. (2013) study the role of credit in business cycles and find that credit intensive expansions are followed by deeper recessions.

This line of research provides significant evidence that financial markets, credit in particular, play an important role in determining the business cycle characteristics. Two main findings are: (1) excessive credit injection during expansions become unsustainable and cause a financial crisis, and (2) financial crashes are followed by deeper recessions and slower recoveries. This is because credit enables the debtor to undertake consumption or investment spending beyond their means. Therefore it allows them to build debt stock beyond their ability to pay. Once agents default, further investments can no longer be funded, hence the economy enters into recession. The main econometric problem of these studies is that many of the analyzed variables including credit are endogenous variables, therefore statistical significance does not provide a causal inference. For instance, credit to GDP accumulates over time endogenously in different theoretical models, as in Gertler and Kiyotaki (2010); Christiano et al. (2010), hence it is endogenously high near the end of expansions. Even if credit has no effect on the turning points, high Credit/GDP ratio or high credit growth before the turning points would lead such models to pinpoint credit as an indicator of recessions[1].

Gadea Rivas and Perez-Quiros (2015) show that the heterogeneity of recessions across these countries is much lower than the heterogeneity of recessions over time. On the one hand, our hierarchical model disentangling recessions (expansions) into further phases into severe and mild recessions (recoveries and secular growth phases) aims to exploit fully this heterogeneity of business cycle fluctuations over time using a rich dataset. On the other hand using this dataset with many countries enables to have a large sample of 'severe recessions' or 'recoveries' which can be considered as rare events. Using a single country, US for example following the convention, would preclude an efficient analysis of these rare events.

The main contributions of this paper are as follows. First, we introduce the HMNM model to the study of business cycles and show that it can detect channels through which financial variables including credit can affect business cycle turning points that commonly used MNM-2 model cannot. Second, using HMNM model with time varying transition probabilities we show that (1) credit/GDP ratio serves as a smoothing mechanism for the economy, not as an independent source of shock, (2) past credit accumulation increases the probability of entering into recessions and depressions, (3) this effect is stronger for countries with high leverage, (4) credit impulse (change in credit growth) only has an effect

[1]For more detailed discussion about this topic, see Gadea Rivas and Perez-Quiros (2015).

when credit markets are not functioning properly.

The rest of the paper is structured as follows: Section 2 introduces the model and estimation methods, section 3 describes the data, section 4 provides the results, section 5 concludes.

2 A Snapshot of Business Cycle Phases in OECD Countries

We start our analysis of business cycle fluctuations with a descriptive yet comprehensive analysis of recessions and expansion experienced in the advanced economies. Therefore, we first construct a longitudinal panel dataset involving a total of 31 OECD countries. The countries in our dataset are Australia, Austria, Belgium, Brazil, Canada, Chile, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Indonesia, Israel, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, South Africa, Spain, Sweden, Switzerland, Turkey, United Kingdom and the United States. The final dataset include the countries with sufficiently developed financial markets together with at least more than 15 years of quarterly observations. The resulting panel is unbalanced due to the fact that for some countries part of the dataset starts only from 1990s whereas for most of the countries it covers a long time span starting from 1970s. Details of the dataset are provided in Appendix A???.

For the measure of economic output we use the real Gross Domestic Product (rGDP) following the convention. Consider the output growth rates of countries, $n = 1, \dots, N$, for the time periods, $t = 1, \dots, T_n$,

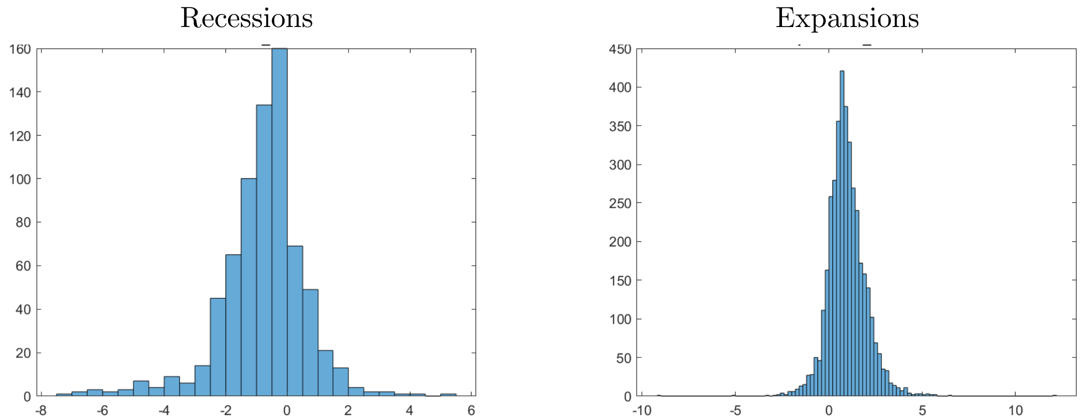
$$\begin{aligned} X &= \{x_{t,n}\}_{t=1:T_n:\ n=1:N} \\ &= \{x_{1,1}, \dots, x_{T_1,1}, \dots, x_{1,n}, \dots, x_{T_n,n}, \dots, x_{1,N}, \dots, x_{T_N,N}\}. \end{aligned}$$

Following Gadea Rivas and Perez-Quiros (2015), we transform the growth rates of the countries as

$$y_{t,n} = \frac{y_{t,n} - \mu_n}{\sigma_n} * \sigma_{\text{global}} + \mu_{\text{global}} \tag{1}$$

This transformation greatly reduces the heterogeneity among country specific business cycle fluctuations and it allow us to focus mainly on the general causes of heterogeneity in cyclical phases of economic fluctuations. We first compute the recession and expansion dates for each of the countries in our sample using the BBQ dating algorithm[2]. We display the the distribution of growth rates in expansion and recession phases in Figure 1.

Figure 1: Histogram of growth rates during business cycle phases



Histogram of the growth rates during recessions (expansions) is displayed on the left (right) panel. As can be understand from the frequencies on the left axis the total number of expansion periods are much larger than the recessions reflecting the asymmetry in business cycle. Figure 1 clearly indicate that for both recessions and expansions, (unconditional) distribution of the growth rates do not follow a normal distribution. Regarding to recessions, displayed in the left panel, the distribution seems to be left skewed with a sizable probability mass for observations of growth rates exceeding values as low as -4%. Left skewness of this distribution provides us many observations of 'severe recessions' which can be considered a rare event for a single country. However, the dataset we use using 31 OECD countries involves many observations of these rare events.**To elaborate further, when we identify 'severe recessions' as recessions that are below the 10^{th} percentile of the distribution we observe 63 severe recessions in these 31 countries.** We also observe a leptokurtotic distribution resulting from fat tails mostly to the left of the distribution. We observe a similar pattern for the distribution of the growth rates during expansions, which seems to be right skewed with relatively more observations

[2]As many of these countries lack a business cycle dating committee as opposed to US (NBER dating committee) we use the dates estimated by the BBQ algorithm as reference recession dates. The BBQ algorithm is a nonparamatric procedure used for dating business cycle turning points based on the definition of a recession as two consecutive quarters decline in economic activity. The algorithm is proposed by Bry and Boschan (1971) and simplified by Harding and Pagan (2002).

larger than 4% of growth rates. Note that we also observe positive (negative) growth rates for recessions (expansions). This is due to the fact that over the course of recessions (expansions) we might observe positive (negative) swings as temporary bounce-backs following successive periods of economic downturn (upturn).

3 Econometric model

In the previous section, our descriptive evidence suggests that the distributions during recessions as well as expansions deviate from normality. Most notably, we observe a left skewed distribution with considerably low values for the growth rates during recessions. A similar pattern much like a mirror image can be observed for the expansions with a right skewed distribution with considerably large values of growth rates. In this section, we present the econometric specification to model these patterns observed in business cycle fluctuations. The model allows for distinct dynamics during the expansionary and recessionary phases of the business cycle. The key feature of the model is that it allows for a more refined picture of the expansions and recession in a hierarchical structure. Specifically, it allows the distributions of these phases to be non-normal by disentangling the recessions further into a deeper and mild recessions and the expansions further into a recovery and secular growth periods. By doing this classification, the parametric specification enables us to have a elaborative focus on the effects of financial variables during these distinct phases of recessions and expansions.

As in the standard specifications of regime switching models, we assume that there are two regimes of the business cycle as expansion and recession that are characterized by different means of y_t. In case of no autoregressive dynamics for the growth rate, the model specification is

$$y_t = \mu_{S_t} + \epsilon_t \quad \epsilon_t \sim N(0, \sigma^2). \tag{2}$$

Here S_t is a latent binomial variable taking the value 1 if y_t is in the recession regime and 0 if y_t is in the expansion regime at time t. Moreover, $\mu_{S_t} = \mathsf{E}[y_t|S_t]$ denotes the mean of the growth rate, y_t, in the regime indicated by S_t, and the error terms follow a multivariate Normal distribution with variance σ^2. We assume that the variable S_t is governed by a first order 2-state homogenous Markov processes with transition probabilities

$$p(S_t = j|S_{t-1} = i) = p_{ij} \quad \text{for } i, j = 0, 1 \tag{3}$$

The model is denoted as Markov Mixture model (MM), which proved to be useful in many applications of empirical business cycle analysis, see for example Frühwirth-Schnatter (2006) for a general exposition. We depart from the conventional MM specification by allowing the mixture components, that is, the distributions of the rGDP growth rate in recession and expansion regimes to be non-normal. We do this by specifying further mixture distributions for these components in a hierarchical manner, see for example, Geweke and Amisano (2011) for an application on US stock returns. Specifically, considering recessions, conditional on that y_t is in the recession regime, i.e. $S_t = 1$, it can further be assigned to the 'mild recession' regime with probability q_1 or a 'deep recession' regime with probability $1 - q_1$. In the other case considering expansions, conditional on that y_t is in the expansion regime, i.e. $S_t = 0$, it can further be assigned to the 'recovery' regime with probability q_0 or a 'secular growth' regime with probability $1 - q_0$. This model is denoted as Hierarchical Markov Mixture model (HMM). Formally, let I_t denote a latent binomial variable indicating the regime dependence of y_t conditional on S_t. Then, the modeling structure implies that $P(I_t = 1|S_t = i) = q_i$ or $P(I_t = 0|S_t = i) = 1 - q_i$ for $i = 0, 1$. The resulting specification implies a 'mild recession' regime for $I_t = 1$ conditional $S_t = 1$, i.e. during a recession and a 'recovery' regime for $I_t = 1$ conditional $S_t = 0$, i.e. during an expansion. Notice that while traditional MM model is a 2-regime specification, the HMM model essentially implies a 4-regime specification with the regimes being as deep recession, mild recession, secular growth and recovery in a parsimonious manner. To elaborate this further, let $\tilde{S}_t$ denote the latent multinomial variable representing the 4 regimes implied by the HMNM model. The matrix of transition probabilities in this case are as follows

$$\tilde{P} = \begin{bmatrix} p_{00}\, q_0 & p_{00}(1-q_0) & (1-p_{00})q_1 & (1-p_{00})(1-q_1) \\ p_{00}\, q_0 & p_{00}(1-q_0) & (1-p_{00})q_1 & (1-p_{00})(1-q_1) \\ (1-p_{11})q_0 & (1-p_{11})(1-q_0) & p_{11}\, q_1 & p_{11}(1-q_1) \\ (1-p_{11})q_0 & (1-p_{11})(1-q_0) & p_{11}\, q_1 & p_{11}(1-q_1) \end{bmatrix}. \tag{4}$$

As conditional on S_t, I_t do not follow a Markov process but these are independent, the transition matrix (4) implied by the HMM specification in a parsimonious way using only four parameters, p_{00}, p_{11}, q_0, q_1, as opposed to 12 parameters of an unrestricted 4-regime MM specification. Therefore, the HMM model facilitates the analysis of the business cycle allowing for quite a rich dynamics in a sensible manner taking the extreme events in the distribution of recession and expansion phases in a tractable way.

As the focal point of the model is to examine the potential differential effects of financial factors on the economic fluctuations, we extend the HMM model to allow for time variation in transition probabilities. An attractive feature of the hierarchical structure is that it allows for analysis of time variation in the transition probabilities of expansions and recessions and in the probabilities for extreme events of recoveries and deep recessions separately. This type of dissection of business cycle phases might be crucial as it allows to examine the effects of potential financial variables such as credit focusing in distinct types of recessions or expansions. On the contrary a 2-regime MM model only permits the effects of financial variables on the recessions which are implicitly assumed to be similar. In this sense, if financial variables are only effective during severe recessions this effect is likely to be averaged out as mild recessions constitute the lion's share of the observations over the course of recessions. For modeling the time variation in the transition probabilities we use a probit type of specification. This implies the following

$$\begin{aligned} p_{ii,t} &= \Phi(p_{ii} + \theta_i' x_{t-1}) \\ q_{i,t} &= \Phi(q_i + \gamma_i' x_{t-1}) \end{aligned} \tag{5}$$

where Φ stands for the standard normal cdf, θ_i is the vector of coefficients measuring the effect of the vector of variables x_{t-1} on the probability $p_{ii,t}$, and γ_i is the vector of coefficients measuring the effect of the vector of variables x_{t-1} on the probability $q_{i,t}$.

