

LEVERAGE AND RATE OF RETURN HETEROGENEITY AMONG U.S. HOUSEHOLDS

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This paper proposes a method to construct household-level panel data on rates of return in the United States for the last 18 years. Fixed effects with empirical Bayesian shrinkage is used to document permanent heterogeneity in returns to wealth among U.S. households. This is shown for returns to assets and wealth for the entire household portfolio as well as for five asset classes: risk-free assets, primary and secondary real estate, private businesses, and public equities. The majority of the permanent heterogeneity in the returns to wealth are driven by heterogeneity in the degree and cost of borrowing. Permanent heterogeneity in leverage across households is also documented. Evidence is provided on how returns, and household-specific returns, differ over the wealth distribution. On average, returns to wealth are declining in total household wealth due to lower leverage of wealthy households. That said, wealthier households are found to have higher permanent returns to assets. The wealthiest households are found to maintain leverage for private business and secondary housing assets. Most, but not all of the permanent heterogeneity in returns are associated with risk. Heterogeneity in returns not associated with risk can be shown to arise from regressivity of after-tax mortgage rates. These insights hold important implications for the study of portfolio allocation, wealth inequality, social mobility, and corresponding policies.

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JEL classification: D14, D31, E21, G11

1 Introduction

The causes and consequences of wealth inequality have long been topics of intense debate. More recently, permanent heterogeneity in returns to wealth has been implicated in explaining wealth inequality (Kesten, 1973; Benhabib et al., 2011; Quadrini, 2000; Piketty, 2014) and the evolution of wealth inequality over recent decades (Gabaix et al., 2016; Khieu and Wälde, 2018). While persistent differences in returns to wealth have been posited in quantitative models, the degree and cause is just beginning to be empirically validated. The lack of evidence is driven by the scarcity of panel data at the individual and household levels leaving key questions unanswered. For example, what is the degree of heterogeneity in returns? How much of the heterogeneity persists? Is return heterogeneity across individuals driven by risk taking? Which asset classes induce persistence in returns? How do returns covary with wealth?

This paper addresses these questions using household-level micro-panel data for the United States. The panel data spans 1999 to 2017 covering various financial cycles. A method is proposed to construct household-level returns to wealth and assets using recent information in the newly revised Panel Study of Income Dynamics (PSID). Returns to wealth versus returns to assets are compared to isolate the role of endogenous borrowing decisions, risk taking, and regressivity in tax-deductible interest obligations. Returns are evaluated for the entirety of the household portfolio as well as broken into five asset classes: risk-free assets, primary and secondary real estate, private equity, and public equity. Empirical evidence is presented on the degree of persistent heterogeneity in returns – “type dependence” – and on how the average returns correlate with wealth – “scale dependence.”

This paper complements recent evidence on scale and type dependence in returns to wealth in the Scandinavian administrative tax data sets of Bach et al. (2016) and Fagereng et al. (2018). However, the analysis differs in key ways. First, this paper is the first to examine persistent heterogeneity in household returns in the United States. Fagereng et al. (2018) is the only other study to document permanent individual returns on assets in Norway. Second, net investment is included in the measure of capital gains for housing and private businesses. This contrasts with all previous studies that exclude net investment. Third, this is the first study to examine persistence in returns to wealth, which allows an isolation of the role of leverage and borrowing cost heterogeneity. Permanent heterogeneity has only been observed for the return to assets by Fagereng et al. (2018), but it is the return to wealth that matters for understanding wealth inequality. Fourth, the analysis is not limited to taxable wealth but includes the entire household portfolio including non-taxable assets and pension wealth as well as other debts (such as family and medical debts) not observed in the tax administrative data of Bach et al. (2016) and Fagereng et al. (2018). Finally, this is the first study to observe returns by asset class to examine which classes induce scale and type dependence.

The main contribution of the paper is to test if and why returns correlate with wealth and display persistent heterogeneity across households. The estimates motivate the structure and calibration of return heterogeneity in studies of wealth inequality.

Household-specific returns to both wealth and assets are estimated using fixed effects with empirical Bayesian shrinkage. The household-specific return to wealth is of primary interest for the literature on wealth inequality (Benhabib et al., 2011; Gabaix et al., 2016). The household-specific return to assets and wealth are compared to examine the contribution of heterogeneity

in borrowing and borrowing costs to the heterogeneity in household-specific returns to wealth. The same method is applied to estimate household-specific leverage to examine whether leverage uniformly or heterogeneously disperses the return to assets.

Returns to assets and to wealth for the total household portfolio as well as within asset classes are then estimated with linear panel data regression models. The regressions include: observable characteristics of the household (lagged wealth, age, marital status, education, and gender); year fixed effects; and financial characteristics (relative volatility, leverage and lagged portfolio allocation). Permanent observable household characteristics such as marital status, race, and education are found to be significantly associated with heterogeneity in returns. That said, the portfolio characteristics of leverage, risk, and allocation explain the majority of the heterogeneity in returns to wealth.

Significant household-specific returns are found for total portfolio returns and for returns within all asset class. This suggests that portfolio allocation alone cannot explain heterogeneity in returns entirely, as returns exhibit persistent heterogeneity even within asset classes. The standard deviation of the household-specific return to total household assets is estimated to be 3.94 percentage points. Primary housing assets exhibit a sizeable permanent component with a standard deviation of 3.19 percentage points.

In contrast, the standard deviation of the household-specific return to wealth is estimated to be 9.34 percentage points. A key finding of the analysis is that the majority of the permanent heterogeneity in returns to wealth, 58 percent, is driven by household specific differences in leverage. Idiosyncratic persistent heterogeneity in leverage explains a high share of the returns to wealth for the leveraged asset classes: 19 percent of the persistent heterogeneity of return to private business equity and 75 percent for primary housing wealth.

Leverage systematically amplifies both the average return and the volatility of the return to wealth. Hence, the component of permanent heterogeneity in returns attributed to leverage is associated with higher risk. The ability in this study to observe encompassing measures of debt is shown to further amplify household-specific returns to wealth. Failing to take secondary household liabilities, such as family debts, into account is shown to underestimate household-specific leverage and returns to wealth.

Leverage is then deconstructed into its household-specific component using fixed effects with empirical Bayesian shrinkage, similar to returns household-specific returns. Substantial household-specific leverage is observed between households for all asset classes and for total household leverage. The majority of the idiosyncratic heterogeneity in leverage for private business and housing assets is found to be household specific. The permanent heterogeneity in leverage suggests that leverage's amplification of the return to assets for the dispersion of the rate of return to wealth is not uniform across households.

Leverage is found to be predicable and significantly correlated with household wealth but also with portfolio allocation, marital status, and education level. A higher share of investment in public equities in the total household portfolio is significantly and positively correlated with more leverage within the housing asset classes.

Regarding scale dependence, returns to wealth are found to be negatively correlated with total household wealth in the United States. This is consistent with the findings for total household wealth in Sweden by [Bach et al. \(2016\)](#) and Norway by [Fagereng et al. \(2018\)](#). The ability to

observe leverage directly as well as returns by asset class allows a detangling of this phenomenon. Rates of return to wealth decline in total household wealth due to deleveraging as household wealth increases. Poorer households maintain higher leverage in housing, whereas wealthier households aggressively pay off primary housing mortgage debts. The exception is the wealthiest private business and secondary homeowners who maintain leverage in those asset classes. These findings cast doubt on the mechanism of increasing returns to wealth driving the recent growth in wealth inequality as proposed by [Gabaix et al. \(2016\)](#).

The negative correlation of returns to wealth with total household wealth also arises from declining marginal returns to non-financial assets. The return to housing assets and private business assets are found to be declining in total household wealth. Moreover, concentration of portfolio allocation within an asset class is significantly found to reduce the return to that asset. This provides evidence against increasing returns from asset specification. The evidence of decreasing marginal returns and decreasing returns to specialization has important implications for optimal taxation, as increasing the wealth of the wealthiest households is unlikely to increase average efficiency when marginal returns are declining ([Conesa et al., 2009](#); [Guvenen et al., 2017](#)).

The household-specific return captures permanent differences in returns across households, including systematic differences in risk taking. Higher values of household-specific returns to assets are found to be closely associated with higher idiosyncratic return volatility, especially for financial wealth and total household assets. Differences in household wealth and the relative realized volatility of the household portfolio return are found to explain the majority of the household-specific return to assets. Combined with the volatility enhancing nature of leverage, this suggests that differences in risk taking account for the majority of the household-specific returns to wealth.

Portfolio allocation in primary and secondary housing is observed to be associated with lower returns to assets within the other asset classes. This suggests that background risk can affect an investor's risk appetite within asset classes and explain part of the persistence in returns within asset classes.

The difference in real mortgage rates between the bottom and top 20 percentiles of wealth is observed to be 2.9 percentage points and 3.6 percentage points for the real after tax mortgage rate. The contribution of borrowing costs and interest deductibility is unique as it shows a systematic component between household wealth and returns to wealth that is not associated with risk taking. This alone contributes to a 1.5 percentage point difference in the return to wealth between the top and bottom 20 percentiles of wealth. Hence, an important part of the relationship between returns and wealth is not explained by risk, but rather, the regressivity of interest deductibility and the lower costs of borrowing for the wealthy.

An advantage of the PSID household-level microdata is the level of detail and encompassing measure of the household portfolio. This allows a deconstruction of the role of risk taking and leverage by asset class. The disadvantages of the PSID include the potential reporting bias and measurement error with survey data. All panel data on returns exhibit forms of measurement error. For example, measures of net investment for the calculation of capital gains for housing and private businesses are reported in the PSID but not in administrative tax data. Fixed effects with empirical Bayesian shrinkage is used to remove the bias of transitory measurement error in the estimation of the household-specific component. The results are thus found to be very robust.

This analysis is able to provide insights into the returns to wealth for the majority of households.

Concerns that the wealthiest percentiles of households are under-represented in the sample are partially mitigated by the use of the most recent waves which have no incidences of top-coding. While much of the focus of studies of return to wealth heterogeneity has been to explain the thick right tail of the wealth distribution, the PSID misses the top two percentiles of the wealth distribution. [Bach et al. \(2016\)](#) documented that the cross-sectional standard deviation of realized returns to wealth begins to increase at the 90th wealth percentile and continues to increase as household wealth increases. Hence, the PSID should capture the shifting risk appetite of the wealthy.

The empirical insights of this paper have important implications for the study of portfolio allocation, wealth inequality, social mobility, and corresponding policies.

Household-specific returns are found for financial assets and are associated with heterogeneity in risk characteristics. Moreover, risk taking in public equity systematically increases with total household wealth. This suggests that the share of equities in the portfolio is not able to identify the degree of risk taking and calls into question using the share of the portfolio invested in stocks as a measure of risk preferences such as in [Brunnermeier and Nagel \(2008\)](#) or [Chiappori and Paiella \(2011\)](#). Instead, the degree of risk taking within an asset class needs to be considered jointly with the share in that asset to identify how risk aversion changes with wealth.

The findings for return heterogeneity can provide the basis for appropriate models of asset income and wealth inequality such as in [Benhabib et al. \(2019\)](#). Both household-specific returns to assets and leverage are needed to capture the degree of return to wealth heterogeneity. Higher household-specific returns to assets should be associated with higher risk taking and the endogenous leverage decisions are critical to match the declining returns to wealth over the life cycle. Unlike the earning heterogeneity literature, age profiles are not found to produce significant life cycle variability in returns but are due to variation in wealth across households. Reduced borrowing costs and risk-return efficiency of the wealthy suggest that risk and returns will not scale linearly with wealth.

The empirical evidence supports the role of return heterogeneity arising from entrepreneurial skill ([Quadrini, 2000](#); [Cagetti and De Nardi, 2008, 2009](#); [Nirei and Aoki, 2016](#); [Benhabib et al., 2011](#)). Returns to private business equity are found to exhibit declining returns to scale which are slightly offset by wealthier households maintaining higher leverage of business assets.

Finally, the findings may have important implications for the design of taxation and redistribution. Adequate explanations for permanent heterogeneity in returns will be important for proper inferences in such investigations. [Shourideh \(2013\)](#) finds that the relative degree of permanent and transitory components of returns informs the degree of capital income taxation progressivity. This paper provides a variance decomposition of the permanent and transitory components that can be used to inform the progressivity of taxation. The permanent component's contribution to the variance of the idiosyncratic return to total assets is found to be 18 percent, close to the estimate of 28 percent for individuals in [Fagereng et al. \(2018\)](#) for Norway. However, the permanent component's contribution to the variance of the idiosyncratic return to total wealth is found to be higher at 36 percent. The evidence also suggests that the permanent component differs by asset class, being particularly low for public equity. This may be used to inform targeted taxation of specific asset classes.

The paper is structured as follows. Section 2 introduces the modelling method. Section 3

introduces the data source summarizes the sample and the main features. Section 4 reports the estimation of scale and type dependence across households and examines robustness. Section 5 summarises the findings and discusses implications.

