Discount Rates and Monetary Policy*

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July 2019

Abstract

We derive the expected stochastic discount factor from a term structure model to infer the interest rate hedging strategies of investors' with different levels of wealth and degrees of risk aversion. Time-varying non-linear risk premia in the U.S. Treasury market, consistent with empirical evidence, helps explain the asymmetric impact of monetary policy on the discount rate and hence investors' cashflow valuations. We find a direct effect, through which risk pricing depends on the shape of the yield curve relative to its historical average; and an indirect effect that amplifies differences in the distribution of investors' risk aversion.

JEL Classification Numbers: E43; C58; G11; G12

Keywords: Interest-rate risk; stochastic discount factor; investment portfolio; term structure model; risk aversion; conditional distributions

*The views expressed in this paper are those of the authors and do not necessarily represent those of the International Monetary Fund or IMF policy. The authors would like to thank participants at the 4th International Workshop on Financial Markets and Nonlinear Dynamics and the Bates College Casey Lecture Fund in Economics Seminar Series for helpful comments and suggestions. We would also like to thank Ricardo Sousa for his insightful discussion.

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I. INTRODUCTION

Financial market participants hedge interest rate risk by taking long or short positions on financial instruments of different maturities. From a traditional dynamic stochastic general equilibrium framework, there is a direct relationship between the discount rate (i_t) and the expected future cashflows of investors, the "stochastic discount factor" or SDF,

$$\frac{1}{1+i_t} = \mathbb{E}_t \left[SDF_{t+1} \right]. \tag{1}$$

Hence monetary policy tightening, would increase the discount rate and lower investors' expected present value of financial instruments. This conclusion follows from an SDF that is positive and monotonically decreasing in economic activity, an argument that holds under the assumption of complete markets (Dybvig, 1988).[1] There is substantial evidence, however, that the expected SDF varies over time and is non-monotonic.[2] It is therefore not clear how monetary policy impacts cashflow valuations and hence investors' attitudes toward interest rate risk as they rebalance their portfolios.

We combine the theoretical insights from standard general equilibrium models with the empirical evidence on time-varying and non-linear discounting to show how investors' hedging positions reflect their attitudes towards maturity risk, contingent on the shape of the yield curve and the mean-reverting properties of interest rates. Our findings suggest that monetary policy has direct and indirect effects on the bond markets. First, we identify changes in the entire yield curve to influence the market prices of risk non-linearly depending on the current value of interest rate risk factors relative to their historical averages. Second, we find that monetary policy also influences portfolio positions by changing the distribution of investors' absolute risk aversion in equilibrium, amplifying the risk exposure of risk-averse or less wealthy investors.

This paper begins by outlining a canonical no-arbitrage term structure model in Section II, in which the U.S. yield curve is described by observable interest-rate risk factors that capture cross-sectional variation in interest rates as in Joslin et al. (2011). The model is parsimonious and econometrically tractable, yet successful at explaining interest rate movements

[1]In standard macroeconomic models, there are limited opportunities to hedge against interest rate fluctuations. Popular models used by Central Banks to understand the effects of monetary policy on the economy are FRB/US (Brayton et al., 2014) at https://www.federalreserve.gov/econresdata/frbus/us-models-about.htm and Euro (Smets and Wouters, 2003, 2007; Christoffel et al., 2008).

[2]See Campbell (2014) for a discussion of the contributions of Eugene Fama, Lars Peter Hansen, and Robert Shiller to this line of research, which is the basis for their 2013 Nobel Memorial Prize in Economic Sciences.

and inferring the market prices of risk that describe how interest-rate risk factors impact the investor's SDF non-linearly, following log-normal probability distributions. Hedging strategies arise from investors' conditional expectations of future interest rate movements, such that they go long on long-term securities when yields are above their historical average and viceversa. Given these hedging positions, investors suffer a capital loss when the future yields move away from their long-term mean.

In Section III, we derive the continuous-time conditional expected SDF, relative to each one of the interest-rate risk factors moving, while everything else held constant, following Chami et al. (2017). The probability density function for the SDF is log-normal, thus introducing time-variation and non-linearity with respect to the risk factors.

In macroeconomic-finance, the equity premium puzzle of Mehra and Prescott (1985) demonstrates that an SDF dependent only on consumption growth is insufficient to explain the excess return on stocks and the risk-free interest rate. Intuitively, small variation of the risk-free interest rate implies small variation in the expected SDF; however, high fluctuations in expected stock returns implies that the SDF must have high variation in its correlation with equity markets. Cochrane (1991) shows that the time variation of the equity premium, predictability of returns, and excess stock volatility are all derived from the same properties of the expected SDF. In financial economics, the pricing kernel puzzle as described by Beare and Schmidt (2016) documents the inconsistency between the Dybvig (1988) conclusion and estimated pricing kernels for various financial instruments.[3] Hens and Reichlin (2012) offer three solutions to the pricing kernel puzzle: (i) Incomplete markets; (ii) alternatives to the risk averse expected utility for investors; and (iii) incorrect beliefs. These alternatives are also used to explain the equity premium puzzle.[4]

Our results are consistent with the substantial evidence against a monotonic SDF or pricing kernel found in the literature. Non-monotonicity in risk premia has been identified in studies of combinations of forward rates (Fama and Bliss, 1987; Cochrane and Piazzesi, 2005), Treasury spreads (Campbell and Shiller, 1991), and equity returns (Parker and Julliard, 2005; Lustig and Van Nieuwerburgh, 2005; Yogo, 2006; Sousa, 2010); and explained by slow-moving habit driven by shocks to aggregate consumption (Campbell and Cochrane, 1999; Wachter, 2006), shocks to inflation (Brandt and Wang, 2003), countercyclicality (Ludvigson

[3]The pricing kernel and SDF are related. For example Beare and Schmidt (2016) define: (i) The SDF is a random variable at a future time $t+1$; (ii) the current price of a financial instrument Y_t is given by $\mathbb{E}_t\left[SDF\ Y_{t+1}\right]$; (iii) $M^* = \mathbb{E}_t\left[SDF\ S_{t+1}\right]$, where S_{t+1} is the price of the market portfolio at time $t+1$; and (iv) the pricing kernel is defined as $M^* = \frac{1}{1+i}\pi_t\left(S_{t+1}\right)$.

[4]Cochrane (2017) surveys this work and proposes that an extra variable that increases risk aversion during bad times is necessary to capture the time-variation of the expected SDF and hence the equity premium.

and Ng, 2009), transitory deviations from the common trend among consumption, aggregate wealth and labor income (Lettau and Ludvigson, 2001), and long-run risk (Bansal and Yaron, 2004), among others. Cochrane (2011) also finds substantial time variation in the discount rate or expected SDF across many financial assets including the expected excess return on longer-term bonds. Thus, macroeconomic-finance and financial economics have found significant evidence that the expected SDF varies over time and is non-linear.

We show that a widening of the term spread (the difference between the long and short end of the yield curve), increases expected cashflow valuations when the yield curve is relatively flat, yet lowers the expected SDF of the investor when the yield curve is steep; an important distinction that arises from the Gaussian properties of the probability density function for the SDF.