3.1 Bayesian inference

In this section we provide details on econometric inference of the hierarchical Markov normal mixture model. We adopt a Bayesian approach for inference of the HMM model with time varying transition probabilities and its special cases. Bayesian inference allows us to handle the hierarchical structure with the time varying probabilities in an efficient way by exploiting the conditional distributions implied by this hierarchical structure. The posterior distribution for the model parameters is proportional to the product of the likelihood function and the prior distributions of these parameters. In Section 3.1.1, we discuss the prior distributions and in Section 3.1.2 we discuss the full posterior conditional distributions together with the resulting simulation scheme.

3.1.1 Prior Distributions

To estimate the models parameters together with the unobserved regimes, we would like to obtain posterior results that are driven by the data rather than by the prior distributions. Therefore, we impose rather diffuse prior specifications for the model parameters. For the transition probabilities we use a uniform distribution on the unit interval (0,1), that is

$$\begin{aligned} f(p_{ii}) &= \mathbb{I}[0 < p_{ii} < 1] \\ f(q_i) &= \mathbb{I}[0 < q_i < 1] \quad \text{for } i = 0, 1. \end{aligned} \tag{6}$$

When this non-informative prior for the transition probabilities is assigned special attention must be paid to the prior specifications of the regime-dependent parameters. This follows from the fact that the complete data likelihood value remains identical if we switch the regime dependent means together with the corresponding transition probabilities. This 'label switching problem' complicates proper posterior analysis as the posterior distributions of the regime dependent parameters become multimodal, see Geweke (2007) for discussion. To circumvent this problem, we define the prior for the regime dependent mean parameters $\mu = (\mu_{00}, \mu_{01}, \mu_{10}, \mu_{11})'$ in such a way that it identifies the regimes as 'deep recession', 'mild recession' and 'secular growth' and 'recovery'. Specifically, we set the prior specification for μ using truncated normal distribution

$$f(\mu) \propto \begin{cases} 1 & \text{if } \mu \in \{\mu \in \mathbb{R}^J | \mu_{00} < \mu_{01} < 0 < \mu_{10} < \mu_{11}\} \\ 0 & \text{elsewhere.} \end{cases} \tag{7}$$

For the variance parameter we take a noninformative prior

$$f(\sigma^2) \propto \sigma^{-2} \tag{8}$$

see Geisser (1965). Finally for the parameters regarding to time varying probabilities, we use a noninformative improper prior of form

$$f(\theta_i) \propto 1 \text{ and } f(\gamma_i) \propto 1 \quad \text{for } i = 0, 1. \tag{9}$$

3.1.2 Likelihood Function and Posterior Simulation

We follow the common practice in the literature to treat the unobserved regimes as parameters to be estimated. For this purpose we derive the complete data likelihood function. We do so for the HMNM model as given in (3) together with (4) and (5). We assume a Normal distribution for the error terms ϵ_t which leads to the Normal conditional density of y_t given the past observations $y^{t-1} = \{y_1, \ldots, y_{t-1}\}$ and given the past and current states $\tilde{S}^t = \{\tilde{S}_1, \ldots, \tilde{S}_t\}$. The complete data likelihood function for model conditional on the first observation equals

$$f(y^T, \tilde{S}^T | y_1, \Gamma) = \left(\prod_{t=2}^{T} \prod_{i=1}^{J} \prod_{j=1}^{J} \tilde{p}_{t,ij} \right) \prod_{t=2}^{T} f(y_t | y^{t-1}, \tilde{S}^t, \mu, \sigma^2), \tag{10}$$

where T denotes the sample size and $\Gamma = (\mu', \sigma^2, p', q', \theta', \gamma')'$ for $i = 0, 1$ represents all the model parameters where $\mu = (\mu_{00}, \mu_{01}, \mu_{10}, \mu_{11})'$, $p = (p_{00}, p_{11})'$, $q = (q_0, q_1)'$, $\theta = (\theta_0', \theta_1')'$ and $\gamma = (\gamma_0', \gamma_1')'$. Finally, $\tilde{P}_t = \{\tilde{p}_{t,ij}\}_{i,j=1}^4$ the matrix with transition probabilities of the HMNM model displayed in (4) together with time variation as modeled in (5). The likelihood function conditional only on the model parameters can be obtained by summing (10) over all the possible states as

$$f(Y^T | Y_1, \theta) = \sum_{\tilde{S}_1=1}^{4} \sum_{\tilde{S}_2=1}^{4} \cdots \sum_{\tilde{S}_T=1}^{4} f(y^T, S^T | y_1, \Gamma). \tag{11}$$

subsectionPosterior Simulation The posterior distribution for the model parameters is proportional to the product of the likelihood function (11) and the joint prior $f(\Gamma)$. The joint prior can be obtained as the product of (6)–(9). To obtain posterior results, we use Gibbs sampling together with data augmentation (see Geman and Geman, 1984; Tanner and Wong, 1987). This leads to the following algorithm to draw from the joint posterior distribution

1. Sample μ from $f(\mu|\sigma^2, \tilde{S}^T)$,
2. Sample σ^2 from $f(\sigma^2|\mu, \tilde{S}^T)$,
3. Sample p and θ from $f(p, \theta|S^T)$,
4. Sample q and γ from $f(q, \gamma|I^T)$,
5. Sample $\tilde{S}^T$ from $f(\tilde{S}^T|\mu, \sigma^2, \tilde{P}_t)$

Details on the derivation of the conditional posterior distributions resulting from the likelihood function and prior distributions are given in the Appendix.

3.2 Details on the Dataset and Model Specifics

As we discussed intensively in the previous section, we use the real Gross Domestic Product (rGDP) for economic output. Our main set of predictive variables involve financial variables with a focus on the credit related variables. These are constructed using the level of credit measured as the total credit to the private non-financial sector. Specifically, we use credit to GDP ratio as measured by the ratio of credit to the GDP. We also use the growth rate of the credits in real terms and the credit impulse computed as the growth rate of the credit growth. Finally, we use real stock market index representing the expectations of the financial markets. For constructing the dataset, we compile the database of Bank of International Settlements, FRED databes of the Federal Reserve Bank at St. Louis and the World Bank.

Prior to analysis, we transform the rGDP growth rates in a specific way to reduce the cross-sectional heterogeneity following the practice in Gadea Rivas and Perez-Quiros (2015). Specifically, for each country we first standardized the country growth rates using the country specific standard deviations and the mean. We then de-standardize the data using the standard deviations and mean computed using the entire sample of countries.

Such a transformation reduces country specific fluctuations and improves the identification of rare events having similar characteristics across countries rather reflecting idiosyncratic nature of specific countries. We display the summary statistics of the variables used in this analysis in Table 1.

Table 1: Summary statistics

Variables	N	mean	s.d.
GDP growth	4392	0.68	1.25
Credit/GDP ratio	4392	117.23	56.23
Real Credit growth	4344	1.12	5.77
Real Credit impulse	4336	-0.011	7.65
Real Stock Return	2851	0.75	13.81

4 Empirical Findings

In this section, we discuss the findings of the hierarchical specification with respect to the capability of financial variables, with the credit related variables as our focal point, on the predictability of the business cycle fluctuations. We closely follow the practice in Gadea Rivas and Perez-Quiros (2015) who focus on the analysis of predictability of the recent recession using credit and related variables using a 2-regime MNM specification in order to facilitate the comparison with earlier findings. We, first, display the findings regarding to in-sample estimation. For each model, we provide two sets of results, one using the data until the onset of the 2007-9 global economic crisis, i.e. until 2007 last quarter and the other using the all available data including the post 2008-9 crisis period, i.e. until the end of 2018. Comparison of these two sets of results would provide evidence of predictability of the recent crisis with a further refinement of the economic recessions defined as either 'severe' or 'mild' recession.

We first provide the findings on the predictive ability of the credit to GDP ratio in predicting economic fluctuations. We, therefore, include the first lag of the credit to GDP ratio in the probit specifications for the transition probabilities in (5).

Table 2: Estimation Results Using Credit Ratio

	MNM-TVTP Full sample	MNM-TVTP Until 2007 q4	HMNM-TVTP Full sample	HMNM-TVTP Until 2007 q4
p_{00}	1.894 (0.137)	1.550 (0.189)	1.446 (0.117)	1.168 (0.164)
$\theta_{0,1}$	0.001 (0.001)	0.002 (0.002)	0.002 (0.001)	0.003 (0.001)
p_{11}	-0.357 (0.270)	0.085 (0.273)	-0.215 (0.189)	-0.170 (0.228)
$\theta_{1,1}$	0.005 (0.002)	0.004 (0.002)	0.005 (0.001)	0.006 (0.002)
q_0			-0.982 (0.140)	-1.266 (0.181)
$\gamma_{0,1}$			-0.007 (0.001)	-0.006 (0.002)
q_1			1.565 (0.222)	1.619 (0.309)
$\gamma_{1,1}$			-0.001 (0.002)	0.003 (0.003)

Note: The table presents posterior means and standard deviations (in parenthesis) of the coefficients for the with time varying transition probabilities, $p_{ii,t,c} = \Phi(p_{ii} + \theta_{i,1} RCG_{t-1})$ for transition from/to recession, $i = 1$, and expansion regimes, $i = 0$, and time varying probabilities, $q_{k,t} = \Phi(q_k + \gamma_{k,1} RCG_{t-1})$ of switching to 'mild' or 'severe' $(1 - q_{k,t})$ recessions in recession regimes, $k = 1$ and 'recovery' or to 'secular growth' $(1 - q_{k,t})$ in the expansion regime, $k = 0$. MNM-TVTP stands for the 2-regime Markov Normal Mixture model with time varying transition probabilities and HMNM-TVTP stands for the Hierarchical Markov Normal Mixture model with time varying probabilities.

Table 2 shows how credit ratio affects the transition probabilities between different phases of the business cycle. First we consider the full sample estimation including the post 2008 periods until the end 2017. The first column displays the results of the 2-regime MNM model. As can be seen from the estimates regarding to $\theta_{0,1}$ credit ratio does not affect the probability of staying in expansion or equivalently the probability of the transition to recessions from expansions. On the other hand, credit to GDP ratio significantly increases the probability of staying in recessions as the estimate of $\theta_{1,1}$ indicates. This implies that the duration of the recessions increase with the increasing credit to GDP ratio. However, this evidence of the relation between the credit to GDP ratio and the recessions cannot be observed if we use the data only until the end of 2007 as displayed in the second column. In this case, zero is inside the 95% HPDI for the distribution of $\theta_{1,1}$, though it is still outside of the 90% HPDI. This finding is similar to Gadea Rivas and Perez-Quiros (2015) who report similar results when the sample is restricted to terminate at the end of 2007. HMNM model with the full sample provides a picture with higher resolution as can been seen in the third column of the Table 2. First, the results indicate that larger credit to GDP ratio lead to longer recessions but also longer expansions. This implies that credit to GDP ratio acts as a smoothing mechanism for the business cycles but not necessarily a negative shock to the real economy. This evidence of smoothing behavior is also supported by the negative effect of credit ratio on the probability of entering into recovery state. An important finding is that this effect of the credit to GDP ratio could also be detected when the sample is restricted to terminate at the end of 2007 before the global economic crisis as can be seen in the fourth column of Table 2. HMNM model's capability of detecting these effects with both full sample and restricted sample implies that the link between credit to GDP ratio and business cycles was evident before the 2008 crisis as well. While it may seem confusing that the 2-regime MNM and HMNM models produce different results we note that the MNM specification involves components with normal distribution whereas HMNM model allows for nonnormality using the mixture structure also for the mixture components leading to more precise estimates.

Modeling rare events of business cycles separately reveals that countries with higher credit/GDP ratios have longer business cycles. This finding is intuitive. Credit ratio is used as a proxy for financial development in various studies (Schularick and Taylor, 2012; Rajan and Zingales, 1998). All else equal, financially developed countries have more advanced mechanisms to absorb shocks. Hence, small shocks that can put financially underdeveloped countries into recession state may be absorbed by the complex financial system of the developed countries, allowing the latter to stay in the expansion state longer (e.g. Fed increasing interest rates is unlikely to put Germany into recession, but it is more likely to cause a short recession in Turkey). Since small shocks are likely to cause short recession in developing countries, countries with high credit ratios also have longer recessions on average.