2 Empirical Model

This section proposes a model to estimate household-specific returns to wealth. The return to wealth is regressed on a set of controls to remove year and age effects. Year effects are removed to account for aggregate effects in the unbalanced panel data. Age effects are removed to isolate age invariant differences in returns. This can capture life cycle portfolio allocation or gains from experience. Specifically, the return on wealth is regressed on a set of year and age indicators, $z'_{j,it}$.

$$r_{j,it}^w = \mathbf{z}_{j,it} \beta_j^z + \tilde{r}_{j,it}^w. \quad (1)$$

The residuals $\tilde{r}_{j,it}^w$ represent the return excluding age and year effects. This is then regressed on a set of household observables and controls for portfolio characteristics, $x'_{j,it}$. Specifically, the return excluding age and year effects for household i in year t is modelled as:

$$\tilde{r}_{j,it}^w = \mathbf{x}_{j,it} \beta_j^w + e_{j,it}^w. \quad (2)$$

The controls measure predictable variation in returns and include lagged portfolio shares (interacted with year fixed effects). Portfolio shares are between 0–1 and rates of return are represented in percent, so the coefficients are interpreted as percentage points. The controls for observable household include indicators for family size, marital status, number of children, region interacted with year fixed effects, outside dependants, and income from a family member other than head or spouse. Controls also include separate indicators if the household reported selling any primary housing, secondary housing, private business, and/or public equity since the last wave. Indicators for the percentile of wealth for household i in the previous wave is also included in the set of controls. The lagged value of wealth is used to avoid spurious correlation of returns and household wealth.

The unexplained component, $e_{j,it}^w$ is represented as the sum of a household-specific return $\epsilon_{j,i}^w$, and the idiosyncratic error term $u_{j,it}^w$.

$$e_{j,it}^w = \epsilon_{j,i}^w + u_{j,it}^w. \quad (3)$$

The standard deviation of $\hat{\epsilon}_{j,i}^w$ is denoted $\hat{\sigma}(\epsilon_j; w)$. The same framework is used to estimate the return to assets $r_{j,it}^a$ to estimate $\hat{\epsilon}_{j,i}^a$ and yield $\hat{\sigma}(\epsilon_j; a)$. This is done for the total household portfolio as well as each asset class $j \in \{ph, oh, b, s, f\}$.

The household-specific return reflects persistent differences in financial knowledge, risk preferences (including portfolio allocation), investment skill (potentially including entrepreneurial ability), persistent differences in borrowing costs, and gains or losses from asset scale. It also subsumes permanent characteristics such as race, educational level, and gender. The contribution of these observable characteristics to the measures of returns is examined in more detail in the later sections. The transitory idiosyncratic error, $u_{j,it}^w$, can be interpreted as the sum of measurement error and the risk of household's i investments from both good and bad luck.

The household-specific return, $\hat{\epsilon}_{j_i}^w$, is estimated with empirical Bayes (EB). The EB procedure adjusts fixed effects estimates by shrinking them toward the mean of the true, underlying return distribution of the household-specific return. This ensures that the transitory term $u_{j,it}^w$ does not introduce bias. This procedure is based on [Morris \(1983\)](#) as operationalized by [Chandra et al. \(2016\)](#). Heteroskedastic-robust standard errors are used to estimate the noise to shrink the fixed effects. A detailed description of this procedure along with Monte Carlo experiments can be found in appendix B.

By comparing the household-specific return to assets and the return to wealth, a measure of the contribution of the standard deviation in the household-specific return to wealth due to differences in borrowing and the cost of borrowing can be calculated, γ_j .

$$\gamma_j = \frac{\hat{\sigma}(\epsilon_j; w) - \hat{\sigma}(\epsilon_j; a)}{\hat{\sigma}(\epsilon_j; w)}, \quad (4)$$

where $\gamma_j = 0$ if $\hat{\sigma}(\epsilon_j; w) = \hat{\sigma}(\epsilon_j; a)$ and $\gamma_j = 1$ if $\hat{\sigma}(\epsilon_j; a) = 0$. Bounds, $\gamma_j \in [0, 1]$, are guaranteed by the requirements of positive asset values and the requirement that both rates of returns to assets and wealth are observed. This measure captures the share of the standard deviation of the household-specific return to wealth due to leverage. Leverage plays a unique role in the dispersion of the returns to wealth, amplifying both the level and volatility of returns to wealth. In contrast, analysing the household-specific return to assets removes the endogenous leverage and borrowing cost decisions and allows for a focused examination on asset risk.

3 Data

The Panel Study of Income Dynamics (PSID) is used to calculate before-tax real returns and log-real wages. The dataset provides household-level unbalanced panel data using surveys conducted every two years between 1999–2017. All returns are real and nominal values are converted into real values using the Federal Reserve consumer price index.

3.1 Measurement

A detailed description of how returns are calculated can be found in appendix A so this section summarizes the main data contributions. The total return to assets is the weighted sum of returns to primary housing, secondary housing, private businesses, public equity, risk-free and other assets. The total return to household assets is given as:

$$r_{a,it} = \sum_{j=b}^J \omega_{j,it} r_{j,it}, \quad (5)$$

where $j \in J$ and $J \in \{b, ph, oh, s, f, o\}$ and $\omega_{j,it}$ is the weight of that asset in total household assets.

The return to primary residential housing includes capital gains, the value of housing services, the costs of maintenance, and rental income. Capital gains use information on the reported value of the house in each period, the sale price if sold, and any net investment in the housing from major renovations or repairs. The return is based on [Flavin and Yamashita \(2002\)](#) but differs in a few ways. Information on net investment were not available and missing for the calculation of capital

gains in [Flavin and Yamashita \(2002\)](#) but are available and included in the current measure. Also, returns in this paper include rental income accrued from renting part of the primary residence.

Asset values for secondary housing and private business assets are available for the last three waves, and asset values for these asset categories prior to the last two waves are calculated using the change in net worth and net investment. The return to business assets is defined as the sum of income from businesses and farms plus capital gains. Only one household reports farm income and the results are not sensitive to their inclusion in the sample. Similar to the return to primary housing, the measure of returns to secondary housing includes capital gains, the value of housing services, the costs of maintenance, and rental income. The PSID includes information on repairs and maintenance, but only beginning in 2005. The average depreciation rate, δ , is set to the average value of repairs and depreciation costs from 2005-2017, 1.7 percent.

The interest income from public holdings is the difference between reported value of bond interest income and the potential return on risk-free assets calculated using the Treasury bill rate. The remainder of the reported interest income is then allocated to individual retirement accounts (IRAs) and direct equity holdings. The return to public equities is the sum of dividends, interest income, and capital gains from direct equity holdings over the value of IRAs and direct equity holdings.

The return to assets represents the pre-tax real return and does not deduct interest payments in the yield. The yield for the return to total wealth is identical to the yield for the return to assets but deduct interest payments. The return to wealth also has lagged wealth in the denominator of the return. Two measures of net household wealth are calculated. The first is total net worth excluding other debts, $d_{o,it}$, such as credit card, student, medical, legal and family debt. The second is inclusive of all debts and serves as the main measure of return to wealth.

The total return to assets used in this paper is the most similar measure to the return to individual “net worth” in [Fagereng et al. \(2018\)](#), who use the asset value in the denominator but exclude interest payments in the numerator. However, wealth in this paper includes information on durable wealth and other valuables such as collections that are reported by the household that would not traditionally be reported as asset income for tax purposes. This is one of several advantages of the PSID over alternative datasets. This includes comprehensive measures of households asset portfolios including non-taxable assets not observed in the European administrative tax data sets ([Bach et al., 2016](#); [Fagereng et al., 2018](#)). It also differs from the European administrative tax data sets in that asset values are observed in each period as opposed to only realized capital gains. In addition, this is the first study to include net investments in measures of capital gains.

Nominal returns to assets and wealth for all asset classes and for total household returns are converted to real returns using the annualized total consumer price index provided by the Federal Reserve (CPI)

Leverage is also examined for the household and for each asset class by measuring the debt-to-asset ratio. Total household leverage excluding other debts, $lev_{ax,it}$, provides a measure of leverage of debts associated with particular asset classes and is measured as:

$$lev_{ax,it} = \frac{d_{b,it} + d_{ph,it} + d_{oh,it}}{a_{b,it} + a_{ph,it} + a_{oh,it} + a_{s,it} + w_{f,it} + w_{ira,it} + w_{o,it} + w_{v,it}}. \quad (6)$$

where $d_{j,it}$ is debt in asset class j , $a_{j,it}$ is the value of assets in asset class j , and $w_{j,it}$ is the wealth

in asset class j . This is contrasted with total leverage which includes other debts, $lev_{a,it}$:

$$lev_{a,it} = \frac{d_{b,it} + d_{ph,it} + d_{oh,it} + d_{o,it}}{a_{b,it} + a_{ph,it} + a_{oh,it} + a_{s,it} + w_{f,it} + w_{ira,it} + w_{o,it} + w_{v,it}}. \quad (7)$$

Similarly, the debt-to-asset leverage ratio is calculated for asset classes that can be leveraged and is measured as:

$$lev_{j,it} = \frac{d_{j,it}}{a_{j,it}}, \quad (8)$$

where $j \in \{b, ph, oh\}$. For the rates of return to wealth and leverage in each asset class, the debts associated with the asset are attributed to that asset class. The measure of the returns and leverage are summarized and modelled in the next sections.

3.2 Sample Selection

The baseline analysis considers households with a continuous marital status. Observations are dropped if there is a change in the head or spouse. By including only households with continuous marital status, no attempt is made to model asset return risk associated with divorce and marriage. Observations are biennial from 1999–2017, as per the survey frequency. The baseline sample excludes Survey of Economic Opportunity (SEO) households. Households are dropped if the head was born prior to 1920. Households are kept only if the head’s age is between 20–70. The mean and median age is 43.

Outliers for wages are treated in a similar way to that of [Blundell et al. \(2008\)](#). A household is dropped if the household’s total labor income is below \$100. Returns are excluded if the value of the asset or net worth in the denominator of returns is less than \$500. This ensures that returns to wealth are finite. Hence, the analysis of returns to wealth should always be interpreted as conditional on positive wealth. The exception is private business assets where the return measure is dropped if the minimum asset value is below \$5000. This selects towards private businesses with significant capital and helps remove extreme asset return values. The robustness of minimum values is examined during the robustness exercises. The requirement that there must be an observed household wage means that there are very few heads who are students or retired in the sample.

For direct holdings of stocks, secondary housing and private businesses, an observation is dropped if the household reported ownership in the last period but the current period’s asset value is zero and the household did not report selling any of the asset. This requirement excludes a small number of households in the bottom tail of the return to assets. Observations are dropped if any demographic data is missing, unknown, or not reported. Returns are calculated as long as all components of asset income, net investment, debt and wealth are reported.

To account for extreme values that could skew the distribution, the top and bottom 5 return observations are dropped. Then, return and wage observations are dropped if the change or level of return to assets is beyond the studentized 99% confidence interval. Out of all the variables used in this study, no observation was found to be top coded or truncated at a high value.

Observations are kept only if there are three consecutive waves of available data on asset income, wealth, and labor income. Hence, there is a minimum requirement of two consecutive return observations. An observation for both the return to assets and return to wealth are required for a household to be included in the sample. The Bayesian shrinkage method is robust to noise from

transitory idiosyncratic errors. This makes the main results robust to all of the above assumptions as shown in section 4.3.

3.3 Data Summary

A comparison of household-level returns to assets versus wealth highlights that returns are heterogeneous within asset classes and that debt disperses the return to assets. The return to assets and wealth for the five asset classes are summarized in Table 1. The total return to assets, r_{it}^a , is described as “*Total Assets*” and has a mean of 3.3 percent and a standard deviation of 10.2 percentage points. In contrast, the mean and standard deviations for returns to total wealth is 7.7 and 27.3 percentage points, respectively. The higher standard deviation on the returns to asset wealth reflects the fact that households’ wealth is leveraged. The return to total household wealth excluding other debts has a mean of 5.6 and standard deviation of 18.2 percentage points, respectively.

Table 1. Returns for all Asset Classes are Heterogeneous Within and Across Households

	Obs	Households	Mean	Std. Dev. Total	Std. Dev. Within	Std. Dev. Between	25p	75p	Skewness	Kurtosis
Total Assets	13076	4061	3.3	10.2	9.2	8.4	-0.3	7.5	1.1	8.1
Private Business	711	354	32.8	88.1	73.5	93.9	-10.7	47.1	3.1	15.7
Primary Hous.	13811	3737	5.1	10.6	9.7	8.2	-0.3	10.2	0.4	5.7
Secondary Hous.	1470	486	8.3	29.1	25.0	22.9	-6.4	14.8	1.6	7.4
Risk Free	18966	4848	-1.7	1.0	0.7	0.7	0.0	0.1	2.4	8.2
Public Equity	6299	2156	9.1	37.9	30.6	30.5	-0.12	8.6	3.3	17.3
Wealth ex. O. Debt	12949	4059	5.6	18.2	18.0	16.6	-0.8	10.7	1.0	15.0
Private Business	711	354	45.5	137.4	142.7	203.0	-13.8	75.5	8.1	91.0
Primary Hous.	13811	3737	13.3	35.7	32.8	34.5	-3.8	18.5	1.4	17.0
Secondary Hous.	1470	486	11.8	50.0	40.8	46.1	-17.6	23.8	2.7	17.1
Total Wealth	13076	4061	7.7	27.3	24.1	24.8	-0.9	11.9	1.1	20.1

Annualized rates of return for the household in percentage points, 1999-2017. Simple averages, excluding outliers and households with missing data. 25 and 75 p refers to the corresponding percentiles. "Wealth ex. O. Debt" refers to total household returns to wealth excluding other debts.