Next, we determine the optimal portfolio allocation decisions of investors given different measures of relative risk aversion in Section IV. Following the optimization problem in Sangvinatsos and Wachter (2005), we find the optimal portfolio rule for an investor that takes as given the linear relation between the expected excess return on bonds and the interest-rate risk factors. These portfolio rules are also linear in the Sharpe ratio of U.S. Treasury securities, a leverage constraint, and the risk factors, and are derived for an investor with a constant relative risk averse utility for terminal wealth.[5]

By multiplying these portfolio rules (which state the percentage of wealth invested in each security) with the investor's wealth in Section V, we obtain the investor's demand for all the maturities of U.S. Treasury securities. We aggregate these demands across all investors with different degrees of absolute risk aversion, and find the total demand for U.S. Treasury securities at all maturities, which is equated to the total supply of government securities made available to the market by the U.S. Treasury department and the Federal Reserve Board.[6] Finally, we obtain the equilibrium expected return on Treasury securities as a linear function of the interest-rate risk factors, the supply of Treasury securities, and the average leverage of investors in the market for Treasuries. The dependence on the interest-rate risk factors reflects Merton's hedging demand. This demand is based on a comparison of the current factors with the expected utility desired by each investor; which is scaled by the variance-covariance matrix for the investor's expected utility and multiplied by the

[5]If the investors also have distinct investment horizon, then the hedging demand strategy will depend on different mean and variance-covariances according to the investor's horizon.

[6]This argument follows Wang (1994)'s model of stock market volume and Vayanos and Vila (2009)'s model of preferred habitat in the market for Treasury securities, which derives the investor's preferred habitat based on the absolute risk aversion of the investor.

beta from regressing the return on Treasury securities on the risk factors. In equilibrium, the coefficients on these factors are an average of the responses by all the investors in the economy, which we can think of as the marginal investor's response to changes in interest rate risk.

Using this equilibrium model of the U.S. Treasury market, we examine both the direct and indirect effects of contractionary monetary policy that raises the level of the yield curve on average. The expected SDF for the market is an envelope of Gaussian distributions, since the equilibrium excess return on Treasury securities is an affine function of the interest-rate risk factors. As a result, the impact of monetary policy depends on whether the factor is on the left or the right of the mean of the factor, which leads to the highest marginal utility of the investor. For more risk averse investors, this change leads to a decrease in their valuation of cashflows; whereas less risk averse investors would experience an increase in the valuation of their cash flows, such that they take the opposite position in longer-term securities. This behavior leads to a second indirect effect of monetary policy through changes in the distribution of the absolute risk aversion of the investors. In particular, the investors with a capital gain have higher wealth in the future and a lower absolute risk aversion, since wealthier individuals are more willing to take on risk. On the other hand, the investors who suffer a capital loss, have a higher absolute risk aversion, making them even more conservative in their portfolio positions. Thus, contractionary monetary policy alters the distribution of the absolute risk aversion for the investors.

Section VI illustrates how contractionary monetary policy impacts the investors in the Treasury markets by investigating the 2017 tightening of monetary policy by the FOMC of the Federal Reserve Bank. We conclude in Section VII with a discussion of questions that can be addressed with this model of the Treasury markets and provide important caveats of the analysis along with possible extensions and policy recommendations.

II. PRICES OF RISK IN THE BOND MARKET

We begin with a canonical term structure model with observable factors as in Joslin et al. (2011). The zero-coupon nominal yield to maturity $r_{\tau,t}$ is driven by an affine process mapping the yield of each maturity to a parsimonious number of underlying factors, Y_t, such that

$$r_{\tau,t}\left(Y_t\right) = A_\tau + B_\tau Y_t, \tag{2}$$

where the time subscript t corresponds to today's date and τ is the maturity date. The parameters A_τ and B_τ for each maturity are set so that there is no arbitrage opportunity for investors in the bond markets.

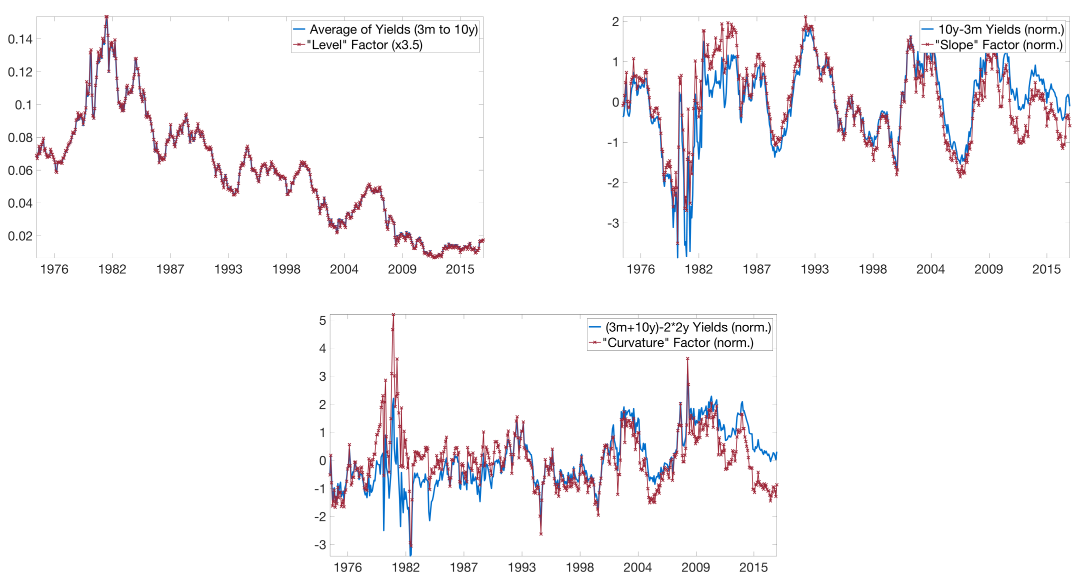
The state vector $Y_t = \begin{bmatrix} Y_{1t} & Y_{2t} & Y_{3t} \end{bmatrix}'$ contains the set of observable "interest-rate risk" factors constructed by principal component analysis as weighted averages of the yields for all the maturities, y_t^{obs}, with weight vector W:

$$Y_t = W y_t^{obs}.$$

Consistent with the literature identifying the first three principal components to account for over 99% of the cross-sectional variation in the yield curve, we refer to these factors as "level," "slope," and "curvature" as in Litterman and Scheinkman (1991).

Figure 1 shows the factors along with their empirical counterparts. The level factor is compared to the average of all yields, the slope factor is shown along with the spread: $y_{10y,t}^{obs} - y_{3m,t}^{obs}$, and the curvature factor is related to $2 * y_{2y,t}^{obs} - level$, following Diebold and Li (2006). These graphs show that the estimated factors in our term structure model track the empirical level, slope, and curvature factors and hence inherit their name.

Figure 1: Estimated Factors versus Empirical Factors Explaining Yields to Maturity



Notes: In blue, the factors are constructed as the geometric representation of the yield curve described in the legend. In red, the factors are constructed by principal component analysis. Factors are normalized when appropriate for scaling purposes.