Credit serves as a propagator of shocks in various theoretical models (Bernanke, 1983; Gertler and Kiyotaki, 2010). After 2008, much emphasis has been made also about its ability to serve as an independent source of shock. We want to study the latter effect. The empirical problem is that real credit growth is an endogenous variable that contains the signals of these two opposing effects. On one hand, a positive credit growth implies that the financial markets are functioning well, which implies that either the economy did not face a negative shock of considerable size recently, or that it did receive a shock but is already recovering from it. In both cases, conditioning on positive RCG, the economy is expected to be in good health. Therefore, we include both the first lag of credit growth RCG_{t-1} and past credit accumulation $ARCG_{t-2}$, where $ARCG_t = \sum_{i=0}^{3} RCG_{t-i}$ in our model. Table 3 provides the results.

Table 3: Posterior results of the model with Annual Real Credit Growth (ARCG) and Real Credit Growth (RCG)

	MNM-TVTP Full sample	MNM-TVTP Until 2007 q4	HMNM-TVTP Full sample	HMNM-TVTP Until 2007 q4
p_{00}	2.305 (0.104)	1.807 (0.112)	1.864 (0.088)	1.478 (0.113)
$\theta_{0,1}$	**-0.026 (0.006)**	0.000 (0.008)	**-0.020 (0.005)**	0.009 (0.007)
$\theta_{0,2}$	**0.073 (0.014)**	**0.084 (0.022)**	**0.049 (0.013)**	**0.053 (0.021)**
p_{11}	0.193 (0.117)	0.576 (0.130)	0.453 (0.096)	0.468 (0.130)
$\theta_{1,1}$	0.004 (0.007)	0.016 (0.010)	0.004 (0.005)	**0.019 (0.009)**
$\theta_{1,2}$	-0.044 (0.031)	0.006 (0.016)	**-0.045 (0.019)**	0.008 (0.013)
q_0			-2.062 (0.117)	-2.081 (0.134)
$\gamma_{0,1}$			**0.017 (0.005)**	**0.015 (0.006)**
$\gamma_{0,2}$			-0.007 (0.011)	-0.019 (0.013)
q_1			1.675 (0.130)	2.401 (0.231)
$\gamma_{1,1}$			**-0.016 (0.007)**	0.006 (0.010)
$\gamma_{1,2}$			**0.043 (0.010)**	**0.077 (0.020)**

Note: The table presents posterior means and standard deviations (in parenthesis) of the coefficients for the with time varying transition probabilities, $p_{ii,t} = \Phi(p_{ii} + \theta_{i,1}ARCG_{t-2} + \theta_{i,2}RCG_{t-1})$ for transition from/to recession, $i = 1$, and expansion regimes, $i = 0$, and time varying probabilities, $q_{k,t} = \Phi(q_k + \gamma_{k,1}ARCG_{t-2} + \gamma_{k,2}RCG_{t-1})$ of switching to 'mild' or 'severe' $(1 - q_{k,t})$ recessions in recession regimes, $k = 1$ and 'recovery' or to 'secular growth' $(1 - q_{k,t})$ in the expansion regime, $k = 0$.

The first two columns show the result of MNM model with full and restricted samples. Looking at the first column, we see that the coefficients of RCG_{t-1} and $ARCG_{t-2}$ have different signs. The first panel indicates that whereas a high credit growth in the last period implies longer expansions, past credit accumulation implies shorter expansions. This is consistent with the discussion in the previous paragraph. Conditioned on a positive credit growth in the last period, the financial system is expected to be in good health, and expansion phase is expected to last longer. In contrast, cumulative credit growth in the previous year implies a shorter expansion period. We argue that this is the negative shock coming from the credit channel. Excessive credit in the economy will not be sustainable, and hence will cause the expansion period to last shorter. It is important to note that, as can be seen in the second column, this evidence was not visible before the 2008 crisis with MNM model which classifies the business cycle fluctuations only as homogeneous recessions and expansions. Whereas the coefficient of the RCG_{t-1} remains positive and significant, the coefficient of past credit accumulation is zero.

Similar to the previous case, HMNM provides richer results, which can be seen in the last two columns of Table 3. First, the full sample HMNM analysis detects the same effects as MNM model does. Additionally, it detects that high RCG_{t-1} signals shorter recession periods as it can be seen in the second panel of the column. MNM also finds a negative estimate, but cannot provide enough precision to find reliable evidence. This is consistent with our intuition. If the credit market is active despite the real economy being in recession, it means that the economy was either not hit by a shock big enough to freeze the credit markets, or it was hit by a significant shock but had enough time to recover from it. In either case, we expect the recession to end shortly. HMNM model also detects that past credit accumulation affects the transitory states of the economy. $ARCG_{t-2}$ increases the probability of both entering to recovery and to depression. In other words, if the economy is fed with credit and can sustain it (i.e. remain in expansion), then the economy is more likely to enter to recovery. If it cannot sustain it, then the economy is more likely to enter into depression. As the 2008 crisis and the subsequent depression originated with a credit boom that gone bust, it is not surprising that this effect was not visible before 2008. Nevertheless, HMNM can still detect that past credit accumulation prolongs the duration of recessions. In other words, **unlike its MNM counterpart, HMNM model can detect the negative shock coming from the credit channel even before the 2008 crisis**. Thus, disentangling the recession further into depressions and mild recessions provides a clearer picture of economic downturns throughout the sample. Furthermore, the probability of entering into depression decreases with positive credit growth in the last period. This is because depressions require an amplification of the initial shock through the credit channel. High RCG_{t-1} implies that credit markets are functioning well, hence depression is unlikely. We also control for credit/GDP ratio, results remain robust (table in the appendix).

If past credit accumulation possesses a negative shock to the economy, we would expect this shock to be higher for countries with higher leverage. To provide more evidence for causality, we define a proxy as a dummy variable that equals to one if credit ratio is above a certain threshold. In particular, we define the variable $75CR_{t-1}$ in the following way:

$$f(75CR_{c,t}) = \begin{cases} 1 & CR_{c,t} > prctile(CR, 75) \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

Because our sample consists of post-1960 OECD countries, we believe the 4th quartile should be a good proxy for high leverage in the country-quarter space. We also repeat this section by defining high leverage as observations that are above the mean and 90^{th} percentile of credit/GDP ratio, results are robust to these different specifications (Tables are in the appendix). Formally, we estimate the following transition probabilities:

$$\begin{aligned} p_{ii,t,c} &= \Phi(p_{ii} + \theta_{i,1} ARCG_{t-2} + \theta_{i,2} 75CR_{t-1,t,c} + \theta_{i,3} ARCG_{t-2} * 75CR_{t,c} + \theta_{i,4} RCG_{t-1,c}) \\ q_{k,t,c} &= \Phi(q_k + \gamma_{k,1} ARCG_{t-2} + \gamma_{k,2} 75CR_{t-1,t,c} + \gamma_{k,3} ARCG_{t-2} * 75CR_{t,c} + \gamma_{k,4} ARCG_{t-1,c}) \end{aligned} \tag{13}$$

Table 4: Posterior results of the model with $75CR_{t-1}$

	MNM-TVTP Full sample	MNM-TVTP Until 2007 q4	HMNM-TVTP Full sample	HMNM-TVTP Until 2007 q4
p_{00}	2.256 (0.105)	1.747 (0.141)	1.766 (0.083)	1.427 (0.099)
$\theta_{0,1}$	-0.021 (0.006)	0.000 (0.010)	-0.012 (0.005)	0.011 (0.007)
$\theta_{0,2}$	0.283 (0.212)	0.310 (0.215)	0.562 (0.263)	0.347 (0.181)
$\theta_{0,3}$	-0.023 (0.014)	0.014 (0.021)	-0.045 (0.017)	0.004 (0.017)
$\theta_{0,4}$	0.070 (0.015)	0.084 (0.023)	0.046 (0.011)	0.048 (0.018)
p_{11}	0.084 (0.153)	0.490 (0.145)	0.346 (0.109)	0.377 (0.133)
$\theta_{1,1}$	0.004 (0.011)	0.019 (0.013)	0.003 (0.005)	0.022 (0.010)
$\theta_{1,2}$	0.339 (0.250)	0.308 (0.290)	0.303 (0.186)	0.359 (0.246)
$\theta_{1,3}$	0.014 (0.023)	0.010 (0.031)	0.003 (0.015)	-0.003 (0.021)
$\theta_{1,4}$	-0.054 (0.033)	0.003 (0.019)	-0.050 (0.019)	0.007 (0.013)
q_0			-1.812 (0.116)	-1.891 (0.158)
$\gamma_{0,1}$			0.017 (0.005)	0.017 (0.006)
$\gamma_{0,2}$			-2.076 (1.254)	-5.071 (2.274)
$\gamma_{0,3}$			-0.009 (0.048)	0.011 (0.098)
$\gamma_{0,4}$			-0.005 (0.010)	-0.020 (0.013)
q_1			1.724 (0.154)	2.556 (0.523)
$\gamma_{1,1}$			-0.018 (0.008)	0.003 (0.014)
$\gamma_{1,2}$			-0.161 (0.259)	2.712 (1.909)
$\gamma_{1,3}$			0.009 (0.017)	-0.018 (0.089)
$\gamma_{1,4}$			0.045 (0.010)	0.108 (0.045)

Note: The table presents posterior means and standard deviations (in parenthesis) of the coefficients for the with time varying transition probabilities, $p_{ii,t,c} = \Phi(p_{ii} + \theta_{i,1}ARCG_{t-2} + \theta_{i,2}75CR_{t-1,t,c} + \theta_{i,3}ARCG_{t-2} * 75CR_{t,c} + \theta_{i,4}RCG_{t-1,c})$ for transition from/to recession, $i = 1$, and expansion regimes, $i = 0$, and time varying probabilities, $q_{k,t,c} = \Phi(q_k + \gamma_{k,1}ARCG_{t-2} + \gamma_{k,2}75CR_{t-1,t,c} + \gamma_{k,3}ARCG_{t-2} * 75CR_{t,c} + \gamma_{k,4}ARCG_{t-1,c})$ of switching to 'mild' or 'severe' $(1 - q_{k,t})$ recessions in recession regimes, $k = 1$ and 'recovery' or to 'secular growth' $(1 - q_{k,t})$ in the expansion regime, $k = 0$.

The coefficient estimates of $ARCG_{t-2}$ displayed in Table 4 show how past credit accumulation affects the transition probabilities for low leverage countries. In full sample, there is strong evidence that credit decreases time spent in expansions for low leverage countries. However, MNM model cannot detect a separate effect for high leverage countries. In contrast, HMNM with full sample finds strong evidence that past credit accumulation decreases time spent in expansions more for high leverage countries. This shows that high credit ratio makes the countries more vulnerable to the shocks coming from the credit channel. HMNM model also detects longer business cycles for countries with high leverage. This is consistent with our results in table 2. Besides the higher precision in the dynamics between the persistent states, HMNM does not detect any evidence for the interaction term in transitory states of the economy with neither full sample nor restricted sample. The rest of the coefficients are consistent with the previous results. Cumulative credit growth still increases the probability of entering into recovery and depression states. High leverage decreases the probabilities of recovery. We also find strong evidence for this effect with restricted sample and weak evidence with full sample. Results are robust to controlling for credit ratio (for all labels, tables are in the appendix).

The observation that past credit accumulation is more detrimental for countries with high leverage provides more evidence for a causal inference. To show further evidence for causality, we use other financial variables to control for the health of the economy (which we argue is done by the RCG_{t-1} in our main specification). In particular, we include the first lag of real stock return to capture the signal about the current health of the economy. If stock return can capture a stronger signal, then its first lag RSR_{t-1} can allow us to capture the negative effect of credit in RCG_{t-1}, the same way RCG_{t-1} allows us to capture the negative effect of past credit accumulation.

