Endogenous Repo Cycles*

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Abstract

This paper presents a simple and tractable equilibrium model of repos, where collateralized credit emerges under limited commitment. We show that even if there is no time variation in fundamentals, repo markets can fluctuate endogenously over time. In our theory, repo market fragilities are associated with endogenous fluctuations in trade probabilities, collateral values, and debt limits. We show that the collateral premium of a durable asset will become the lowest right before a recession and the highest right after the recession, and that secured credit is acyclical.

Keywords: collateral; search; endogenous credit market fluctuations

JEL Classification Numbers: E30 E50 C73

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1 Introduction

The recent financial panic started with a run in the sale and repurchase (repo) market. Financial crises are typically preceded by credit booms.[1] As suggested by Gorton and Ordoñez (2014), a theory of crises should also explain credit booms .

The objective of this paper is to explore a model of financial fragilities. In particular, we are interested in understanding the occurrence, and the consequence, of fluctuations – ups and downs – in repo markets. For this purpose, we construct a simple and tractable equilibrium model which features precisely the following characteristics of repos. First, a borrower borrows money and provides an asset as collateral to a lender. The lender takes physical possession of the collateralized asset. Second, upon a contemporaneous agreement, the borrower repurchases the collateral on a specified future date, which is eventually returned to the borrower when he repays the amount borrowed plus the (repo) interest rate to the lender. If one party defaults, then the agreement is terminated and the non-defaulting party can keep the money or the collateral depending on their position.

Our notion of fluctuations is related to sunspot equilibria developed by Cass and Shell (1983), Gramond (1985), and Woodford (1992). In our framework, individuals do not know ex ante whether their counter-party would be willing to trade at all in any given day. We assume that a probability of trade is determined by sunspots. The trading probability is a part of strategies and so is endogenous. We show that the borrower's value of collateral will play the key role in shaping equilibrium trade. If individuals suspect that the trading probability is low in the following period, then a lower future value is expected from holding collateral. Hence, the borrower finds it more profitable to renege today. Anticipating that the borrower is less likely to respect the repurchase agreement, the lender lends only a smaller amount of the production good. Such a self-fulfilling belief on the value of collateral gives rise to endogenous fluctuations in repo markets. We show that in a non-stationary equilibrium, cycles occur with endogenous time-variant probabilities of trade even with no time variation in fundamentals. We think our approach captures an important aspect of financial fragilities because, as emphasized by Gorton and Ordoñez (2014), the recent crisis was not the result of

[1] After the banking panic in September 2008, lending volume in the forth quarter of 2008 was 47% lower than it was in the prior quarter and 79% lower than at the peak of the credit boom in the second quarter of 2007 (Ivashina and Scharfstein, 2010).

a large shock.

One implication of our theory is that assets have a role in facilitating exchange, just like in Kiyotaki and Moore (2005, 2019), Duffie et al. (2005, 2008), and many other papers surveyed in Nosal and Rocheteau (2017) and Lagos, Rocheteau and Wright (2018), which commonly emphasize that assets are valuable not only for their return, but also for their liquidity services. We can capture it vividly by defining a *collateral premium*, which is a premium or net trade surplus that is created by an asset used as collateral. Along our cyclical equilibrium, if tomorrow is a recession (meaning that the trading probability is low), an economic downturn is expected and so the value of holding collateral and the amount of lending are both low today. This forward-looking nature of lending and borrowing implies that the collateral premium of a durable asset will become the lowest right before a recession and the highest right after it. This finding is consistent with the empirical evidence (see e.g., Gorton and Metrick, 2012) that the run in repo markets corresponded to a drastic increase in the degree of over-collateralization.

In their seminal work, Kiyotaki and Moore (1997) show that collateral constraints are the channel through which fluctuations in fundamentals are amplified. As a complementary effort, we show that collateralized credit can fluctuate endogenously even when fundamentals are time-invariant. Awaya, Fukai, and Watanabe (2017) focus attention on stationary equilibria in a related model but with a narrower class of repo-strategies. In the current paper we allow individual trading probabilities to take any number between zero and one. This allows us to develop the interesting dynamic equilibria described above.[2]

Azariadis, Kaas and Wen (2016) find that for the U.S. economy over the period 1981-2012, unsecured debt is strongly procyclical, while secured debt is acyclical. They argue that their finding challenges the conventional view of Kiyotaki and Moore (1997) which suggests that secured debt should be procyclical. Our equilibrium outcome could reconcile the empirical finding that secured credit is acyclical with the main insight of the Kiyotaki-Moore credit constraint.

In the literature of monetary economics (see, e.g., Kiyotaki and Wright, 1989; Kocherlakota, 1998), several related papers derive endogenous fluctuations in markets with credit or money which pays a dividend. Lagos and Wright (2003) show that a fluctuation can occur in

[2]Awaya, Fukai, and Watanabe (2017) addresses the payment puzzle raised by Lagos (2011), which is not the issue of the current paper. See also Madison (2018).

consumption in a version of the Lagos–Wright (2005) model. Ferraris and Watanabe (2011) introduce capital and show that monetary factors can interact with consumption and capital accumulation via collateral constraints. Rochetau and Wright (2013) use a variant of the model with entry decisions. Gu, Mattesini, Monnet, and Wright (2013) take into account the borrowers' limited commitment following the Kehoe and Levine (1993) tradition but without collateral.

Other models which are related but do not study collateral nor endogenous fluctuations are, for instance, Albuquerque and Hopenhayn (2004) on some credit relationship using a dynamic contract approach, and Kocherlakota (2001) on risky collateral as a mechanism to enforce contracts. Papers on other related topics are Parlatore (2017) on repos as the borrower's financing choice problem, Dang, Gorton, and Holmstrom (2013) on over-collateralization and repo runs as the adverse selection problem with a borrower's default risk, Gottardi, Maurin, and Monnet (2017) on the role of repos as insuring against price fluctuations, and Infante (2017) on the intermediary's problem in repo markets that brings together lenders and borrowers.

The rest of the paper is organized as follows. Section 2 presents the basic setup. Section 3 describes the stationary equilibria. Section 4 develops the cyclical equilibrium, and Section 5 concludes. Omitted proofs are collected in the Appendix.

2 The Model

Time is discrete, lasts forever, and is indexed by $t = 0, 1, 2, \ldots$. There is a continuum of individuals. Each individual is either a *borrower* or a *lender*. Each of the two parties has a unit measure. All individuals are long-lived and have a common discount factor $\delta \in (0, 1)$.