These factors are assumed to follow an autoregressive process of order one under the actual or physical distribution $\mathbb{P}$,

$$Y_{t+1} = K_0^{\mathbb{P}} + \left(K_1^{\mathbb{P}} + I\right) Y_t + \Sigma \varepsilon_{t+1}^{\mathbb{P}}, \tag{3}$$

where $K_0^{\mathbb{P}}$ and $K_1^{\mathbb{P}}$ are constants, I is the identity matrix, and Σ is a lower triangular matrix obtained from the variance covariance of the innovations to the factors $\left(\varepsilon_{t+1}^{\mathbb{P}}\right)$, which are i.i.d. This stochastic process is mean reverting to $\left(K_1^{\mathbb{P}}\right)^{-1} K_0^{\mathbb{P}}$, as long as all the eigenvalues of $K_1^{\mathbb{P}}$ are strictly negative. If all the eigenvalues are zero, then Equation (3) is a random walk (with drift if $K_0^{\mathbb{P}} \neq 0$). In this case, there is no change in the conditional expected future yields and hence no benefit to hedging against interest-rate risk. On the other hand, under a mean reverting stochastic process with $K_1^{\mathbb{P}} < 0$, investors expect the factors (and hence yields) to move back to their long-term unconditional means and therefore adopt hedging positions to take advantage of these expectations.

The risk neutral distribution $\mathbb{Q}$ of the factors satisfies

$$Y_{t+1} = K_0^{\mathbb{Q}} + \left(K_1^{\mathbb{Q}} + I\right) Y_t + \Sigma \varepsilon_{t+1}^{\mathbb{Q}}. \tag{4}$$

Equation (4) adjusts the mean of the physical distribution for the price of risk per unit of volatility.[7] Thus, bonds of any maturity can be priced as if investors in the bond markets were risk neutral.

The risk-free interest rate, r_t, is also a linear function of the factors:

$$r_t\left(Y_t\right) \equiv r_t = \rho_0 + \rho_1 Y_t, \tag{5}$$

such that the constant ρ_0 and the vector ρ_1 are independent of time.

To determine the price of zero-coupon bonds, the SDF, M_{t+1}, is assumed to have the following exponential quadratic form

$$M_{t+1} = \exp\left\{-r_t - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\varepsilon_{t+1}^{\mathbb{P}}\right\}, \tag{6}$$

so the price of risk that characterizes investors' attitude toward risk, Λ_t, is affine in the

[7] Following Beare and Schmidt (2016) as in footnote 4, suppose $Y_{t+1} = f(S_{t+1})$ is the payoff on a contingent security. Thus, $Y_t = \mathbb{E}_t\left[SDF\ f(S_{t+1})\right] = \frac{1}{1+r}\int_0^\infty f_t(x)q_t(x)dx$. $q(x)$ is the risk neutral distribution. $Y_t = \mathbb{E}_t\left[\pi_t(S_{t+1})\ f(S_{t+1})\right] = \frac{1}{1+r}\int_0^\infty f_t(x)\pi_t(x)p_t(x)dx$. $p_t(x)$ is the physical distribution. This means $\pi_t(x) = \frac{q_t(x)}{p_t(x)} = M^*(x)(1+r)$.

factors,

$$\Sigma\Lambda_t = K_0^{\mathbb{P}} - K_0^{\mathbb{Q}} + \left(K_1^{\mathbb{P}} - K_1^{\mathbb{Q}}\right) Y_t. \tag{7}$$

The adjustment for risk in the SDF, $-\frac{1}{2}\Lambda_t'\Lambda_t$, follows from the shocks to the interest-rate risk factors being a log-normal probability distribution. This risk adjustment is given by

$$-\frac{1}{2}\Lambda_t'\Lambda_t = -\frac{1}{2}\left[K_0^{\mathbb{P}\prime} - K_0^{\mathbb{Q}\prime} + Y_t'\left(K_1^{\mathbb{P}} - K_1^{\mathbb{Q}}\right)\right]\left(\Sigma'\Sigma\right)^{-1}\left[K_0^{\mathbb{P}} - K_0^{\mathbb{Q}} + \left(K_1^{\mathbb{P}} - K_1^{\mathbb{Q}}\right) Y_t\right]. \tag{8}$$

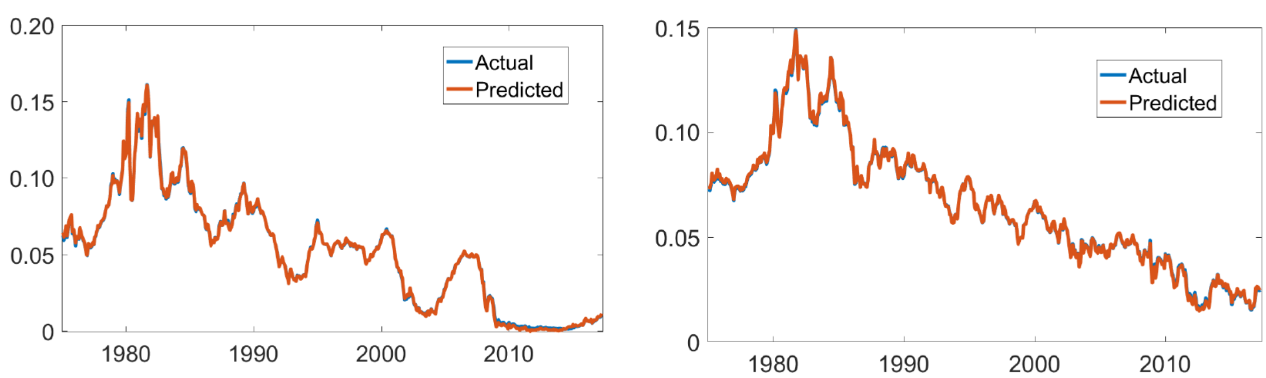
Importantly, the logarithm of the SDF exhibits a quadratic shape. This property is a consequence of time-varying interest-rate risk, implying a Gaussian bond risk premium – the adjustment for maturity risk in bond markets.

We estimate the model parameters using monthly U.S. Treasury yields data from January 1975 to March 2017 as in Yung (2017). In the estimation, we use 12 maturities: 3 and 6 months, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 years. An advantage of the two-stage procedure of Joslin et al. (2011) is that Equation (3) can be separately estimated by OLS and all other parameters can be rotated such that the maximum likelihood algorithm immediately converges to the global optimum.

In Figure 2, we plot the actual yield to maturity relative to the estimated values from the model for the one- and ten-year bonds. As is standard in the term structure literature, the model captures the movement in yields over time really well, both in the short and the long end.

Figure 2: Model Interest-Rate Fit (in percentage)

(a) 1-Year Yield (b) 10-Year Yield



Notes: The one- and ten-year yields in annualized percentage from the data (in blue) are compared to the model-implied yields (in red) from Equation (2).