Formally, we estimate the following time varying transition probabilities.

$$\begin{aligned} p_{i,t} &= \Phi\Big(p_{ii} + \theta_{i,1} ARCG_{t-2} + \theta_{i,2} RCG_{t-1} + \theta_{i,3} RSR_{t-1}\Big) \\ q_{k,t} &= \Phi\Big(q_k + \gamma_{k,1} ARCG_{t-2} + \gamma_{k,2} RCG_{t-1} + \gamma_{k,3} RSR_{t-1}\Big) \end{aligned} \tag{14}$$

Table 5: Posterior results of the model with real stock returns

	MNM-TVTP Full sample	MNM-TVTP Until 2007 q4	HMNM-TVTP Full sample	HMNM-TVTP Until 2007 q4
p_{00}	2.739 (0.143)	2.105 (0.226)	2.374 (0.196)	1.808 (0.184)
$\theta_{0,1}$	-0.031 (0.007)	-0.006 (0.012)	-0.031 (0.008)	0.005 (0.011)
$\theta_{0,2}$	0.048 (0.017)	-0.040 (0.024)	-0.034 (0.016)	-0.069 (0.031)
$\theta_{0,3}$	0.044 (0.006)	0.082 (0.016)	0.087 (0.012)	0.075 (0.016)
p_{11}	-0.111 (0.208)	5.453 (1.452)	0.987 (0.159)	1.510 (0.329)
$\theta_{1,1}$	0.003 (0.011)	0.086 (0.032)	-0.015 (0.009)	0.016 (0.013)
$\theta_{1,2}$	0.023 (0.031)	0.064 (0.099)	0.054 (0.026)	0.056 (0.030)
$\theta_{1,3}$	-0.058 (0.017)	-0.628 (0.178)	-0.085 (0.016)	-0.156 (0.035)
q_0			-2.596 (0.238)	-7.889 (2.792)
$\gamma_{0,1}$			0.006 (0.009)	-0.155 (0.113)
$\gamma_{0,2}$			0.011 (0.028)	0.256 (0.135)
$\gamma_{0,3}$			0.025 (0.013)	-0.018 (0.061)
q_1			2.250 (0.207)	9.617 (2.494)
$\gamma_{1,1}$			-0.011 (0.007)	-0.032 (0.039)
$\gamma_{1,2}$			0.032 (0.015)	0.345 (0.150)
$\gamma_{1,3}$			0.036 (0.006)	0.050 (0.042)

Note: The table presents posterior means and standard deviations (in parenthesis) of the coefficients for the with time varying transition probabilities, $p_{i,t} = \Phi(p_{ii} + \theta_{i,1}ARCG_{t-2} + \theta_{i,2}RCG_{t-1} + \theta_{i,3}RSR_{t-1})$ for transition from/to recession, $i = 1$, and expansion regimes, $i = 0$, and time varying probabilities, $q_{k,t} = \Phi(q_k + \gamma_{k,1}ARCG_{t-2} + \gamma_{k,2}RCG_{t-1} + \gamma_{k,3}RSR_{t-1})$ of switching to 'mild' or 'severe' $(1-q_{k,t})$ recessions in recession regimes, $k = 1$ and 'recovery' or to 'secular growth' $(1-q_{k,t})$ in the expansion regime, $k = 0$.

Looking Table 5, we see that zero is out of 99% HPID for real stock return in all four specifications. A high stock return in the previous period signals longer expansions and shorter recessions. This is expected as stock returns are forward-looking. Controlling for RSR, in MNM model with full sample we don't see a difference in the signs of the estimates of coefficients. RCG_{t-1} still signals longer expansion periods and $ARCG_{t-2}$ signals shorter expansions. However, the coefficient of RCG_{t-1} changes sign with restricted sample, it no longer captures the health signal of the economy. As MNM model inherently models the recession observations with a normal distribution, observations from the far left tail of the distribution such as the 2008 crisis can drastically change the results. HMNM model supports this idea. This shows that disentangling the recessions into depressions and mild recessions not only buys precision but also prevents false inference. We also see that in full sample RCG_{t-1} still conveys information about the probability of depressions. Whereas RSR captures the signal about the general health of the economy, RCG_{t-1} still contains additional information about the tail risks. Controlling for RSR, the economy becomes more likely to enter to depression if the credit growth is negative.

Looking at rows 3 and 4, we see that the sign of the lagged credit growth is negative and zero is outside the 95% HPID. Once again, HMNM model captures dynamics in the persistent states that MNM model cannot. Looking at the transitory states, we see that in full sample RCG_{t-1} still conveys information about the probability of depressions. Whereas RSR captures the signal about the general health of the economy, first lag of credit growth still contains additional information about the tail risks. Controlling for real stock return, the economy becomes more likely to enter to depression if the credit growth is negative.

We add real stock return as a control variable to our previous study of high leverage countries as well (tables are in the appendix). We observe similar results. RSR proxies well for the current health of the economy and as a result the coefficient estimate of RCG_{t-1} changes. Our conclusions regarding the effect of cumulative credit growth and high leverage countries being more affected by credit growth remain robust.

Our results indicate that credit growth both depends on and can affect different phases of the economy. A natural extension is to see what kind of relationship there is between the business cycles and the change in the growth rate of credit. In an attempt to shed light on the creditless recoveries, Biggs et al. (2009) document that credit impulse (i.e. the change in the credit growth) explains expansions better than the credit growth. For reasons already discussed in the introduction, we believe analyzing these endogenous variables in an MS setting can provide more reliable answers. Formally, we estimate the following time varying transition probabilities.

$$
\begin{aligned}
p_{i,t} &= \Phi\Big(p_{ii} + \theta_{i,1} negCG_{t-1} + \theta_{i,2} negCG_{t-2,t-1} + \theta_{i,3} RCI_{t-1} + \theta_{i,4} RCI_{t-1} * negCG_{t-1})\Big) \\
q_{k,t} &= \Phi\Big(q_k + \gamma_{k,1} negCG_{t-1} + \gamma_{k,2} negCG_{t-2,t-1} + \gamma_{k,3} RCI_{t-1} + \gamma_{k,4} (RCI_{t-1} * negCG_{t-1})\Big)
\end{aligned} \tag{15}
$$

where $negCG_{t-1}$ is a dummy variable that equals to 1 if real credit growth was negative at time $t-1$, $negCG_{t-2,t-1}$ is a dummy variable that equals to 1 if real credit growth was negative at times $t-1$ and $t-2$, RCI_{t-1} is the first lag of real credit impulse, and the last variable is the interaction term between real credit impulse and sign of first lag of credit growth. We use this model specification because of several reasons. First, we want to understand whether RCI affects the business cycles differently in different phases of the credit cycle. If the credit impulse has an effect only when credit markets are not functioning properly, then this would appear in the results as an insignificant coefficient for RCI_{t-1} and a positive and significant coefficient for the interaction term.[3] Second, positive RCI and negative RCG at time $t-1$ imply that credit growth has been negative for two consecutive quarters. To isolate the effect of RCI, we need to control for $negCG_{t-2,t-1}$. Third, whether credit cannot grow for one period or two consecutive periods can interact differently with the transition probabilities. Whereas the former can be more relevant for the persistent states of the economy, the latter can be more relevant for the depression states. Table 6 provides the results.

Considering Table 6, we see that HMNM model detects strong evidence for the effect of $negCG_{t-1}$ in both phases of the economy while MNM model can only find weak evidence in expansion with full sample, and in recession with restricted sample. If the economy is in expansion despite having a negative credit growth in the last period, then the expected length of the expansion is longer. This might be because being in expansion state despite having negative credit growth in the last period signals that the economy is in the early stages of the business cycle. Hence the expected length of the expansion is longer. Notice that the results do not imply that negative credit growth is good for the economy. Negative credit growth only signals longer expansion periods if the economy is in expansion. If the economy is in recession, negative credit growth in the previous period signals longer recession.

[3] e.g. The burst of an asset bubble can freeze financial markets. But a positive RCI in the following period would signal that the credit markets are recovering, which can explain the positive relationship between the credit impulse and expansion periods Biggs et al. (2009) found.

Table 6: Posterior results of the model with real credit impulse (RCI)

	MNM-TVTP Full sample	MNM-TVTP Until 2007 q4	HMNM-TVTP Full sample	HMNM-TVTP Until 2007 q4
p_{00}	2.079 (0.089)	1.883 (0.116)	1.646 (0.078)	1.648 (0.115)
$\theta_{0,1}$	0.561 (0.315)	0.635 (0.526)	0.620 (0.255)	0.409 (0.331)
$\theta_{0,2}$	-0.404 (0.300)	-0.191 (0.690)	-0.028 (0.316)	0.117 (0.414)
$\theta_{0,3}$	0.041 (0.016)	-0.015 (0.023)	0.020 (0.014)	-0.026 (0.014)
$\theta_{0,4}$	0.035 (0.023)	0.106 (0.041)	0.063 (0.023)	0.103 (0.025)
p_{11}	-0.526 (0.213)	0.269 (0.190)	0.059 (0.140)	0.201 (0.157)
$\theta_{1,1}$	11.597 (7.259)	12.345 (7.481)	0.927 (0.428)	1.356 (1.234)
$\theta_{1,2}$	14.679 (7.822)	-11.727 (7.520)	-0.253 (0.458)	-0.962 (1.201)
$\theta_{1,3}$	0.030 (0.018)	-0.002 (0.023)	0.019 (0.014)	0.008 (0.016)
$\theta_{1,4}$	0.494 (0.310)	0.096 (0.219)	-0.015 (0.023)	0.059 (0.064)
q_0			-1.820 (0.096)	-1.843 (0.129)
$\gamma_{0,1}$			-0.677 (0.273)	-0.412 (0.302)
$\gamma_{0,2}$			-1.504 (0.910)	-2.071 (1.441)
$\gamma_{0,3}$			-0.035 (0.012)	-0.037 (0.016)
$\gamma_{0,4}$			-0.015 (0.023)	0.001 (0.026)
q_1			1.959 (0.206)	2.397 (0.374)
$\gamma_{1,1}$			0.245 (0.354)	5.486 (4.943)
$\gamma_{1,2}$			-1.287 (0.320)	-6.117 (4.606)
$\gamma_{1,3}$			-0.019 (0.016)	0.049 (0.046)
$\gamma_{1,4}$			0.058 (0.022)	0.157 (0.157)

Note: The table presents posterior means and standard deviations (in parenthesis) of the coefficients for the with time varying transition probabilities, $p_{i,t} = \Phi(p_{ii} + \theta_{i,1}negCG_{t-1} + \theta_{i,2}negCG_{t-2,t-1} + \theta_{i,3}RCI_{t-1} + \theta_{i,4}RCI_{t-1} * negCG_{t-1}))$ for transition from/to recession, $i = 1$, and expansion regimes, $i = 0$, and time varying probabilities, $q_{k,t} = \Phi(q_k + \gamma_{k,1}negCG_{t-1} + \gamma_{k,2}negCG_{t-2,t-1} + \gamma_{k,3}RCI_{t-1} + \gamma_{k,4}(RCI_{t-1} * negCG_{t-1}))$ of switching to 'mild' or 'severe' $(1 - q_{k,t})$ recessions in recession regimes, $k = 1$ and 'recovery' or to 'secular growth' $(1 - q_{k,t})$ in the expansion regime, $k = 0$.

Considering the effects of negative credit growth, we see that having two consecutive negative credit growth rate does not provide additional information regarding the length of expansions and recessions. However, HMNM model with full sample shows that negative credit growth in two consecutive periods Granger causes depressions, whereas only one period of negative credit growth does not affect depressions. This is consistent with the financial accelerator model (Bernanke, 1983). For depression to occur, credit markets must freeze for a significant amount of time, which works as an amplifier for the negative shock. It is assuring that HMNM model can detect this phenomenon, which further shows that our model is a good fit to study business cycles. HMNM model with restricted sample detects the correct signs but is not precise enough to find reliable evidence.

Considering the effects of the real credit impulse we see that RCI has an effect only when credit growth is negative. This is consistent with the previous discussion. MNM model with full sample does not find evidence for the interaction term. It does find evidence for the effect of RCI, but it cannot differentiate this effect for negative credit growth periods and positive credit growth periods. Furthermore, MNM model provides absurdly high coefficient estimates for $negCG_{t-1}$ and $negCG_{t-2,t-1}$. This is because these variables effect the tail events as seen in the HMNM model. MNM model's Gaussian assumption in persistent states, together with variables that affect the transitory tail phases, results in considerable imprecision and unrealistic estimates.

The positive sign of the interaction term in HMNM models (full sample and restricted sample) sheds more light onto the relationship between credit and business cycles. Let an economy be in expansion phase at time $t-2$. A negative demand shock arrives at time time $t-2$, which causes a negative credit growth in time $t-1$. both $RCt-1$ and $RCIt-1$ are negative, hence the coefficient in the interaction variable implies that even if the economy is not in recession yet, it is likely to enter into recession state at time t. This is expected as credit markets are frozen, which will aggregate the effect the shock. The real sector without credit is likely to enter into recession phase. One period later at time t, let the economy still be in expansion, and let credit growth be less negative. As RCI is now positive, the risk of entering into recession decreases. This is because the positive rebound in the credit growth signals that the credit markets are starting to recover from the shock. With no amplifying shock coming from the credit channel, the effect of the shock will disappear. Notice that RCI is only making up for two consecutive negative credit growth periods. It is better for the economy if the credit growth becomes positive after the shock, which is captured by the coefficient of $negCG1$. Only by conditioning on credit growth remaining negative does credit impulse have an effect. In other words, it is of second order importance, whereas credit growth is of first-order importance. Unlike Biggs et al. (2009), our results suggest that credit impulse is not more directly related with expansion periods than credit growth. It is only when credit cannot grow does the credit impulse have an effect.

