Interest Rate Uncertainty and Sovereign Default Risk *

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Abstract

Global factors, usually associated with uncertainty, are primary drivers of fluctuations in sovereign spreads. We build a model of sovereign default in which shocks to the level and to the volatility of the world interest rate help to account for this phenomenon. We calibrate the model parameters to Argentine data and estimate a process for the world interest rate using US treasuries. Time variation in the world interest rate interacts with default incentives in the model and leads to state contingent effects on borrowing and sovereign spreads. We find that shocks to the level and volatility of the world interest rate (i.e. uncertainty shocks) cause the model to predict an average sovereign spread that is 280 basis points larger and 200 basis points more volatile than a model with a constant world interest rate. The model generates a time-varying volatility in the spread that is about 77 percent of that seen in Argentine data. The model also predicts that countries will prefer a longer maturity for their debt when facing a time-varying world interest rate. Welfare gains from eliminating uncertainty about the world interest rate amount up to a 1 percent permanent increase in consumption.

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Keywords: Sovereign Debt, Sovereign Default, Interest Rate Spread, Time-varying Volatility, Uncertainty Shocks.

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1 Introduction

The emerging economy business cycle literature has shown that shocks to country spreads play an important role in accounting for domestic business cycles.[1] In addition, a large body of empirical work has traced variation in these yields to both domestic and global factors.[2] Motivated by this work, the sovereign default literature provides a framework in which time varying default probabilities generate endogenous variation in sovereign yields.[3] However, this literature has emphasized the role of domestic factors with little attention to the global interlinkages highlighted by the empirical work. Our paper addresses this gap by focusing on the relationship between uncertainty in the world interest rate and sovereign default risk, while also retaining a role for domestic factors. Our focus on variation in the world interest rate as a global factor is consistent with several studies. For example, González-Rozada and Levy Yeyati (2008) finds that movements in US treasuries as well as in proxies for global risk explain about half of the long run volatility in emerging economy interest rates.[4]

Some recent episodes highlight the importance of the behavior of US interest rates for world debt markets. A notorious example, usually referred to as the "taper tantrum," occurred in May of 2013 when former US Fed chairman Ben Bernanke suggested the possibility of a reduction in future bond purchases by the Fed. This triggered a sharp market adjustment in emerging market economies featuring a reversal in capital flows and a spike in government bond yields. On average, sovereign yields across emerging economies rose by 1% (Rai and Suchanek, 2014). An example of policy makers' dislike of uncertainty about the world interest rate occurred in 2015, as summarized by the following quotes reported on the Financial Times (September 9, 2015):

[1] See, for example, Neumeyer and Perri, 2005 or Uribe and Yue, 2006.

[2] See early work in Edwards, 1984, Cantor and Packer, 1996 and Eichengreen and Mody, 2000. For some recent examples of studies that highlight global factors, see Hilscher and Nosbusch (2010), González-Rozada and Levy Yeyati (2008) Akıncı (2013), and Maltritz (2012)

[3] We use the terms "sovereign" and "government" interchangeably throughout the paper.

[4] These findings are re-iterated using a variety of empirical methods and proxies for global risk, different time periods, and different countries in other work. For example, Akıncı (2013) uses a structural VAR on a panel of emerging economies while Maltritz (2012) uses Bayesian model averaging on a panel of European nations and both replace the high yield spreads used by González-Rozada and Levy Yeyati (2008) (as a measure of gloabl risk) with corporate bond spreads captured by BAA bonds. Hilscher and Nosbusch (2010) add VIX as a measure of global uncertainty and find that it is statistically significant in explaining credit default swap (CDS) spreads of Mexico, Turkey, and Korea.

"We think US monetary policymakers have got confused about what to do. The uncertainty has created the turmoil."

Mirza Adityaswara, Sr. Deputy Governor, Indonesia Central Bank.

"The uncertainty about when the Fed hike will happen is causing more damage than the Fed hike will itself."

Julio Velarde, Governor, Peru Central Bank.

Motivated both by the empirical evidence on the importance of global factors in the movement of emerging economies' sovereign spreads as well as recent events and policy maker concerns, we develop an equilibrium model of sovereign default (in the tradition of Eaton and Gersovitz, 1981) to study the relationship between endogenous country spreads and movements in both the level and the volatility of the world interest rate. To do so, we introduce stochastic volatility into the world interest rate process (as modeled by Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe, 2011) in an otherwise standard quantitative model of long term sovereign debt (Hatchondo and Martinez, 2009). We use the model to separate out the role that shocks to the level of the interest rate play from the role that time-varying volatility in the world interest rate plays in explaining the long run dynamics of the sovereign spread, the borrowing levels, and the ex-ante optimal maturity of sovereign debt.

A calibrated version of our model generates the following main findings: (i) introducing uncertainty about world interest rate fluctuations (measured by both interest level and volatility shocks) increases the mean of the spread by roughly 280 bps and the standard deviation of the spread by 200 bps; (ii) the time-varying volatility of the world interest rate alone is responsible for roughly two-thirds of the increase in the mean and standard deviation of spreads; (ii) the welfare gains of eliminating world interest rate uncertainty are sizable, in some cases amounting up to a 1% permanent increase in consumption; (iv) the presence of interest rate uncertainty changes the ex-ante optimal maturity of sovereign debt, inducing governments to extend the duration of their portfolios; (v) the volatility coming from the world interest rate propagates through the model and produces higher time-varying volatility in the endogenous spread. Moreover, our model matches well other business cycle moments observed in the data.

The rest of the paper is organized as follows. Section 2 briefly reviews the related literature. Section 3 describes the model and defines the equilibrium. Section 4 discusses the numerical solution and the calibration. Section 5 presents the results and section 6 concludes.

2 Related Literature

There is ample evidence that movements in the international risk-free rate (usually proxied by the T-bill rate) have macroeconomic consequences for emerging economies. Neumeyer and Perri (2005) report that real country interest rates in emerging economies are strongly countercyclical and tend to lead the cycle. They also find that exogenous interest rate shocks can account for up to 50 percent of the volatility of output in Argentina. Uribe and Yue (2006) report that a strong relation exists between the world interest rate, the country spread and emerging market fundamentals. In particular, they show that US interest rate shocks and country spread shocks can explain the large movements seen in the aggregate activity of emerging economies. García-Cicco, Pancrazi and Uribe (2010) also find that the country spread shock is one of the most important drivers of emerging economies business cycles. They show that an exogenous country spread shock and a preference shock can explain a large fraction of aggregate fluctuations in Argentine business cycles. All these papers take the country spread as an exogenous variable with a time-invariant volatility, while our work endogenizes both the level and the time-varying volatility of the spread (as a result of default incentives on the part of the sovereign).[5]

Fernández-Villaverde et al. (2011) study the impact of exogenous time-varying volatility on the macroeconomic dynamics of a small open economy. They examine the effects on the business cycles of four emerging economies: Argentina, Ecuador, Venezuela, and Brazil. We follow Fernández-Villaverde et al. (2011)'s approach to modeling the stochastic behavior of the world interest rate, while departing from their approach to modeling the country spread:

[5] In a recent study, Reyes-Heroles and Tenorio (2017) document the existence of two regimes in the volatility of interest rates at which emerging economies borrow and show that these regimes are closely related to the occurrence of sudden stops in these economies.

as already noted above, our model is one of endogenous spreads. Then, we explore the mechanism by which world interest rate uncertainty affects the country spread and default risk in emerging economies. We see our work to be complementary to theirs.

Within the quantitative literature on sovereign defaults (following Eaton and Gersovitz, 1981, Aguiar and Gopinath, 2006, and Arellano, 2008) our paper is particularly related to two other studies. The first one is by Seoane (Forthcoming). He studies how changes in aggregate income volatility affect sovereign spreads of four European economies: Greece, Italy, Portugal, and Spain. He presents a model in the spirit of Arellano (2008) and incorporates time-varying volatility of the income process which generates substantial variability in spreads. Our work complements his: we keep the income process with a time-invariant volatility and introduce time-varying volatility in the world interest rate process. The second paper in the default literature to which our work relates is the one by Pouzo and Presno (2016). These authors study the problem of a small open economy that can default on its obligations in the presence of model uncertainty. In their model, lenders fear that the probability model of the underlying state of the borrowing economy is misspecified and hence may demand higher returns on their investments. Even though our paper tackles a different type of uncertainty (i.e. time-varying volatility of the world interest rate) the results are consistent: more uncertainty leads to higher and more volatile spreads.[6]

Finally, our paper is also related to the literature on uncertainty shocks in macroeconomic models.[7] For instance, Justiniano and Primiceri (2008) and Bloom (2009) study the effect of changes in the volatility of technology shocks in general equilibrium models for closed economies. Justiniano and Primiceri (2008) study the changes in volatility in postwar US data by estimation of a large-scale dynamic stochastic general equilibrium model (DSGE) allowing for time variation in the structural innovations. They find that shocks specific to investment are mostly responsible for the observed "great moderation." Bloom (2009), on the other hand, shows that uncertainty shocks can generate short sharp recessions and recoveries.