There are three kinds of goods – *production*, *consumption*, and *durable* goods. We assume that the consumption and production goods are perishable, while the durable good is durable. We also assume that the consumption and production goods are perfectly divisible, while the durable good is indivisible. In each period, each lender is endowed with one unit of the production good. Each borrower, on the other hand, is not endowed with the production good. In each period, a lender (resp. a borrower) can costlessly produce 1 unit (resp. a units) of the consumption good from each unit of the production good. We assume that $a > 1$ so that borrowers are better at producing the consumption good. It is impossible to produce the production and durable goods.[3] Individuals derive utility from the consumption good.

At start of the initial period, each borrower owns one unit of the durable good. Any individual who holds the durable good at the end of each period derives utility flow of $y > 0$.

In each period, borrowers and lenders engage in random pairwise meetings. At start of each period, all borrowers and lenders are unmatched. Each unmatched borrower (resp. lender) will be matched with a random lender (resp. borrower) with probability one. After matches are formed, each pair decides whether to trade or not. Importantly, the history of past actions is not public and individuals cannot commit to future actions. Each individual only observes her partners' actions, but does not observe past actions of any other individual.

In the main part of the paper, we assume that random pairwise meetings are the only possible opportunity to trade goods. In particular, we assume that there is no centralized market for the durable good. However, this assumption is innocuous. In Appendix B, we show that the presence of a market for the durable good does not alter our results.

If a pair decide to trade, trade occurs through three subperiods and goes as follows:

Subperiod 1 The lender decides how much of the production good to lend to the borrower. Let $q \in [0, 1]$ denote the amount lent to the borrower. The lender uses the remaining $1 - q$ by herself. The lender may ask the borrower for the durable good when she lends the production good. The transfer of the production and durable goods is simultaneous and occurs only if both parties agree to the terms of trade.

Subperiod 2 The borrower then produces aq units of the consumption good by using q units of the production good and chooses how much of the consumption good to give to the lender. Let $r \in [0, aq]$ denote the amount of the consumption good given to the lender. The borrower may ask the lender for the durable good in exchange for the consumption good. The transfer of the consumption and durable goods is simultaneous and occurs only if both parties agree to the terms of trade.

Subperiod 3 Each pair exogenously separate. Whoever holds the durable good at this point of time enjoys flow utility $y > 0$ from holding it.

If a lender lends q units of the production good and a borrower gives her r units of the

[3] Our results hold if production costs of the production and durable goods are sufficiently high.

consumption good, then the payoff to the lender for this period is $(1-q)+r$ and the payoff to the borrower is $aq-r+y$, provided that the durable good is returned to the original holder (i.e., the borrower). The repayment r from a borrower to a lender does not affect the sum of the payoffs to the two, which is $1+(a-1)q+y$. However, because a lender always has an option to use the production good by herself, repayment has to be high enough to meet the lender's incentive to lend.

Following the convention of exchange, a transfer of goods *within* each subperiod occurs at the same time. For example, in the first subperiod, once a borrower and a lender agree with lending the production good for the durable good, the lender cannot escape without giving the production good after receiving the durable good from the borrower. The same is true in the second subperiod. Note, however, that we assume that there is *no* commitment *across* subperiods. For example, in the first subperiod, a pair *cannot* write down a contract that specifies the terms of trade in the second subperiod.

We assume that the durable good holdings are private information to each individual, but prior to trade, individuals in each pair can ask each other to report their durable good holdings. It will be clear that, in equilibrium, a borrower never has an incentive to hide the durable good if he has one, whereas a lender always has an incentive to hide it if she has taken one in the past. As a result, it follows that, in effect, a lender can know a borrower's true holdings of the durable good, and a borrower cannot know a lender's.

The equilibrium notion is *sequential equilibrium* (simply, *equilibrium* henceforth), as is standard in an economy in which the history of actions is not public. The per-period payoff is linear in consumption. The lifetime payoff is

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t u_t$$

where u_t is utility in period t.

When there is no durable good in the economy, it is easy to show that the only equilibrium outcome is no trade, i.e., $(q,r)=(0,0)$ every period. On the other hand, any efficient allocation must satisfy $q=1$ (i.e., full lending) in (almost) every match. In our economy, the durable good is necessary to achieve a positive amount of lending in an equilibrium.

The exogenous parameters are the discount factor δ, borrowers' productivity a, and the durable good's return y. As we will see below, they affect individuals' incentives to trade in different ways. For the existence of non-stationary equilibria we are interested in, we make assumptions on these parameters. Throughout the paper, we focus on the following parameter values:

Assumption.

$$1 > \delta a \tag{1}$$

and

$$y \in \left[\underline{y}, \bar{y}\right) \tag{2}$$

where $\underline{y} := \frac{1-\delta^3 a^2}{1+\delta(1+\delta)a}$ *and* $\bar{y} := \frac{1-\delta^3-\delta^2(1+\delta)(a-1)}{1+\delta+\delta^2}$.

Assumption (1) is a joint restriction on the discount factor δ and borrowers' productivity a. Assumption (1) holds if individuals are sufficiently impatient and/or if borrowers are not too productive. In contrast, Assumption (2) puts a restriction on the return of the durable good as well as the other two. When δ or a increases, both $\bar{y}$ and $\underline{y}$ become smaller, that is, Assumption (2) is satisfied for smaller y. Also, Assumption (1) implies that $1 - \delta^3 a^2 > 0$. Thus, from Assumption (2), the fundamental value of the durable good y must be positive.

One can show that $\bar{y} > \underline{y}$ if and only if $\delta(a-1)(1-\delta^3 a) > 0$. Moreover, under Assumption (1), we have $1 - \delta^3 a > 0$. This and the fact that $a > 1$ together imply that Assumptions (1) and (2) are compatible with each other – there is an open set of parameter values (δ, a, y) that satisfy both of them.