Following the previous example, as the shock hits the economy at time $t-1$ when economy is already in recession, then it is more likely to enter into depression. At time t, if the credit growth is still negative, the coefficient of $negCG_{t-1,t-2}$ means that likelihood of depression is further increased. This is where the interaction variable provides valuable information. If the credit growth plummets further, then this signals that the economy in entering into a vicious circle. Credit markets are completely frozen, which will amplify the shock, hence the economy is more likely to enter into depression. In contrast, if the credit growth becomes less negative, then the positive RCI signals that financial markets are recovering from the shock, hence depression becomes less likely. By disentangling the recession further into depression and mild recessions, we can examine the anatomy of the financial accelerator and provide empirical evidence for its role in the modern business cycles. This provides further evidence that our model is a good fit to study business cycles.

5 Conclusion

Distribution of growth rates in expansion and recession periods are not normal: they are left and right skewed. Commonly used two state markov normal mixture models who typically assume a Gaussian distribution for the error terms do not fit the data well, and therefore cannot detect certain effects of credit on business cycles. In this paper, we apply Hierarchical Markov Normal Mixture (HMNM) model, which models tail episodes of expansion and recession phases (i.e. recoveries and depressions) explicitly, to provide robust evidence that past credit accumulation causes recessions. We employ various specifications to show that the credit channel is not only an early warning indicator but also has a causal effect on time spent in expansions. We also document the order of importance between the first derivative and the second derivative of credit. Our results are consistent with financial accelerator models and also imply that ignoring the different dynamics of tail events and "normal times" both costs precision and can lead to wrong inference.

Our results are of interest for the direction of future research in business cycles. The vast majority of the empirical literature analyzes the "normal" times of business cycles, probably because there is no consensus on the difference between normal recessions and depressions except for two events. HMNM model allows for both an endogenous determination of rare phases and a tractable analysis of time varying transition probabilities. This is why we believe it is a logical extension that future research can follow upon.

References

Albert, J. H. and S. Chib (1993). Bayesian Analysis of Binary and Polychotomous Response Data. *Journal of the American Statistical Association 88*(422), 669–679.

Amisano, G. and G. Fagan (2013). Money growth and inflation: A regime switching approach. *Journal of International Money and Finance 33*, 118–145.

Bernanke, B. S. (1983). Nonmonetary Effects of the Financial Crisis in Propagation of the Great Depression. *American Economic Review 73*(3), 257–276.

Biggs, M., T. Mayer, A. Pick, M. Biggs, T. Mayer, and A. Pick (2009). Credit and economic recovery.

Bry, G. and C. Boschan (1971). *Cyclical Analysis of Time Series: Selected Procedures and Computer Programs*. National Bureau of Economic Research, Inc.

Christiano, L., M. Rostagno, and R. Motto (2010). Financial factors in economic fluctuations. *Working Paper Series*.

Eichengreen, B. and C. Arteta (2001). Banking Crises in Emerging Markets: Presumptions and Evidence. *Macroeconomics*.

Frühwirth-Schnatter, S. (2006). *Finite Mixture and Markov Switching Models*. Springer Series in Statistics. Springer New York.

Gadea Rivas, M. D. and G. Perez-Quiros (2015). The Failure to Predict the Great Recession—A View through the Role of Credit. *Journal of the European Economic Association 13*(3), 534–559.

Geisser, S. (1965). A bayes approach for combining correlated estimates. *Journal of the American Statistical Association* (60), 602–607.

Geman, S. and D. Geman (1984). Stochastic relaxations, gibbs distributions, and the bayesian restoration of images. *IEEE Transaction on Pattern Analysis and Machine Intelligence 6*, 721–741.

Gertler and Mark (1988). Financial Structure and Aggregate Economic Activity: An Overview. *Journal of Money, Credit and Banking 20*(3), 559–588.

Gertler, M. and N. Kiyotaki (2010). Financial Intermediation and Credit Policy in Business Cycle Analysis. *Handbook of Monetary Economics 3*, 547–599.

Geweke, J. (2007, April). Interpretation and inference in mixture models: Simple mcmc works. *Computational Statistics & Data Analysis 51*(7), 3529–3550.

Geweke, J. and G. Amisano (2011). Hierarchical Markov normal mixture models with applications to financial asset returns. *Journal of Applied Econometrics 26*(1), 1–29.

Gourinchas, P.-O. and M. Obstfeld (2012). Stories of the Twentieth Century for the Twenty-First. *American Economic Journal: Macroeconomics 4*(1), 226–265.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the Econometric Society*, 357–384.

Harding, D. and A. Pagan (2002). A comparison of two business cycle dating methods. *Journal of Economic Dynamics and Control 27*(9), 1681–1690.

Jorda, O., M. Schularick, and A. M. Taylor (2013). When Credit Bites Back. *Journal of Money, Credit and Banking 45*(s2), 3–28.

Kaminsky, G. L. and C. M. Reinhart (1999). The Twin Crises: The Causes of Banking and Balance-of-Payments Problems. *American Economic Review 89*(3), 473–500.

Kim, C.-J. and C. R. Nelson (1999). *State-space models with regime switching : classical and Gibbs-sampling approaches with applications.* MIT Press.

Laeven, L. and F. Valencia (2012). Resolution of banking crises. In *Handbook of Safeguarding Global Financial Stability*, pp. 231–258. Elsevier.

Mendoza, E. and M. Terrones (2008). An Anatomy Of Credit Booms: Evidence From Macro Aggregates And Micro Data. Technical report, National Bureau of Economic Research, Cambridge, MA.

Mendoza, E. and M. Terrones (2012). An Anatomy of Credit Booms and their Demise. Technical report, National Bureau of Economic Research, Cambridge, MA.

Mishkin, F. S. (1978). The Household Balance Sheet and the Great Depression. *The Journal of Economic History 38*(04), 918–937.

Rajan, R. G. and L. Zingales (1998). Financial Dependence and Growth. *The American Economic Review 88*(3), 559–586.

Reinhart, C. M. and K. S. Rogoff (2009). *This time is different : eight centuries of financial folly.* Princeton University Press.

Schularick, M. and A. M. Taylor (2012). Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises. *American Economic Review 102*(2), 1029–1061.

Tanner, M. A. and W. H. Wong (1987). The calculation of posterior distributions by data augmentation. *Journal of the American Statistical Association 82*, 528–550.

A Bayesian Inference of the HMNM model

Following Kim and Nelson (1999), using arbitrary starting values for the parameters of the model, the following three steps are repeated until convergence.

Step 1: Generate $\tilde{s_t}$ by using $g(\tilde{s_t}|\mu, h^{-1}, p, r, \tilde{y_T})$ using Multimove Gibbs Sampling

Step 2: Generate the transition probabilities p, r from $g(p, r|\tilde{s_t})$

Step 3: Generate μ and h^{-1} from $g(\mu, h^{-1}|\tilde{y_T}\tilde{s_t})$

A.1 Simulation of the discrete state variables

Following the derivations in Kim and Nelson (1999),

$$g(\tilde{s_t}|\tilde{y_T}) = g(s_t|\tilde{y_T}) \prod_{t=1}^{T-1} g(s_t|S_{t+1}, \tilde{y_T}) \tag{16}$$

Equation 16 suggests the following two steps to generate $\tilde{s_t}$

Step 1: Run Hamilton's filter to obtain filtered probabilities $g(s_t|\tilde{y}_t$, t=1,2, ..., T, and save them (Hamilton, 1989). In order to generate s^1 independently of s^2, we integrate out s^2 from the filtered probabilities:

$$g(s_t^1|y_t) = \sum_{i=0}^{1} g(s_t^1, s_t^2 = i|y_t) \tag{17}$$

This can be done by simply adding the columns that represent states s_{00}, s_{01} and s_{10}, s_{11}, which creates the filtered probabilities of s^1. The last iteration of the filter provides $g(s_t|\tilde{y_T})$, from which s_t^1 is generated.

Step2: The generate s_t^1 conditioned on s_{t+1}^1 and y_t, $t = T-1, \ldots, 1$, I employ the following result

$$
\begin{aligned}
g(s_t^1|\tilde{y}_t, s_{t+1}^1) &= \frac{g(s_t^1, s_{t+1}^1|\tilde{y}_t)}{g(s_{t+1}^1|\tilde{y}_t)} \\
&= \frac{g(s_{t+1}^1|s_t, \tilde{y}_t)g(s_t^1, |\tilde{y}_t)}{g(s_{t+1}^1|\tilde{y}_t)} \\
&= \frac{g(s_{t+1}^1|s_t^1)g(s_t^1, |\tilde{y}_t)}{g(s_{t+1}^1|\tilde{y}_t)} \\
&\propto g(s_{t+1}^1|s_t^1)g(s_t^1|\tilde{y}_t)
\end{aligned} \tag{18}
$$

where $g(s_{t+1}^1|s_t^1)$ is the transition probability, and $g(s_t^1|\tilde{y}_t)$ has been calculated during step 1. Equation 18 suggests that the smoothed probabilities can be calculated as:

$$
Pr(s_t^1 = 0|s_{t+1}^1, \tilde{y}_t) = \frac{g(s_{t+1}^1|s_t = 0)g(s_t^1 = 0|\tilde{y}_t)}{\sum_{j=0}^{1} g(s_{t+1}^1|s_t^1 = j)g(s_t^1 = j|\tilde{y}_t)} \tag{19}
$$

Once s_t^1's are generated, we generate the s_t^2's by:

$$
g(s_t^2|s_t^1, y_t) = \frac{g(s_t^1, s_t^2|y_t)}{g(s_t^1|y_t)} = \frac{g(s_t^1, s_t^2|y_t)}{\sum_{i=0}^{1} g(s_t^1, s_t^2 = i|y_t)} \tag{20}
$$

where $g(s_t^1, s_t^2)$ is already saved from step 1.

Initializing the filter

A special problem arising in this model is how to initialize the filter. The common method is initialize the filter from the steady state probabilities, but since the probabilities are time varying depending on the matrix of early waring indicators x, defining a steady state is not straight forward. Following Amisano and Fagan (2013) we let the indicator variable assume its unconditional mean:

$$
x_t^{(0)} = \bar{x} \tag{21}
$$

We then compute

$$
P^{(0)} = \begin{bmatrix} \Phi(p_0 + \theta_0'\bar{x}) & 1 - \Phi(p_0 + \theta_0'\bar{x}) \\ 1 - \Phi(p_1 + \theta_1'\bar{x}) & \Phi(p_1 + \theta_1'\bar{x}) \end{bmatrix} \tag{22}
$$

$$R^{(0)} = \begin{bmatrix} \Phi(r_0 + \gamma_0'\bar{x}) & 1 - \Phi(r_0 + \gamma_0'\bar{x}) \\ 1 - \Phi(r_1 + \gamma_1'\bar{x}) & \Phi(r_1 + \gamma_1'\bar{x}) \end{bmatrix} \tag{23}$$

Using $P^{(0)}$ and $R^{(0)}$, we calculate $P^{*,0}$ and calculate the steady state probabilities to start the filter.

Drawing indicator functions

We generate the states from the smoothed probabilities. Following the derivations in equation (18), we know that:

$$g(s_t^1|\tilde{y}_t, s_{t+1}^1) \propto g(s_{t+1}^1|s_t^1)g(s_t^1|\tilde{y}_t) \tag{24}$$

This means that we can draw s_t^1 by drawing s_T^1 from the filtered distribution and iterate backwards using the probabilities in 24 which depend only on the filtered probabilities and the transition probabilities that are known.

Once s_t^1's are generated, we generate the s_t^2's by:

$$g(s_t^2|s_t^1, y_t) = \frac{g(s_t^1, s_t^2|y_t)}{g(s_t^1|y_t)} = \frac{g(s_t^1, s_t^2|y_t)}{\sum_{i=0}^1 g(s_t^1, s_t^2 = i|y_t)} \tag{25}$$

A.1.1 Simulation of the parameters

Let us collect the parameters of the model in two subsets:

1. the parameters describing the conditional mean and variance of the dependent variable μ and h

2. the parameters determining the transition probabilities $\Gamma = \{p_{00}, p_{11}, \theta, r_0, r_1, \gamma\}$

We then use a 2 block Gibbs sampling algorithm to draw from each block of parameters. Conditional of the state variables, parameters $\{\mu, h\}$ are independent of Γ. We assume independent normal-gamma priors for μ $(\underline{\mu}, \underline{V})$ and h $(\underline{s}^{-2}, \underline{v})$. We take consecutive draws from:

$$\mu | y, h \sim N(\bar{\mu}, \bar{V})$$
$$h | y, \mu \sim G(\bar{s}^{-2}, \bar{v}) \tag{26}$$

where

$$\bar{v} = N + \underline{v}$$
$$\bar{s}^2 = \frac{(y - \mu_{s_t})'(y - \mu_{s_t}) + \underline{vs}^2}{\bar{v}} \tag{27}$$

For Γ, we follow a slightly adjusted version of Albert and Chib (1993). Assume that the model is given by:

$$Pr(s_t = 1) = \Phi(X'\beta) \tag{28}$$

In this case there is only one process that determines $s_t \in \{0, 1\}$, one can simulate latent vector of sign variables:

$$\underline{s}_T^* = \{s_t^*, t = 1, 2, ..., T\}$$
$$s_t^* \sim N(X'\beta, 1) \tag{29}$$
$$s_t^* > 0 \iff s_t = 1$$

In this case, $s_t = 1$ has an interpretation of "success". The way s_t^* is defined ensures that $pr(s_t^* > 0) = pr(s_t = 1)$. Conditioned on $\underline{s}_T$, s_t^* has a truncated normal distribution, truncated either to the left if $s_t = 0$ or to the right if $s_t = 1$. Once s^* are generated, β is estimated via linear regression.