[6] Yet another study in the sovereign debt literature that deals with time-varying volatility is Gu and Stangebye (2017). They study costly information acquisition in a model of defaultable debt and show how this can create time-varying volatility in the spread.

[7] Fernández-Villaverde and Rubio-Ramírez (2013) and Bloom (2014) provide thorough accounts of the growing literature dealing with uncertainty shocks and time-varying volatility in macroeconomics.

3 Model

We consider a small open economy populated by a continuum of households. The economy trades long-duration non-state-contingent bonds with a mass of competitive foreign lenders and has no commitment to repaying its debts. The world interest rate (which matters for bond prices) is time-varying. Time is discrete and goes on forever: $t = 0, 1, 2, ...$

3.1 Domestic Economy

There is a single tradable good. As is standard in the sovereign default literature, the economy receives a stochastic endowment stream of this good y_t, where

$$log(y_t) = \rho_y log(y_{t-1}) + \varepsilon_{y,t} \tag{1}$$

with $|\rho_y| < 1$, and $\varepsilon_{y,t} \sim N(0, \sigma_\varepsilon^2)$. The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2}$$

where $\mathbb{E}$ denotes the expectation operator, $\beta \in (0, 1)$ denotes the subjective discount factor, and the $u(\cdot)$ is a period utility function which satisfies $u' > 0$, $u'' < 0$.

Each period, the government makes two decisions. First, it decides whether to default. Second, it chooses the number of bonds that it purchases or issues in the current period.

The government has access to an international financial market where it trades long-duration non-contingent bonds with competitive foreign investors at a price q_t. As in Hatchondo and Martinez (2009), we assume that a bond issued in period t promises an infinite stream of coupons, which decrease at a constant rate δ. In particular, a bond issued in period t promises to pay one unit of the good in period $t+1$ and $(1-\delta)^{s-1}$ units in period $t + s$, with $s \geq 2$. Let b_t (b_{t+1}) denote the number of outstanding coupon claims at the beginning of the current (next) period. A positive value of b_t implies that the government was a net issuer of bonds in the past. The number of bonds issued by the government is given by $[b_{t+1} - (1 - \delta)b_t]$. The resource constraint for the repayment case is then given by:

$$c_t + b_t = y_t + q_t \left[b_{t+1} - (1-\delta) b_t \right]. \tag{3}$$

If the government declares a default, it is excluded from financial markets and remains in financial autarky for a stochastic number of periods. While the government is in default, it cannot issue debt and domestic aggregate income is reduced by $\phi(y)$. As in Arellano (2008) and Chatterjee and Eyigungor (2012), we assume that it is proportionally more costly to default in good times ($\phi(y)/y$ is increasing in y).[8] Following most studies of sovereign default, the income-cost of defaulting is not a function of the size of the default.[9] Thus, when the government defaults, it does so on all current and future debt obligations. As argued in Hatchondo, Martinez and Sosa-Padilla (2016), this is consistent with the behavior of defaulting countries.[10] Following previous studies, we also assume that the recovery rate for debt in default is zero. The resource constraint for the default case is given by:

$$c_t = y_t - \phi(y_t) \tag{4}$$

3.2 Foreign Lenders

Foreign creditors are risk averse and their stochastic discount factor is given by:

$$m_{t,t+1} = e^{-r_{t+1} - \kappa\left(\varepsilon_{y,t+1} + 0.5\kappa\sigma_{\varepsilon}^2\right)}, \quad \text{with} \quad \kappa \geq 0. \tag{5}$$

This formulation introduces a positive risk premium because bond payoffs are more valuable to lenders in states in which the government defaults (i.e., in states in which income shocks in the domestic economy, ε_y, are low). Here, r is the time-varying world interest rate,

[8] Arellano (2008) and Chatterjee and Eyigungor (2012) show that this property is important in accounting for the dynamics of the sovereign debt interest rate spread. Mendoza and Yue (2012) show that this property of the cost of defaulting arises endogenously in a setup in which defaults affect the ability of local firms to acquire a foreign intermediate input good.

[9] See Sosa-Padilla (2018) for a model of endogenous default costs, where the output-cost-of-default is a function of the amount of debt that is defaulted upon.

[10] Sovereign debt contracts often contain acceleration and cross-default clauses. These clauses imply that after a default event, future debt obligations become current. The type of acceleration clauses depend on the details of each bond contract and on the jurisdiction under which the bond was issued (see IMF (2002)). For instance, in some cases it is necessary that creditors holding a minimum percentage of the value of the bond issue request their debt to be accelerated for their future claims to become due and payable. In other cases, no such qualified majority is needed.

and κ is the parameter governing the magnitude of the risk premium. A higher value of κ can be seen as capturing how correlated the small open economy is with respect to the lenders' income process, or alternatively, the degree of diversification in foreign lenders' portfolios.[11]

Bonds are priced in a competitive market inhabited by a large number of identical lenders, which implies that bond prices are pinned down by a zero expected profit condition. The price per bond is then given by:

$$q_t = \mathbb{E}_t \left\{ m_{t,t+1} \left(1 - d_{t+1}\right) \left[1 + (1-\delta) q_{t+1}\right] \right\} \tag{6}$$

where d_{t+1} and q_{t+1} represent the government's default decision and equilibrium bond price in period $t+1$, respectively.

3.3 Law of Motion for the World Interest Rate

Following Fernández-Villaverde et al. (2011) we specify the international risk-free rate faced by investors as:

$$r_t = \bar{r} + \varepsilon_{r,t} \tag{7}$$

where $\bar{r}$ is the mean of world risk-free real rate, and $\varepsilon_{r,t}$ represents deviations from this mean. In particular, we assume the following AR(1) behavior for $\varepsilon_{r,t}$:

$$\varepsilon_{r,t} = \rho_r \varepsilon_{r,t-1} + e^{\sigma_{r,t}} u_{r,t} \tag{8}$$

where $u_{r,t}$ is a normally distributed shock with mean zero and unit variance. The crucial ingredient in this stochastic process is that the standard deviation ($\sigma_{r,t}$) is not constant but time-varying and itself follows another (independent) AR(1) process:

$$\sigma_{r,t} = (1 - \rho_{\sigma_r})\bar{\sigma}_r + \rho_{\sigma_r} \sigma_{r,t-1} + \eta_r u_{\sigma_r,t} \tag{9}$$

where $u_{\sigma_r,t}$ is a normally distributed shock with mean zero and unit variance. We further assume that $u_{r,t}$ and $u_{\sigma_r,t}$ are independent of each other. The parameters $\bar{\sigma}_r$ and η_r measure

[11] This modeling of risk-averse foreign lenders is borrowed from Arellano and Ramanarayanan (2012).

the degree of mean volatility and stochastic volatility in the international risk free rate. A high $\bar{\sigma_r}$ corresponds to a high mean volatility and a high η_r corresponds to a high degree of stochastic volatility in the international risk free rate.

3.4 Timing

The timing of events, for a government that is not excluded from financial markets, is as follows. The government starts with an initial bond position b_t and observes the realizations of the income level (y_t), the world interest rate level (r_t) and the interest rate volatility ($\sigma_{r,t}$), and then decides whether to repay its outstanding debt. If it decides to repay, it chooses b_{t+1} subject to the resource constraint, taking the bond price schedule $q_t(b_{t+1}; y_t, r_t, \sigma_{r,t})$ as given. Finally, consumption takes place.

On the other hand, if the government decides to default it gets excluded from financial markets and suffers a direct income loss. In case of default, there is no other decision to be made as the level of consumption equals the (reduced) income level. The government will re-access financial markets in the following period with probability μ (and it will remain excluded from financial markets with probability $1-\mu$).

3.5 Recursive Equilibrium

We now turn to recursive notation, where *primes* denote next-period value of the variables. Let $\mathbf{s} = \{y, r, \sigma_r\}$ denote the aggregate exogenous state. Given a number of outstanding coupon claims at the beginning of the next period b' and a realization of $\mathbf{s}$, the price of a bond satisfies:

$$q(b', \mathbf{s}) = \mathbb{E}_{\mathbf{s}'|\mathbf{s}} \left\{ m(\mathbf{s}', \mathbf{s})(1-d') \Big[1 + (1-\delta) q(b'', \mathbf{s}') \Big] \right\} \tag{10}$$

where d' is the next-period default decision, and b'' is the next-period debt choice. The optimal default decision is taken as:

$$v^0(b; \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ (1-d) v^c(b; \mathbf{s}) + d v^d(\mathbf{s}) \right\} \tag{11}$$

where d equals 1 (0) if the government chooses to (not to) default. Under no-default, the government solves the following problem:

$$v^c(b;\mathbf{s}) = \max_{b'} \left\{ u\left(y + q(b';\mathbf{s})(b' - (1-\delta)b) - b\right) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}} \left[v^0(b';\mathbf{s}') \right] \right\} \tag{12}$$

Under default, the value function is given by:

$$v^d(\mathbf{s}) = u(y - \phi(y)) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}} \left[\mu v^0(0;\mathbf{s}') + (1-\mu) v^d(\mathbf{s}') \right] \tag{13}$$

where, in order to keep the environment as simple as possible, we assume that when the government gains re-access to financial markets it does so with no debt obligations (i.e. it gets a "fresh start").[12] Next, we define the recursive equilibrium of this economy.