Returns are skewed right and display more kurtosis than a normal distribution. This is particularly true for business and financial assets. Skewness and kurtosis are amplified when leverage is accounted for in the return to wealth. The right skewness reflects the limited downside risk arising from the natural limit of zero asset values and the option to default or sell the asset.

The mean return to business assets is the highest at 32.8 percent and has a total standard deviation of 88.1 percentage points. In contrast, the mean and standard deviations for returns to business wealth are 45.5 and 137.4 percentage points, respectively, which reflects the fact that business wealth is highly leveraged. The between household standard deviation is found to be sizeable for all asset classes.

The heterogeneity within asset classes is greater than the heterogeneity in returns found in comparable aggregate benchmarks. The return to primary housing assets has a mean of 5.1 percentage points and a standard deviation of 10.6 percentage points. The standard deviation is lower than

the 14 percentage points from 1968–1992 calculated by [Flavin and Yamashita \(2002\)](#). Other than the sample period, this reflects the inclusion of individualized tax rates, net investment, and rental income in the calculation of the return to primary housing. The mean and total standard deviation of the secondary housing assets are 8.3 and 29.1 percentage points.

The Case and Shiller index to which [Flavin and Yamashita \(2002\)](#) compare their primary housing returns had a standard deviation of 7.7 percentage points between 1998–2014. Similarly, the standard deviation of the Freddie Housing index from 1998–2014 is 6.4 percent. The finding in [Flavin and Yamashita \(2002\)](#) that housing indexes underestimate the household return risk on housing is confirmed in this paper for the later sample period. This highlights a major advantage of the PSID in which asset values are reported in every wave. Relying on period-by-period returns extrapolated from realized returns and aggregate series may understate the degree of household heterogeneity in returns.

The mean return to public equity is 9.1 percentage points with a standard deviation of 37.9 percentage points. In contrast, the real return and standard deviation on the Standard and Poor’s (S&P) 500 index, including dividends, over the same period was smaller with a mean return of 4.6 and a standard deviation of 11.6 percentage points. Like housing, realized household returns to stocks exhibit more variation compared to their aggregate benchmarks. The mean return to risk-free assets is -1.7 percentage points with a standard deviation of 100 basis points. The negative real return reflects the fact that most risk-free assets are held in checking and savings accounts earning very little interest, especially during the second half of the period when the U.S. Treasury rate was close to zero.

4 Estimates of Household-Specific Returns

Attention is now turned to estimates of the household-specific returns using fixed effects with empirical Bayes shrinkage as per equation (3). The household-specific returns capture permanent differences across households which may arise from differences in financial knowledge or entrepreneurial ability as well as persistent differences in portfolio allocation and risk characteristics. It also subsumes other permanent observable household characteristics such as gains from pooling assets in marriage. The estimate of the standard deviation of the household-specific return is reported for each asset class in Table 2.

The estimates of the standard deviation of the fixed effects are significant at the one percent level and found for the returns to total household wealth and assets, as well as within every asset class. The standard deviation for the return on total household assets is 3.94 percentage points for the household-specific return and 8.43 percentage points for the transitory idiosyncratic risk. The standard deviation for the household-specific return to total wealth is 9.34 percentage points and 21.98 percentage points for transitory idiosyncratic risk. The key finding is that the majority of type dependence arises from heterogeneity in leverage, γ , which explains 58 percent of the fixed effects standard error for the total return to household wealth. Leverage amplifies the return to assets and is the most important factor driving the persistent heterogeneity in returns to wealth.

Leverage accounts for the majority of the household-specific return on wealth for housing returns. For example, the standard deviation of the household-specific return to primary housing is 3.2 percentage points, but 12.7 percentage points for the return to primary housing wealth.

Table 2. Leverage Explains 58% of Household-Specific Returns to Wealth

	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_u$	$\hat{\sigma}_\epsilon^2 / \hat{\sigma}_e^2$	γ
Total Assets	3.94	8.43	0.18	
Private Business	104.32	67.81	0.70	
Primary Hous.	3.19	9.00	0.11	
Secondary Hous.	8.86	25.97	0.10	
Risk Free	0.39	0.70	0.23	
Public Equity	4.04	32.84	0.01	
Wealth ex. O. Debt	7.25	14.72	0.20	46%
Private Business	128.71	100.78	0.62	19%
Primary Hous.	12.68	29.28	0.16	75%
Secondary Hous.	18.36	43.01	0.15	52%
Total Wealth	9.34	21.98	0.15	58%

Notes: Estimates of the fixed effects estimated using observable controls excluding leverage. Returns for the household in percentage points. "Wealth ex. O. Debt" refers to returns to total household wealth excluding other debts.

Hence, 75 percent of the household-specific return to primary housing wealth is due to the role of heterogeneity in the degree of borrowing and interest costs. This includes both the effects of household-specific borrowing and the role that leverage plays in amplifying both the volatility and average return. The role of leverage is also found to be important for household-specific returns to private business wealth accounting for 19 percent of the household-specific heterogeneity in the return on wealth.

The presence of other debts further amplifies the dispersion of the total return to wealth. This suggests that ignoring other household liabilities, such as family and credit card debt, would underestimate households' leverage and total return to wealth. The other debts amplify the fixed effects, increasing the share of the fixed effects explained by leverage from 46 percent to 58 percent.

The variance decomposition of the relative importance of the household-specific return in the idiosyncratic variation is presented Table 2. Specifically, the share of the variance explained by the household-specific return is given by $Var(\epsilon_j^x)/Var(e_j^x)$ for assets and wealth explicitly in $x \in \{a, w\}$. Shourideh (2013) has shown that the relative importance of $Var(\epsilon_j^x)$ and $Var(u_j^x)$ informs the degree of progressivity of optimal capital income taxation. In this paper, permanent differences in returns to total wealth across households accounts for 15 percent of the idiosyncratic return. For the return on assets, the permanent component represents 18 percent of the idiosyncratic component, slightly lower than the 29 percent found in Fagereng et al. (2018).

The variance decomposition can be examined for each asset class. The fixed effects compose the highest share of idiosyncratic returns to private business assets with a value of 0.62 for returns to wealth and 0.70 for the return to assets. The household-specific component for primary housing represents 0.11 for the return to assets and 0.16 for the return to wealth. The household-specific component variance share is the lowest for return to public equity at 0.01.

The household-specific return measures permanent differences in returns across households

which captures differences in wealth, portfolio shares and investment skill. To isolate the role of persistent differences in household-specific returns from portfolio composition, wealth, and risk, modified versions of equation 1 and 2 are estimated. Specifically, in the first stage, equation 1, portfolio composition, wealth, and relative portfolio volatility are successively regressed on the household return. Household’s relative portfolio volatility is defined as $sd(r_{j,it}^a)/sd(r_{jm}^a)$, where $sd(r_{j,it}^a)$ is the standard deviation of households i 's return on asset j and $sd(r_{jm}^a)$ is the within-household cross-sectional standard deviation of returns of asset j . Then, equation 2 is re-estimated with those controls excluded. The purpose is to isolate the contribution of portfolio shares, wealth, and risk from the household-specific component. These results are summarized in table 3.

Table 3. Wealth Contributes more than Portfolio Allocation

	Year	Year, Age	+Portfolio	+Wealth	+Port. & Wealth	+Port., Wealth and Risk
Total Assets	4.22	3.94	3.69	1.72	1.07	0.93

Notes: Standard deviation of the household specific return excluding the contributions of portfolio shares, household wealth, and relative portfolio risk. Returns for the household in percentage points. For example, "+Port., Wealth, and Risk" removes the contribution of year, age, portfolio shares, wealth, and relative risk.

Perhaps surprisingly, persistent differences in portfolio allocation contribute very little to household-specific returns. This aligns with the substantial household-specific returns within each asset class presented in Table 2. Instead, the wealth of the household represents the largest contribution of the household-specific return to assets. When both wealth and portfolio allocation are accounted for, the standard deviation of the household-specific return falls from 3.94 to 1.07 percentage points. When the contribution of wealth, portfolio allocation, and relative volatility are excluded from the household-specific return to assets, the standard deviation of the household-specific return falls to 0.93 percentage points. This provides evidence of the presence of household-specific heterogeneity in returns that could be attributed to investor skill.

These results reject homogeneous returns to wealth across households and motivate the inclusion of return heterogeneity in studies of wealth inequality. However, most of the permanent heterogeneity in returns to wealth is driven by differences in leverage and borrowing costs across households. Moreover, for the household-specific return to assets heterogeneity is associated with the wealth and relative volatility of the household asset portfolio. That said, even when wealth, portfolio allocation, or the relative volatility of the household portfolio are accounted for, there still remains a component of the household-specific return.

These insights provide an important contribution to the debate on how household-specific returns should be understood and modelled arising from the different results documented in the Scandinavian administrative tax data set (Bach et al., 2016; Fagereng et al., 2018). Fagereng et al. (2018) document fixed effects for returns on assets across households, but this partly reflects the persistent differences in portfolio risk characteristics across households. That said, even though persistent differences in risk characteristics drive the majority of the heterogeneity in return, as in Bach et al. (2016), risk and portfolio characteristics cannot explain all of the heterogeneity in household-specific returns to wealth.

4.1 Explaining Returns to Assets and Wealth

The household-specific return may subsume permanent household characteristics such as race, educational level, and gender. As found in the last section the household-specific return also subsumes differences in wealth, portfolio shares and investment skill. This section examines how these observable characteristics are related to the average returns to assets and wealth.

Specifically, the return to assets is modelled using linear panel regressions:

$$r_{j,it}^a = \mathbf{X}_{j,it} \beta_j^X + \zeta_{j,it}^a \quad (9)$$

where the return to assets for household i in year t , $r_{j,it}^a$ is regressed on a vector of controls $\mathbf{X}_{j,it}$. The controls in $\mathbf{X}_{j,it}$ include the controls of equation 1 and 2 as well as controls for race, educational level, and gender. Three variations on the controls are examined. All three include lagged portfolio shares, year fixed effects, and observable household characteristics. Observable household controls include indicators for the head of household’s age, gender, along with dummy variables for family size, being single, number of children, race, educational attainment, region of residence interacted with year fixed effects, outside dependants, and the presence of income from a family member other than head or spouse. Attainment of a post-secondary degree or above proxies for financial sophistication (Campbell, 2006). Controls also include separate indicators if the household reported selling any primary housing, secondary housing, private business, and/or public equity since the last wave. Indicators for the percentile of wealth for household i in the previous wave is also included in the set of controls. The lagged value of wealth and portfolio shares is used to avoid spurious correlation of returns and household wealth. Wealth can capture scale dependence or participation costs (Guiso and Sodini, 2013; Guvenen, 2009), or financial sophistication (Kacperczyk et al., 2018).

A second specification includes a measure of the relative volatility of household’s i portfolio as a control. The third specification interacts portfolio shares with the time fixed effects.

Estimates of the three versions of the equation (9) are estimated for the total return to household assets and presented in Table 4. The first column reports the estimates when portfolio shares are not interacted with time fixed effects and when the relative risk of the portfolio is excluded. The regression has an adjusted R^2 of 0.094. Only the share of the asset portfolio in housing assets is found to significantly increase the return to total household assets at the 1 percent level. A household with all of the portfolio allocated to primary housing, ceteris paribus, would have a 5.02 percentage point higher return to total assets compared to a household with no primary housing in the portfolio. The portfolio share on business assets is not found to be significantly higher than the portfolio share on public equities at the 5 percent level.

The presence of an advanced education degree increases the total rate of return on assets by a significant 1.68 percentage points. Single and male heads of household have statistically insignificant coefficients for the returns to assets. A head of the household that identifies as African-American is associated with a negative 131 basis point reduction in the return on assets. As in Fagereng et al. (2018), we find that year fixed effects are significant (although not reported here) in explaining the returns to total household assets and wealth.