Because in our case there are two processes that determine the likelihood of states, the definition of 'success' is slightly different:

$$s_t^* > 0 \iff s_{t-1}^1 = s_t^1 \tag{30}$$

This is because the transition probabilities that affect the outcome of s_t^1 are determined by the state at time t-1, in particular, by $\theta_i, p_i, s_{t-1}^1 = i$. Therefore, $\mathbf{s*}$ is generated according to:

$$
\begin{aligned}
&if \quad s^1_{t-1} = 0 \,\&\, s^1_t = 0,\ s^*_t \sim N(p_0 + \theta_0 x_{t-1}) 1(s^*_t > 0) \\
&if \quad s^1_{t-1} = 0 \,\&\, s^1_t = 1,\ s^*_t \sim N(p_0 + \theta_0 x_{t-1}) 1(s^*_t \leq 0) \\
&if \quad s^1_{t-1} = 1 \,\&\, s^1_t = 0,\ s^*_t \sim N(p_1 + \theta_1 x_{t-1}) 1(s^*_t \leq 0) \\
&if \quad s^1_{t-1} = 1 \,\&\, s^1_t = 1,\ s^*_t \sim N(p_1 + \theta_1 x_{t-1}) 1(s^*_t > 0)
\end{aligned} \tag{31}
$$

Once s^*_t is generated, we have a linear model with IID standard Gaussian disturbances. We use noninformative priors in this case, which is equivalent to OLS.

Since s^*_t depends on the states s^1_t and s^1_{t-1}, defining s^*_1 requires us to generate s^1_0. We generate s^1_0 from the steady state probabilities by using the unconditional mean of $\mathbf{x}$.

Similarly, generating $\mathbf{r}$ and γ also requires simulating the latent vector of sign variables:

$$
\begin{aligned}
&if \quad s^1_t = i \,\&\, s^2_t = i,\ s^*_t \sim N(r_i + \gamma_i x_{t-1}) 1(s^*_t > 0) \\
&if \quad s^1_t = i \,\&\, s^2_t = j,\ s^*_t \sim N(r_i + \gamma_i x_{t-1}) 1(s^*_t \leq 0)
\end{aligned} \tag{32}
$$

A.2 Label switching

Likelihood in Markov Switching Models are invariant to permutations in the labeling of the discrete states (**?**). Whereas this is not a problem for the estimation, it complicates assigning the discrete states a structural interpretation. Since we interpret the regimes as "recovery", "expansion", "recession", and "depression", we impose the constraint:

$$
\mu_{00} > \mu_{01} > 0 > \mu_{10} > \mu_{11} \tag{33}
$$

B Table Appendix

Table 7: Results for credit growth (RCG) lags 1, 2, 3, 4

RCG_{t-2}		RCG_{t-3}		RCG_{t-4}				RCG_{t-1}		RCG_{t-2}		RCG_{t-3}		RCG_{t-4}	
δ_0	δ_1	γ_0	γ_1	δ_0	δ_1	r_0	r_1	α_0	α_1	δ_0	δ_1	γ_0	γ_1	δ_0	δ_1
MNM TIME-VARYING PROBABILITY															
.021	**-0.105**	**-0.035**	**0.066**	-0.024	0.012										
012)	**(0.033)**	**(0.015)**	**(0.030)**	(0.013)	(0.027)										
MNM TIME-VARYING PROBABILITY (2007-4)															
.038	-0.025	0.000	0.043	0.015	0.034										
017)	(0.029)	(0.020)	(0.029)	(0.016)	0.043										
HMNM TIME-VARYING PROBABILITY															
.029	-0.028	**-0.029**	0.007	0.001	**0.042**	-2.075	2.009	-0.003	**0.051**	**0.041**	**0.030**	0.005	**-0.042**	0.002	**-0.080**
013)	(0.017)	**(0.014)**	(0.017)	(0.012)	**(0.018)**	(0.109)	(0.171)	(0.012)	**(0.011)**	**(0.012)**	**(0.012)**	(0.010)	**(0.018)**	(0.010)	**(0.021)**
HMNM TIME-VARYING PROBABILITY (20074)															
.027	-0.002	0.006	0.026	0.023	0.045	-2.080	2.696	-0.016	**0.079**	**0.038**	0.038	0.002	-0.032	0.001	0.009
020)	(0.022)	(0.016)	(0.023)	(0.016)	(0.031)	(0.128)	(0.474)	(0.015)	**(0.027)**	**(0.014)**	(0.024)	(0.013)	(0.031)	(0.013)	(0.037)

Table 19: CR1 with duration dependence

		CR1		DD				CR1	
p_{00}	p_{11}	α_0	α_1	γ_0	γ_1	r_0	r_1	α_0	α_1
MNM TIME-VARYING PROBABILITY									
8.6719	0.4110	0.0007	**0.0071**	-1.6737	**-0.4305**				
4.8597	0.4026	0.0010	**0.0026**	1.2084	**0.1547**				
MNM TIME-VARYING PROBABILITY 2007-4									
8.2239	6.4260	0.0003	0.0053	-1.5857	-1.6385				
6.3127	3.6121	0.0015	0.0027	1.5801	0.9121				
HMNM TIME-VARYING PROBABILITY									
19.3155	3.6153	-0.0007	0.0023	**-4.3510**	-0.8635	1.8802	1.9551	0.0005	0.0005
4.8637	4.0904	0.0008	0.0012	**1.2157**	1.0309	0.2506	0.3724	0.0019	0.0028
HMNM TIME-VARYING PROBABILITY 2007-4									
5.065	10.342	0.000	0.0031	-0.804	**-2.522**	2.034	2.091	0.004	0.004
2.394	4.096	0.001	0.0018	0.601	**1.032**	0.446	0.562	0.004	0.005

Table 12: Results for high leverage analysis (50CR, controlling for CR)

		$ARCG_{t-2}$		$50CR_{t-1}$		$ARCG_{t-2}$*$50CR_{t-1}$		RCG_{t-1}		CR_{t-1}				$ARCG_{t-2}$		$50CR_{t-1}$		$ARCG_{t-2}$*$50CR_{t-1}$		RCG_{t-1}		CR_{t-1}	
p_{00}	p_{11}	α_0	α_1	δ_0	δ_1	γ_0	γ_1	δ_0	δ_1	θ_0	θ_1	r_0	r_1	α_0	α_1	δ_0	δ_1	γ_0	γ_1	δ_0	δ_1	θ_0	θ_1
MNM TIME-VARYING PROBABILITY																							
2.187	-0.469	**-0.016**	-0.005	0.429	0.346	**-0.024**	0.043	**0.069**	-0.054	-0.001	0.003												
0.184	0.345	**0.006**	0.010	0.233	0.395	**0.011**	0.023	**0.013**	0.035	0.002	0.003												
MNM TIME-VARYING PROBABILITY (2007-4)																							
1.810	0.149	0.008	0.014	**0.806**	0.361	-0.004	0.028	**0.071**	-0.005	-0.004	0.002												
0.305	0.406	0.010	0.014	**0.289**	0.433	0.017	0.039	**0.022**	0.023	0.003	0.005												
HMNM TIME-VARYING PROBABILITY																							
1.622	-0.124	-0.009	-0.004	**0.438**	0.451	**-0.024**	**0.026**	**0.052**	**-0.038**	0.000	0.002	-1.582	1.830	**0.011**	-0.017	**-1.393**	0.020	-0.007	0.000	-0.001	**0.044**	0.003	-0.001
0.179	0.222	0.007	0.006	**0.203**	0.277	**0.011**	**0.012**	**0.012**	**0.018**	0.002	0.002	0.169	0.278	**0.005**	0.010	**0.308**	0.357	0.016	0.016	0.010	**0.011**	0.002	0.003
HMNM TIME-VARYING PROBABILITY (20074)																							
1.284	-0.178	**0.013**	0.012	**0.604**	0.151	-0.006	0.010	**0.047**	0.001	-0.001	0.005	-2.095	5.896	0.012	0.005	**-1.713**	**6.935**	-0.019	-0.009	-0.011	**0.122**	**0.009**	**-0.042**
0.231	0.311	**0.006**	0.009	**0.255**	0.336	0.012	0.015	**0.017**	0.012	0.003	0.004	0.244	1.945	0.007	0.019	**0.392**	**2.595**	0.017	0.062	0.012	**0.035**	**0.003**	**0.019**

Table 20: Annual Cumulative CG with duratilon dependence

		ARCG2		RCG1		DD				ARCG2		RCG1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1
MNM TIME-VARYING PROBABILITY													
17.428	0.906	**-0.026**	-0.001	**0.075**	-0.055	-3.769	**-0.309**						
7.811	0.362	**0.006**	0.008	**0.015**	0.038	1.955	**0.146**						
MNM TIME-VARYING PROBABILITY 2007-4													
15.716	4.396	0.000	0.025	**0.087**	0.020	**-3.459**	-0.954						
6.350	2.595	0.008	0.013	**0.019**	0.028	**1.588**	0.653						
HMNM TIME-VARYING PROBABILITY													
11.209	1.335	-0.007	0.006	0.014	-0.015	**-2.355**	-0.173	2.187	2.243	-0.020	-0.019	**0.059**	0.052
4.377	0.658	0.004	0.004	0.008	0.010	**1.095**	0.173	0.171	0.203	0.010	0.013	**0.018**	0.027
HMNM TIME-VARYING PROBABILITY 2007-4													
11.507	6.711	0.004	0.009	0.009	0.006	**-2.442**	**-1.496**	2.815	2.839	0.004	0.004	**0.071**	0.067
4.558	2.827	0.004	0.006	0.009	0.012	**1.139**	**0.711**	0.329	0.358	0.019	0.021	**0.030**	0.039

Table 21: Add caption

		ARCG2		RCG1		CR1		DD				ARCG2		RCG1		CR1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	δ_0	δ_1	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	δ_0	δ_1
MNM TIME-VARYING PROBABILITY																	
23.775	0.329	**-0.025**	-0.001	**0.075**	-0.050	0.000	**0.007**	**-5.366**	**-0.466**								
3.375	0.448	**0.005**	0.013	**0.014**	0.039	0.001	**0.003**	**0.848**	**0.205**								
MNM TIME-VARYING PROBABILITY 2007-4																	
7.0219	22.2858	0.0004	**0.0288**	**0.0798**	0.0383	0.0004	0.0036	**-1.2980**	**-5.5347**								
2.3122	9.6108	0.0081	**0.0132**	**0.0197**	0.0293	0.0016	0.0027	**0.5725**	**2.4001**								
HMNM TIME-VARYING PROBABILITY																	
8.109	1.072	**-0.008**	0.005	**0.015**	-0.018	-0.001	**0.002**	-1.545	-0.222	2.331	2.376	-0.021	-0.020	**0.062**	**0.057**	-0.001	-0.001
5.224	0.522	**0.004**	0.005	**0.007**	0.010	0.001	**0.001**	1.301	0.145	0.311	0.415	0.011	0.013	**0.020**	**0.029**	0.002	0.003
HMNM TIME-VARYING PROBABILITY 2007-4																	
17.2444	26.0967	0.0036	0.0093	0.0082	0.0077	-0.0003	0.0027	-3.8553	-6.4398	2.7864	2.8223	0.0125	0.0122	**0.0929**	**0.0883**	0.0037	0.0035
8.1882	4.5659	0.0042	0.0065	0.0093	0.0133	0.0011	0.0019	2.0447	1.1419	0.8870	0.9520	0.0204	0.0222	**0.0359**	**0.0449**	0.0077	0.0084