3 Stationary Equilibria

In this section, we study stationary equilibria, where none of the endogenous objects described below depends upon time t. We consider a class of strategies where an endogenous fraction $m \in (0, 1)$ of all pairs engages in (a positive amount of) lending. Such decision to trade is determined in the following way. Consider a pair of borrower i and lender j. Right after the match is formed, they commonly observe a random variable m_{ij} that is drawn from the uniform distribution on $[0, 1]$. The draws are independent across pairs and periods. Borrower i and lender j independently agree to trade if and only if $m_{ij} \leq m$. We emphasize that m_{ij} is a sunspot, i.e., it does not affect economic fundamentals, and that the trading probability m is part of individuals' strategies. Trade between borrower i and lender j occurs if and only if they both agree to trade. Because m_{ij} is drawn uniformly, borrower i and lender j agree to trade with probability m. We consider symmetric strategies, where all pairs of borrowers and lenders share the same cutoff m and agree to trade with probability m.

Now consider the following *repo strategies*: If $m_{ij} > m$, then borrower i and lender j choose not to trade and they simply separate. If $m_{ij} \leq m$, in which case we shall call them *trading pairs*, they play the following strategies:

Subperiod 1 If the borrower has the durable good, the lender lends $q \in [0, 1]$ portion of the production good in exchange for the borrower's durable good.

Otherwise, they make no trade.

Subperiod 2 If trade occurred in the first subperiod, the borrower gives $r \in [0, aq]$ units of the consumption good to the lender in exchange for the durable good in the lender's hand.

Otherwise, they make no trade.

Subperiod 3 Each pair separates.

We further restrict our attention to the repo strategy that satisfies the following additional features: Within each trading pair, (i) the amount of lending is *trade-efficient* in the sense that q is as high as possible, subject to individuals' incentive constraints, and (ii) the borrower chooses r which gives himself the highest possible payoff subject to the lender's incentive constraints, which implies that the lender is indifferent between trading and not trading. Notice that, rather than lending q units of the production good to the borrower, the lender can utilize it by herself and produce q units of the consumption good. In contrast, a loan to the borrower guarantees the lender r units of the consumption good at the end of the second subperiod (provided that the borrower's incentives to repay are met). The lender is indifferent between trade and no trade when $r = q$. Thus, we consider the repo strategy for which $r = q$.

Hereafter, slightly abusing notation, we denote by q and r the amount of lending and repayment in an equilibrium we focus on.

In the following proposition, we show that a positive amount of lending can occur in equilibrium if individuals adopt the repo strategy specified above. We shall refer to it as a *stationary equilibrium* because the same set of actions is repeated over time.

Proposition 1. *For a sufficiently small trading probability $m \in (0,1)$, a stationary equilibrium exists with $q \in (0,1)$. Moreover, q is increasing in m.*

A low probability m of trade is in itself a source of inefficiency – as a positive fraction of population do not engage in trade. Proposition 1 suggests that a low probability m of trade causes an additional inefficiency in the model: it can also decrease the amount of lending q in the trading pairs.

But why does the probability m of trade affect equilibrium lending? Lending q units of the production good creates a surplus of $(a-1)q$, which is a benefit of aq created by a borrower's production of the consumption good using q units of the production good minus the opportunity cost of q when all the consumption good is produced by a lender only. The repo strategy specifies that this surplus goes to the borrower. Because trade occurs with probability m in every future period, a borrower, by getting back the durable good from a lender, expects to receive a (lifetime) payoff of $\delta m(a-1)q$ from future trades. In addition, the borrower receives a (lifetime) payoff of y from the durable good returned at the end of every period including today. Therefore, under the specified repo strategy, the borrower's lifetime payoff from getting back the durable good is $y + \delta m(a-1)q$, which is even bigger than its fundamental value y. The opportunity cost of getting back the durable good is the repayment $(1-\delta)q$. The expression

$$y + \delta m(a-1)q - (1-\delta)q$$

can thus be thought of as the *value of collateral for a borrower.* Notice that this measures exactly how tight the borrower's incentive constraint is. The value is increasing in m. When m is small, the value of collateral for a borrower is small. So, a smaller m makes it relatively more profitable for the borrower to escape with the consumption good produced. Anticipating that the borrower will not respect the repurchase agreement when the probability m of future trade is small, the lender lends only a small amount of the production good today.

4 Cycles

In this section, we demonstrate that under the Assumption, the credit market can exhibit endogenous cycles in equilibrium. To this end, we allow for a non-stationary class of strategies and study, in particular, a class of repo strategies in which endogenous variables exhibit a three-period cycle.[4] We first construct an equilibrium and then outline its important features.

To simplify notation, we assume that the cycle begins in period 0 and ends in period 2. Hence, we relabel the time modulo 3 as $t \in \{0, 1, 2\}$.

Like in the case of stationary equilibrium, in each period, decision to trade in each pair goes as follows. At start of period t, a pair of borrower i and lender j draw a random variable (sunspot) m_{ij} from the uniform distribution on $[0, 1]$. The draws are independent across pairs and periods. Borrower i and lender j agree to trade in period $t \in \{0, 1, 2\}$ if and only if $m_{ij} \leq m_t$. Trade between borrower i and lender j occurs if and only if they both agree. We again focus on symmetric strategies in which all pairs agree to trade with probability m_t in period t. Thus, the fraction of trading pairs in period t is m_t. The rest of the pairs choose not to trade, and simply separate. In particular, we look for cyclical equilibria in which $0 < m_0 < 1 = m_1 = m_2$.

As in the case of stationary equilibrium, a lender lends the production good for a borrower's durable good in Subperiod 1. Then, the borrower gives the consumption good to the lender in exchange for the durable good in the lender's hand in Subperiod 2. Again, within each trading pair, we focus on trade in which (*i*) the amount of lending is trade-efficient in the sense that q_t is as big as possible, subject to individuals' incentive constraints, and (*ii*) the borrower obtains the highest possible payoff, subject to the lender's incentive constraints. Like in Section 3, the borrower receives the highest payoffs when $r_t = q_t$. We also focus on the repo strategy such that $q_t < 1$ for exactly one $t \in \{0, 1, 2\}$, and $q_t = 1$ for the rest of periods. As in Section 3, q_t and r_t are the amount of lending and repayment in the equilibrium we focus on.

In the following proposition, we show that the repo strategy with the above features constitutes an equilibrium if the trading probability m_0 is sufficiently low. We shall call it a *cyclical equilibrium*. Like in the case of the stationary strategy, if the return y of the durable good is too low, a borrower does not follow the repo strategy specified above: the borrower's benefit from escaping with all the consumption good outweighs the cost of losing the durable good (which, in turn, includes a loss of future trades). Also, if y is sufficiently high, full lending, i.e., $q_t = 1$ for all t, can be sustained in equilibrium, and thus any $q_t < 1$ cannot be trade-efficient.