The second column of Table 4 includes the relative volatility of the portfolio and the adjusted R^2 increases slightly. The relative volatility of the portfolio is found to be highly significant in explaining the return to assets. If a household’s volatility of asset return doubles compared to that

Table 4. Persistent Returns to Assets can be Partly Explained

	(1)	(2)	(3)
Private business share	3.19*	-0.73	
	(1.83)	(1.84)	
Primary housing share	5.02***	3.85***	
	(0.31)	(0.31)	
Secondary hous. share	5.27***	3.20**	
	(1.41)	(1.42)	
Public equity share	0.80	-0.12	
	(0.66)	(0.64)	
Advanced degree	1.68***	2.43***	2.52***
	(0.43)	(0.42)	(0.42)
Single	-0.19	-0.46	-0.48
	(0.49)	(0.48)	(0.48)
African-American	-1.31***	-1.29***	-1.23***
	(0.35)	(0.34)	(0.34)
Male	0.05	-0.07	-0.17
	(0.37)	(0.36)	(0.36)
Relative volatility		2.81***	2.87***
		(0.25)	(0.25)
N	13076	13076	13076
Adj. R ²	0.094	0.124	0.145

Notes: OLS regressions excluding the fixed effects for the return to total household assets. Returns in percentage points. All regressions control for dummies for year interacted with region, lagged wealth, age, observable household characteristics, and indicators if assets were sold in that period. Column 3 includes interactions between time effects and lagged portfolio shares. Asset shares are values from 0-1. Wild bootstrap standard errors reported in parentheses. ***p<0.01, **p<0.05, *p<0.1.

of the within-household cross section volatility, the average return to assets would increase by 2.8 percentage points. This evidence suggests that, even when controlling for the wealth and portfolio share of the household, heterogeneity in the reported volatility of the returns to assets can partially explain differences in average return to assets across households.

The observable household characteristics and the relative volatility of the portfolio continue to maintain their significance when the portfolio shares are interacted with the year fixed effects in column 3 of Table 4.

The exercise is now repeated for the rate of return to total household wealth, estimated as per equation 9, and presented in Table 5. The first column reports the estimate when portfolio shares are not interacted with time fixed effects and when the relative volatility of the portfolio is excluded. The regression has an adjusted R^2 of 0.082. In contrast to the rate of return to assets, all coefficients for the wealth portfolio shares are statistically significant and larger for the rate of

Table 5. Persistent Returns to Wealth can be Partly Explained

	(1)	(2)	(3)	(4)
Private business share	13.73*** (2.46)	7.68*** (2.48)	3.09 (2.39)	
Primary housing share	13.66*** (1.40)	11.10*** (1.40)	9.95*** (1.38)	
Other housing share	11.73*** (2.45)	7.83*** (2.49)	6.29** (2.53)	
Public equity share	7.05*** (1.38)	5.26*** (1.36)	3.91*** (1.37)	
Advanced degree	8.15*** (1.14)	9.32*** (1.14)	8.45*** (1.14)	8.55*** (1.14)
Single	-4.17*** (1.26)	-4.50*** (1.25)	-3.47*** (1.24)	-3.52*** (1.24)
African-American	-2.52** (1.11)	-2.52** (1.10)	-2.61** (1.09)	-2.45** (1.09)
Male	-0.86 (0.86)	-1.05 (0.85)	-0.65 (0.84)	-0.85 (0.84)
Relative volatility		4.59*** (0.50)	5.20*** (0.51)	5.24*** (0.51)
Leverage: business debt			32.95*** (7.94)	32.77*** (7.97)
Leverage: housing debt			5.91*** (1.01)	6.03*** (1.01)
Leverage: other debt			12.74 (13.74)	14.87 (14.38)
N	12889	12889	12889	12889
Adj. R ²	0.082	0.096	0.104	0.115

Notes: OLS regressions for the return to total household wealth excluding fixed effects. Returns in percentage points. All regressions control for dummies for year, lagged wealth, age, region, observable household characteristics, and indicators if assets were sold in that period. Regressions 3- 4 include lagged debt-to-asset leverage ratios. Regression 4 includes interactions between time effects and lagged portfolio shares. Asset shares are values from 0-1. Wild bootstrap standard errors reported in parentheses. ***p<0.01, **p<0.05, *p<0.1.

return to wealth. A household with all of the portfolio allocated to private business assets, ceteris paribus, would have a 13.7 percentage point higher return to total wealth compared to a household with no private business assets in the portfolio. The coefficient of the portfolio share on private business assets is found to be higher than the return to public equity at the 0.1 percent significance level.

The second column of Table 5 includes the relative volatility of the portfolio. The relative volatility of the portfolio is found to be significant in explaining the return to wealth. The presence of an advanced degree is found to have a positive and significant increase in the return to wealth. Single and African-American heads of households have significantly lower returns to wealth.

Observable household characteristics continue to maintain their significance when the lagged leverage within each asset class is included in column 3 of Table 5 or in column 4 when year fixed effects are interacted with the portfolio shares. However, importantly, the coefficients on the contribution of the portfolio shares are lower, and lose statistical significance for private business wealth. This suggests that ignoring leverage for the return to wealth omits an important explanatory variable and overstates the importance of portfolio allocation.

Table 6. Explaining Persistent Returns to Assets Within Asset Classes

	Business	Prim. Housing	Otr. Housing	Public Equity	Risk Free
If sold	-5.02 (19.8)	-5.07*** (0.49)	-9.89*** (3.30)	1.21 (1.65)	
Private business share	-113.57*** (29.7)	-1.49 (1.11)	-3.79 (10.53)	-3.58 (5.60)	-0.08 (0.057)
Primary hous. share	-38.80 (32.9)	-6.79*** (0.89)	-5.63 (10.50)	-9.10*** (2.85)	-0.05** (0.025)
Other hous. share	-64.08* (34.3)	-4.63*** (1.11)	-39.84*** (8.86)	-12.82*** (4.09)	-0.21*** (0.045)
Public equity share	-42.14 (35.1)	-1.12 (0.88)	-7.14 (9.28)	-32.33*** (3.44)	0.18*** (0.046)
Advanced degree	5.61 (36.6)	2.25*** (0.54)	-0.23 (7.22)	3.65 (3.79)	0.18*** (0.033)
Single	-10.48 (20.0)	0.26 (0.59)	-4.29 (7.09)	0.71 (2.80)	-0.14*** (0.035)
African-American	-1.81 (26.0)	-0.45 (0.48)	9.54* (4.95)	-0.57 (2.44)	-0.16*** (0.020)
Male	-16.67 (15.8)	-0.19 (0.51)	-7.34 (6.02)	-0.94 (2.10)	0.00 (0.029)
Relative volatility	39.45*** (7.0)	1.22*** (0.25)	10.66*** (2.08)	18.01*** (1.19)	-0.41*** (0.020)
N	710	12546	1286	6745	17277
Adj. R^2	0.367	0.155	0.195	0.211	0.356

Notes: OLS regressions excluding fixed effects for the return to assets for each asset class. Returns in percentage points. All regressions control for dummies for year, lagged wealth and portfolio shares, age, region, observable household characteristics, and indicators if assets were sold in that period. Asset shares are values from 0-1. Wild bootstrap standard errors reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

This exercise is now repeated for the return to assets within each asset class and presented in Table 6. The portfolio shares are not interacted with time fixed effects so the coefficients for portfolio shares can be reported. The relative volatility of the portfolio is included. The regressions for all asset classes have adjusted R^2 measures that range from 0.155 for secondary housing to 0.367 for private business assets.

All estimates include an indicator if that asset was sold since the last wave. This may help capture reporting bias, underestimated commissions and costs from selling, or address the timing of liquidation. Selling of primary and secondary housing assets is found to reduce the return to

primary housing assets significantly by 5.1 and 9.89 percentage points, respectively. Interestingly, the selling of private business assets or public equities is not found to significantly explain returns.

The ability to examine each asset class enables an understanding of which asset classes contribute to the effects observed in the return to total household assets. For example, a single or African-American head of household is found to only be significantly negative for only risk-free assets. An advanced degree is found to have a significant positive effect only for the return to primary housing and risk-free assets. The gender of the head of the household does not have a significant explanatory effect on any of the returns.

Instead, asset specialization, represented by the portfolio share of assets in total assets, is consistently an important negative and significant factor in explaining the return to assets for each asset class. This provides evidence against increasing returns to asset specialization arising from say knowledge acquisition (Lusardi et al., 2017; Kacperczyk et al., 2018).

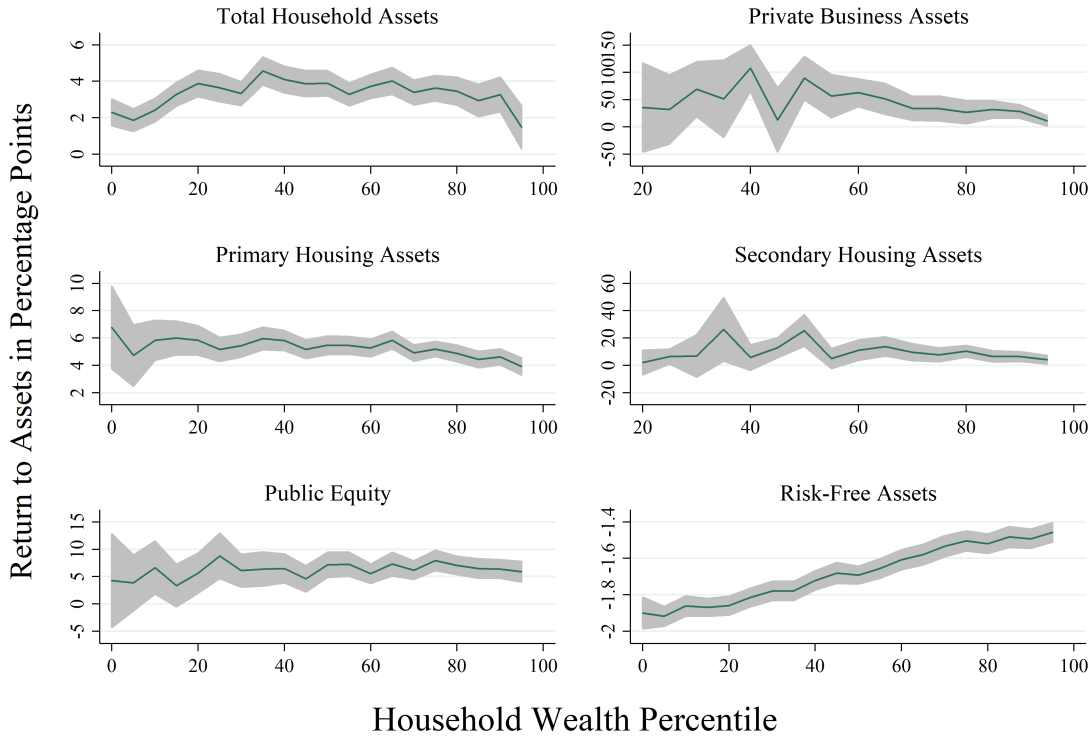
Background risk from holding other assets in the portfolio has negative effects on the level of return for some asset classes. For the return to public equity, holding a share of the portfolio in housing assets is associated with a significant and negative effect on the level of return of public equities. Flavin and Yamashita (2002) posit that background risk from primary housing would reduce the share of that households' portfolio share in stocks compared to bonds which was empirically confirmed in the PSID sample by Palia et al. (2014) and Cocco (2004). The results from this analysis suggest secondary housing background risk also has a significant and larger negative effect

In addition to persistent heterogeneity in returns to match the wealth tail, Gabaix et al. (2016) show that one way to capture the speed of changes in tail inequality observed in the data is to allow for a positive correlation of returns with wealth, also called "scale dependence." Attention is now turned to how returns covary with household wealth and age. The predicted margins for wealth and age are plotted from equation 9 that interacts the lagged portfolio shares with the year fixed effects and includes the measure of relative volatility.

The predicted margin for the return to total household assets and the five asset classes at each of the 5 percentiles of household wealth percentile are illustrated in Figure 1. The main finding is that only a measure of risk-free assets would display a positive relation between return to assets and total household wealth consistent with the findings of financial returns in Bach et al. (2016) and Fagereng et al. (2018). Notably, the return to housing and business assets shows some evidence of declining returns. The return to total household assets increases until the 40th percentile of household wealth and then is relatively stable up until the very top percentile of total household wealth. The decline in the return for the wealthiest percentiles is due to decreasing returns to private business and secondary housing assets concentrated in the portfolios of the wealthiest households. The declining returns to assets for the other non-financial asset classes are a novel finding and drive the stable return to total assets.

While life-cycle age profiles are important for the persistent component of household earnings in the study of wealth inequality, the age profiles for the return to assets are much less precise. For all asset classes other than risk-free assets, the predicted margin of asset returns for 5 year age bins are quite stable, see Figure 2. For example, the return to total household assets is quite stable between 3 and 4 percentage points. The implication is that the permanent household-specific returns and not lifetime age profiles, as commonly assumed earnings, should be used in quantitative studies of

Figure 1. Only Returns to Risk-Free Assets Increase in Total Household Wealth



Predictive Margins with 95 % CIs.