Definition 1. *The recursive equilibrium for this economy is characterized by*

1. *a set of value functions v^0, v^c, and v^d,*
2. *a default policy rule d and a borrowing policy rule b',*
3. *a bond price function q,*

such that:

(a) *given the default and borrowing policy functions, v^0, v^c, and v^d satisfy equations (11) – (13) when the government can trade bonds at q;*

(b) *given the default and borrowing policy functions, the bond price function q is given by equation (10);*

(c) *the default and borrowing policy functions d and b' solve the dynamic programming problem defined by equations (11) – (13) when the government can trade bonds at q.*

[12] For studies with positive recovery rates and renegotiation between sovereigns and lenders, see for example Yue (2010), D'Erasmo (2011), and Hatchondo et al. (2016).

4 Numerical Solution

We solve the model numerically using value function iteration with a discrete state space.[13] We focus on Markov-perfect equilibria. We use Tauchen (1986)'s method to discretize the income shock, the interest rate level shock and the interest rate volatility shock.[14] We solve for the equilibrium of the finite-horizon version of our economy, and we increase the number of periods of the finite-horizon economy until value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium objects as the infinite-horizon economy equilibrium objects.

The functional form for the period utility is:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{14}$$

where γ is the coefficient of relative risk aversion. As in Chatterjee and Eyigungor (2012), we assume a quadratic loss function for income during a default episode:

$$\phi(y) = \max\{0, d_0 y + d_1 y^2\} \tag{15}$$

As explained by Chatterjee and Eyigungor (2012), this functional form for the income loss $\phi(y)$ is flexible enough to accommodate many cases. If $d_0 > 0$ and $d_1 = 0$, then the cost is proportional to income; if $d_0 = 0$ and $d_1 > 0$, then the cost increases more than proportionately with income; if $d_0 < 0$ and $d_1 > 0$, then the cost is zero in a region $(0 < y < -d_0/d_1)$ and then increases faster than income (for $y > -d_0/d_1$). This last case is very similar to Arellano (2008)'s cost-of-default function.

4.1 Calibration

We define the "full model" as one in which all the shocks are present. This full model is calibrated to a quarterly frequency using data for Argentina from the period 1983.Q4 -

[13] The algorithm computes and iterates on two value functions: v^c and v^d. Convergence in the equilibrium price function $q(\cdot)$ is also assured.

[14] Related work using shocks to the level of the interest rate includes Uribe and Yue (2006), Neumeyer and Perri (2005), Hatchondo et al. (2016), and Bianchi, Liu and Mendoza (2016), among others.

Table 1: Parameters of Full Model Economy

Household risk aversion	γ	2	Standard value
Household's discount factor	β	0.96	Standard value
Mean int'l risk-free rate	$\bar{r}$	0.01	Standard value
Income auto-correlation coefficient	ρ_y	0.9317	Argentina's GDP
Std. dev. of income innovations	σ_ε	0.037	Argentina's GDP
Prob. of re-entry	μ	0.0385	Chatterjee and Eyigungor (2012)
Coupon decay rate	δ	0.0341	Average debt duration
Lenders' risk aversion	κ	4.0	Calibrated to fit targets
Default cost parameter	d_0	-1.57	Calibrated to fit targets
Default cost parameter	d_1	1.60	Calibrated to fit targets

2001.Q4.[15] Table 1 summarizes the parameter values.

We estimate equation (1) using quarterly real GDP per capita for Argentina ranging from 1983.Q4 till 2001.Q4. The data counterpart of $log(y)$ is the deviation of the natural logarithm of GDP per capita from its trend (computed using HP-filter, with smoothing parameter $1,600$). The re-entry probability μ is set to 0.0385 according to Chatterjee and Eyigungor (2012), which implies an average period of 6.5 years of financial exclusion.[16]

We assume the representative agent in economy has a coefficient of relative risk aversion γ of 2, the typical value in the literature. The average risk-free rate and the domestic discount factor ($\bar{r} = 0.01$ and $\beta = 0.96$) are standard in quantitative business cycle and sovereign default studies. We set $\delta = 3.41\%$. With this value and the targeted level of sovereign spread, sovereign debt has an average duration of 4 years in the simulations, which is roughly the average duration of Argentine bonds according to Cruces, Buscaglia and Alonso (2002).[17]

We are left with three parameters to assign values to: the parameter controlling the risk premium, κ, and the coefficients of the default cost function, d_0 and d_1. We calibrate these three parameters to match an average debt-to-income ratio of 43%, a mean spread of 7.4%,

[15] As is common in the sovereign default literature, we focus on time series that do not include defaults. Argentina defaulted in 2002.Q1 and so we use data until 2001.Q4.

[16] Benjamin and Wright (2008) report Argentina as being in a state of default between 1982 and 1992, and between 2001 and 2005. The average exclusion period is 7.5 years by these measures. Gelos, Sahay and Sandleris (2011) report an average exclusion of 4 years for emerging economies.

[17] We use the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = (1+i^*)/(\delta+i^*)$, where i^* denotes the constant per-period yield delivered by the bond. Using a sample of 27 emerging economies, Cruces et al. (2002) find an average duration of 4.77 years, with a standard deviation of 1.52 years.

Table 2: Estimates of the World Interest Rate Process

Autocorrelation risk-free rate	ρ_r	0.908
Mean volatility of int'l risk-free rate	$\bar{\sigma}_r$	−6.2869
Autocorrelation interest vol. shock	ρ_{σ_r}	0.8742
Stochastic vol. of int'l risk-free rate	η_r	0.2632

and a standard deviation for the spread of 4.4%. The spreads statistics compare well with their empirical counterparts (EMBI Global spreads). The data counterpart for the debt ratio is from Cowan, Levy-Yeyati, Panizza and Sturzenegger (2006), who report a median debt-to-GDP ratio for Argentina of 43% for the period 1990–2004.

Table 2 presents the parameterization of the stochastic processes that govern the behavior of the world interest rate. We estimate equations (8) and (9) using data on the real international risk free rate.[18] We obtain this rate by subtracting expected inflation from the quarterly US T-bill rate. Following Neumeyer and Perri (2005) and Fernández-Villaverde et al. (2011), we compute expected inflation as the average of the US CPI inflation in the current quarter and in the 3 preceding quarters. Parameter values in Table 2 correspond to the median of the posterior estimates. These posterior estimates imply annualized average standard deviations for the risk-free interest rate of 74 basis points (with only mean volatility) and 97 basis points with both mean and stochastic volatility.

5 Results

In this section we present the main results of our paper. First, we show the ability of the full model to account for salient features of the Argentine business cycle. Second, we study the effects of introducing time variation in both the level and the volatility of the world interest rate. Third, we compare our main results to the dynamics obtained in an otherwise identical model but with a constant world interest rate (we call this model the "basic model"). Fourth, we disentangle the effects of stochastic volatility (shocks to the volatility of the world interest rate) and mean volatility (shocks to the level of the world interest rate) by introducing

[18] We use the `stockvol` R package, which implements an efficient algorithm for Bayesian estimation of stochastic volatility models via MCMC methods. See Kastner (2016) for more details on the estimation procedure.

Table 3: Business Cycle Statistics

	Data	Full Model
$E(b/y)$ (in %)	43	43
$E(R_s)$ (in %)	7.4	7.4
$sd(R_s)$ (in %)	4.4	4.4
$sd(c)/sd(y)$	1.1	1.2
$corr(c, y)$	0.8	0.9
$corr(tb/y, y)$	-0.9	-0.4
$corr(R_s, y)$	-0.7	-0.6

Note: The mean and the standard deviation of a variable x are denoted by $E(x)$ and $sd(x)$, respectively. The correlation between two variables x and z is denoted as $corr(x, z)$. All variables are logged (except those that are ratios) and then de-trended using the Hodrick-Prescott filter, with a smoothing parameter of 1,600. We report deviations from the trend. R_s stands for the sovereign spread. The data for sovereign spreads is from J.P. Morgan's EMBI+, which represents the difference in yields between Argentine and US government bonds of similar maturity.

an "intermediate" version of our model, where only level (but not volatility) shocks affect the world interest rate. Fifth, we present a measure of the welfare cost of interest rate uncertainty. Sixth, we study the effect that world interest rate's uncertainty has on the ex-ante optimal maturity of government debt. Finally, we present empirical measures of the degree of time-varying volatility on the time series of the sovereign spread (which is endogenous in our model), and find that our model functions as an amplification mechanism for the time-varying volatility inherited from the world interest rate.