[4] Our departure from the usual two-period cycle is motivated by the fact that a three-period cycle can distinguish a high period which has a low period tomorrow from a high period which does not have a low period tomorrow. Absence of this distinction would blur some of our implications.

However, the Assumption rules out these extreme cases.

Proposition 2. *For a sufficiently small trading probability* $m_0 \in (0,1)$ *in period* 0*, a cyclical equilibrium exists with* $q_0 = q_1 = 1$ *and* $q_2 \in (0,1)$*. Moreover,* q_2 *is increasing in* m_0*.*

Why $q_2 < 1$ while $q_0 = q_1 = 1$? It is because the value of collateral for a borrower is highest in period 0 (a recession) and is lowest in period 2 (the end of a boom). Roughly speaking (see the following analysis for a formal derivation), both borrowers and lenders anticipate that period 0 is the worst time with the lowest trading probability m_0, and that trading probabilities thereafter are only rising towards period 2. Because it includes gains from future trades, the value of collateral for a borrower is highest in period 0, right before the beginning of a credit boom, and is lowest in period 2, right before a bust. Therefore, given a low value of collateral for a borrower in period 2, the borrower is not willing to repay a sufficiently high quantity, i.e. $r_2 = q_2 < 1$. Indeed, this is more so with lower values of m_0, and so q_2 is monotonically increasing in m_0.

It is worth mentioning that Proposition 2 implies acyclical secured credit, in the sense that the lowest lending occurs not in period 0 (a recession) but in period 2 (right before a recession); i.e. $q_2 < q_t = 1$ for $t \neq 2$. See Figure 1.[5] This contrasts sharply with the original Kiyotaki-Moore collateral constraint that predicts cyclical credit.

The Value of Collateral: Borrowers. By the construction of the repo strategy, borrowers hold the durable good at the end of each period in equilibrium. We study how the value of collateral for a borrower changes over time.

Let U_t be the expected lifetime payoff at the beginning of periods $t \in \{0,1,2\}$ (*prior to* realization of m_{ij}) using the repo strategy with the amount of lending q_t and repayment r_t. Recall that the repo strategy gives the highest payoffs to the borrower. Such highest payoffs were earlier shown to be attained when $r_t = q_t$. Thus, we have

$$U_t = (1-\delta)[m_t(a-1)q_t + y] + \delta U_{t+1}$$

where $t \in \{0,1,2\}$ (modulo 3). In (11), we solve for U_t explicitly.

Now in the second subperiod, with the repo strategy, the borrower receives a continuation payoff of $(1-\delta)[(a-1)q_t + y] + \delta U_{t+1}$. In contrast, if he deviates and does not repay, he

[5] Figures 1-3 are all depicted for $a = 1.2, \delta = 0.8, y = 0.098, m_0 = 0.05$.

consumes aq_t units of the consumption good today, but at the same time, he loses the durable good. A loss of the durable good precludes future trades. Therefore, the borrower's payoff from the deviation is $(1-\delta)aq_t$. Such a deviation is not profitable if and only if $(1-\delta)[(a-1)q_t+y]+\delta U_{t+1} \geq (1-\delta)aq_t$.

Define

$$v_t^B(q_0, q_1, q_2) := (1-\delta)(y-q_t) + \delta U_{t+1} \tag{3}$$

In (12) of Appendix A.2, we derive v_t^B explicitly as a function of (q_0, q_1, q_2) and (m_0, m_1, m_2).[6] By definition, (q_0, q_1, q_2) must satisfy the borrower's incentive constraint, $v_t^B(q_0, q_1, q_2) \geq 0$ for all t. Note $v_t^B(q_0, q_1, q_2)$ defined in (3) represents the value of collateral for a borrower. In a trading pair, a borrower receives a benefit of $(1-\delta)y + \delta U_{t+1}$ from getting back the durable good, whereas the cost of doing so is the repayment $(1-\delta)q_t$. Similarly to the stationary case, the value of collateral for a borrower in period t represents exactly how tight the borrower's incentive constraint in period t is. For $t, t' \in \{0, 1, 2\}$, $v_t^B > v_{t'}^B$ means that the durable good has a higher value of collateral to the borrower in period t than in period t'.

Corollary 1. *In the cyclical equilibrium, the value of collateral for a borrower satisfies*

$$v_0^B(1, 1, q_2) > v_1^B(1, 1, q_2) \geq v_2^B(1, 1, q_2) = 0$$

The proof of Corollary 1 follows from the proof of Proposition 2 (in particular, the borrower's incentive part in the proof of Lemma A.2 in Appendix A.2).

The Value of Collateral: Lenders. We now derive the value of collateral for a lender, denoted by v_t^L, where

$$v_t^L = y - (1-\delta)q_t$$

Notice that when the durable good is in the lender's hand, she will receive a lifetime payoff of y from confiscating it. On the other hand, if she does so, she will lose a payoff from the repayment $(1-\delta)q$ today. Thus, v_t^L is the value of the durable good for a lender. Note that, unlike the one for a borrower, the value of collateral for a lender is a negative sign of the lender's incentive constraint, i.e. $v_t^L \leq 0$.

From Proposition 2, we know that $q_2 < q_1 = q_0 = 1$. Hence, we have

[6] To simplify notation, we omit arguments (m_0, m_1, m_2).

Corollary 2. *In the cyclical equilibrium, the value of collateral for a lender satisfies*

$$v_2^L > v_1^L = v_0^L \tag{4}$$

Corollary 2 is a mirror image of Corollary 1 in the sense that given that the value of collateral for a borrower is lowest in period 2, quantity q_2 is lowest. This implies that the value of collateral for a lender is highest in period 2. Figure 2 depicts the movement of the value of collateral for a borrower and for a lender.

The Collateral Premium. In our model, the durable good is beneficial in facilitating exchange. We capture this by deriving a collateral premium, denoted by x_t, where

$$x_t = m_t(a-1)q_t$$

The value of trade is given by $(a-1)q_t$ units of the consumption good produced within each trading pair. Because there is a fraction m_t of trading pairs, net total production increased by use of the durable good as collateral is $m_t(a-1)q_t$.