III. INVESTORS' EXPECTED SDF

We derive the future SDF of investors, M_{t+k}, for any month $k = \{1, ..., K\}$ by substituting an iterated version of Equations (5), (7) and (8) into Equation (6), such that M_{t+k} can be expressed as a function of time-t factors and model parameters. For each k, the future interest-rate risk factors are given by

$$Y_{t+k} = \sum_{i=1}^{k} \left(K_1^{\mathbb{P}} + I\right)^{i-1} K_0^{\mathbb{P}} + \left(K_1^{\mathbb{P}} + I\right)^{k} Y_t + \sum_{i=1}^{k} \left(K_1^{\mathbb{P}} + I\right)^{i-1} \Sigma \varepsilon_{t+k+1-i}^{\mathbb{P}}.$$

Given the properties of the log normal probability distribution, the expected SDF for investments paying off in k periods and conditional on information at time t, i.e., $Y_t = Y$, is given by

$$\mathbb{E}_t[M_{t+k}|Y_t] \equiv \mathcal{M}(k, Y) = \mathcal{M}(k) \exp\left\{ -\frac{1}{2}\Big(Y - \mu_{\mathcal{M}}(k)\Big)' \left(\sigma_{\mathcal{M}}(k)\right)^{-1} \Big(Y - \mu_{\mathcal{M}}(k)\Big)\right\}. \quad (9)$$

$\mu_{\mathcal{M}}(k)$ is the mean of the expected SDF and $\sigma_{\mathcal{M}}(k)$ is the variance-covariance matrix for a time horizon of k periods.[8] The unanticipated shock to interest-rate risk factors is log-normally distributed. Taking the conditional expectation converts the shock into a time horizon-dependent term only, which we include in the constant $\mathcal{M}(k)$ to simplify the notation.

The discrete model for the factors can be transformed into a continuous stochastic process allowing the factors to coincide at each date of observation, i.e. $X(s) = Y_{t+j-1}$ for $j = 1, \cdots, T$, and $s = j - 1$. This mapping allows us to derive the probability distribution for the interest-rate risk factors over any given time horizon.

The continuous time stochastic process for the factors is

$$dX(s) = \left(\gamma^{\mathcal{P}} - A^{\mathcal{P}} X(s)\right) ds + \Sigma_X d\epsilon_s,$$

where ϵ_s is a Brownian motion. The mapping from the discrete to the continuous time model is given by

$$A^{\mathcal{P}} = -\ln\left(K_1^{\mathbb{P}} + I\right), \quad \gamma^{\mathcal{P}} = A^{\mathcal{P}} \left(I - e^{-A^{\mathcal{P}}}\right)^{-1} K_0^{\mathbb{P}}, \text{ and}$$

[8] See Cosimano and Yung (2019) for the discrete-time derivation and Cosimano and Ma (2018) for the continuous-time counterpart.

$$\int_0^{\tau} e^{-A^{\mathcal{P}}(\tau-s)} \Sigma_X d\epsilon_s = \sum_{i=0}^{j-1} \left(K_1^{\mathbb{P}} + I\right)^i \Sigma \varepsilon_{t+j-i}^{\mathbb{P}}.$$

In addition, the continuous-time equivalent price of risk is now given by

$$\Lambda\left(X(s)\right) = \left(\Sigma_X\right)^{-1}\left(\gamma^{\mathcal{P}} - \gamma^{\mathcal{Q}}\right) - \left(\Sigma_X\right)^{-1}\left(A^{\mathcal{P}} - A^{\mathcal{Q}}\right) X(s), \tag{10}$$

where $\gamma^{\mathcal{Q}}$ and $A^{\mathcal{Q}}$ are risk-adjusted parameters such that the variance-covariance matrix of the residuals, $\Sigma_X \Sigma_X^{'}$, is invariant across both distributions, following the diffusion invariance principle (Girsanov, 1958). The excess holding period return on a zero coupon security with maturity τ, where $P_{\tau,s}$ is the price, is given by

$$\begin{aligned} \frac{dP_{\tau,s}}{P_{\tau,s}} - r(s) =& b_\tau \left((\gamma^{\mathcal{P}} - \gamma^{\mathcal{Q}}) - (A^{\mathcal{P}} - A^{\mathcal{Q}})X(s)\right) ds + b_\tau \Sigma_X d\epsilon_s \\ =& \left(\mu_\tau(s) - r(s)\right) ds + b_\tau \Sigma_X d\epsilon_s. \end{aligned} \tag{11}$$

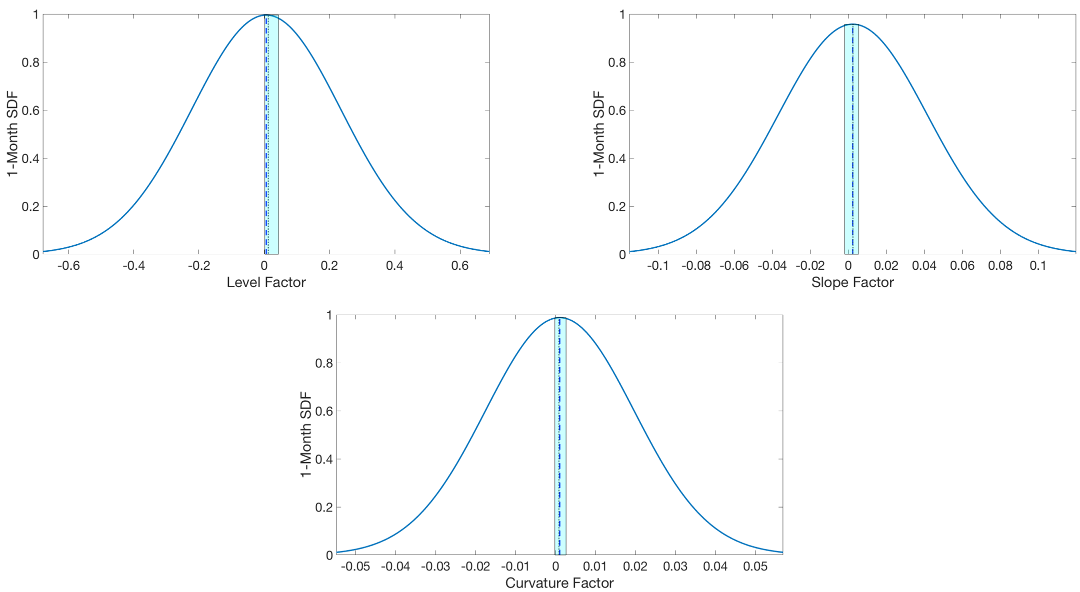
b_τ is a constant proportional to the bond pricing coefficient, B_τ, $r(s)$ is the continuous-time risk-free rate and $\mu_\tau(s)$ is the expected return on a maturity-τ bond.

Figure 3 shows the model-implied one-month expected SDF for an investor, $\mathcal{M}(1, Y)$, relative to the level, slope, or curvature, while all other factors are held constant. The blue vertical dashed line in each plot corresponds to the value at which the expected SDF is at its conditional mean, $\mu_{\mathcal{M}}(1)$. The vertical gray line indicates the factor's unconditional mean estimated by Equation (3), $\bar{Y} = (0.01200, 0.00241, 0.00085)$, and the blue shaded area provides the range of historical values for each factor over the 1975–2017 time period.