5.1 Business Cycle Moments of the Full-Model Economy

Table 3 reports moments in the data and in our simulations of the full model. As in previous studies, we report results for pre-default simulation samples. We simulate the model for 1,500 samples of 3,000 periods each. We then discard the initial 1,500 periods of each sample as a burn-in and from the remaining periods we extract 1,000 samples of 32 consecutive quarters before a default.

The moments reported in Table 3 are chosen to illustrate the ability of the full model to replicate distinctive business cycle properties of economies with sovereign risk. The second column of the table shows that the full model approximates well the moments used as

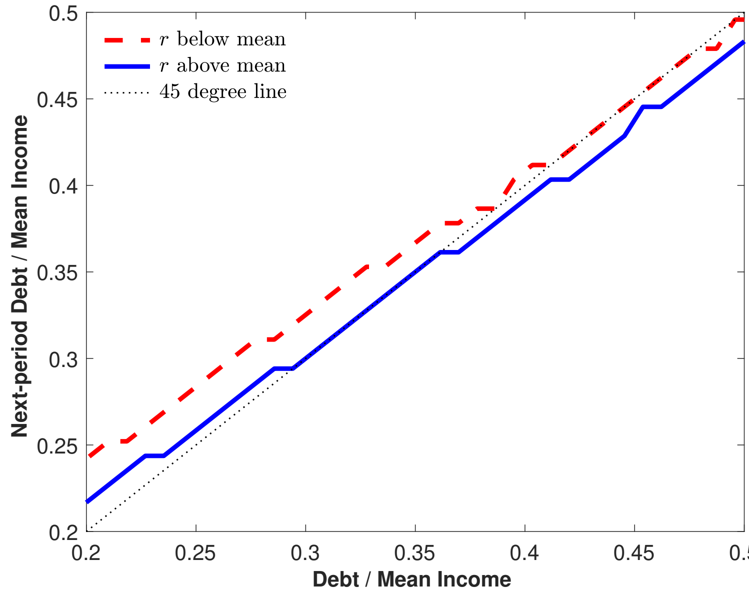


Figure 1: Borrowing policy functions for different levels of r. The solid blue line corresponds to the case in which the level of r is 1 std. dev. above mean, and the red dashed line to the case of 1 std. dev. below mean. Both lines are for the average volatility of the world interest rate and the mean income level.

targets (the debt-to-income ratio, and the average and standard deviation of the sovereign spread) and it is broadly consistent with non-targeted moments in the data: consumption is procyclical and more volatile than income; the trade balance is countercyclical; and the sovereign spread is also countercyclical (as is often the case in economies facing sovereign risk, see Neumeyer and Perri, 2005 and Uribe and Yue, 2006).

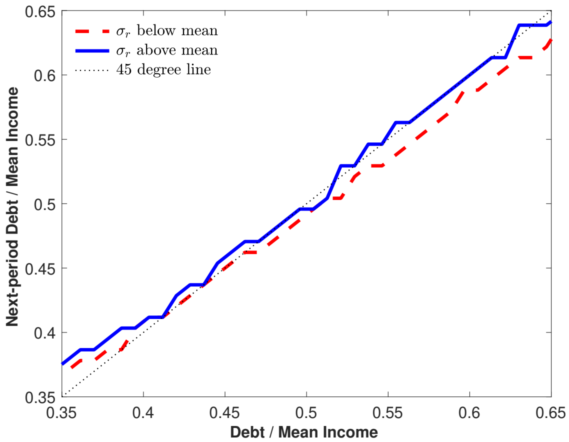
5.2 Effects of Uncertainty about the World Interest Rate

Since our model introduces two sources of time variation to the world interest rate (level and volatility shocks) into an otherwise standard sovereign default model, we start our discussion of the results by studying how these features interact with the dynamics typically studied in this literature. In particular, we examine the effects on borrowing decisions, default incentives, and sovereign spreads.

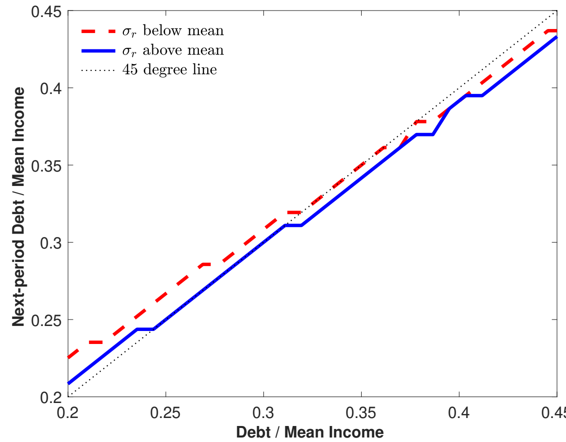
Borrowing decisions. Figure 1 shows the effect of different levels of the world interest rate on borrowing decisions (for the average volatility). It is clear from this figure that when the small open economy faces a low world interest rate, it prefers to increase borrowing: the b' function for the low r level lies consistently above the one for high r level.[19] It is not surprising that when facing better terms the government's reaction is to lever up and take advantage of the lower rates. The figure also highlights a well known aspect of borrowing decisions in this class of models. At low levels of debt, the policy rules lie above the 45 degree line and at high levels they lie below the 45 degree line. This occurs because spread-debt menus offered by lenders increase in the amount of debt chosen. The sovereign realizes this and optimally chooses to lower debt when it is relatively high. The debt level at which borrowing is curtailed is lower in the solid blue line when interest rates are high.

Figure 2 presents the effect of the volatility of r on borrowing choices. Panels (a) and (b) of this figure show the borrowing policy functions for two different values of the volatility of the world interest rate (σ_r), conditional on facing a low level of r (panel a) or a high level of r (panel b). The main takeaway from the two panels is that the effect of σ_r on borrowing choices is highly state contingent. With low r, a higher volatility state involves relatively more borrowing than the low volatility state: the solid blue line sits above the dashed red line at most points in debt to mean income space. We note that the difference in the two lines is especially pronounced at high levels of debt. We also note that the red dashed line sits below the 45 degree line when debt to mean income is .45 or higher, while the solid blue line is roughly at the same level. In other words, the economy wants to lower debt, from current high levels, in the low volatility state but not when volatility is high. Recall that debt issued today involve a promise of a long term stream of coupon payments, far into the future. As a result, the economy must take into account current interest rate conditions relative to expected future conditions. When the interest rate is unusually low, there is a strong incentive to borrow and lock in the "good times". This desire is tempered by the realization that bond yields are increasing in debt. When the volatility state is high, agents realize that it is more likely that in the future the economy will transition to a high world interest rate state and vice versa. As a result, the sovereign prefers to lower borrowing in low volatility states (and benefit from lower spreads) but prefers to "lock in the good rates" and

[19] This figure was created for the average level of r volatility, however a similar pattern is observed for different volatility levels.



(a) Low r



(b) High r

Figure 2: Effect of volatility on the borrowing policy functions. The left panel is for r below mean and the right panel is for r above mean. The solid blue line corresponds to the case in which σ_r is 1 std. dev. above mean, and the red dashed line to the case of 1 std. dev. below mean. Both panels are for the mean income level.

not lower borrowing in high volatility states. When debt levels are really low, the sovereign "locks in" the low interest rate and also increases borrowing at both volatility levels.

In the right panel of Figure 2, the state contingent nature of the sovereign's decisions become further clarified. The impact of debt levels on the borrowing decision is similar to our earlier discussion. The presence of a high level of r shifts the borrowing rules to the right, making the sovereign more likely to lower debt than in the left panel for both volatility regimes. Turning to the impact of volatility, we see that the location of the two lines is reversed: the red dashed line corresponding to the low volatility state sits mostly above the solid blue line corresponding to the high volatility state. Focusing in on the moderate debt levels around .35, the sovereign chooses to lower debt in the high volatility state but not in the low volatility state. At higher debt levels, both volatility states lead to lower borrowing but the effect is more pronounced in high volatility states. The explanation for this is similar to the previous paragraph: the sovereign sees that rates are unfavorable today so a high volatility regime implies a higher chance (given the high σ_r) that rates will decrease in the near future compared to the chance in a low volatility state. As a result, it is optimal for the sovereign to wait for better times and consequently to inter-temporally substitute current borrowing for future borrowing, when interest rates are lower. Coupled with the