Corollary 3. *In the cyclical equilibrium, the collateral premium satisfies*

$$x_1 > x_2 > x_0$$

While the quantity loss occurs in period 2, i.e., $q_2 < 1$, the low trading probability m_0 determines the premium to be lowest in period 0. Still, the quantity loss in period 2 leads to $x_2 < x_1$. In other words, the collateral premium is highest in period 1, which is immediately after the current recession or is the most remote from the next recession in the future, as illustrated in Figure 3. This finding is consistent with the existing empirical evidence that the run in repo markets accompanied with a drastic increase in the degree of over-collateralization.

5 Conclusion

This paper developed a dynamic model of repos. Even in the frictional world with limited commitment, collateralized credit can occur endogenously. We showed that repo markets can fluctuate endogenously over time, even without time variation in fundamentals, leading to acyclical secured credit. The collateral premium of a durable asset becomes the lowest right before a recession and the highest right after the recession.

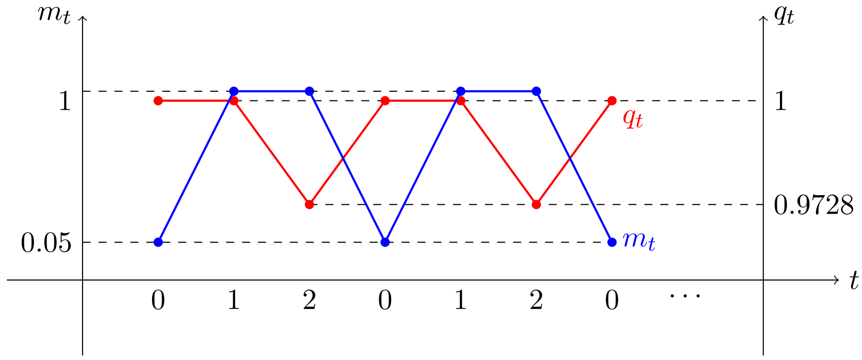

Figure 1: The Cyclical Equilibrium.

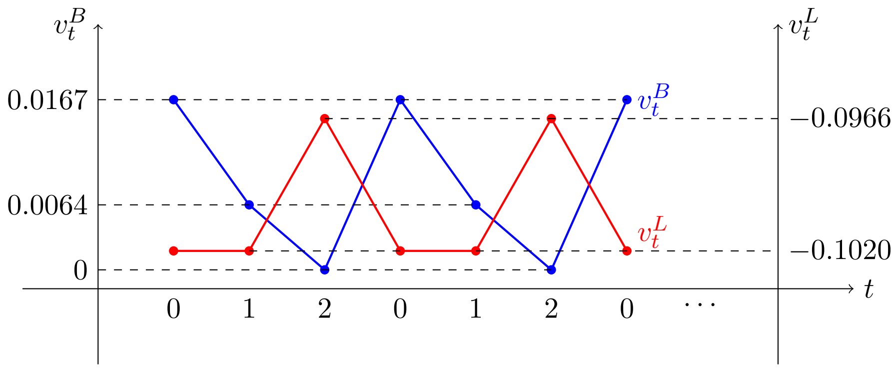

Figure 2: The Values of Collateral for a Borrower and for a Lender.

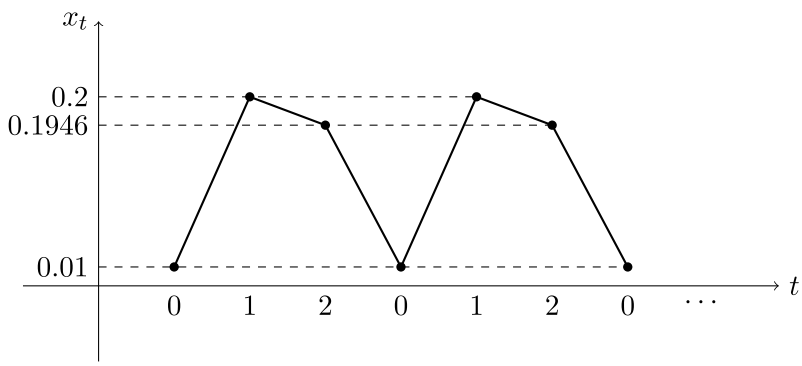

Figure 3: The Collateral Premium.

As an extension of the model, we could study the occurrence and consequence of rehypothecation as an equilibrium outcome. During the life of a repo, since the lender holds legal title to the securities, she could use the collateralized asset for her own interest, aiming at a higher return but with some risk of default. The dynamic/stability implication of this practice, and the theoretical effect of policies that regulate it are not obvious.

A Appendix: Proofs

A.1 Proof of Proposition 1

To prove the statement, we take steps of the proof as follows. For each $m \in (0,1)$, we find a condition for durable good return y for which the repo strategy with $q \in (0,1)$ constitutes an equilibrium. Taking the union of these sets with respect to m across $[0,1]$, we find a condition for durable good return y for which the repo strategy with $q \in (0,1)$ constitutes an equilibrium for *some* trading probability $m \in (0,1)$. The last step is to show that the condition is implied by the Assumption.

Fix a trading probability $m \in (0,1)$. Incentives of individuals in non-trading pairs are simple. When a borrower chooses not to trade, a lender is indifferent between choosing to trade and not, and vice versa. Thus, choosing not to trade is mutually optimal for a borrower and a lender.

Hereafter, we check incentives of individuals in trading pairs. First, we check a lender's incentives within a traing pair. A lender can deviate from the repo strategy in two ways. She can either refuse to trade in the first subperiod or escape with the durable good in the second subperiod.

Consider a deviation of a lender in the second subperiod. If the lender escapes with the durable good, she receives a lifetime payoff of y from taking the durable good. Instead, she loses a payoff of r that she would otherwise receive from a borrower today, or a lifetime payoff of $(1-\delta)r$. The actions and durable good holdings are private information in each pair, and hence, other borrowers do not know the fact that the escaping lender has the durable good. Therefore, after the deviation above, the lender's future (expected) payoffs are the same as the ones from following the repo strategy. Hence, the lender does not deviate in the second subperiod if and only if $(1-\delta)r \geq y$.

Consider a deviation of a lender in the first subperiod. If the lender refuses to trade in the first subperiod, she produces the consumption good by herself, which gives her a payoff of 1 in that period. In contrast, if the lender follows the repo strategy, she produces $(1-q)$ units of the consumption good by herself and receives r units from the borrower in the second subperiod. As in the case of a deviation in the second subperiod, the lender's behavior today does not affect her future payoffs because actions and durable good holdings are private information.