In the case of the level, for example, the one-month ahead expected SDF is at its maximum of 0.9988 whenever the level factor is at $\mu_{\mathcal{M}}^{level}(1) = -0.0017$. From Equation (1), this value corresponds to a one-month discount rate of 0.12%. An increase in the level of the yield curve increases investors' cashflow valuations only when the level change occurs within the left of the conditional mean, i.e. $\Delta Y_{t \to t+1}^{level} < -0.0017$. If the level increases within the right of the conditional mean, $\Delta Y_{t \to t+1}^{level} > -0.0017$, then investors' expected SDF actually decreases. Notice that during this period, the level ranges from 0.002 to 0.044, so that the level of the yield curve was historically to the right of the conditional mean.

In practical terms, this framework can be used to quantify how changes in the level, slope, and curvature impact investors' expected SDF. Two takeaways are worth emphasizing. First, the effect on expected cashflows depends on the current state of the yield curve and hence the economy. A steepening of the yield curve increases investors' expected SDF if the yield curve is very flat to begin with (to the left of the conditional mean). However, if it steepens by the same magnitude but during times when the yield curve is already quite steep (to the right of the conditional mean), then the effect on expected future cashflows is actually negative. Second, monetary policy tightening affects investors' expected SDF through movements in the entire maturity spectrum; hence decomposing the impact of interest rates by one factor at a time helps understand the net effect of different yield curve shapes on bond valuation through the non-linear risk adjustment in investors' expected SDF.

Figure 3: Expected SDF versus Level, Slope and Curvature



Notes: Each plot shows the one-month expected SDF conditional on each factor moving, while the others are held constant, as given by Equation (9), where the blue dashed line represents the value at which the expected SDF is at its conditional mean $\mu_{\mathcal{M}}(1)$. The shaded region accounts for the historical range in which each respective factor has moved during the 1975–2017 period, with the gray vertical line indicating the factors unconditional mean.

IV. PORTFOLIO ANALYSIS: INVESTOR'S RISK AVERSION

Let investors be grouped into J investment buckets. Without loss of generality, each bucket of investors $j = 1, ..., J$ chooses how to optimize a portfolio with four U.S. Treasury securities of different maturities $i = \{1, 2, 3, 4\}$, subject to a liquidity constraint that limits the percentage invested in these securities to ξ^j. Investors have a constant relative risk aversion coefficient,

γ^j, and seek to maximize expected lifetime utility over a fixed terminal wealth at time horizon τ^j, where W^j is the sum of total investment by all investors in bucket j.

The total demand for Treasury securities, $D(t)$, is

$$D(t) = \sum_{j=1}^{J} \omega^j(t) W^j, \tag{12}$$

where $\omega^j(t)$ is the optimal percentage of wealth invested:

$$\omega^j(t) = \frac{1}{\gamma^j}\Bigg[\underbrace{\omega_1\left(\mu(s) - \mu_\tau(s)\iota\right)}_{\text{Sharpe Ratio}} + \underbrace{\omega_1\omega_2\xi^j}_{\text{Leverage}} + \underbrace{\omega_1\omega_3\gamma^j\left(\sigma_j(\tau^j)\right)^{-1}\left[X - \mu_j(\tau^j)\right]}_{\text{Hedging Interest-Rate Risk}}\Bigg], \tag{13}$$

for $\omega_1^j(t) = \xi^j - \iota'\omega^j(t)$, where $\iota' = (1 \quad 1 \quad 1)$.

The terms $\omega_1, \omega_2, \omega_3$ are constants defined as follows:

$$\omega_1 \equiv \left[b\Sigma_X\Sigma_X'b' + \iota\iota'b_\tau\Sigma_X\Sigma_X'b_\tau' - 2b\Sigma_X\Sigma_X'b_\tau'\iota'\right]^{-1},$$

$$\omega_2 \equiv 2\left(b\Sigma_X\Sigma_X'b_\tau' - \iota b_\tau\Sigma_X\Sigma_X'b_\tau'\right),$$

$$\omega_3 \equiv \left(b - \iota b_\tau\right)\Sigma_X\Sigma_X'.$$

$b' = (b_{2\tau} \quad b_{3\tau} \quad b_{4\tau})$ is a vector of bond price elasticities with respect to the interest-rate risk factors, such that $b' - \iota b_\tau = \begin{pmatrix} b_{2\tau} - b_\tau & b_{3\tau} - b_\tau & b_{4\tau} - b_\tau \end{pmatrix}$ captures the elasticity of the $2^{nd}, 3^{rd}, 4^{th}$ bonds relative to the elasticity of the 1^{st} bond, b_τ.

The first term in portfolio rule Equation (13) is the traditional Sharpe ratio adjusted for investors' coefficient of relative risk aversion γ^j. The expected return on the $2^{nd}, 3^{rd}, 4^{th}$ bonds, $\mu(s)' = (\mu_{2\tau}(s) \quad \mu_{3\tau}(s) \quad \mu_{4\tau}(s))$ relative to the 1^{st} bond, $\mu_\tau(s)$, is given by

$$\mu(s) - \mu_\tau(s) \equiv (b - \iota b_\tau)\left[(\gamma^{\mathcal{P}} - \gamma^{\mathcal{Q}}) - (A^{\mathcal{P}} - A^{\mathcal{Q}})X(s)\right]. \tag{14}$$

Notice that the excess return on bonds is measured relative to the 1^{st} Treasury security in Equation (14), rather than the risk-free rate as in Equation (11). Consequently, the price of risk $(\gamma^{\mathcal{P}} - \gamma^{\mathcal{Q}}) - (A^{\mathcal{P}} - A^{\mathcal{Q}})X(t)$ from Equation (10) is multiplied by the relative bond elasticity parameters, $b - \iota b_\tau$. The ω_1 term in the Sharpe ratio adjusts the variance-covariance of the last three bonds, $b\Sigma_X\Sigma_X'b'$, by the variance of the first bond, $b_\tau\Sigma_X\Sigma_X'b_\tau'$ and the covariance of the last three bonds with the first, $b\Sigma_X\Sigma_X'b_\tau'$.

The second term in Equation (13) is an adjustment to ensure that the portfolio weights add up to ξ^j, so that leverage is limited. The term $\omega_1\omega_2$ represents the coefficient from regressing the excess return for the last three bonds against the return for the first bond.