Table 22: Add caption

		ARCG2		RCG1		RSR1		DD				ARCG2		RCG1		RSR1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	δ_0	δ_1	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	δ_0	δ_1
MNM TIME-VARYING PROBABILITY																	
2.632	-0.009	**-0.032**	0.002	**0.048**	0.022	**0.043**	**-0.057**	0.033	-0.061								
1.525	0.529	**0.008**	0.011	**0.018**	0.033	**0.007**	**0.017**	0.382	0.246								
MNM TIME-VARYING PROBABILITY 2007-4																	
3.540	7.906	0.003	0.023	-0.055	**0.125**	**0.089**	**-0.394**	-0.346	**-1.221**								
1.307	2.273	0.014	0.027	0.030	**0.049**	**0.018**	**0.074**	0.320	**0.489**								
HMNM TIME-VARYING PROBABILITY																	
6.323	0.756	-0.008	0.001	-0.007	0.005	**0.019**	**-0.020**	-1.123	-0.026	4.205	4.242	**-0.059**	**-0.058**	0.008	0.010	**0.116**	**0.114**
2.591	0.448	0.005	0.006	0.010	0.014	**0.004**	**0.006**	0.648	0.132	0.681	0.693	**0.018**	**0.021**	0.031	0.043	**0.026**	**0.028**
HMNM TIME-VARYING PROBABILITY 2007-4																	
10.574	3.224	0.003	0.007	-0.014	0.017	**0.019**	**-0.023**	-2.220	-0.639	4.447	4.458	-0.026	-0.026	0.088	0.087	0.069	0.069
8.195	2.725	0.006	0.007	0.014	0.017	**0.006**	**0.008**	2.057	0.704	0.817	0.837	0.032	0.033	0.066	0.076	0.038	0.042

Table 23: Add caption

		ARCG2		50CR		ARCG2*50CR		RCG1		DD				ARCG2		50CR		ARCG2*50CR		RCG1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$
MNM TIME-VARYING PROBABILITY																					
3.628	0.651	**-0.019**	-0.009	0.335	**0.944**	-0.020	0.028	**0.071**	-0.056	-0.357	**-0.439**										
2.554	0.419	**0.007**	0.010	0.175	**0.377**	0.011	0.020	**0.013**	0.038	0.638	**0.198**										
MNM TIME-VARYING PROBABILITY 2007-4																					
5.794	3.193	0.006	0.012	0.387	0.572	-0.003	0.026	**0.079**	0.006	-1.040	-0.761										
4.093	3.722	0.012	0.015	0.198	0.399	0.016	0.030	**0.023**	0.032	1.012	0.916										
HMNM TIME-VARYING PROBABILITY																					
15.737	1.431	-0.005	0.003	-0.003	**0.284**	-0.008	0.007	**0.016**	-0.017	**-3.471**	-0.269	2.087	2.148	-0.017	-0.016	0.274	0.254	-0.007	-0.006	**0.056**	0.051
4.517	0.710	0.005	0.006	0.102	**0.143**	0.007	0.009	**0.008**	0.011	**1.127**	0.188	0.171	0.238	0.011	0.015	0.282	0.376	0.021	0.028	**0.019**	0.028
HMNM TIME-VARYING PROBABILITY 2007-4																					
7.0914	17.2485	0.0038	0.0082	0.0632	0.1935	0.0011	0.0044	0.0081	0.0074	-1.3376	**-4.1716**	2.8082	2.8422	0.0113	0.0101	**2.7042**	**2.6746**	-0.0048	-0.0029	**0.0991**	**0.0956**
2.9046	7.1361	0.0050	0.0085	0.1136	0.1739	0.0087	0.0126	0.0093	0.0128	0.7290	**1.7872**	0.5300	0.5616	0.0247	0.0273	**0.8108**	**0.8632**	0.0570	0.0607	**0.0393**	**0.0486**

Table 24: Add caption

		ARCG2		50CR		ARCG2*50CR		RCG1		CR1		DD				ARCG2		50CR		ARCG2*50CR		RCG1		CR1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1
MNM TIME-VARYING PROBABILITY																									
12.7973	0.4779	**-0.0181**	-0.0081	0.5074	0.4844	-0.0208	0.0275	**0.0690**	-0.0707	-0.0020	0.0043	-2.6081	**-0.4847**												
6.8985	0.5608	**0.0067**	0.0154	0.2657	0.5636	0.0116	0.0236	**0.0129**	0.0379	0.0021	0.0044	1.7230	**0.2341**												
MNM TIME-VARYING PROBABILITY 2007-4																									
8.3071	4.4357	0.0052	0.0141	**0.7443**	0.3931	-0.0031	0.0226	**0.0765**	0.0118	-0.0049	0.0016	-1.5724	-1.0826												
5.4767	4.2789	0.0109	0.0156	**0.2947**	0.4924	0.0150	0.0240	**0.0207**	0.0315	0.0029	0.0047	1.3597	1.0620												
HMNM TIME-VARYING PROBABILITY																									
9.5539	1.2806	-0.0058	0.0023	0.0952	0.1663	-0.0083	0.0070	**0.0160**	-0.0176	-0.0011	0.0013	-1.8991	-0.2711	2.5457	2.5838	-0.0172	-0.0169	0.8153	0.7647	-0.0115	-0.0101	**0.0645**	0.0580	-0.0052	-0.0048
4.3843	0.6265	0.0046	0.0057	0.1528	0.2468	0.0069	0.0096	**0.0078**	0.0106	0.0013	0.0021	1.0933	0.1689	0.3949	0.5150	0.0129	0.0159	0.5409	0.6857	0.0233	0.0292	**0.0234**	0.0327	0.0043	0.0057
HMNM TIME-VARYING PROBABILITY 2007-4																									
7.4367	11.5925	0.0048	0.0066	0.3071	-0.0266	0.0000	0.0047	0.0081	0.0062	-0.0029	0.0029	-1.3779	**-2.8172**	4.2163	4.2049	0.0012	0.0009	3.1465	3.0613	-0.0215	-0.0206	**0.0990**	**0.0941**	-0.0159	-0.0153
3.8463	5.5069	0.0051	0.0079	0.1846	0.3023	0.0086	0.0120	0.0094	0.0127	0.0019	0.0032	0.9639	**1.3784**	1.1808	1.2572	0.0257	0.0285	1.7049	1.7683	0.0565	0.0599	**0.0365**	**0.0456**	0.0149	0.0158

Table 25: Add caption

		ARCG2		50CR		ARCG2*50CR		RCG1		RSR1		DD				ARCG2		50CR		ARCG2*50CR		RCG1		RSR1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1
MNM TIME-VARYING PROBABILITY																									
2.864	0.026	**-0.026**	-0.006	-0.054	0.413	-0.023	0.025	**0.047**	0.025	**0.046**	**-0.070**	0.008	-0.239												
1.403	0.754	**0.010**	0.020	0.254	0.491	0.015	0.029	**0.020**	0.043	**0.007**	**0.027**	0.344	0.429												
MNM TIME-VARYING PROBABILITY 2007-4																									
4.487	11.148	-0.023	0.054	-0.461	0.421	0.024	0.175	-0.039	0.129	**0.102**	**-0.895**	-0.461	-0.928												
2.012	4.580	0.019	0.059	0.354	1.538	0.024	0.093	0.023	0.127	**0.018**	**0.347**	0.489	0.886												
HMNM TIME-VARYING PROBABILITY																									
9.190	0.744	-0.005	-0.005	-0.141	0.041	-0.008	0.012	**-0.008**	0.005	**0.020**	**-0.020**	-1.813	-0.039	5.993	6.024	**-0.094**	**-0.095**	-1.101	-1.102	0.049	0.051	0.007	0.008	**0.169**	**0.167**
6.740	0.463	0.006	0.008	0.119	0.160	0.009	0.012	**0.010**	0.015	**0.004**	**0.006**	1.683	0.137	1.280	1.296	**0.031**	**0.034**	0.590	0.666	0.041	0.046	0.037	0.048	**0.050**	**0.052**
HMNM TIME-VARYING PROBABILITY 2007-4																									

Table 26: Add caption

		ARCG2		75CR		ARCG2*75CR		RCG1		DD				ARCG2		75CR		ARCG2*75CR		RCG1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$
MNM TIME-VARYING PROBABILITY																					
5.9266	0.8963	**-0.0215**	-0.0027	0.3042	0.4339	-0.0264	0.0055	**0.0727**	-0.0640	-0.9055	**-0.3824**										
3.3366	0.4252	**0.0062**	0.0094	0.2305	0.3185	0.0152	0.0234	**0.0150**	0.0412	0.8366	**0.1907**										
MNM TIME-VARYING PROBABILITY 2007-4																					
6.288	3.518	-0.003	0.022	0.218	0.388	0.020	0.028	**0.088**	0.016	-1.107	-0.766										
2.586	2.407	0.009	0.015	0.215	0.352	0.020	0.037	**0.020**	0.028	0.647	0.605										
HMNM TIME-VARYING PROBABILITY																					
9.3433	1.3114	-0.0048	0.0051	-0.0012	0.1578	-0.0139	0.0013	0.0146	-0.0168	**-1.8783**	-0.1976	2.2227	2.2764	-0.0224	-0.0216	0.0066	0.0036	0.0076	0.0079	**0.0605**	0.0553
2.9240	0.5646	0.0041	0.0050	0.1177	0.1510	0.0082	0.0104	0.0079	0.0103	**0.7307**	0.1522	0.1877	0.2302	0.0117	0.0148	0.3759	0.4511	0.0261	0.0327	**0.0194**	0.0292
HMNM TIME-VARYING PROBABILITY 2007-4																					
7.100	9.510	0.003	0.010	0.008	0.138	0.005	0.000	0.007	0.007	-1.332	**-2.216**	2.825	2.852	0.001	0.001	1.687	1.656	0.002	0.003	**0.091**	**0.087**
3.560	4.110	0.005	0.007	0.129	0.188	0.010	0.013	0.009	0.012	0.895	**1.035**	0.379	0.414	0.020	0.022	1.308	1.355	0.079	0.082	**0.035**	**0.044**

Table 27: Add caption

		ARCG2		75CR		ARCG2*75CR		RCG1		DD				ARCG2		75CR		ARCG2*75CR		RCG1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$
MNM TIME-VARYING PROBABILITY																					
5.9266	0.8963	**-0.0215**	-0.0027	0.3042	0.4339	-0.0264	0.0055	**0.0727**	-0.0640	-0.9055	**-0.3824**										
3.3366	0.4252	**0.0062**	0.0094	0.2305	0.3185	0.0152	0.0234	**0.0150**	0.0412	0.8366	**0.1907**										
MNM TIME-VARYING PROBABILITY 2007-4																					
6.288	3.518	-0.003	0.022	0.218	0.388	0.020	0.028	**0.088**	0.016	-1.107	-0.766										
2.586	2.407	0.009	0.015	0.215	0.352	0.020	0.037	**0.020**	0.028	0.647	0.605										
HMNM TIME-VARYING PROBABILITY																					
9.3433	1.3114	-0.0048	0.0051	-0.0012	0.1578	-0.0139	0.0013	0.0146	-0.0168	**-1.8783**	-0.1976	2.2227	2.2764	-0.0224	-0.0216	0.0066	0.0036	0.0076	0.0079	**0.0605**	0.0553
2.9240	0.5646	0.0041	0.0050	0.1177	0.1510	0.0082	0.0104	0.0079	0.0103	**0.7307**	0.1522	0.1877	0.2302	0.0117	0.0148	0.3759	0.4511	0.0261	0.0327	**0.0194**	0.0292
HMNM TIME-VARYING PROBABILITY 2007-4																					
7.100	9.510	0.003	0.010	0.008	0.138	0.005	0.000	0.007	0.007	-1.332	**-2.216**	2.825	2.852	0.001	0.001	1.687	1.656	0.002	0.003	**0.091**	**0.087**
3.560	4.110	0.005	0.007	0.129	0.188	0.010	0.013	0.009	0.012	0.895	**1.035**	0.379	0.414	0.020	0.022	1.308	1.355	0.079	0.082	**0.035**	**0.044**