Hence, the deviation is not profitable for the lender if and only if $1-q+r \geq 1$, or equivalently, $r \geq q$. Because we focus on the repo strategy with $r = q$, we conclude that neither deviation is profitable for the lender if and only if $(1-\delta)q \geq y$.

Next, we check a borrower's incentives. A borrower also has two possible deviations: (i) to refuse to trade in the first subperiod and (ii) to escape with the consumption good in the second subperiod. Consider a deviation of a borrower in the first subperiod. If a borrower deviates in the first subperiod, he gives up gains from trade today. If the borrower follows the repo strategy, he produces aq units of the consumption good and gives r units of it to the lender in the second subperiod. Because we focus on the repo strategy with $r = q$, gains from trade to a borrower are thus $aq - r = (a-1)q \geq 0$. Hence, the deviation in the first subperiod is never profitable.

To check a deviation of a borrower in the second subperiod, first let U be the borrower's lifetime expected payoff from following the repo strategy, evaluated at the beginning of a period (prior to realization of m_{ij}, i.e., a trading decision). Then, we have

$$U = (1-\delta)(m(aq-r)+y) + \delta U$$

With probability m, the pair is a trading pair, and the borrower receives gains from trade $aq - r$. Recalling that the lender returns the durable good to the borrower at the end of trade, the durable good is in the borrower's hand at the end of the day whether there was trade or there was not. The borrower thus receives y from holding it. Hence, $m(aq-r)+y$ is the expected payoff for a day.

Solving the Bellman equation for U, we have

$$U = m(aq-r) + y \tag{5}$$

If a borrower follows the repo strategy in the second subperiod, he exchanges the durable good for r units of the consumption good. Moreover, he receives a continuation payoff of U. So, his lifetime payoff from giving r to the lender is $(1-\delta)(aq-r+y)+\delta U$. On the other hand, if he deviates and does not give r, he will consume all the consumption good he produced, but instead, will lose the durable good. The perishable production good yields $(1-\delta)aq$ in that period and nothing more afterwards. Because the repo strategy requires the borrower to hold the durable good to borrow, losing the durable good precludes future trades. Thus, his continuation payoff is 0 after the deviation. The deviation is not profitable for the borrower if and only if $(1-\delta)(aq - r + y) + \delta U \geq (1-\delta)aq$. Substituting (5) into this inequality, we have

$$r \leq \frac{y + \delta maq}{1 - \delta + \delta m}$$

Because we assume $r = q$, this inequality becomes

$$y - (1 - \delta - \delta(a-1)m)q \geq 0$$

Let $v^B(q) := y - (1 - \delta - \delta(a-1)m)q$. The repo strategy with q is incentive compatible for a borrower if and only if $v^B(q) \geq 0$.

Recall that the repo strategy specifies q to be the highest value that is compatible with individuals' incentives. Suppose that q is the highest value that is compatible with borrowers' incentives. To have $q \in (0,1)$, borrowers' incentives must be binding, i.e., $v^B(q) = 0$. From the definition of $v^B(\cdot)$, it follows that

$$q = \frac{y}{1 - \delta - \delta(a-1)m} \tag{6}$$

Note that if $y > 0$, then $q > 0$. Moreover, if $v^B(1) := y - (1 - \delta - \delta(a-1)m) < 0$, then $q < 1$. Now $v^B(1) < 0$ implies that $y < 1 - \delta - \delta(a-1)m$. Hence, given the trading probability m, the repo strategy with $q \in (0,1)$ is compatible with borrowers' incentives if and only if

$$y \in (0, 1 - \delta - \delta(a-1)m) \tag{7}$$

Next, we verify that the amount of lending q given by (6) is also compatible with lenders' incentives. Recall that a lender follows the repo strategy if and only if $(1-\delta)q \geq y$. Plugging (6) into this inequality, we have

$$\delta(a-1)my \geq 0$$

Any y satisfying (7) satisfies this inequality. In other words, any y satisfying borrowers' incentive constraints and $q \in (0,1)$ also satisfies lenders' incentive constraints. Therefore, we conclude that, given a trading probability $m \in (0,1)$, the repo strategy with $q \in (0,1)$ constitutes an equilibrium if and only if

$$y \in (0, 1 - \delta - \delta(a-1)m) \tag{8}$$

Now, we have found the condition on durable good return y for the repo strategy with $q \in (0,1)$ to constitute an equilibrium for a given $m \in (0,1)$. To find a condition on y for the repo strategy with $q \in (0,1)$ to constitute an equilibrium for *some* $m \in (0,1)$, we take the union of intervals in (8) with respect to m across $m \in [0,1]$. Thus, there exists a trading probability $m \in (0,1)$ such that the repo strategy with $q \in (0,1)$ constitutes an equilibrium if and only if

$$y \in (0, 1-\delta) \tag{9}$$

Finally, we need to show that the condition (9) is implied by the Assumption. From Assumption (2), we need to show that $1-\delta^3 a^2 \geq 0$ and $\frac{1-\delta^3-\delta^2(1+\delta)(a-1)}{1+\delta+\delta^2} \leq 1-\delta$. The first inequality follows from Assumption (1). To see that the second inequality holds, notice that

$$\frac{1-\delta^3-\delta^2(1+\delta)(a-1)}{1+\delta+\delta^2} = 1-\delta-\frac{\delta^2(1+\delta)(a-1)}{1+\delta+\delta^2} < 1-\delta$$

where the strict inequality follows from $\delta > 0$ and $a > 1$. This completes the proof. ■

A.2 Proof of Proposition 2

The proposition is a direct implication of Lemma 1 below. In Lemma 1, we impose Assumption (1), and, fixing $m_0 \in (0,1)$, we derive a necessary and sufficient condition for the repo strategy with $q_0 = q_1 = 1 > q_2 > 0$ to constitute an equilibrium when every individual believes that trade will occur with probability m_0 in period 0 and with probability one in the other periods. For each $m_0 \in (0,1)$, we derive the set of durable good return y for which the above repo strategy constitutes an equilibrium. Proposition 2 says that for a sufficiently small m_0, the set of durable good return y in Lemma 1 and Assumption (2) have a non-empty intersection.