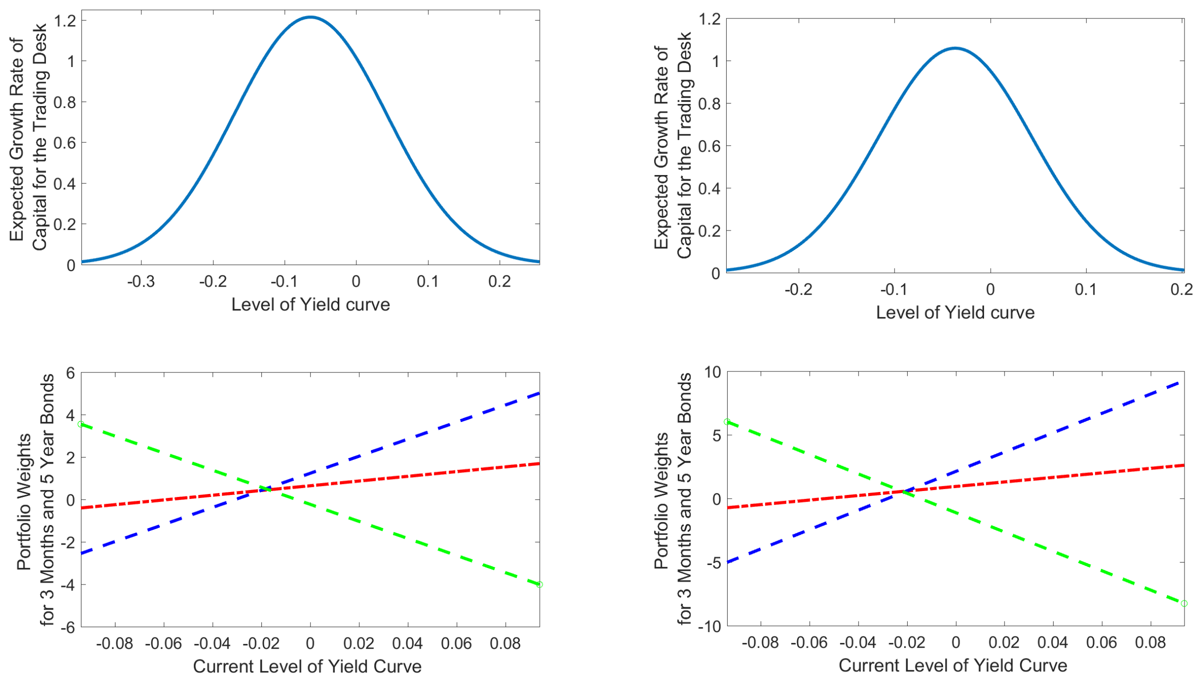
The last term in portfolio rule Equation (13) is the hedging demand for Treasury securities from Merton (1969, 1971), which arises under mean reverting factors, as discussed in Section II. The term $\omega_1\omega_3$ is the ratio of the covariance between excess returns and the interest-rate risk factors relative to the variance-covariance of excess returns. As a result, it can be interpreted as the beta coefficient from regressing expected excess returns on bonds on the lifetime utility of the investor. $\mu_j(\tau^j)$ and $\sigma_j(\tau^j)$ represent investors' expected utility mean and standard deviation, respectively, such that $\gamma^j\left(\sigma_j(\tau^j)\right)^{-1}\left[X - \mu_j(\tau^j)\right]$ captures the sensitivity of the expected lifetime utility for investor j with respect to the factors. This latter term can be interpreted as the risk adjusted duration of investors' portfolio for horizon τ^j. If the factor X is equal to the expected mean, $\mu_j(\tau^j)$, then there is no benefit to hedging interest-rate risk, whereas the hedging demand is positive for $X > \mu_j(\tau^j)$.

To sum up, the investment strategy's expected return is a function of the interest-rate risk factors, such that monetary policy shocks impact the decisions of the investor as she rebalances her portfolio of Treasury securities. To evaluate how changes in the stance of monetary policy affect investors' capital gains as the yield curve moves, we first consider the case for a particular investor bucket ($j = 1$) with absolute risk aversion coefficient $\frac{\gamma^1}{W^1} = 10$. The investor chooses the optimal allocation of her wealth between a 3-month and a 5-year bond to maximize expected lifetime utility for an investment horizon of $\tau^1 = 1$ year. We assume a discount rate of 5% and leverage ratio $\xi^1 = 1$ for simplicity and estimate all parameters for the January 1999–December 2007 period. The investor's expected lifetime utility is a Gaussian function with conditional mean $\mu_1(\tau^1) = -0.0639$ and standard deviation $\sigma_1(\tau^1) = 0.1065$ for changes in the level of the yield curve, while holding all other factors constant.

Figure 4(a) shows the investor's gross rate of return on wealth on the top and her portfolio allocation strategy on the bottom, conditional on the level of the yield curve. During this time period, the range of the level is between –0.11 and 0.06, so the bottom chart focuses on that range in the horizontal axis. When the current level of the yield curve is at its historical mean $\bar{X}^{level} = -0.0177$, the investor is indifferent between a 3-month bond (green) and a 5-year bond (blue), and both lines intersect.

Figure 4: Gross Rate of Return for Investors' Wealth

(a) Absolute Risk Aversion $\frac{\gamma^1}{W^1} = 10$ (b) Absolute Risk Aversion $\frac{\gamma^2}{W^2} = 5$



Notes: The top graphs provide the expected gross growth rate of investor's wealth, while the bottom graphs correspond to the portfolio rules of each investor. The red dotted line represents the investor's hedging demand, the green dashed line is the investor's percentage of wealth placed in three-month bonds, and the blue dashed line illustrates the investor's percentage of wealth in 5-year bonds, conditional on the level of the yield curve moving over time while all other factors are held constant.

If the level is higher than average, $X^{level} > -0.0177$, anticipated mean reversion implies that the investor expects the level of the yield curve to fall, and hence longer-duration bonds would lead to a larger capital gain. As a result, the portfolio is long on five-year bonds and short on three-month bonds during high interest-rate average periods. If the random change in the level is, however, positive then the portfolio would suffer a larger capital loss. The portfolio position is reversed in a low interest rate environment, $X^{level} < -0.0177$, since mean reversion implies that the investor expects the level of the yield curve to move back to its stationary value. If the random change in the level is negative and hence declines even further, then the portfolio would suffer a capital loss.

The hedging demand (red dotted line) is zero (in the vertical axis) whenever the level is at the conditional mean of the expected lifetime utility $X^{level} = -0.0639$. Given that in this case, it is possible for the level factor to be above or below the mean of the expected lifetime utility, the investor's hedging demand can be positive or negative, respectively. Over the December 2016–March 2017 period, for example, the expected SDF fell from 0.9982 to 0.9790, leading to a capital loss of 1.98% per month in the investor's portfolio.

Figure 4(b) corresponds to the bucket of less risk averse investors with the same parameters but with absolute risk aversion coefficient $\frac{\gamma^2}{W^2} = 5$. This difference in investors' risk profiles can result from a lower aversion to risk or a higher level of wealth for the investors in the second bucket relative to the first. These two types of investors could be representative, for example, of the investment positions of small and large banks, respectively, since the smaller banks would have less wealth and a higher risk aversion.

The investor with a lower aversion to risk increases the magnitude of the bet on the level of the yield curve reverting to its long-term mean. This implies that the less risk averse investor will choose a portfolio with higher duration, relative to the investor that is more risk averse. Interestingly –and perhaps, initially, more surprising– the expected gross growth rate of capital for the less risk averse investor is lower than that of the more risk averse investor. How is that possible? The answer is that, first, the higher duration portfolio of the less risk averse investor is more susceptible to interest rate volatility, and, second, the no-arbitrage condition rules out profiting from such volatility. As a result, the expected gross growth rate of wealth for the more risk averse investor is higher.