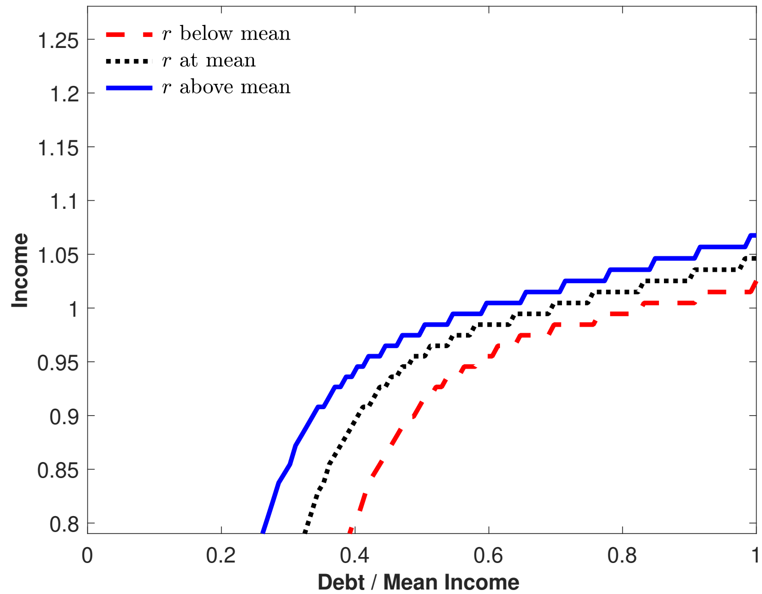
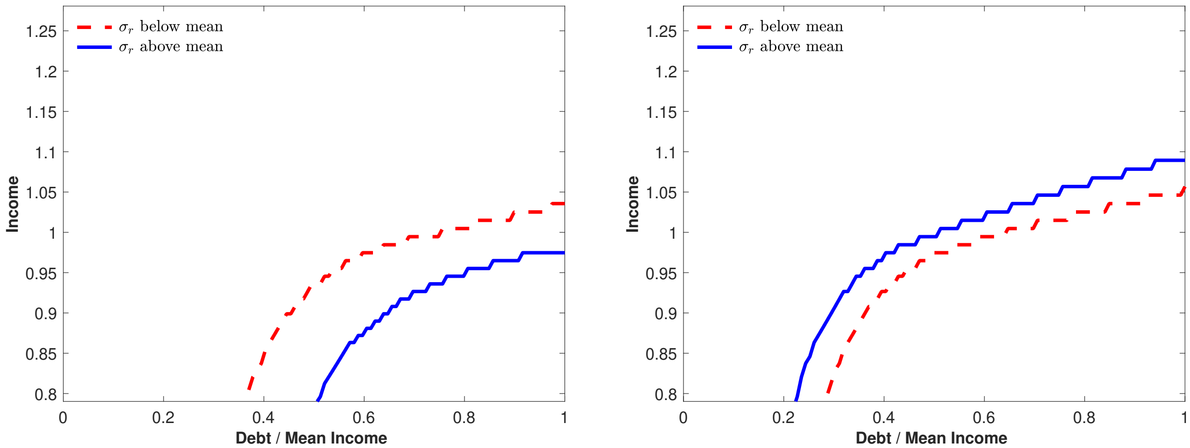


Figure 3: Default sets. The solid blue line corresponds to the case in which the level of r is 1 std. dev. above mean, the black dotted line is for r at its mean and the red dashed line is for the case of r 1 std. dev. below mean. Each line is the respective default set contour: the government defaults *south* of the line. The figure assumes the average volatility of the world interest.

desire to avail of lower spreads, the sovereign lowers borrowing more in the high volatility state.

Default incentives. Figure 3 shows the effect of different levels of r on default incentives, holding the volatility state at its average. The graph shows combinations of income and debt to income states, and divides the space into two regions. To the north of the default contour, the sovereign chooses to repay outstanding claims while it chooses to default at or south of the contour. We plot three contours for three levels of the world interest rate. The message from the figure is clear: other things equal, the default set is increasing in the level of the world interest rate. Two forces drive this result. First, the sovereign would wish to repay claims maturing in the next period in order to have continued access to bond markets in order to smooth consumption. Higher interest rates make this option value more expensive since more consumption must be sacrificed in order to roll-over debt, ceteris paribus. Second, if r is high today, then it is likely that it will be high tomorrow as well since r follows a persistent process. This makes the threat of financial exclusion less severe, as the periods in which the sovereign would be unable to borrow would likely be periods of high world interest



(a) Low r (b) High r

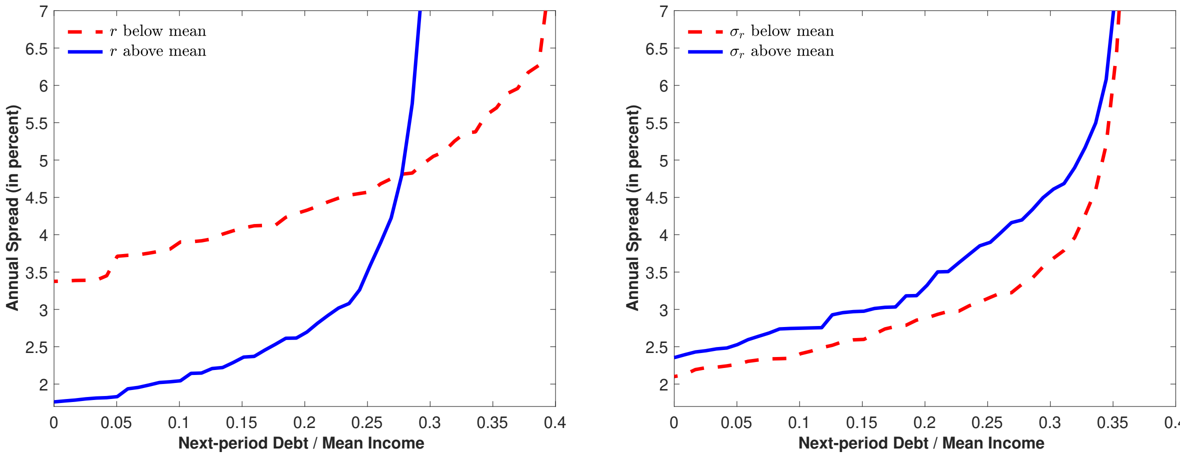
Figure 4: Effect of volatility on default incentives. The left panel is for r below mean and the right panel is for r above mean. The solid blue line corresponds to the case in which σ_r is 1 std. dev. above mean, and the red dashed line to the case of 1 std. dev. below mean. Each line is the respective default set contour: the government defaults *south* of the line.

rates. Both forces go in the same direction and make the default set larger as r increases.

Figure 4 presents the effect of shifts in the volatility state of r on default incentives. Panels (a) and (b) of this figure show the default set for two different values of the volatility of the world interest rate (σ_r), conditional on facing either a low level of r (panel a) or a high level of r (panel b). Here we again see that the effect of volatility is state contingent. As explained above, when rates are low (panel a) it is cheap to borrow and the government wants access to the markets, for which it needs to repay – these incentives are stronger if there is more volatility because in this case it is more likely that rates are going to increase in the near future. As a result, it is less willing to default. When rates are high today (panel b), the opposite is true, and volatility now enlarges the default set since it is a good time to default, ceteris paribus.

Sovereign spreads. The lessons learned from the effects of time-varying interest rates on borrowing and default decisions manifest themselves in the equilibrium spread-debt menus facing the small open economy. Figure 5 shows the menu of spreads and next-period debt level combinations from which the sovereign can choose.[20] Panel (a) of this figure keeps

[20]Both panels in this figure are done for an income level 2 std. dev. below its mean, which helps visualize the effects. The qualitative results are unchanged if we fix income at its mean.



(a) For σ_r at mean

(b) For r at mean

Figure 5: Spread-debt menus. The left panel keeps σ_r at its mean and the right panel keeps r at its mean. The solid blue line corresponds to values 1 std. dev. above the mean (either level or volatility of r), and the red dashed line to the case of 1 std. dev. below mean. Both panels are for a low income level.

the volatility of the world interest rate at its mean and varies the current level of r. The behavior of the spread-debt menu reveals a number of forces at play in our model. First, we see that even for low next-period debt levels (even close to zero), spreads are high (near or above 200bps). This is due to the presence of long-duration bonds in our model and the debt-dilution phenomenon – lenders not only price one-period ahead default probability, but all future default probabilities which may be quite high in certain future states.[21] Second, for low borrowing levels, when r is below its mean, the spreads facing the sovereign are *higher* than when r is above its mean. At first glance, this may seem odd but it is in fact precisely what you would expect given our previous intuition – the persistence of the interest rate process implies that a low rate today is most likely followed by low rates in the near future. This, in turn, will make the sovereign borrow more in the future (see discussion of Figure 1) which implies that lenders will charge them higher spreads today (the 'dilution premium'). Third, high levels of r lead to lower debt capacity: the endogenous debt limit implied by the steeply rising part of the spread-debt menu is lower (moves to the left in the solid blue line) for the case in which r is above its mean.

Panel (b) of Figure 5 illustrates the effects of the volatility of r on the spread-debt menus.

[21] See Hatchondo et al. (2016) for a study on debt dilution and sovereign default risk.

Table 4: Simulation Results: Full, Intermediate and Basic Models

	Full Model	Intermediate Model	Basic Model
$E(b/y)$ (in %)	43	42	39
$E(R_s)$ (in %)	7.4	5.6	4.6
$sd(R_s)$ (in %)	4.4	3.0	2.4
$sd(c)/sd(y)$	1.2	1.2	1.1
$corr(c, y)$	0.9	1.0	1.0
$corr(tb/y, y)$	-0.4	-0.5	-0.7
$corr(R_s, y)$	-0.6	-0.7	-0.7
CVR	3.15	2.82	2.32

Note: the moments' computation, de-trending method, and data sources are the same as those described in the footnote to Table 3. CVR refers to the "Crisis Volatility Ratio", as defined in equation (19). Only the full model is calibrated to match $E(b/y)$, $E(R_s)$ and $sd(R_s)$. Parameters are kept unchanged across models (except for those that control shocks to the world interest rate).