Lemma 1. *Suppose that Assumption (1) holds. Given a trading probability*

$$m_0 < \bar{m} := \frac{1-\delta^3 a-\delta^2(a-1)-(1+\delta+\delta^2)y}{\delta(a-1)} \in (0,1)$$

the repo strategy with $q_2 \in (0,1)$ and $q_0 = q_1 = 1$ constitutes an equilibrium if and only if

$$y \in \left[\underline{y}_c, \bar{y}_c\right)$$

where $\bar{m}, \underline{y}_c, \bar{y}_c$ are defined in the proof of Lemma 1.

Moreover, in such an equilibrium,

$$q_2 = \frac{(1+\delta+\delta^2)y+\delta(a-1)m_0+\delta^2(a-1)}{1-\delta^3 a} \tag{10}$$

Proof of Lemma 1

Throughout this proof, q_t denotes a quantity of trade (not an equilibrium quantity as in the main text) whereas q_t^* denotes an equilibrium quantity.

Borrowers' incentives. Suppose that lenders follow the repo strategy. Consider a borrower's incentive constraints. Because $a > 1$, any positive amount of lending creates a strictly positive surplus. In addition, the repo strategy we focus on allocates all the gains from trade to borrowers. Because refusing to trade yields a borrower a payoff of zero in that period, we conclude that the borrower does not have a profitable deviation in the first subperiod.

Recall that given (q_0, q_1, q_2), a borrower has no profitable deviation in the second subperiod of period t if and only if $v_t^B(q_0, q_1, q_2) := (1-\delta)(y - q_t) + \delta U_{t+1} \geq 0$, as defined in (3). Recall that, denoting by U_t the continuation payoff to a borrower from period t onward, we have the Bellman equation $U_t = (1-\delta)[m_t(a-1)q_t + y] + \delta U_{t+1}$. This gives us, by iterating the Bellman equation,

$$U_t = y + \frac{(a-1)(m_t q_t + \delta m_{t+1} q_{t+1} + \delta^2 m_{t+2} q_{t+2})}{1 + \delta + \delta^2} \tag{11}$$

Substituting the above expression into $v_t^B(q_0, q_1, q_2)$, we get

$$v_t^B(q_0, q_1, q_2) = y - (1-\delta)q_t + \frac{\delta(a-1)}{1 + \delta + \delta^2}(m_{t+1} q_{t+1} + \delta m_{t+2} q_{t+2} + \delta^2 m_t q_t) \tag{12}$$

Here, notice that m_t and q_t are repeated every three periods, that is, $m_t = m_{t+3}$ and $q_t = q_{t+3}$ for each t. It is easily checked that (12) directly leads to Corollary 1.

Observe that for $t = 2$,

$$v_2^B(q_0, q_1, q_2) = y - (1-\delta)q_2 + \frac{\delta(a-1)}{1 + \delta + \delta^2}(m_0 q_0 + \delta m_1 q_1 + \delta^2 m_2 q_2)$$

Because $m_0 < m_1 = m_2 = 1$, we must have $v_2^B(1, 1, \bar{q}) = 0$ for some $\bar{q}$. Solving for $\bar{q}$ from the above expression for v_2^B, we have

$$\bar{q} = \frac{(1 + \delta + \delta^2)y + \delta(a-1)m_0 + \delta^2(a-1)}{1 - \delta^3 a}$$

We have $\bar{q} < 1$ if and only if

$$y < \frac{1 - \delta^3 - \delta^2(1+\delta)(a-1) - \delta(a-1)m_0}{1 + \delta + \delta^2} =: \bar{y}_c$$

For this tuple of quantities $(q_0, q_1, q_2) = (1, 1, \bar{q})$, a borrower has an incentive to follow the strategy at $t = 2$ (provided that he follows the repo strategy in the other periods).

Now consider a borrower's incentive at $t = 1$. Remember that to show that $(q_0, q_1, q_2) = (1, 1, \bar{q})$ satisfies all remaining incentive constraints of a borrower, $v_0^B(1, 1, \bar{q}) \geq 0$ and $v_1^B(1, 1, \bar{q}) \geq 0$ must hold. It can be easily shown that for any $q_2 < 1$, $v_0^B(1, 1, q_2) > v_1^B(1, 1, q_2)$ because $m_0 < 1 = m_1 = m_2$. Therefore, we only need to show $v_1^B(1, 1, \bar{q}) \geq 0$.

Using $q_1 = q_0 = 1$, we get

$$v_1^B(1, 1, q_2) = \frac{1}{1 + \delta + \delta^2} \left[(1 + \delta + \delta^2)y + \delta(a - 1)q_2 + \delta^2(a - 1)m_0 - (1 - \delta^3 a) \right]$$

Substituting $q_2 = \bar{q}$ into the above equation, we have

$$\begin{aligned} v_1^B(1, 1, \bar{q}) = \frac{1}{1 + \delta + \delta^2} \Bigg(&(1 + \delta + \delta^2)y + \frac{(1 + \delta + \delta^2)\delta(a - 1)y}{1 - \delta^3 a} \\ &+ \delta^2(a - 1)m_0 + \frac{\delta^2(a - 1)^2 m_0}{1 - \delta^3 a} - (1 - \delta^3 a) + \frac{\delta^3(a - 1)^2}{1 - \delta^3 a} \Bigg) \end{aligned}$$

We now simplify this expression using the following steps. Consider the first two terms inside the parentheses:

$$\begin{aligned} (1 + \delta + \delta^2)y + \frac{(1 + \delta + \delta^2)\delta(a - 1)y}{1 - \delta^3 a} &= \frac{(1 + \delta + \delta^2)}{1 - \delta^3 a}(1 - \delta^3 a + \delta(a - 1))y \\ &= \frac{1 - \delta^3}{1 - \delta^3 a}(1 + \delta(1 + \delta)a)y \end{aligned}$$

where, to get the last equality, we just take $1 - \delta$ out of parentheses. Consider the third and forth terms:

$$\begin{aligned} \delta^2(a - 1)m_0 + \frac{\delta^2(a - 1)^2 m_0}{1 - \delta^3 a} &= \frac{\delta^2(a - 1)}{1 - \delta^3 a}(1 - \delta^3 a + (a - 1))m_0 \\ &= \frac{(1 - \delta^3)\delta^2 a(a - 1)}{1 - \delta^3 a} m_0 \end{aligned}$$