Table 28: Add caption

		ARCG2		75CR		ARCG2*75CR		RCG1		CR1		DD				ARCG2		75CR		ARCG2*75CR		RCG1		CR1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1
MNM TIME-VARYING PROBABILITY																									
11.7963	-0.1978	**-0.0212**	-0.0020	0.3000	-0.8289	-0.0280	0.0069	**0.0766**	-0.0400	0.0004	**0.0130**	-2.3837	**-0.5148**												
5.9028	0.5575	**0.0064**	0.0171	0.2995	0.5765	0.0151	0.0280	**0.0139**	0.0325	0.0019	**0.0053**	1.4712	**0.2289**												
MNM TIME-VARYING PROBABILITY 2007-4																									
11.3273	6.4244	-0.0047	0.0229	0.3679	-0.0388	0.0250	0.0362	**0.0915**	0.0310	-0.0021	0.0051	-2.3081	-1.6144												
6.7992	4.5898	0.0094	0.0175	0.3193	0.7186	0.0216	0.0530	**0.0216**	0.0337	0.0028	0.0065	1.6892	1.1269												
HMNM TIME-VARYING PROBABILITY																									
6.7264	1.2915	-0.0059	0.0039	0.0589	-0.2742	-0.0144	0.0019	0.0160	-0.0183	-0.0005	0.0043	-1.1922	-0.3420	**3.2289**	**3.2616**	**-0.0317**	-0.0311	1.3303	1.3000	-0.0059	-0.0057	**0.0998**	0.0926	-0.0074	-0.0072
3.2222	1.1697	0.0042	0.0058	0.1753	0.2871	0.0084	0.0118	0.0084	0.0116	0.0013	0.0023	0.8097	0.2972	**1.0019**	**1.0845**	**0.0158**	0.0191	1.3945	1.4844	0.0468	0.0513	**0.0459**	0.0523	0.0073	0.0084
HMNM TIME-VARYING PROBABILITY 2007-4																									
40.156	23.309	0.003	0.010	0.111	-0.187	0.005	-0.002	0.008	0.008	-0.001	0.004	**-9.561**	**-5.773**	3.382	3.414	0.002	0.002	2.063	2.061	-0.010	-0.009	**0.112**	**0.108**	-0.002	-0.003
11.567	4.686	0.005	0.008	0.194	0.321	0.011	0.014	0.009	0.013	0.002	0.003	**2.890**	**1.166**	1.231	1.335	0.024	0.026	1.430	1.537	0.061	0.065	**0.042**	**0.051**	0.012	0.013

Table 29: Add caption

		ARCG2		75CR		ARCG2*75CR		RCG1		RSR1		DD				ARCG2		75CR		ARCG2*75CR		RCG1		RSR1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1
MNM TIME-VARYING PROBABILITY																									
6.9077	0.2729	**-0.0228**	0.0048	0.4596	0.2514	**-0.0605**	-0.0544	**0.0468**	0.0165	**0.0448**	**-0.0752**	-1.0371	-0.3064												
4.7367	0.6671	**0.0080**	0.0149	0.4124	0.4211	**0.0233**	0.0445	**0.0199**	0.0379	**0.0072**	**0.0235**	1.1777	0.3333												
MNM TIME-VARYING PROBABILITY 2007-4																									
2.884	12.082	0.004	0.048	**-0.831**	-5.104	0.003	0.147	**-0.073**	0.132	**0.086**	**-0.547**	-0.118	-1.942												
1.575	5.974	0.015	0.045	**0.320**	4.045	0.023	0.163	**0.032**	0.070	**0.019**	**0.222**	0.387	1.084												
HMNM TIME-VARYING PROBABILITY																									
15.0842	0.8078	-0.0045	-0.0005	-0.0623	0.1611	-0.0174	0.0054	-0.0064	0.0041	**0.0200**	**-0.0198**	**-3.2960**	-0.0666	5.9167	5.9471	**-0.0865**	**-0.0866**	-0.4940	-0.4984	0.0153	0.0162	-0.0037	-0.0033	**0.1812**	**0.1798**
5.7125	0.4768	0.0055	0.0071	0.1408	0.1848	0.0107	0.0149	0.0106	0.0154	**0.0044**	**0.0060**	**1.4260**	0.1441	0.8707	0.8866	**0.0216**	**0.0253**	0.7363	0.8321	0.0564	0.0655	0.0343	0.0475	**0.0359**	**0.0380**
HMNM TIME-VARYING PROBABILITY 2007-4																									

Table 30: Add caption

		ARCG2		90CR		ARCG2*90CR		RCG1		DD				ARCG2		90CR		ARCG2*90CR		RCG1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$
MNM TIME-VARYING PROBABILITY																					
16.440	1.193	**-0.021**	-0.003	1.127	0.417	**-0.101**	-0.022	**0.067**	**-0.077**	**-3.534**	**-0.472**										
6.054	0.450	**0.006**	0.010	0.583	0.377	**0.033**	0.039	**0.013**	**0.037**	**1.512**	**0.202**										
MNM TIME-VARYING PROBABILITY 2007-4																					
7.737	6.394	-0.005	**0.027**	0.669	0.303	0.116	0.066	**0.090**	0.030	-1.456	-1.464										
3.309	3.888	0.009	**0.013**	0.564	0.619	0.070	0.085	**0.022**	0.029	0.827	0.975										
HMNM TIME-VARYING PROBABILITY																					
8.2232	1.2666	-0.0062	0.0048	-0.0312	0.3078	**-0.0275**	0.0113	**0.0156**	-0.0172	-1.5904	-0.1999	2.1089	2.1727	-0.0186	-0.0176	1.2653	1.2376	0.0345	0.0318	0.0571	0.0510
3.8217	0.5209	0.0038	0.0047	0.1633	0.1940	**0.0129**	0.0170	**0.0079**	0.0103	0.9557	0.1449	0.1608	0.2009	0.0105	0.0131	1.0295	1.0981	0.0726	0.0825	0.0186	0.0286
HMNM TIME-VARYING PROBABILITY 2007-4																					
8.458	14.534	0.002	0.012	0.006	0.164	0.015	-0.010	0.007	0.010	-1.656	**-3.476**	**5.010**	**5.028**	0.031	0.031	4.607	4.585	-0.213	-0.212	**0.174**	**0.170**
4.596	3.871	0.004	0.007	0.201	0.287	0.016	0.020	0.009	0.013	1.148	**0.970**	**1.379**	**1.379**	0.032	0.033	4.190	4.229	0.200	0.203	**0.079**	**0.085**

Table 31: Add caption

		ARCG2		90CR		ARCG2*90CR		RCG1		CR1		DD				ARCG2		90CR		ARCG2*90CR		RCG1		CR1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1
MNM TIME-VARYING PROBABILITY																									
4.995	0.203	**-0.021**	-0.003	0.983	-0.501	**-0.097**	-0.033	**0.073**	-0.050	0.001	**0.010**	-0.705	**-0.644**												
3.831	0.644	**0.006**	0.016	0.627	0.615	**0.034**	0.049	**0.013**	0.040	0.002	**0.005**	0.954	**0.310**												
MNM TIME-VARYING PROBABILITY 2007-4																									
6.833	2.314	-0.001	0.019	2.325	-0.161	0.286	0.055	**0.087**	0.003	0.001	0.005	-1.284	-0.572												
3.073	2.207	0.010	0.013	1.747	0.604	0.170	0.073	**0.022**	0.026	0.002	0.004	0.762	0.546												
HMNM TIME-VARYING PROBABILITY																									
25.152	1.084	-0.007	0.004	-0.046	0.140	**-0.028**	0.013	**0.018**	-0.020	0.000	0.002	**-5.812**	-0.238	2.680	2.702	-0.019	-0.018	**22.012**	**21.895**	0.286	0.284	**0.059**	0.052	-0.005	-0.005
5.101	0.527	0.004	0.005	0.207	0.293	**0.013**	0.018	**0.008**	0.011	0.001	0.002	**1.276**	0.150	0.427	0.548	0.011	0.014	**7.478**	**7.510**	0.291	0.293	**0.019**	0.029	0.003	0.004
HMNM TIME-VARYING PROBABILITY 2007-4																									

Table 32: Add caption

		ARCG2		90CR		ARCG2*90CR		RCG1		RSR1		DD				ARCG2		90CR		ARCG2*90CR		RCG1		RSR1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	Θ_0	Θ_1
MNM TIME-VARYING PROBABILITY																									
22.936	1.634	**-0.033**	0.004	0.303	-1.849	-0.052	**-0.489**	**0.055**	0.022	**0.040**	**-0.139**	-5.036	**-1.115**												
13.271	0.950	**0.007**	0.018	0.655	1.026	0.041	**0.174**	**0.019**	0.060	**0.007**	**0.044**	3.319	**0.517**												
MNM TIME-VARYING PROBABILITY 2007-4																									
5.422	21.156	0.001	0.057	-0.365	**14.236**	0.024	**-1.015**	-0.063	**0.196**	**0.090**	**-0.882**	-0.778	**-3.551**												
3.858	5.642	0.016	0.054	0.362	**4.631**	0.034	**0.318**	0.033	**0.091**	**0.019**	**0.210**	0.950	**1.054**												
HMNM TIME-VARYING PROBABILITY																									
10.416	0.792	-0.009	0.001	-0.135	0.289	-0.012	-0.009	-0.003	0.003	**0.019**	**-0.020**	-2.127	-0.071	6.491	6.516	**-0.101**	**-0.101**	**2.999**	2.977	0.238	0.237	-0.002	0.001	**0.209**	**0.207**
4.336	0.497	0.005	0.007	0.191	0.242	0.018	0.023	0.010	0.015	**0.004**	**0.006**	1.086	0.150	1.753	1.764	**0.038**	**0.040**	**1.474**	1.583	0.202	0.210	0.041	0.052	**0.060**	**0.062**
HMNM TIME-VARYING PROBABILITY 2007-4																									

Table 33: Add caption

		negCG1		negCG12		RCI		RCI*negCG1		DD				negCG1		negCG12		RCI		RCI*negCG1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$
MNM TIME-VARYING PROBABILITY																					
18.888	0.416	0.382	3.701	-0.322	**8.919**	**0.046**	0.028	0.027	0.110	**-4.180**	**-0.356**										
5.429	0.393	0.291	3.219	0.275	**3.212**	**0.019**	0.018	0.025	0.130	**1.358**	**0.151**										
MNM TIME-VARYING PROBABILITY 2007-4																					
8.766	0.707	0.896	**13.992**	-0.445	**-12.288**	-0.024	0.000	**0.127**	0.471	-1.701	-0.120										
4.617	0.666	0.753	**5.913**	0.851	**5.172**	0.021	0.024	**0.047**	0.334	1.153	0.202										
HMNM TIME-VARYING PROBABILITY																					
10.857	1.368	0.211	0.167	-0.073	0.040	0.005	0.003	0.026	-0.011	-2.262	-0.242	2.451	2.470	0.246	0.205	**-1.351**	-1.137	-0.019	-0.017	0.058	0.048
6.656	0.583	0.149	0.223	0.176	0.249	0.009	0.011	0.014	0.018	1.665	0.155	0.246	0.300	0.420	0.564	**0.427**	0.642	0.021	0.028	0.030	0.045
HMNM TIME-VARYING PROBABILITY 2007-4																					
10.244	11.885	0.054	0.036	-0.028	-0.041	-0.011	0.008	0.031	-0.012	-2.087	**-2.830**	3.003	3.008	2.194	2.143	**-3.104**	-2.990	0.028	0.027	0.045	0.041
6.153	5.290	0.195	0.294	0.229	0.328	0.010	0.015	0.017	0.024	1.540	**1.329**	0.413	0.476	1.483	1.571	**1.518**	1.625	0.045	0.052	0.085	0.095

Table 34: Add caption

		negCG1		negCG12		RCI		RCI*negCG1		DD				negCG1		negCG12		RCI		RCI*negCG1	
p_{00}	p_{11}	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$	γ_0	γ_1	r_0	r_1	α_0	α_1	β_0	β_1	θ_0	θ_1	δ_0	$\delta 1$
MNM TIME-VARYING PROBABILITY																					
18.888	0.416	0.382	3.701	-0.322	**8.919**	**0.046**	0.028	0.027	0.110	**-4.180**	**-0.356**										
5.429	0.393	0.291	3.219	0.275	**3.212**	**0.019**	0.018	0.025	0.130	**1.358**	**0.151**										
MNM TIME-VARYING PROBABILITY 2007-4																					
8.766	0.707	0.896	**13.992**	-0.445	**-12.288**	-0.024	0.000	**0.127**	0.471	-1.701	-0.120										
4.617	0.666	0.753	**5.913**	0.851	**5.172**	0.021	0.024	**0.047**	0.334	1.153	0.202										
HMNM TIME-VARYING PROBABILITY																					
10.857	1.368	0.211	0.167	-0.073	0.040	0.005	0.003	0.026	-0.011	-2.262	-0.242	2.451	2.470	0.246	0.205	**-1.351**	-1.137	-0.019	-0.017	0.058	0.048
6.656	0.583	0.149	0.223	0.176	0.249	0.009	0.011	0.014	0.018	1.665	0.155	0.246	0.300	0.420	0.564	**0.427**	0.642	0.021	0.028	0.030	0.045
HMNM TIME-VARYING PROBABILITY 2007-4																					
10.244	11.885	0.054	0.036	-0.028	-0.041	-0.011	0.008	0.031	-0.012	-2.087	**-2.830**	3.003	3.008	2.194	2.143	**-3.104**	-2.990	0.028	0.027	0.045	0.041
6.153	5.290	0.195	0.294	0.229	0.328	0.010	0.015	0.017	0.024	1.540	**1.329**	0.413	0.476	1.483	1.571	**1.518**	1.625	0.045	0.052	0.085	0.095