Keeping the level of r at its mean, we see that an increase in volatility leads to an uniformly worse spread-debt menu: at all levels of debt-to-income an increase in the volatility of the world interest rate causes international debt markets to demand higher interest rates from the small open economy. This is reminiscent of the rise in interest rates experienced by emerging economies during the "taper tantrum" episode discussed in the Introduction.

5.3 Keeping r Constant: Basic Model

In order to isolate the effects of shocks to the world interest rate, we define a "basic model" in which this interest rate is constant (i.e. we set $u_r = u_{\sigma_r} = 0$ in the basic model while leaving all other parameters unchanged at their values in the full model). We simulate and compute statistical moments from the basic model in the same way as we did for the full model. To ease comparison across models, Table 4 reports moments generated by the basic and full models (as well as the 'intermediate model', which will be defined in the next subsection).

As we showed in section 5.2, uncertainty about the world interest rate can in some cases increase borrowing and decrease it in others. A similar result was found for default incentives and sovereign spreads. Comparing across the columns for the full and basic models, our simulations indicate that on average the full model can sustain more debt but at a higher

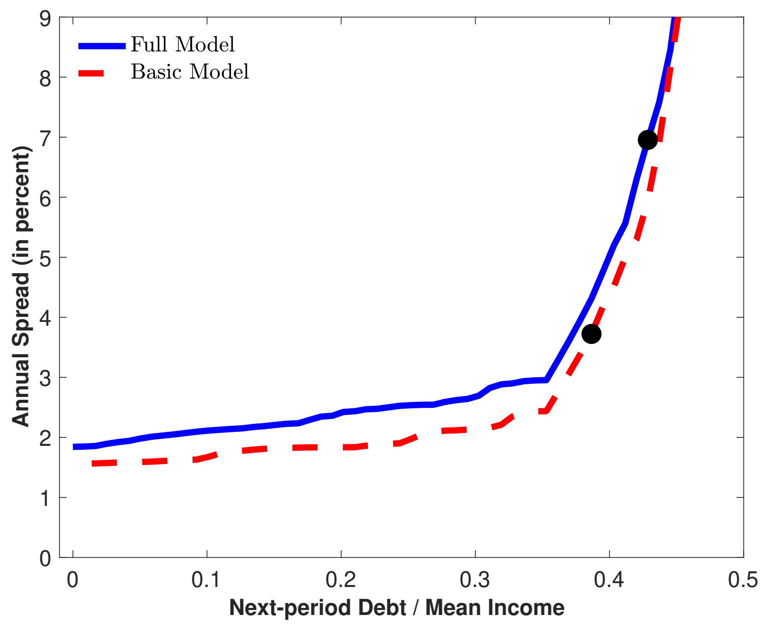


Figure 6: Spread-debt menus. The solid blue line is for the full model, and the red dashed line is for the basic model. Dark dots represent the average equilibrium choices in both models. Both lines are for the mean income level. The full model line assumes average r level and volatility.

and more volatile spread. Shocks to the world interest rate lead to an average spread in the full model that is 61% higher than in the basic model with no interest rate shocks. Similarly, the volatility of the spread is 83% higher in the full model compared to the basic model. These interest rate shocks also produce an average debt level that is 4 percentage points larger than what the model would produce with a constant world interest rate.

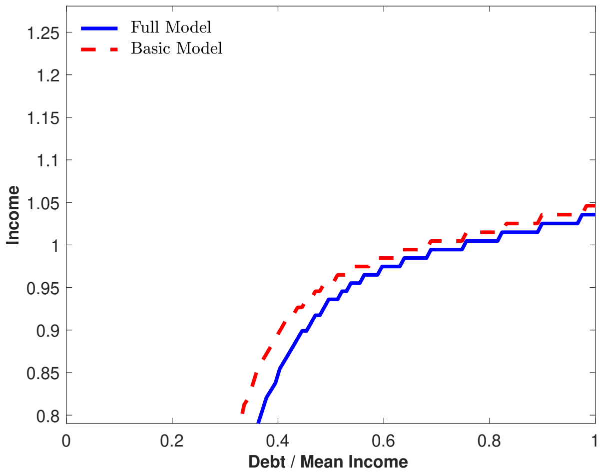
Comparing borrowing opportunities. Figure 6 illustrates an implication of uncertainty in the world interest rate that is related to the findings in section 5.2 except that it compares the spread-debt menus between the full model and the basic model. The figure also provides an illustration of the main differences between the two models highlighted in Table 4. For the mean equilibrium debt level in the full model (roughly 43% of income), the equilibrium spread level would be about 100 basis points lower in the basic model without world interest rate shocks even when both the level and the volatility states are at the mean. The figure also shows the equilibrium choices of the sovereign in the two models. Despite facing a worse spread-debt menu, the sovereign chooses more debt and higher spreads than in the basic model.

Comparing default sets. Figure 7 plots the default sets for both the basic and the full models, and for different points in the state space (i.e. for different realizations of r and σ_r). Looking across the four quadrants in Figure 7 brings out a number of points worth mentioning.

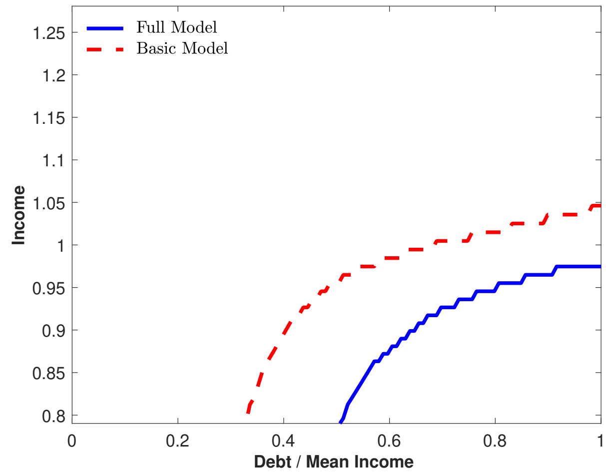
(i) The effect of interest rate uncertainty on default incentives is clearly state-dependent: there are configurations of the state space for which uncertainty about the world interest rate does not make the government in the full model default differently than in the basic model, but there are others in which it does. When r is at its mean level (middle row, both graphs) the volatility state leaves the default set virtually the same. Looking down the first column, we see the effect of increasing the level of the world interest rate, holding volatility at a low level. As the level of r rises, as expected, defaults go from being less likely in the full model to more likely since the sovereign is much more keen to keep market access when borrowing is cheap than when it is expensive. The second column shows that these effects are magnified in the high volatility state.

(ii) Why does the full model display larger spreads on average than the basic model when next period default probabilites may be the same or lower at some points in the state space? As discussed before, by virtue of having long-term debt, the bond price reflects not only one-period ahead default probabilities (those captured by each of these plots), but the expectation of all future default probabilities, which involves (among other things) the likelihood of moving across the panels as time goes by. Hence, spread differences are large on average because there are several configurations of the world (i.e. points in the state space) where the default incentives are clearly larger in the full model (with the full model being more prone to default). The presence of these points in the state space, imply high spreads in the full model even when there is no imminent likelihood of default occurring.

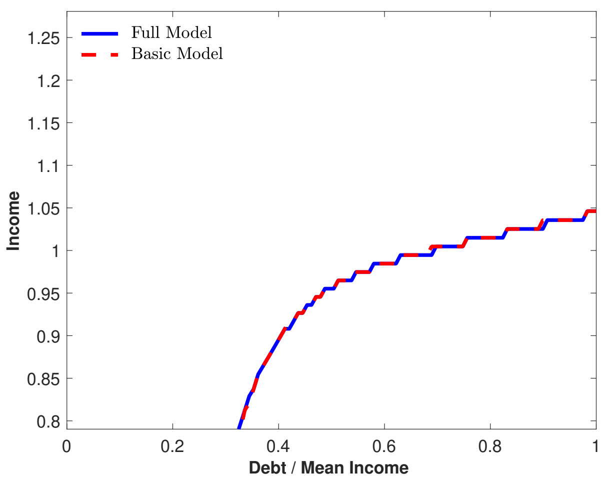
(iii) Each of these default sets has the expected shape: for a given level of debt-to-income ratio, the country is more likely to default when it gets low realizations of income; for a given level of income, the country is more likely to default when facing higher indebtedness.