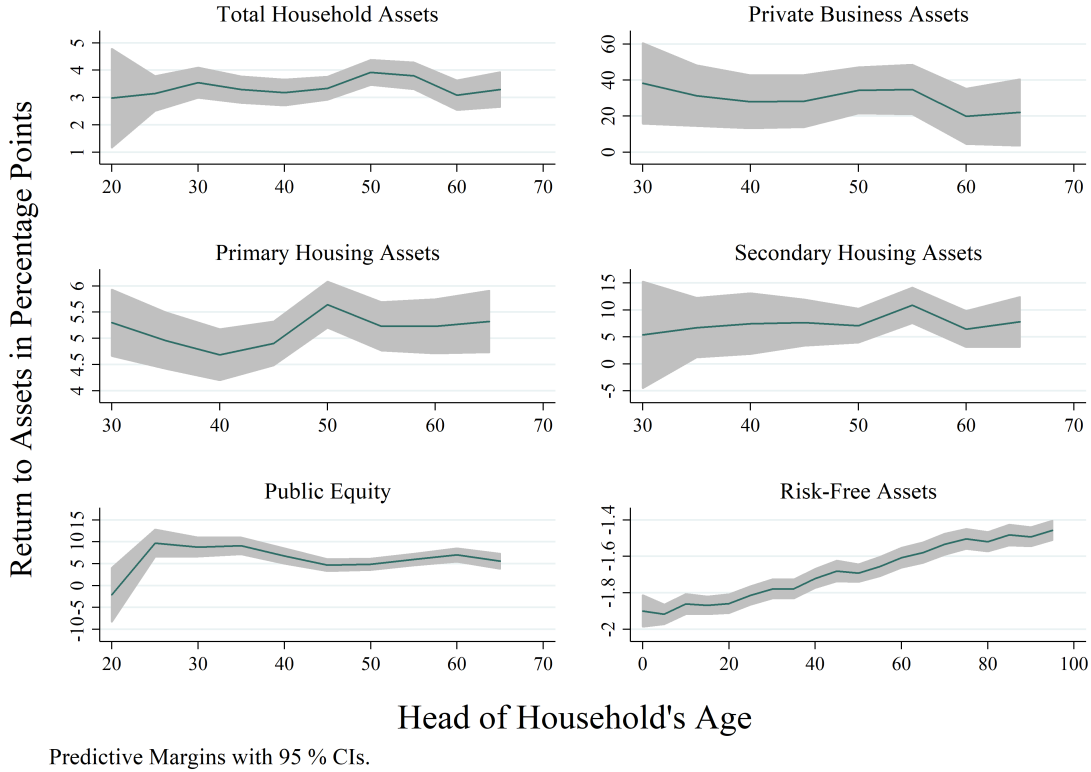
wealth inequality with return heterogeneity.

The above results have focused on the return to assets and its relationship with the total household wealth. However, for scale dependence to explain the change in the wealth distribution over time, as suggested by [Gabaix et al. \(2016\)](#), the return to wealth would need to be positively correlated with wealth. Figure 3 illustrates the predicted margin of the returns to wealth by the total wealth of the household with the 95 percent confidence interval.

Notably, there is a strong negative correlation between returns to wealth and total household wealth. This relationship is especially robust for primary housing assets. The declining return to wealth is due to the combined effect of the declining marginal returns to non-financial assets and lower average leverage as wealth increases. This result is consistent with the findings for the return to total household wealth in [Bach et al. \(2016\)](#) and [Fagereng et al. \(2018\)](#) (when primary housing wealth is included).

For primary housing assets, the role of borrowing constraints and the progressivity of mortgage payment tax deductibility can be shown. The real after-tax mortgage rate and real mortgage rate on primary housing are illustrated in Figure 4. Generally, mortgage rates are high for the lowest 40th percentile of household wealth, then increasing slightly again at the 70th percentile of wealth. The difference between the bottom and top 20 percentiles of wealth is 2.9 percentage points for the real mortgage rate and 3.6 percentage points for the real after tax mortgage rate. The contribution of borrowing costs to the return to wealth can be calculated in an exercise setting tax rates and borrowing costs to the average across households. In this case, the return to wealth

Figure 2. Total Returns Vary Little with Household Age



for the bottom and top 20 percentiles of wealth would be further increased by an additional 1.5 percentage points. The contribution of borrowing costs and interest deductibility is unique as it shows a systematic component between household wealth and returns to wealth not associated with risk taking. Wealthier households benefit from lower borrowing costs and the progressivity of mortgage payment tax deductibility.

It has been documented that the relative volatility of the households realized returns has strong predictive power for all measures of returns. Moreover, in the estimation of household-specific returns in Table 3, accounting for differences in household wealth were found to decrease the standard deviation of the household-specific return to assets to 1.72 from 3.94 percentage points. However, this may arise given differences in risk taking across wealth. To this end, Figure 5 reports the relative volatility of the households portfolio over the wealth distribution. As can be seen, the wealthy do have more volatile portfolios but within asset classes this is only true of public equities.

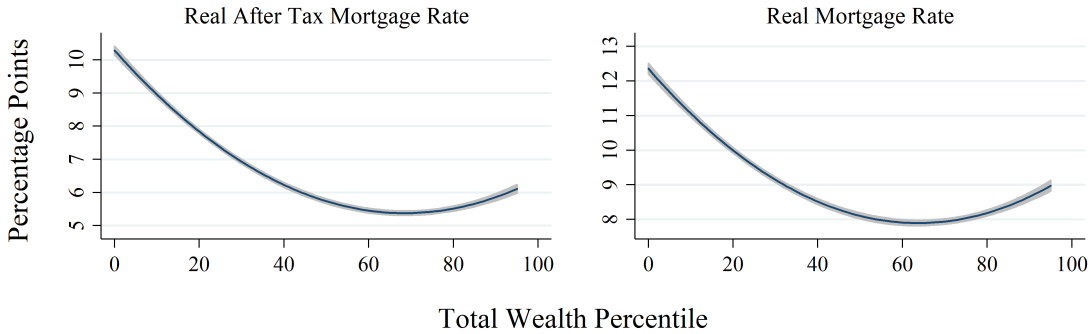
To illustrate how wealth correlates with household-specific returns, the quadratic prediction between household-specific return to assets, $\epsilon_{j,i}^a$, and total household wealth is presented in Figure 6. The measure of household-specific return to assets is the baseline measure that removes contribution of year and age effects in Table 3. Household-specific returns positively covary with total household wealth for all asset classes. The relationship is very strong and is exhibited across the wealth distribution. It is important to note that this relationship may be endogenous, as households with permanently higher returns are more likely to be wealthier. Analysis of how much of this arises endogenously would require a structural model, is beyond the scope of this paper, and is left for

Figure 3. Total Rates of Return to Wealth Decline with Wealth



Predictive Margins with 95 % CIs.

Figure 4. Primary Housing Borrowing Cost Decline in Wealth



Quadratic prediction line in blue with 95 % CIs.

future research.

4.2 Heterogeneity in Leverage

Given the importance of leverage for the return to wealth, this section asks how much of the leverage is household-specific. The leverage in asset j of household i at time t , $lev_{j,it}$, is modelled using linear panel regressions similar to the returns to assets and wealth in equation 1 and 2. The unexplained component of leverage, $e_{j,it}^l$ from the second stage regression is represented as the sum

Figure 5. Relative Volatility Correlates with Wealth Only for Financial Assets



Quadratic prediction line in blue with 95 % CIs.

of household-specific leverage $\epsilon_{j,i}^l$, and the idiosyncratic error term $u_{j,it}^l$.

$$e_{it}^l = \epsilon_i^l + u_{it}^l \quad (10)$$

The standard deviation of $\epsilon_{j,i}^l$ is denoted $\hat{\sigma}(\epsilon_j^l)$. The household-specific leverage, ϵ_i^l is estimated using fixed effects with EB shrinkage.

Table 7. Estimates of Household-Specific Leverage

	$\hat{\sigma}_{\epsilon^l}$	$\hat{\sigma}_{u^l}$	$\hat{\sigma}_{\epsilon^l}^2 / \hat{\sigma}_{e^l}^2$
Total ex. O. Debt	16.6	15.3	0.54
Total	33.5	58.8	0.24
Private Business	37.3	74.2	0.20
Primary Hous.	23.8	15.5	0.70
Secondary Hous.	22.5	11.1	0.80

Notes: Estimates of the fixed effects for leverage in percent using all observable controls. "Total ex. O. Debt" refers to total household leverage excluding other debts.

Table 7 displays the estimates of the household-specific leverage for the measure of total house-

Figure 6. Household-Specific Asset Returns Correlate with Wealth



Quadratic prediction line in blue with 95 % CIs.

hold leverage, total household leverage excluding other debts, and each leveraged asset class. The main finding is that household-specific leverage is sizeable. Household-specific leverage represents a particularly high share of the overall heterogeneity in leverage for housing assets. This confirms that much of the household-specific return to wealth is explained by household-specific differences in leverage. The variance decomposition for total leverage excluding other debts suggests that 54 percent of the heterogeneity in leverage is household-specific. The inclusion of other debts does reduce the estimate of total household leverage to 24 percent.

Table 8. Total Household-Specific Leverage Excluding Contributors

	Year, Age	+Portfolio	+Wealth	+Port., Wealth and Risk
Total ex. O. Debt	16.6	16.2	17.3	15.4
Total	33.5	33.6	34.0	33.3

Notes: Standard deviation of the household specific leverage excluding the contributions of portfolio shares and the household wealth. Debt-to-asset ratio for the household in percentage points. "+Port., Wealth and Risk" removes the additional contribution of both portfolio shares and wealth as well as the relative riskiness of the household portfolio.

Similar to the estimates of the household-specific returns, to isolate the role of portfolio composition and wealth to household-specific leverage, a modified version of equation 1 and 2 is estimated. Specifically, in the first stage, equation 1, portfolio composition, household wealth percentile, and then both portfolio composition and wealth are regressed on household leverage. Then, equation 2 is re-estimated with those controls excluded. The purpose is to isolate the contribution of portfolio shares and wealth that can permanently differ across household from the household-specific component. These results are summarized in Table 8.

The results suggest that persistent differences in portfolio allocation and wealth contribute very little to household-specific leverage. Excluding both the contribution of portfolio shares and wealth to household-specific leverage reduces the estimates of the household-specific return from 16.6 percentage point standard deviation to 15.4 percentage points. These results motivate substantial household-specific heterogeneity in leverage in studies of wealth inequality.

Table 9. Leverage Heterogeneity Characteristics are Partly Observable

	(1)	(2)	(3)
Asset return fixed effect	-0.76** (0.30)	-0.65** (0.29)	-0.69** (0.31)
Private business share	47.29*** (4.20)	50.02*** (4.20)	
Primary housing share	51.06*** (2.41)	51.75*** (2.38)	
Other housing share	54.99*** (3.13)	56.39*** (3.09)	
Public equity share	29.14*** (4.03)	29.72*** (4.00)	
Advanced degree	17.90*** (4.84)	17.28*** (4.84)	17.24*** (4.84)
Single	-12.86*** (3.84)	-12.82*** (3.83)	-12.49*** (4.04)
Male	-0.96 (4.56)	-0.88 (4.56)	-0.36 (4.75)
Relative volatility		-1.98*** (0.64)	-1.99*** (0.66)
N	12572	12572	12572
Adj. R ²	0.096	0.097	0.101

Notes: OLS regressions of the debt to asset ratio in percent excluding the fixed effects. All regressions control for dummies for year, lagged wealth and portfolio shares, age, location, observable household characteristics, and indicators if assets were sold in that period. Asset shares are values from 0-1. Wild bootstrap standard errors reported in parentheses. ***p<0.01, **p<0.05, *p<0.1.

To examine how observable characteristics are related to leverage, the exercise of section 4.1 is

now repeated for the measures of leverage as per equation 9. The only difference is the inclusion of the household-specific return to assets in the controls. The ordinary least square estimates of the debt-to-asset ratio for the measure of total household leverage (including other debts) are summarized in Table 9. The first column does not include the relative volatility of the portfolio or the interaction of the portfolio shares with the time fixed effect. The second column controls for the relative risk of the portfolio to measure the role of risk taking on the leverage decision. The third column reports the estimates when the portfolio shares are interacted with time fixed effects.

A higher household-specific return to total assets is significantly and negatively correlated with a decrease in total asset-specific leverage. One possible explanation is that households with higher returns to assets use asset income to pay down debt.

The allocation of the asset portfolio has a sizeable and highly significant correlation with the leverage of households. Leverage is higher for households concentrated in private business and housing assets. Perhaps most interesting is that concentration of the portfolio share in public equities is also associated with higher household leverage. This provides some evidence that some of the debt held by households is used to finance public equities.

Interestingly, the leverage ratio significantly increases by approximately 17 percentage points for heads of household who hold an advanced degree. This suggests that part of the higher returns to wealth for holders of advanced degrees can be explained by holding higher leverage. One possible explanation is that, since debt is widely held, an advanced education may help a household choose cheaper debt, as suggested by Campbell (2006). It could also arise from a higher willingness to lend to more educated households if they are seen as less risky borrowers. Another possible explanation is that more educated households maintain higher debt levels to achieve higher return to wealth. For example, during the later half of the sample when borrowing rates were low, a household that used savings to invest in risky assets rather than pay off their primary mortgages would have reaped much larger returns to wealth. More education may have contributed to the financial knowledge to make such an allocation decision.