V. PORTFOLIO ANALYSIS: MARKET EQUILIBRIUM

Let $S(t)$ be a vector for the supply of Treasury securities in the bond markets at each maturity provided by the decisions of the U.S. Treasury and the Federal Reserve Board. Consequently, the equilibrium condition in the market for Treasury securities is given by

$$D(t) = S(t).$$

For simplicity, suppose the inverse of ω_1 exists, so that the number of independent securities is the same as the number of interest-rate risk factors (otherwise, we would have to use its pseudo inverse). Then from Equations (12) and (13), we find that the expected excess return on bonds is dependent on the behavior of all the investors in the Treasury market in equilibrium:

$$\mu(s) - \mu_\tau(s) = \frac{1}{\sum_{j=1}^{J} \frac{W^j}{\gamma^j}} \Bigg\{ (\omega_1)^{-1} S(t) + \omega_3 \sum_{j=1}^{J} \gamma^j (\sigma_j(\tau))^{-1} \mu_j(\tau) \frac{W^j}{\gamma^j} - \omega_2 \sum_{j=1}^{J} \xi^j \frac{W^j}{\gamma^j} - \omega_3 \sum_{j=1}^{J} \gamma^j (\sigma_j(\tau))^{-1} \frac{W^j}{\gamma^j} X \Bigg\}.$$

$\sum_{j=1}^{J} \frac{W^j}{\gamma^j}$ is the sum of the inverse of the absolute risk aversion coefficients across all investors, such that $\frac{1}{\sum_{j=1}^{J} \frac{W^j}{\gamma^j}} \sum_{j=1}^{J} (\sigma_j(\tau))^{-1} \mu_j(\tau) \frac{W^j}{\gamma^j}$ is the weighted average of the desired Sharpe ratio and $\frac{1}{\sum_{j=1}^{J} \frac{W^j}{\gamma^j}} \sum_{j=1}^{J} (\sigma_j(\tau))^{-1} \frac{W^j}{\gamma^j}$ is the weighted average of the portfolio's standard deviation. Define $\theta \equiv \frac{1}{\sum_{j=1}^{J} \frac{W^j}{\gamma^j}}$ to be one divided by the sum of the inverse of the absolute risk aversion of all investors, while $\theta^j \equiv \frac{W^j}{\gamma^j}\theta$ is the individual investor's contribution to this value. The price of risk coefficients from Equation (14) in equilibrium are as follows:

$$(b - \iota b_\tau)(\gamma^{\mathcal{P}} - \gamma^{\mathcal{Q}}) = \theta (\omega_1)^{-1} S(t) - \omega_2 \sum_{j=1}^{J} \theta^j \xi^j + \omega_3 \sum_{j=1}^{J} \theta^j \gamma^j (\sigma_j(\tau))^{-1} \mu_j(\tau), \tag{15}$$

$$(b - \iota b_\tau)(A^{\mathcal{P}} - A^{\mathcal{Q}}) = \omega_3 \sum_{j=1}^{J} \theta^j \gamma^j (\sigma_j(\tau))^{-1}. \tag{16}$$

Equation (15) is the constant in risk pricing Equation (14). This term is positively related to the quantity of Treasury securities made available to the markets, weighted by total absolute risk aversion. This constant is also negatively related to the leverage ratio of all investors, weighted by absolute risk aversion relative to its value for the marginal investor. We can think of these weights as the probability distribution of the inverse of the absolute risk aversion for each group of investors in Treasury markets. Finally, the last term in Equation (15) is a weighted average of the Sharpe ratio for the expected lifetime utility for each bucket of investors in the market. This effect is multiplied by the covariance between the expected excess return on bonds and the interest-rate risk factors, ω_3. Thus, this last term captures how much a change in interest-rate risk factors influences the risk adjusted return on the investor's expected lifetime utility; and can hence be interpreted as the amount of risk acceptable to the investor when factors are at their long-term mean.

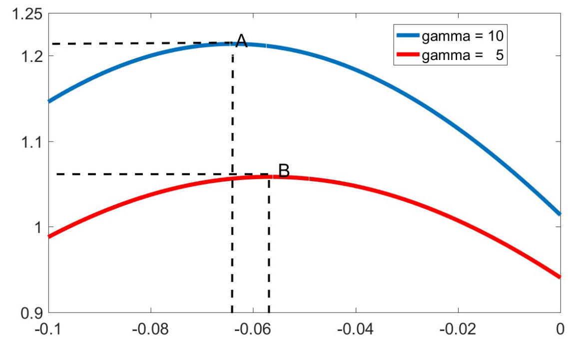
Equation (16) represents the slope of the price of risk in the Treasury market, which captures how changes in interest-rate risk factors impact expected excess returns through the marginal investor's standard deviation of her expected utility. This term is amplified by ω_3 and the absolute risk aversion for investors in each bucket.

For the market, we can find the equilibrium return as the fixed point of Equations (15) and (16) with the two types of investors. In Figure (5), we overlay the top two curves in Figure (4) and limit the range of the level of the yield curve to emphasize the difference in the position of the two investors at a fixed point. Each investor has a maximum expected gross rate of return on wealth under their optimal portfolio strategy, which occurs at the level of the yield curve $\mu^1_{\mathcal{M}}(1) = -0.0639$ and $\mu^2_{\mathcal{M}}(1) = -0.0567$. These maxima are 1.2138 for the more risk averse investor (in blue) at point A, and 1.0584 at point B for the more risky investor (in red). From Figure 4, we know that the duration of the portfolio for the more risk averse investor is always lower relative to the less risk averse investor. In addition, the Gaussian functions of the gross rate of return on wealth have a standard deviation $\sigma^1_{\mathcal{M}}(1) = 0.1065$ and $\sigma^2_{\mathcal{M}}(1) = 0.1167$, so that the dispersion is smaller for the more risk averse group.

We can now discuss properties of the equilibrium in the bond markets. Consider a market in zero net supply, so that the market equilibrium cannot be to the right of point B, since both investors would be long on the long-term bond and short on the short-term bond by Figure 4. As a result, the short-term bonds are not in equilibrium, since the total demand is negative and the supply is zero. Similarly, the market equilibrium cannot be to the left of point A, since both investors would go short on long-term bonds and the demand for long-term bonds would be negative. For the excess returns in the Treasury market to be in equilibrium, it must then be the case that the distribution across the two investors implies that one investor is shorter and the other is longer on longer-term Treasury securities. Thus, a market equilibrium must be between points A and B. In particular, we see in Figure (5) that if the equilibrium is between points A and B, the more risk averse investor is longer on the long-term Treasury security. If $S(t)$ is positive, both investors will be longer on the long-term bond, to the right of point B, but the more risk averse investor would be even longer on those securities.

Notice that while the portfolio position of investors is dependent on their constant relative risk aversion, the price of risk for the Treasury market is also a function of their wealth and hence, absolute risk aversion. In addition, if the coefficient of relative risk aversion is fixed, then the wealthier investor takes on a portfolio of Treasury securities that shorts the longer-term Treasury security. This means that the more risk averse investor will have a larger capital loss when the the level of the yield curve raises, so that her change in wealth would be actually larger, $\Delta W^1 > \Delta W^2$.

Figure 5: Finding the Equilibrium Return with Two Investors



Notes: The blue line represents the expected gross growth rate for an investor with high risk aversion and the re line, for an investor with low risk aversion, as in the top row of Figure 4. Point A and B represent their mean, respectively, conditional on the level of the yield curve.