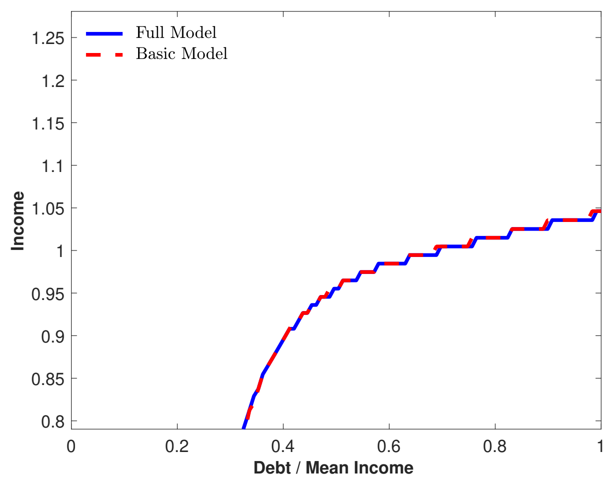
(a) r below mean, σ_r below mean



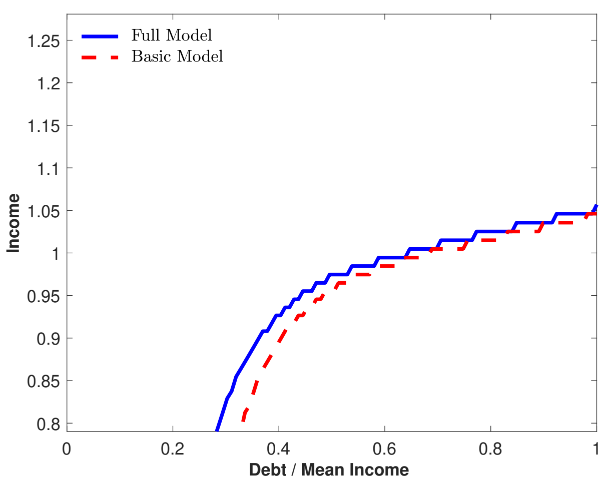
(b) r below mean, σ_r above mean



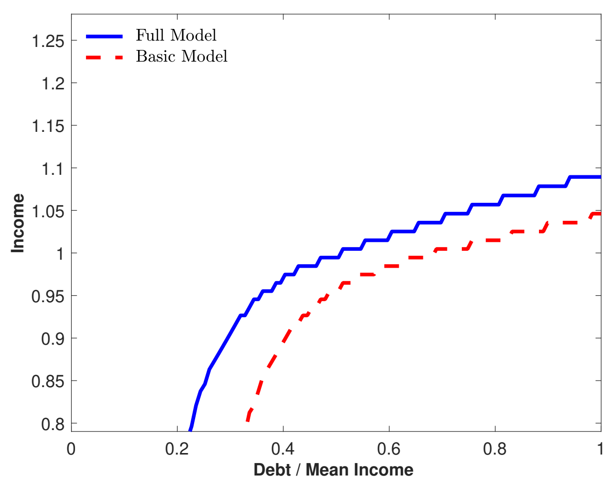
(c) $r = \bar{r}$, σ_r below mean



(d) $r = \bar{r}$, σ_r above mean



(e) r above mean, σ_r below mean



(f) r above mean, σ_r above mean

Figure 7: Default sets. Each line is the respective default set contour: the government defaults *south* of the line. The solid blue line is for the full model and the red dashed line is for the basic model. Panels (a)–(f) denote different combinations of (r, σ_r) for the full model.

5.4 Disentangling the Shocks: Intermediate Model

How important is the contribution of stochastic volatility shocks (i.e., the shocks to the volatility of the world interest rate) over and above mean volatility shocks (i.e. deviations from mean of the world interest rate)? In other words, we want to disentangle the results of the full model to study the relative contribution of u_r vs u_{σ_r}.

Table 4 also presents simulations results for the intermediate model.[22] We can see that, as expected, the intermediate version of our model (in which there are interest rate level shocks but not volatility shocks, $u_{\sigma_r} = 0$) produces business cycle statistics that are in between the basic and the full model.[23]

From studying this table we can measure the incremental effect of volatility shocks to the world interest rate: in particular, we see that including volatility shocks increases the mean spread by 180 basis points and the volatility of the spread by 140 basis points; while keeping the debt-to-income ratio roughly unchanged (it increases by 1%). Put in different terms, volatility shocks alone are responsible for roughly two-thirds of the increase in the mean and the standard deviation of sovereign spreads.[24]

5.5 Welfare Gains of Eliminating Interest Rate Uncertainty

Following the previous results it is then natural to ask: what is the welfare cost of being exposed to shocks to the world interest rate? Or equivalently, what are the welfare gains of getting rid of the world interest rate uncertainty? We compute these welfare gains as follows:

$$\left(\frac{v^0_{\text{basic}}(b, \mathbf{s})}{v^0_{\text{alternative}}(b, \mathbf{s})}\right)^{\frac{1}{1-\gamma}} - 1 \tag{16}$$

where alternative $= \{\text{full}, \text{intermediate}\}$. Equation (16) measures the welfare gains of moving to the basic-model economy (where the world interest rate is constant). The gain is expressed as the constant proportional change in consumption that would leave a consumer

[22] In the interest of avoiding unnecessary clutter, Table 4 does not repeat the column with Argentine data.

[23] The stochastic modeling of the world interest rate process is simple and parsimonious, so it is reassuring that feeding an intermediate version of it into our sovereign default model produces in fact 'intermediate' results.

[24] In the spirit of brevity we do not include graphs with the equilibrium functions for the intermediate model. As already argued, they lie "in between" those of the full and basic models.

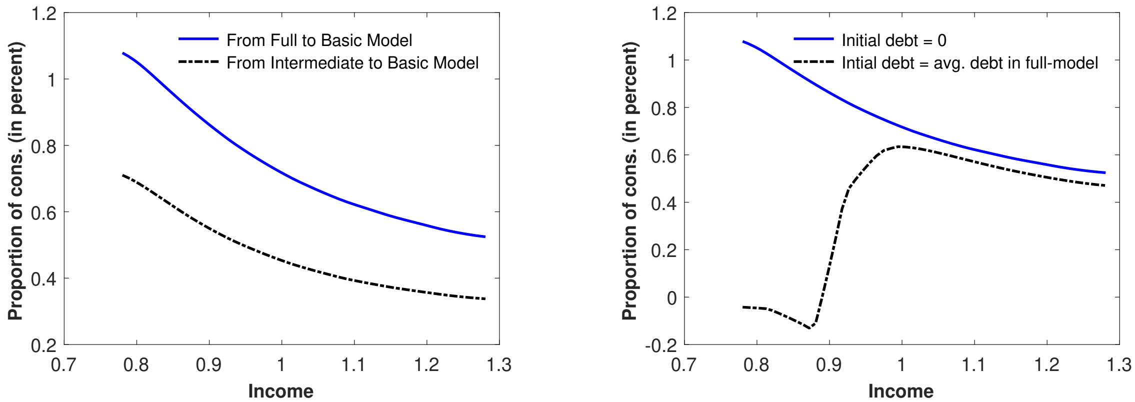


Figure 8: Welfare gains of moving to the basic-model economy. The left panel assumes a zero initial debt and plots the gains of moving to the basic-model economy from the full model (solid line) and from the intermediate model (dashed line). The right panel shows the gains of moving from the full-model economy for two initial debt levels: zero (solid line) and the average in the simulations (dashed line).

indifferent between continuing living in the alternative economy (either full or intermediate) and moving to the basic-model economy.

Figure 8 plots these gains as a function of the income level. The left panel in Figure 8 shows welfare gains when the initial debt is zero. In both cases (moving from the full to the basic economy, and moving from the intermediate to the basic economy) gains are positive and sizable (in some states being larger than a 1% permanent increase in consumption). They are also decreasing functions of the income level: interest rate uncertainty is costlier at low levels of income (as expected). The average welfare gains of moving to the basic-model economy are 0.73% (from the full model) and 0.46% (from the intermediate model).

The right panel in Figure 8 presents welfare gains of moving from the full-model economy to the basic-model economy for two different levels of initial debt: zero and the average debt level observed in the full model (43% of mean annual income). The figure gives interesting insights into the welfare gains. Eliminating all the interest rate uncertainty is less valuable when the indebtedness level is high and the income is low, this is because in those states the government will likely default anyway and the value of defaulting ($v^d(\mathbf{s})$) under no interest rate uncertainty is not dramatically higher than with uncertainty. However, for intermediate levels of income, the welfare gains are higher: it is precisely in those states where not being exposed to uncertainty makes the government able to repay and also to borrow at cheaper rates. The average welfare gain of eliminating all uncertainty about the world interest rate in this case (with initial debt equal to the mean level observed in the simulations) is equal to a .55% constant increase in consumption.

One could also argue that our measures of welfare gains from eliminating uncertainty in the world interest rate are low because there is no production in our setup, and therefore, we cannot capture the effects of the level and volatility of interest rates on aggregate income (Mendoza and Yue, 2012; Sosa-Padilla, 2018). Several studies find evidence of significant effects of the level of interest rates on aggregate productivity (through the allocation of factors of production) and of a significant role of interest rate fluctuations in the amplification of shocks (Uribe and Yue, 2006; Neumeyer and Perri, 2005; Mendoza and Yue, 2012).