A single household is found to hold significantly less leverage compared to a married household. A possible explanation includes a lower willingness to lend to single households if they are seen as more risky borrowers. It could also reflect lower risk tolerance of single households or less ability to smooth income from the absence of a secondary income earner. Male heads of household are not found to hold significantly less debt. This is consistent with the evidence that that gender is not found to be significantly correlated with any other return measures. Finally, the relative risk of the household portfolio is found to be significantly and negatively correlated with total household leverage.

To examine if these features hold only for specific asset classes, the exercise is repeated for the debt-to-asset ratio for each asset class and presented in Table 10. The regressions do not interact year fixed effects with portfolio shares, so that the effect of portfolio shares can be reported, but this has little effect on the qualitative or quantitative findings.

A higher household-specific return to total assets is significantly and negatively correlated with a decrease in total asset-specific leverage only for primary housing assets. Advanced degree holders are found to hold significantly more debt only for primary housing. This presents an interesting puzzle, as lower debt costs or higher willingness to lend to advanced degree holders should have a similar effect on secondary housing leverage. Being single is also significantly and negatively related

Table 10. Leverage Characteristics Vary By Asset Types

Lagged Asset Share	Private Business	Prim. Housing	Secondary Hous.
Asset return fixed effect	0.11 (0.11)	-1.67*** (0.19)	-0.16 (0.35)
Private business share	67.94*** (17.45)	52.80*** (3.20)	45.11*** (8.23)
Primary housing share	38.30*** (12.69)	42.45*** (2.37)	27.79*** (8.03)
Secondary housing share	10.53 (22.35)	51.55*** (3.08)	61.12*** (6.95)
Public equity share	16.27 (22.91)	42.48*** (2.47)	27.63*** (7.42)
Advanced degree	-4.37 (8.27)	20.00*** (1.48)	5.17 (8.81)
Single	9.31 (28.57)	-12.65*** (1.76)	-7.71 (5.34)
Male	24.21 (33.64)	-1.10 (1.43)	-1.38 (3.57)
Relative volatility	-5.64*** (1.95)	-4.70*** (0.56)	-12.14*** (1.14)
N	710	12546	1259
R ² -Adj.	0.124	0.391	0.292

Notes: OLS regressions excluding the fixed effects for the debt-to-asset ratio in percent. All regressions control for dummies for year, two-year lagged wealth, age, location, observable household characteristics, and indicators if assets were sold in that period. Asset shares are values from 0-1. Wild bootstrap standard errors reported in parentheses. ***p<0.01, **p<0.05, *p<0.1.

only to the leverage in primary housing.

Portfolio specialization measured by the share of the asset portfolio within each asset class significantly contributes to higher debt for all asset classes. The exception is secondary housing and public equity shares for private business leverage. Holding a higher portfolio share in stocks is associated with higher leverage only for housing. This provides evidence that households may be using only mortgage debt to finance public equity investment.

The wealth of the household is significantly associated with lower leverage. Figure 7 illustrates the predicted margins presented with the 95 percent confidence intervals for the household debt-to-asset ratio at 5 percentile increments of household wealth. As households become wealthier, they predictably reduce their overall leverage. This effect mainly arises from primary housing leverage. In contrast, only the wealthiest percentiles of households are found to maintain lower leverage in private businesses and secondary housing.

Figure 7. Wealthy Aggressively Pay Down Primary Housing Debt



Predictive Margins with 95 % CIs.

To summarize, an important share of household-specific returns to wealth are due to the contribution of household-specific leverage. Leverage is predictable, being highly related to the wealth of the household, portfolio composition, and other household characteristics. Leverage exhibits persistent heterogeneity across households with the majority of the idiosyncratic component of housing leverage explained by household-specific leverage.

4.3 Robustness of Household-Specific Asset Returns

The estimate of the household-specific returns is quite robust to the underlying data and estimation assumptions. Table 11 reports the estimates of the household-specific return to assets for various baseline assumptions. All household-specific returns are estimates with fixed effects with empirical Bayes shrinkage as per equation (3). The estimates of the standard deviation of the household-specific return are all significant at the five percent level and hence are not reported for brevity. The first row reports the baseline estimation of the standard deviation of the household-specific return, the transitory component, variance decomposition and number of household-year observations.

The first set of rows in Table 11 report the estimates for alternative assumptions of the minimum number of consecutive observations required for the household to be included in the sample. The estimates are close to the baseline estimate of two years minimum for all cases. The largest deviation is for the 1 year minimum where the estimate of the household-specific return increases from 3.9 to 5.4. However, this is consistent with the evidence in the Monte Carlo simulations presented in

appendix B. The simulations show that empirical Bayesian shrinkage works best with at least two consequence conservations. This also highlights the importance of panel data in the estimation of household-specific returns.

Table 11. Robustness of Household-Specific Returns to Assets

	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_u$	$\hat{\sigma}_\epsilon^2 / \hat{\sigma}_e^2$	N
Min. 1 year	5.39	8.19	0.30	14871
Min. 2 year (Baseline)	3.94	8.43	0.18	13076
Min. 3 year	3.51	8.37	0.15	10058
Min. 4 year	3.86	8.29	0.18	7303
Min. 5 year	3.77	8.20	0.17	5013
Individuals	4.18	8.44	0.20	23078
Not req. r_{ait}^w	3.94	8.53	0.18	13359
No min. asset value	3.93	8.47	0.18	13154
\$5000 min. assets	3.92	8.48	0.18	12221
Ex. Homeowners	2.46	4.76	0.21	1799
Ex. Bus. Owners	3.74	8.00	0.18	11961

Notes: Estimates of the household-specific return to asset in percentage points under various sample restriction assumptions.

The row “Individuals” in Table 11 reports the estimates for individual-specific returns instead of household-specific returns. This is done as in Fagereng et al. (2018) by counting married households twice. In this case, the estimate of the household-specific return to assets increases slightly to 4.18 percentage points, but is still lower than the 5.36 percentage points estimated for individuals in Norway by Fagereng et al. (2018). The remaining rows show the estimates for a minimum asset value of \$5000 2010 USD, no minimum (or greater than zero) asset value, and the case where the sample is not restricted to requiring an observation of the rate of return to wealth. In all three of these cases do the estimates of the household-specific return to assets remain very consistent with the baseline estimates. The overall robustness of the estimates is due to the use of EB procedure which makes the estimates immune to transitory measurement error.

The last two rows of Table 11 report the estimates of the household-specific return when primary homeowners and private business owners are dropped from the sample. Excluding private business owners and primary homeowners reduce the estimate of the standard deviation of the household-specific return to assets to 2.46 and 3.74, respectively. This highlights that permanent heterogeneity in returns are not merely limited to participation within these asset classes.

5 Implications of Findings

This paper provides empirical evidence on the scale- and type-dependence in returns to wealth. A method is proposed to calculate the first panel dataset on household level returns in the United States using recent information in the newly revised PSID. Using the observed realized returns, several theories regarding return heterogeneity are tested.

This is the first paper to document permanent heterogeneity in returns between U.S. households and for returns to wealth in any country. Household-specific returns are observed for total household portfolio as well as within all asset classes. The majority of the persistent heterogeneity in the return to assets is associated with households' risk taking and wealth. A comparison of household-specific returns to assets and wealth reveals that leverage on wealth explains 58 percent of the permanent heterogeneity in the return to wealth. Leverage increases both the mean and standard deviation of returns to wealth and is found to exhibit a sizeable household-specific component.

Returns to wealth are found to be negatively correlated with total household wealth due to reduced leverage of the wealthy. Only for financial assets is the return to assets found to be increasing in total household wealth, and this is associated with more risk taking by the wealthy. All asset classes exhibit declining returns to specialization. However, two important components of household-specific returns that vary with wealth but are not associated with risk are borrowing costs and regressive mortgage interest tax deductibility.

The empirical evidence illuminates the magnitude and drivers of type and scale dependence in returns to wealth. The results support the finding of [Bach et al. \(2016\)](#), who suggest that most of the heterogeneity in returns across households are driven by differences in portfolio risk characteristics. The presence of household-specific returns in the United States supports the findings of the fixed effects observed in Norway by [Fagereng et al. \(2018\)](#) and used in quantitative models explaining wealth heterogeneity in [Benhabib et al. \(2019\)](#). In the PSID, the estimate of the standard deviation of the individual-specific return to total household assets is 4.2 percentage points, similar to the 5.4 percentage point return to total observable assets for individuals found in Norway by [Fagereng et al. \(2018\)](#). The similarities in the estimates occur despite the differences in the measures of household return and the country being examined. The estimates in this paper and [Fagereng et al. \(2018\)](#) reject homogeneous return to assets. Novel to this paper, household-specific leverage is documented and shown to increase the dispersion of household-specific returns to wealth. The standard deviation of the fixed effects for the return to household wealth is estimated to be 9.34 percentage points. These findings have important implications for the literature of portfolio theory, optimal taxation, and wealth inequality.

The first applicability of the results regards the study of asset income inequality in explaining wealth inequality. The unexplained differences in leverage, and their role in scale dependence in returns need to be taken into account. For example, preference heterogeneity ([Krusell and Smith, 1998](#); [Hendricks, 2007](#); and [De Nardi, 2004](#)) should be re-examined in the presence of leverage and asset return heterogeneity to examine its contribution to return to wealth heterogeneity. Constraints to household balance sheets, permanent earning profiles, and heterogeneity in preferences may be behind the substantial idiosyncratic permanent heterogeneity in leverage across households. Household specific leverage is observed for all asset classes. The fact that the wealthy households maintain leverage in private business and secondary housing asset classes suggests unique features of these asset classes.

The role of reduced borrowing constraints of the wealthy is also noted. Reduced mortgage rates compound with higher interest deductibility compound to systematically provide higher returns to wealth as household wealth increases, and for the same degree of leverage, are not associated with higher risk. The results suggest a key role of leverage in determining scale- and type-dependence in returns in studies of wealth inequality, such as [Benhabib et al. \(2019\)](#). If the degree of risk

and leverage is not taken into account, models that impose persistence in returns to wealth may overstate the degree of exogeneity in returns. The return to assets is found to be increasing in the wealth percentile only for financial assets, whereas the return to wealth is found to be decreasing in total household wealth. Reduced borrowing constraints and risk-return efficiency of the wealthy suggest that the permanent component and risk will not scale linearly with wealth.

The empirical evidence supports the role of return heterogeneity arising from entrepreneurial skill (Quadrini, 2000; Cagetti and De Nardi, 2008, 2009; Benhabib et al., 2011). Entrepreneurial returns are found to exhibit declining returns to scale. Household-specific returns to private business assets are heterogeneous across the wealth distribution and positively associated with the wealth of the household. That said, removing private business owners from the sample only reduces the estimate of the household-specific return to assets to 3.74 from 3.94. Thus, wealth inequality arising from heterogeneity in household-specific returns is not limited to entrepreneurial activity,

The second applicability of the results is for the study of portfolio choice and risk preferences. The evidence in this paper and Fagereng et al. (2018) demonstrates that the share of public equities in the portfolio is insufficient to identify the degree of portfolio risk. This highlights a pitfall of using the share of the portfolio invested in stocks to identify preferences related to risk aversion such as in Brunnermeier and Nagel (2008) or Chiappori and Paiella (2011). Instead, the degree of risk within an asset class needs to be considered jointly with the share in that asset to identify relative risk aversion, such as in Palia et al. (2014).

The third applicability of the results regards redistribution and taxation of wealth and capital income. Shourideh (2013) finds that the relative degree of permanent and transitory components of returns informs the degree of capital income taxation progressivity. The decomposition of the permanent and transitory component displayed in Table 2 can be used to inform the progressivity of taxation. The variance decomposition of the permanent component in returns to total wealth, $Var(\epsilon_a^w)/Var(e_a^w)$ is found to be 0.15 for households in the United States. This is slightly higher than the estimate of the variance decomposition of the permanent component for the return on assets in Fagereng et al. (2018) for Norway. The permanent component in idiosyncratic returns is also found to differ by asset class, being particularly high for private business assets and low for public equity. The relative importance of the permanent component across asset classes suggests that potential efficiency gains could be made to optimal capital and wealth taxation by targeting taxation of specific asset classes.

Regarding optimal redistribution, inequality has been shown to be efficient if the wealthy are better investors, for example, from entrepreneurial skills (Guvenen et al., 2017). If wealth is allocated to more productive households (if the returns do not stem from monopoly rents), and those investments are scalable, inequality is efficient. Guvenen et al. (2017) show that holding revenue constant, moving from a capital income tax to a wealth tax shifts the burden of taxation disproportionately to lower wealth households. In such a setting, efficiency can be improved if wealth is allocated to more efficient (high return) individuals. However, these efficiency gains depend on the degree of heterogeneity of returns across households' wealth levels and whether returns are household specific and scalable. The evidence from the PSID suggests that, on average, returns are not scalable but exhibit decreasing marginal returns. In this case, allocating more wealth to the wealthy is unlikely to produce consistently high returns to private equity.