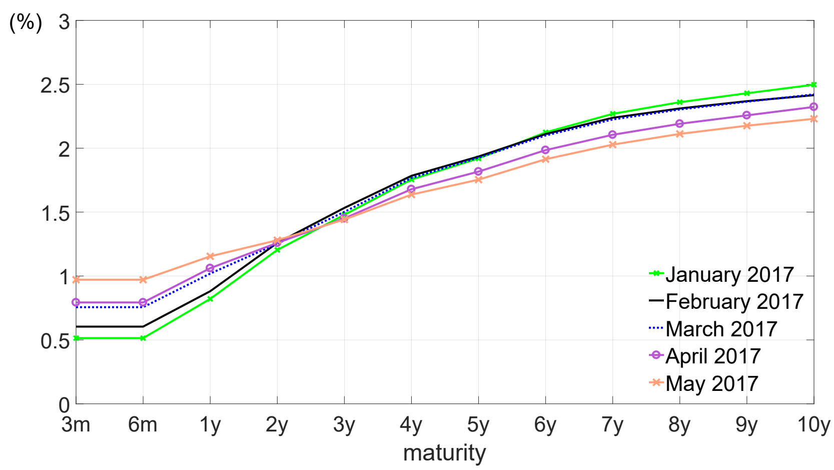
If a is the proportion of more absolute risk aversion, then the mean conditional on the level of the yield curve is $\mu^2_{\mathcal{M}}(1) - a(\mu^2_{\mathcal{M}}(1) - \mu^1_{\mathcal{M}}(1)) = -0.0603$ with $a = 1/2$. Thus, the expected gross growth rate on the marginal investor is given by a Gaussian function between the expected values for the more risk averse investor and the less risk averse investor, at the maximum expected value of 0.1334 for the current level of the yield curve. This opens an indirect impact of monetary policy through changes in the distribution of absolute risk aversion of investors.

Now consider a tightening of monetary policy that increases the level of the yield curve. In this case, investors with a higher level of absolute risk aversion (investors $j = 1$) would be longer on long-term Treasury securities relative to the less risk averse investors (investors $j = 2$). As a result, $j = 1$ investors would suffer a capital loss, while $j = 2$ investors would experience a capital gain. First, the absolute risk aversion increases for the more risk averse investors, while it decreases for the less risk averse investors. Second, the increase in absolute risk aversion for $j = 1$ investors is larger, widening the spread across investor types. If investors have the same constant relative risk aversion, then the tightening of monetary policy leads to more conservative behavior by less wealthier investors in the Treasury market and more risky behavior by the wealthier investors. In particular, if the investors in Treasury securities are banks, the smaller banks would become more conservative and the larger banks would engage on more risky behavior under tighter monetary policy.

VI. APPLICATION: THE 2017 TIGHTENING OF MONETARY POLICY

We now consider the tightening of monetary policy in March 2017, and the anticipated tightening of monetary policy in June 2017. We start with the actual yield curves over the previous five months from January to May 2017 in Figure 6. Over this period, the 3- and 6-month rates went up by 0.5 percentage points (p.p.), while the longer-term rates came down about 0.25 p.p., reflecting an increase in demand for longer-term U.S. Treasury securities.

Figure 6: Yield Curves from January 2017 to May 2017



Source: Bloomberg L. P.

The level of the yield curve was about 1 p.p. below its mean over the longer period January 1975 till March 2017, and about 1.25 standard deviations lower than its historical average. The slope in the current period was also more than 1.33 standard deviations below the long-run mean. Translating the interest rate hike into interest-rate risk factor movements, the level of the yield curve actually decreased by 3 basis points; the slope was reduced by 3 basis points as well; and the curvature of the yield curve changed by less than one basis point over the period January to May 2017. How does this particular change in the level, slope, and curvature of the yield curve affect the portfolio of investors? The increase in short-term rates by 0.25% in March 2017 and anticipated increase in June 2017, influences the expected SDF of the marginal investor in the U.S. Treasury security markets, as in Section III.

In this context, the one-month expected SDF was 0.997007 in January 2017, corresponding to a 29 basis-point return over the following month. Given the documented movement in the level of the yield curve, the expected SDF declined to 0.996996 in May 2017, which is a 0.1 basis point decline in the present discounted value of cash flows. This decline follows from the decrease in the level of the yield curve, since the level factor was below the level associated with the maximum value of the conditional SDF. The one-year expected SDF in January 2017 was 0.8691 and decreased to 0.8690 or 1.2 basis points. As a result, while the decrease in the expected SDF from January to May 2017 was only 1.2 basis points, the annual expected capital gain on investments in the U.S. Treasury markets associated with this tightening episode was estimated to be 14%.

A similar analysis can be done for a change in the slope and curvature of the yield curve reflecting the tightening of monetary policy. This analysis also leads to a decrease in the expected SDF at 1 and 12 months, since the decrease in slope and curvature are to the left of the mean of the slope and curvature associated with the maximum conditional distribution of the expected SDF. Thus, the tightening of monetary policy led to a decrease in the expected value of cash flows received by investors in the bond market. Chami et al. (2017) demonstrate that this decrease in present values would lead banks to reduce lending. Yet, it is not because the level of the yield curve decreased during the monetary tightening, but because it was originally below the level associated with the maximum value of the expected SDF.

VII. CONCLUSION

Using a no-arbitrage asset pricing model for the U.S. Treasury market, we describe how the critical nature of the variation in discount rates matters for the transmission of monetary policy across investors' portfolio positions. Whenever interest-rate risk factors move away from the conditional mean of the expected SDF, the discount rate increases, leading to a decline of all expected future cashflows. During the tightening of monetary policy in the spring of 2017, we find that both the level and slope of the yield curve fell. This lead to a decline of the expected SDF, since the level and slope were below the points at which the investor's conditional density reaches its highest point. Thus, the anticipated interpretation of the impact of monetary policy on financial markets must account for the current state of the yield curve at each particular point in time.

We illustrate the impact of the variation of discount rates on individual decisions, by solving for the optimal portfolio of investors that base their decisions on the expected SDF derived from the no-arbitrage model. When the interest-rate risk factors are above their long-term mean, investors go long on longer-term bonds, since they anticipate a future fall in the factors leading to a bigger capital gain. These portfolio positions are reversed when the factors are below their means as investors anticipate increases in the factors. Chami et al. (2017) use these insights to examine the critical role of bank-holding companies in transmitting monetary and financial sector shocks to the rest of the economy. As discussed using the level, slope, and curvature of the yield curve, a tightening of the short-term rate may lead to a decrease in cash flow valuations, which could eventually disrupt the bank lending channel. However, this result is dependent on the current state of these interest-rate risk factors relative to their means.

Finally, we show that the estimated no-arbitrage model of the term structure is an equilibrium in which all investors follow the optimal portfolio rules with different degrees of absolute risk aversion. In this equilibrium, the more risk averse investors have a longer position in long-term bonds relative to the less risk averse investors. Thus, the more risk averse investor suffers a larger capital loss from the tightening of monetary policy, such that they have a larger increase in the absolute risk aversion.

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