5.6 Effects on Ex-Ante Optimal Maturity

We have shown that, on average, the economy faces higher borrowing costs when subject to uncertainty on the world interest rate. It is then intuitive to think of shocks to r_t as roll-over shocks: they can suddenly make it more costly for the sovereign to borrow and hence more painful to roll-over its debt position. One would expect, then, that an optimizing sovereign will prefer to extend the maturity of the debt portfolio and in that way hedge against this "roll-over risk." In this section we quantify exactly this incentive by solving for the ex-ante optimal coupon decay rate (δ in our notation) both in the basic and full models.[25]

We find that in the basic model (with a constant world interest rate) the government prefers a δ that is close to 1, which implies a debt duration of one quarter. When facing world interest rate uncertainty the government maximizes ex-ante welfare if it could choose a δ close to .49, which implies a debt duration of roughly two quarters. So, we find quantitative support for the intuition laid out in the previous paragraph: facing higher roll-over risk (i.e. interest rate uncertainty) the government prefers to extend the duration of its debt portfolio (in this case, it chooses to double the ex-ante optimal maturity).[26]

[25]The ex-ante optimal coupon decay rate (δ^*) is computed as the one that satisfies:

$$\delta_i^* = \underset{\delta}{\operatorname{argmax}}\, \mathbb{E}\left\{v^0(0;\mathbf{s})\right\} \text{ for } i = \{\text{basic, full}\}.$$

[26]This exercise gives a value for the ex-ante optimal δ that implies a much shorter duration than our calibration. This is exactly what one would expect in light of the results in Hatchondo et al. (2016). Long

5.7 Time-Varying Volatility in the Sovereign Spreads

Fernández-Villaverde et al. (2011) find that changes in the volatility of the interest rate at which small open economies borrow have an important effect on the business cycle. In this section, we study the ability of our model to act as a transmission and amplification mechanism for the time-varying volatility inherited from the world interest rate. In particular, we are interested in studying how much time-varying volatility is present in the sovereign spread generated by our model. We use two nonparametric measures of 'time variation in volatility' and both metrics give us a similar result: our sovereign default model generates time-varying volatility in the sovereign spread, over and above the one inherited from the world interest rate.

We first present a simple and often-used measure of volatility, realized volatility (also called historical volatility). We measure realized volatility (rv) as:

$$rv_t = \sqrt{\frac{1}{w-1}\sum_{\tau=t-w}^{t-1}(spread_\tau - \mu_t^{spread})^2}, \tag{17}$$

with

$$\mu_t^{spread} = \frac{1}{w}\sum_{\tau=t-w}^{t-1} spread_\tau. \tag{18}$$

As the previous two equations show we just select rolling windows of length w and calculate realized volatility as the sample standard deviation of spreads over a given period w from time $t-w$ to $t-1$. We compute rv_t for 400 samples of 300 periods prior to a default and for a window of 100 periods ($w = 100$). The left panel of Figure 9 shows the median rv_t across samples. We see that, as expected, $rv^{full} > rv^{int} > rv^{basic}$. More importantly, we also see that rv_t is time-varying: in particular it increases in the eve of a default (and this is true across models). The right panel of Figure 9 shows the kernel density of rv and confirms its time-varying nature. Moreover, the higher dispersion of the full-model's density makes it clear that this version of the model has more time-varying volatility than the other versions.

duration bonds create a dilution problem, and hence the sovereign prefers to lower its exposure to dilution by choosing a shorter duration. The contribution of our exercise is to quantify *how much longer* the government prefers its debt when facing uncertainty about the world interest rate, in an environment where dilution risk is already present.

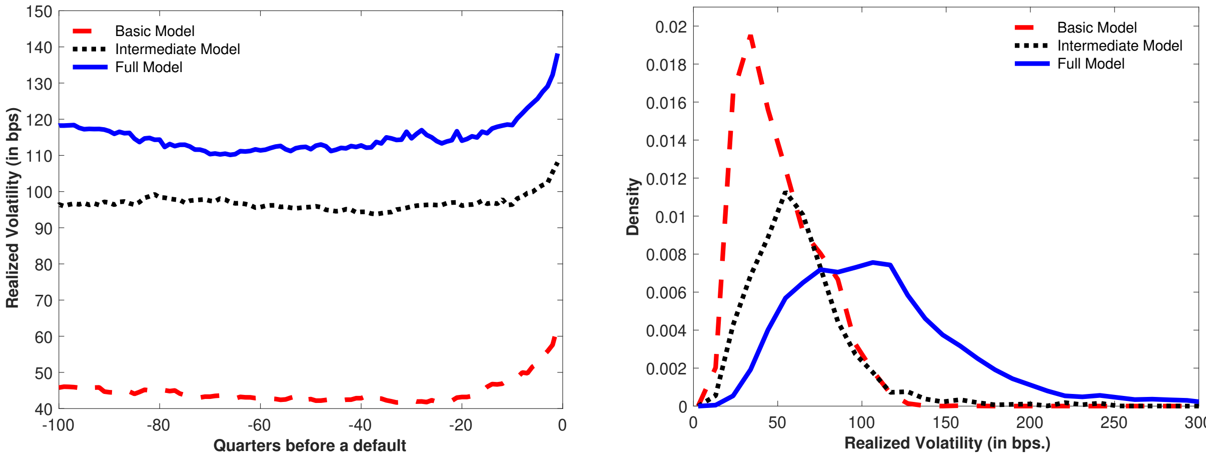


Figure 9: Realized volatility across models. The left panel shows the medians across sample for the realized volatility. The right panel shows the kernel densities of realized volatility. In both panels the solid blue line corresponds to the full model, the black dash-dotted line is for the intermediate model and the red dashed line is for the basic model.

We present a second nonparametric measure of time-varying volatility in spreads. Gu and Stangebye (2017) recently proposed a model-free metric for time-varying volatility in the spreads, which they called "Crisis Volatility Ratio" (CVR). The CVR is defined as follows: in a time series, let $\hat{T}$ denote the set of all periods in which the change in the spread is above the 97.5 percentile of its distribution. Let's call those periods "crises". With this definition of a crisis, the CVR is computed as:

$$CVR = \frac{1}{|\hat{T}|} \sum_{t \in \hat{T}} \frac{\hat{\sigma}_{t+1:t+\omega}}{\hat{\sigma}_{t-\omega:t-1}}, \tag{19}$$

where $\hat{\sigma}_{m:n}$ is the sample standard deviation of the spread computed using the periods from m to n and $|\hat{T}|$ is the number of elements in set $\hat{T}$. As Gu and Stangebye (2017) point out, this ratio compares the volatility of spreads in a window of ω periods immediately before and after a crisis, without including the crisis itself. If $CVR > 1$, then the crisis periods tend to be more volatile than non-crisis periods.[27] Following Gu and Stangebye (2017) we set $\omega = 5$ and compute the spread's CVR for all versions of our model as well as for the Argentine data.[28] The CVR in the Argentine data is 4.1 (or roughly four times of what an

[27] Gu and Stangebye (2017) explain that a process following an AR(1), for example, will have a CVR that is exactly 1.

[28] The bottom row of Table 4 reports the CVR for the different versions of our model.

AR(1) would feature) and suggests that time-varying volatility of the spread is a feature of the data. We also see that our full-model feature a CVR of 3.15, which is roughly 77% of what's observed in the data. Finally, we can also see that the full-model has CVR that is roughly 35% larger than the basic-model's CVR.

6 Conclusions

We have introduced time-varying volatility in the world interest rate in a standard sovereign default framework with long term debt. The process for the world interest rates follows the work of Fernández-Villaverde et al. (2011) and includes both mean volatility (i.e. shocks to the level of the interest rate) and stochastic volatility (i.e. shocks to the volatility of the interest rate). Time variation in the world interest rate interacts with default incentives and its effect on borrowing and sovereign spreads is state contingent. We measure the effects of the increased uncertainty by comparing the simulations of this model with the ones of the "basic model" (which has a constant risk-free world interest rate). We find that introducing uncertainty about the world interest rate in the model produces a mean sovereign spread that is 280 bps larger and 200 bps more volatile than the basic model. This suggests that variation in the level and volatility of the world interest rate may be an important aspect of the global factor isolated by empirical research into the determinants of sovereign default. It may also help to explain why sovereign spreads of many nations tend to rise together. The model also predicts that countries will prefer a longer maturity for their debt when facing a time-varying world interest rate. Our model acts as a propagation device for volatility in the world interest rate - the model with time-variation in the volatility of the world interest rate generates an endogenous time-varying volatility in the spread that is larger than in the basic model and about 77 percent of that observed in Argentine data.

The welfare gains from eliminating uncertainty about the world interest rate amount up to a 1 percent permanent increase in consumption. Put differently, our work helps to understand the concerns expressed by policy makers in the face of an increase in uncertainty about the path of the world interest rate.

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