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Appendices (not intended for publication)

Appendix A: Data Description and Selection

The redesigned Panel Study of Income and Dynamics (PSID) data is the main dataset for the calculation of household-level real rates of return. For the purpose of this paper, the main innovation of the PSID was the regular and detailed collection of asset income, wealth, and net investment. Households were surveyed every two years from 1999–2017. Rates of return are annualized and available between 2000–2016. The year 1998 or the initial household observation is lost due to the calculation of the return.

5.1 Assets and Wealth

The PSID provides detailed socio-economic information on sex, age, marital status, educational attainment, employment status, and geography. Data on labor and asset income are retrospective to the year prior, whereas wealth in assets and debt are reported at the time of the interview. Interviews are conducted early in the year (around March). The head of the household is defined as a person over the age of 15 with the most financial responsibility for the household.

The reported total net worth of household i at time t , w_{it} , includes wealth in several asset classes less other debts held by the households. The wealth in asset classes refers to the amount the household would receive if they sold the asset and paid off all debts associated with the asset. This includes the wealth in all vehicles $w_{v,it}$ (including boats and motor homes), private equity in businesses and farms $w_{b,it}$ (almost all businesses so henceforth private businesses), and the wealth in the primary residence, $w_{ph,it}$, as well as any secondary housing equity such as rental properties or cottages, $w_{oh,it}$. Wealth in risk-free assets, $w_{f,it}$, that are not in employer-based pensions or Individual Retirement Accounts (IRAs) is also reported and includes checking or savings accounts, money market funds, certificates of deposit, government savings bonds, or Treasury bills. The value of direct holdings in public stocks (that are not in employer-based pensions or IRAs) is also reported, $w_{s,it}$, and includes direct holdings in publicly held corporations, mutual funds, or investment trusts. Households' wealth in private annuities and employer-based pensions or IRAs are reported $w_{ira,it}$. Finally, all other assets are reported, $w_{o,it}$, including any other savings or assets, such as bond funds, cash value in a life insurance policy, a valuable collection for investment purposes, or rights in a trust or estate. The total of other debt, $d_{o,it}$, held by the household and not associated with any specific asset class is reported and includes credit card, student, medical, legal and family debt. Total wealth of a household, w_{it} , is thus defined as follows:

$$w_{it} = w_{b,it} + w_{ph,it} + w_{oh,it} + w_{f,it} + w_{s,it} + w_{v,it} + w_{o,it} + w_{ira,it} - d_{o,it}. \quad (11)$$

5.2 Imputation of Capital Gains

A difficulty in previous studies calculating return is that asset income is reported as a total for a year, whereas wealth is observed at a point in time. Wealth can be put into or removed from a particular asset category, for example through accumulation of capital gains in wealth. In the Scandinavian tax database used by [Bach et al. \(2016\)](#) and [Fagereng et al. \(2018\)](#), wealth is reported at the end of the year, income is reported for the year, and capital gains are reported when realized.

However, in the PSID for private businesses, primary and secondary housing assets, and stocks, both asset values at the time of the survey as well as the flow investment and income between the two surveys is reported in every wave. Thus, unlike previous studies, capital gains can be observed for these assets in every period. All capital gains are annualized.

For each asset class, the wealth in the asset is defined as the value of the asset less debt associated with the asset. The asset value and wealth are net of fees and commissions. For example, the wealth in the primary residence, $w_{ph,it}$ is defined as difference of the reported value of the primary residence, $a_{ph,it}$, less primary mortgage debt, $d_{ph,it}$: $w_{ph,it} = a_{ph,it} - d_{ph,it}$.

For the primary residence, capital gains are defined as the change in the reported value of the primary residence, $a_{ph,it} - a_{ph,it-1}$, between the two years if the house was not sold, or the difference from the selling price, $a_{ph,it}^*$, on the last reported value if the primary residence was sold, $a_{ph,it}^* - a_{ph,it-1}$ less the value of renovations and upgrades, $i_{ph,it}$. Capital gains are measured between the waves and then annualized to match asset income flows. Capital gains on primary housing, $yg_{ph,it}$ are:

$$yg_{ph,it} = (1_{\{sold=1\}}a_{ph,it}^* + 1_{\{sold=0\}}a_{ph,it} - a_{ph,it-1} - i_{ph,it})/2. \quad (12)$$

Capital gains to stocks, $yg_{s,it}$, private businesses, $yg_{b,it}$, and secondary housing wealth, $yg_{oh,it}$ are defined as the difference in the reported price asset value, $a_{j,it}$, net of investment, $i_{j,it}$:

$$yg_{j,it} = (a_{j,it} - a_{j,it-1} - i_{j,it})/2, \quad (13)$$

for $j \in \{s, b, oh\}$.

Asset values are available for every period for holdings of public equity and for the primary residence. Asset values are available for private businesses and secondary housing are available starting in the 2011 wave. Prior to 2011, net worth but not asset values are reported for secondary housing and private business assets. Fortunately, however, net worth and net investment were reported. Thus, it is proposed that the asset values for secondary housing and private businesses can be imputed prior to 2011 using simple accounting. The value of secondary housing asset, $a_{oh,it}$ can be imputed using the change in net wealth $\Delta w_{oh,it+1}$, and net investment $i_{oh,it+1}$ as follows:

$$a_{oh,it} = a_{oh,it+1} - \Delta w_{oh,it+1} - i_{oh,it+1}. \quad (14)$$

for $j \in \{b, oh\}$. This imputation implies that wealth accumulation from principal payments is included in net investment. This is confirmed in 2012–2014 when asset values, net investment, and change in net worth is observed. Capital gains for business and secondary housing assets use the imputed asset values in the calculation of capital gains prior to 2011.

5.3 A New Measure of Returns

The returns proposed in this study are pre-tax real returns to assets and wealth. In addition to returns to total household assets and wealth, returns are analysed for five asset categories: risk-free assets, primary and secondary housing, private businesses, and public equity. Observing and comparing returns to assets versus wealth allows to parsing the role of borrowing costs and leverage in the heterogeneity of returns to wealth. Merely focusing on the return to wealth ignores that

leverage is an endogenous decision of the household.

The return to primary housing includes capital gains, the value of housing services, the costs of maintenance, and rental income. Let the dividend value from residence in housing be denoted by, DIV_{it} , where

$$DIV_{it} = (rr + \delta)a_{ph,it-1} + ptax_{ph,it}, \quad (15)$$

and rr is the real interest rate, δ is the depreciation rate, and $ptax_{ph,it}$ is value of property taxes. Following [Flavin and Yamashita \(2002\)](#) it is assumed that $rr = 0.05$. The cost of ownership is given by:

$$COST_{it} = \delta a_{ph,it-1} - (1 - \tau_{it})ptax_{ph,it} \quad (16)$$

where τ_{it} is the marginal income tax rate. It is assumed that the cost of maintenance and repairs from depreciation are equal for both landlords and homeowners which implies a constant physical condition of the house. Finally, households can rent a fraction of their primary residence, RNT_{it} , which accrues rental income $y_{ph,it}$, less reduced flow consumption and the additional cost of utilities, $utils_{ph,it}$:

$$RNT_{it} = y_{ph,it} - \kappa_{ph,i}(aph_{it-1}rr + utils_{ph,it}) \quad (17)$$

where $\kappa_{ph,i}$ is the share of the primary residence rented out. Rental income is reported for all housing assets. Rental income is attributed to primary residence, $y_{ph,it}$, if the household does not own a secondary property, and to secondary income, $y_{oh,it}$, if the household owns a secondary property. Absent direct observations of the share of the residence rented, it is assumed that $\kappa_{ph,i} = 0.5$ if rental income is accrued and $\kappa_{ph,i} = 0$ if no rental income is accrued. For ease of exposition, let the net income from primary and secondary, the numerators or $r_{ph,it}^a$, and $r_{oh,it}^a$, excluding capital gains, be denoted by $yt_{ph,it}$ and $yt_{oh,it}$, respectively. The total return to the primary residence is thus:

$$\begin{aligned} r_{ph,it}^{n,a} &= \frac{yg_{ph,it} + DIV_{it} - COST_{it} + RNT_{it}}{a_{ph,it-1}} \\ &= \frac{a_{ph,it-1}rr(1 - \kappa_{ph,it}) + y_{ph,it} + \tau_{it}ptax_{ph,it} - \kappa_{ph,it}utils_{ph,it} + yg_{ph,it}}{a_{ph,it-1}} \\ &= \frac{yt_{ph,it} + yg_{ph,it}}{a_{ph,it-1}}. \end{aligned} \quad (18)$$

The return to the primary housing asset differs from [Flavin and Yamashita \(2002\)](#) in three ways. First, the tax rate is household and year specific and calculated using the National Bureau of Economic Research tax simulator ([Feenberg and Coutts, 1993](#)). Second, capital gains are net investment, which includes major improvements and upgrades. This data was not available for the sample covered by [Flavin and Yamashita \(2002\)](#). [Fagereng et al. \(2018\)](#) impute housing values based on aggregate housing prices and use the average imputed house price between years in the denominator of the rate of return. Third, rental income is acknowledged as a source of income. Failure to acknowledge rental income can understate the return to housing. These three differences are also true of the return to housing in [Fagereng et al. \(2018\)](#).

The return to secondary housing is modelled allowing for the property to be owner-occupied, rented full time, or rented intermittently. Specifically, the asset return to secondary housing, $r_{oh,it}^a$, is given by:

$$r_{oh,it}^{n,a} = \begin{cases} (a_{oh,it-1}rr + \tau_{it}ptax_{oh,it} + yg_{oh,it})/a_{oh,it-1}, & \text{if occupied} \\ (y_{oh,it} - a_{oh,it-1}\delta - ptax_{oh,it} + yg_{oh,it})/a_{oh,it-1}, & \text{if rented} \end{cases} \quad (19)$$

where $ptax_{oh,it}$ are the property taxes on the secondary housing. It is assumed that the tenant pays utilities. The PSID includes information on repairs and maintenance of the primary residence each year, but this information is only available beginning in 2005. To incorporate this information, the average depreciation rate, δ , is set to the average value of repairs and depreciation costs for the years observed, 1.7 percent. For the baseline sample, 10.3 percent of homeowners have secondary properties, and 42.2 percent of secondary properties are rented.

The PSID contains detailed information on mortgage rates for primary housing. The nominal mortgage interest rate, $r_{ph,it}^n$, is the debt weighted average of the first and second mortgage. The real after-tax mortgage rate, $r_{ph,it}^m$, is measured as

$$r_{ph,it}^m = \frac{1 + (1 - \tau_{it})r_{ph,it}^n}{1 + \pi_t} - 1,$$

where π_t is the inflation rate as measured by the CPI. The interest costs of the primary housing are denoted $m_{ph,it}$. The interest payments utilize the information on monthly mortgage payments, current interest rate on the loan, the year the mortgage was obtained, and the years left to pay, following the TAXSIM recommendations to calculate mortgage deductibility.

Returns to wealth for housing are calculated as the sum of capital gains and flow income net of non-tax deductible interest payments over lagged wealth. The return to primary housing wealth, net of interest costs is thus:

$$r_{ph,it}^{n,w} = \frac{yt_{ph,it} + yg_{ph,it} - (1 - \tau_{it})m_{ph,it}}{a_{ph,it-1} - d_{ph,it-1}}, \quad (20)$$

and the return to secondary housing wealth is

$$r_{oh,it}^{n,w} = \frac{yt_{oh,it} + yg_{oh,it} - (1 - \tau_{it})m_{oh,it}}{a_{oh,it-1} - d_{oh,it-1}}, \quad (21)$$

where $a_{oh,it-1} - d_{oh,it-1} = w_{oh,it-1}$ and $a_{ph,it-1} - d_{ph,it-1} = w_{ph,it-1}$ are the lagged wealth in other and primary housing, respectively.

If an individual in the household actively participates in the business, the PSID assigns half of business income to assets and half to labor. If an individual reports business income but does not actively participate, the PSID assigns all business income to business asset income. If the household reports a loss in total business income, then the loss is attributed only to business asset income. The PSID does not distinguish between labor and asset income from farming so it is assumed that farm owners actively contribute labor to farm activities and farm income is thus split evenly between labor and asset income, as for the case of businesses. The flow profits from businesses and farming activity is denoted $y_{b,it}$. The return to business assets is defined as the sum of income from business and farms plus capital gains:

$$r_{b,it}^{n,a} = \frac{y_{b,it} + yg_{b,it}}{a_{b,it-1}}, \quad (22)$$

where the return to business wealth is defined over the wealth of the business in the last period,

$$r_{b,it}^{n,w} = \frac{y_{b,it} + yg_{b,it}}{a_{b,it-1} - d_{b,it-1}}. \quad (23)$$

The PSID lacks reporting on net investment into risk-free assets. The value of the risk-free asset is thus calculated following [Fagereng et al. \(2018\)](#) by assuming that the wealth is the average between the two observations surrounding the reporting. We superscript measures with a bar when the average value is used, and the average value of assets in risk-free assets is $\bar{w}_{it} = (w_{f,it} + w_{f,it-1})/2$.

Interest income is reported by the household but not allocated to a particular asset category. Interest income from bonds, $y_{c,it}$, is allocated between direct holdings and safe assets and distinguished using the 3-month U.S. Treasury bill rate, $r_{tres,t}^{n,a}$. The interest income from bonds associated with risk-free assets is the smaller value of the Treasury bill rate times the value of risk-free assets or the value reported from bond interest income. That is:

$$y_{c,it} = \begin{cases} y_{c,it}, & \text{if } r_{tres,t}^{n,a} \bar{w}_{f,it} \leq y_{c,it} \\ r_{tres,t}^{n,a} \bar{w}_{f,it}, & \text{otherwise} \end{cases} \quad (24)$$

The remainder of the reported interest income, $y_{i,it} = y_{c,it} - y_{f,it}$, is then allocated to IRAs and direct public equity holdings. The return to risk-free assets, $r_{f,it}^a$, is thus defined as:

$$r_{f,it}^{n,a} = \frac{y_{f,it}}{\bar{a}_{f,it}}. \quad (25)$$

Similarly, the return to public equity, $r_{s,it}$, is the sum of dividends and interest income, $y_{i,it}$, and capital gains from stocks, $yg_{s,it}$, over the value of IRAs $\bar{w}_{ira,it}$ and direct holdings of public stock, $w_{s,it-1}$:

$$r_{s,it}^{n,a} = \frac{y_{s,it} + y_{i,it} + yg_{s,it}}{\bar{w}_{ira,it} + w_{s,it-1}}. \quad (26)$$

It is assumed that households do not leverage wealth in public equities or risk-free assets and thus their return to wealth and assets is equivalent, $r_{s,it}^a = r_{s,it}^w$. There are two main differences with the current measure of the stocks, other than the country, in comparison to the data sets of the [Bach et al. \(2016\)](#) and [Fagereng et al. \(2018\)](#). The first is that pension assets are included in the value of financial assets. The second is that capital gains are computed per period, in contrast to imputed realized capital gains.

Total household asset income includes the return to primary and secondary housing $yt_{ph,it}$ and $yt_{oh,it}$, private business income $y_{b,it}$, dividends $y_{s,it}$, interest income from bond coupon $y_{c,it}$, and trusts $y_{t,it}$. Let total asset income, excluding capital gains be denoted by $y_{a,it}$:

$$y_{a,it} = yt_{ph,it} + yt_{oh,it} + y_{b,it} + y_{s,it} + y_{c,it} + y_{o,it} + y_{t,it}. \quad (27)$$

Similarly let total capital gains be denoted by, $yg_{a,it}$:

$$yg_{a,it} = yg_{ph,it} + yg_{oh,it} + yg_{s,it} + yg_{b,it}. \quad (28)$$

The total return to assets, r_{it}^a , includes flow income excluding capital gains from all assets plus

the capital gains from primary housing, secondary housing, public and private equity:

$$r_{it}^{n,a} = \frac{y_{a,it} + yg_{a,it}}{a_{b,it-1} + a_{ph,it-1} + a_{oh,it-1} + a_{s,it-1} + \bar{w}_{f,it} + \bar{w}_{ira,it} + \bar{w}_{o,it} + \bar{w}_{v,it}}. \quad (29)$$

The return to assets represents the pre-tax return not including deductibility of interest payments. Thus, the measure is the exogenous return to the asset if the asset was fully paid off by the household. This distinction is made to isolate the role of leverage, which includes the endogenous amplification of the return from leverage and the reduction in the yield from non-tax deductible interest payments. The total return to wealth which excludes other debts, r_{it}^{wx} is measured as.

$$r_{it}^{n,wx} = \frac{y_{a,it} + yg_{a,it} - (1 - \tau_{it})(m_{ph,it} + m_{oh,it})}{w_{b,it-1} + w_{ph,it-1} + w_{oh,it-1} + w_{s,it-1} + \bar{w}_{f,it} + \bar{w}_{ira,it} + \bar{w}_{o,it} + \bar{w}_{v,it}}. \quad (30)$$

The total return to wealth excluding other debt is closely related to the measure reported by [Bach et al. \(2016\)](#) and [Fagereng et al. \(2018\)](#) in the Scandinavian tax administrative data sets. However, their measure of return to wealth excludes asset income and wealth from sources that are not taxable. The measure of return to wealth in this paper includes information on durable wealth and other valuables such as collections that are reported by the household that would not traditionally be reported as asset income for tax purposes. This includes the asset return from other family members, $y_{o,it}$, and from royalties and trusts, $y_{t,it}$. Wealth is also included from vehicles, $w_{v,it}$ and wealth not reported elsewhere which includes rights in a trust or estate, cash value in a life insurance policy, or a valuable collection for investment purposes, $w_{o,it}$. The total return to assets used in this paper is the most similar measure to the return to household “net worth” in [Fagereng et al. \(2018\)](#), who use the asset value in the denominator but exclude interest payments in the numerator.

The return to total wealth, $r_{it}^{n,w}$, inclusive of all debts, serves as the main measure of return to wealth. This measure is the same as r_{it}^{wx} but is net of other debts, $d_{o,it}$, held by the household such as credit card, student, medical, legal and family debt:

$$r_{it}^{n,w} = \frac{y_{a,it} + yg_{a,it} - (1 - \tau_{it})(m_{ph,it} + m_{oh,it})}{w_{b,it-1} + w_{ph,it-1} + w_{oh,it-1} + a_{s,it-1} + \bar{a}_{f,it} + \bar{a}_{ira,it} + \bar{w}_{o,it} + \bar{w}_{v,it} - d_{o,it-1}}. \quad (31)$$

Finally, nominal returns to assets and wealth, $x \in \{a, w\}$ respectively, for all asset classes $j \in \{b, ph, oh, s, f\}$ and for total household returns are converted to real returns using the annualized total consumer price index provided by the Federal Reserve (CPI):

$$r_{j,it}^x = \frac{1 + r_{j,it}^{n,x}}{1 + \pi_t} - 1.$$

Total household real wages, W_{it} , are calculated as real total household labor income, Y_{it} , over total hours worked, H_{it} ,

$$W_{it} = Y_{it}/H_{it}. \quad (32)$$

Household labor income includes total head’s and spouse’s labor income and household labor income from businesses. Non-business labor income is the sum of total labor income including salaries, hourly work, bonuses, tips, etc.

Appendix B: Empirical Bayes

Overview

This appendix describes the empirical Bayes shrinkage of the fixed effects estimates used in this paper to estimate household specific returns. It also explores the estimator using Monte Carlo simulation. The empirical Bayes shrinkage procedure is based on Morris (1983).

The household specific return of household i is given by ϵ_i . Let \hat{e}_i be the observed return which is equal to the household specific return plus measurement error η_i .

$$\hat{e}_{it} = \epsilon_i + \eta_{it}$$

The empirical Bayes procedure adjusts the estimate of the household specific return so that transitory term does not introduce bias into the estimates. This is accomplished by shrinking the estimate of the household specific return towards the true underlying household specific return distribution. True household specific returns are not observable, but distribution is estimable.

Implementation

The method of Morris (1983) is implemented as follows. Equation 3 produces the estimated household specific return \hat{e}_{it} and the corresponding standard error $\hat{\pi}_e$. The standard error is squared, $\hat{\pi}_e^2$. Then from Morris (1983) the following is estimated:

$$\hat{\beta} = (X'WX)^{-1}X'R$$

$$\hat{\sigma}_\epsilon^2 = \max \left\{ 0, \frac{\sum_i W_i \left\{ \left(\frac{N_i}{N_i - N_x} \right) (\hat{e}_{it} - x_i \hat{\beta})^2 - \hat{\pi}_e^2 \right\}}{\sum_i W_i} \right\}$$

$$W_i = \frac{1}{\hat{\pi}_e^2 + \hat{\sigma}_\epsilon^2}$$

Where X is the stacked x_{it} , W is a diagonal matrix of the W_i , and R is the stacked \hat{e}_i . N_i is the number of households and N_x is the dimensionality of x_i .

$\hat{\beta}$ is a WLS regression of the \hat{e}_i on x_i . $\hat{\sigma}_\epsilon^2$ is the weighted average of the squared deviations of \hat{e}_i from $x_i \hat{\beta}$ less the weighted average of $\hat{\pi}_e^2$. The weights W_i , give more weight to observations with less measurement error. The max operator ensures that $\hat{\sigma}_\epsilon^2$ is always nonnegative in finite samples.

$\hat{\beta}$ and $\hat{\sigma}_\epsilon^2$ are simultaneously determined and are estimated by iterating over the following procedure. Begin by fixing $W_i = 1 \forall i$, then iterate the following to convergence:

1. Compute $\hat{\beta}$ and then a new estimate $\hat{\sigma}_\epsilon^2$
2. If $\hat{\sigma}_\epsilon^2$ has converged, exit. Otherwise, fix new weights W_i and return to step 1

The feasible best estimate of the posterior mean ϵ_i^f is:

$$\epsilon_i^f = (1 - \hat{B}_i) \hat{e}_i + \hat{B}_i x_i \hat{\beta}$$

$$\hat{B}_i = \left(\frac{N_i - N_x - 2}{N_i - N_x} \right) \left(\frac{\hat{\pi}_e^2}{\hat{\pi}_e^2 + \hat{\sigma}_\epsilon^2} \right)$$

The variance of the household specific return on covariates, called, $\hat{\zeta}_i^2$ is given by:

$$\hat{\zeta}_i^2 = \max \left\{ 0, \frac{\sum_i W_i \left\{ \left(\frac{N_i}{N_i - 1} \right) (\hat{e}_i - \bar{R})^2 - \hat{\pi}_e^2 \right\}}{\sum_i W_i} \right\}$$

$$\bar{R} = \frac{\sum_i W_i \hat{e}_{it}}{\sum_i W_i}$$

The empirical Bayes procedure produces the feasible empirical Bayes household specific returns ϵ_i^f and the unconditional estimated variance of the household specific returns $\hat{\zeta}_i^2$. The root of $\hat{\zeta}_i^2$ provides the standard deviation of the household specific returns $\hat{\zeta}_i$.

Monte Carlo Simulations

The properties of the empirical Bayes procedure is now illustrated using a Monte Carlo exercise. Let the simulated return for household i in time t , r_{it} , be the sum of the household-specific return $\epsilon_i \sim N(0, \sigma_\epsilon^2)$ and a transitory idiosyncratic component $u_{it} \sim N(0, \sigma_u^2)$, $r_{it} = \epsilon_i + u_{it}$. The relative values of the household specific return and the idiosyncratic transitory error is set similar to that found in the data: $\sigma_\epsilon = 5$ and $\sigma_u = 10$.

A random variable is simulated, $s_i \sim U[0, 1]$ to construct a observable x_{it} that is either unrelated to ϵ_{it} or is linearly correlated:

$$x_{it} = \begin{cases} s_i, & \text{if no correlation} \\ |\alpha + \epsilon_i| s_i, & \text{if linear correlation} \end{cases} \quad (33)$$

The simulations are run with $\alpha = 0.3$, to produce a noisy correlated x_{it} with ϵ_{it} . Then the household specific return is estimated with:

$$r_{it} = \beta x_{it} + \epsilon_i + u_{it}.$$

Table 12 summarizes the estimated standard deviation of the household specific return $\hat{\sigma}_\epsilon$ using fixed effects (FE), random effects (RE), and the fixed effects with empirical Bayesian shrinkage (BS). There are 500 simulations of $N \in [350, 4000]$ households. The number of time periods, T , is either $T = 9$ or unbalanced with $N/2$ household missing the last half of their observations.

As can be seen in Table 12, in the absence of a correlation between the regressor and the household specific return, the random effects and the empirical Bayes estimates are quite similar and close to the true value. In contrast the fixed effect estimate is upward biased. When there is a linear correlation of the regressor the household specific return, the fixed effects estimator remains the same but the random effects estimates becomes downward biased. The empirical Bayes estimates are similar and close to the true value.

Table 12. Monte Carlo Simulations

Periods	N	No Correlation			Linear Correlation		
		FE	RE	BS	FE	RE	BS
9	350	6.13 (0.45)	5.01 (0.45)	4.98 (0.45)	6.13 (0.45)	3.06 (0.41)	5.04 (0.45)
9	4000	6.10 (0.14)	4.99 (0.14)	4.99 (0.14)	6.10 (0.14)	3.04 (0.12)	5.03 (0.14)
9 (Unbal.)	350	6.68 (0.53)	4.93 (0.54)	5.07 (0.54)	6.68 (0.53)	2.99 (0.45)	5.09 (0.54)
9 (Unbal.)	4000	6.67 (0.15)	4.99 (0.15)	5.13 (0.16)	6.67 (0.15)	3.03 (0.13)	5.13 (0.16)

Monte Carlo estimates for the standard deviation of the household specific return with a true value of 5. Estimators include random effects, fixed effects, and fixed effects with empirical Bayes shrinkage. 500 simulations. Standard deviations reported in brackets. “No Correlation” refers to orthogonal household-specific returns and the independent variable.