Consider the fifth and sixth terms:

$$\begin{aligned} -(1 - \delta^3 a) + \frac{\delta^3(a - 1)^2}{1 - \delta^3 a} &= \frac{1}{1 - \delta^3 a}(-(1 - \delta^3 a)^2 + \delta^3(a - 1)^2) \\ &= \frac{1}{1 - \delta^3 a}(-(1 - \delta^3) + \delta^3 a^2(1 - \delta^3)) \\ &= -\frac{1 - \delta^3}{1 - \delta^3 a}(1 - \delta^3 a^2) \end{aligned}$$

Collecting all these terms, we can now rewrite v_1^B as

$$v_1^B(1,1,\bar{q}) = \frac{1-\delta^3}{(1-\delta^3 a)(1+\delta+\delta^2)}\left[(1+\delta(1+\delta)a)y + \delta^2 a(a-1)m_0 - (1-\delta^3 a^2)\right]$$

From Assumption (1), we have $1-\delta^3 a > 0$. From the above expression, it follows that $v_1^B(1,1,\bar{q}) \geq 0$ if and only if

$$y \geq \frac{1-\delta^3 a^2 - \delta^2 a(a-1)m_0}{1+\delta(1+\delta)a} =: \underline{y}_c \tag{13}$$

All in all, we conclude that the borrower does not have a profitable deviation from the repo strategy with $q_0^* = q_1^* = 1 > q_2^* = \bar{q}$ if and only if $y \in \left[\underline{y}_c, \bar{y}_c\right)$. It is routine to verify that the range of y specified here and Assumption (2) have a non-empty intersection. We can also check that with the Assumption, the upper bound of m_0, which is $\frac{1-\delta^3 a-\delta^2(a-1)-(1+\delta+\delta^2)y}{\delta(a-1)}$, is positive and is less than one.

Finally, we show that $(q_0, q_1, q_2) = (1, 1, \bar{q})$ is trade-efficient. Notice under Assumption (1), $v_t^B(\cdot)$ is strictly decreasing in q_t for each $t \in \{0,1,2\}$. That is, for any $q_2 > \bar{q}$, $v_2^B(1,1,q_2) < 0$. That is, for any such q_2, a borrower has an incentive to deviate.

Lenders' participation incentives. We now check lenders' participation incentives. Suppose that a lender does not escape with the durable good in every period. In period t, if the lender agrees to trade, she receives a payoff of $(1-q_t^*) + r_t^*$ today and a continuation payoff, denoted by U_{t+1}^l, tomorrow. In contrast, if she refuses to trade, she receives a payoff of 1 in the period (from producing the consumption good by herself) and a continuation payoff of U_{t+1}^l tomorrow. Notice that the lender's decision today does not affect what she will receive from tomorrow onward. Therefore, the lender's continuation payoff from not trading is same as the one from trading. So, the lender finds it optimal to trade if and only if $r_t^* \geq q_t^*$ for each $t \in \{0,1,2\}$. Because we consider a strategy in which all trade surplus goes to borrowers, i.e., $r_t^* = q_t^*$, the lender's participation constraint is met.

Lenders' incentives to return the durable good. Next, we verify that a lender does not have an incentive to escape with the durable good in the second subperiod. Recall that the lender's past actions and the durable good holdings are private information. So, the lender's future partners will not know whether the lender has escaped with the durable good in the past.

Therefore, her continuation payoff from following the repo strategy is the same as the one from escaping with the durable good.

If the lender escapes with the durable good, she receives a payoff of y in every period onward. Moreover, she receives a continuation payoff of U^l_{t+1} from future trades. Thus, the lifetime expected payoff after the deviation is $y + \delta U^l_{t+1}$. In contrast, if she follows the repo strategy, she receives a payoff of q^*_t today and a continuation payoff of U^l_{t+1} from future trades. Thus, the lifetime payoff from the repo strategy is $(1-\delta)q^*_t + \delta U^l_{t+1}$. The lender does not deviate in period t if and only if $(1-\delta)q^*_t \geq y$. The profitability of the deviation depends on the amount of lending q^*_t. Recall that $q^*_2 < q^*_1 = q^*_0$. Thus, the lender does not deviate in any period if and only if $(1-\delta)q^*_2 \geq y$. Below, we will show that for any q^*_t that is incentive compatible with borrowers' incentives, lenders' incentive constraints are satisfied.

Recall that $q^*_2 = \frac{(1+\delta+\delta^2)y+\delta(a-1)m_0+\delta^2(a-1)}{1-\delta^3 a}$. Then, it follows that $(1-\delta)q^*_2 \geq y$ if and only if

$$y \geq -\frac{(1-\delta)(m_0+\delta)}{\delta^2} \tag{14}$$

Hence, to show that $q^*_2 = \frac{(1+\delta+\delta^2)y+\delta(a-1)m_0+\delta^2(a-1)}{1-\delta^3 a}$ is the highest amount of lending that is compatible with *both* borrowers' and lenders' incentives, we need to show that inequality (14) is implied by (13), or

$$-\frac{(1-\delta)(m_0+\delta)}{\delta^2} \leq \frac{1-\delta^3 a^2 - \delta^2 a(a-1)m_0}{1+\delta(1+\delta)a}$$

Rearranging terms, we have

$$(1-\delta^3 a)(\delta(1-m_0) + (1+\delta a)m_0 + \delta^2 a) \geq 0$$

Under Assumption (1), $1-\delta^3 a > 0$, and hence, the above inequality is satisfied for any m_0. This implies that borrowers follow the repo strategy with $q^*_0 = q^*_1 = 1$ and $q^*_2 = \frac{(1+\delta+\delta^2)y+\delta(a-1)m_0+\delta^2(a-1)}{1-\delta^3 a}$. ■

A.3 Proof of Corollary 3

It is obvious that x_1 is the highest because $q_1 = 1$ and $m_1 = 1$. Thus, we only need to compare x_0 with x_2.

From Lemma 1, we know that m_0 satisfies

$$m_0 < \frac{1-\delta^3 a - \delta^2(a-1) - (1+\delta+\delta^2)y}{\delta(a-1)} \tag{15}$$

and given this m_0, $q_0 = q_1 = 1$ and $q_2 = \frac{(1+\delta+\delta^2)y+\delta(a-1)m_0+\delta^2(a-1)}{1-\delta^3 a}$.

Using the definition of x_t, it is easy to verify that $x_0 < x_2$ if and only if $m_0 < q_2$. Using the fact that $q_2 = \frac{(1+\delta+\delta^2)y+\delta(a-1)m_0+\delta^2(a-1)}{1-\delta^3 a}$, it follows that $x_0 < x_2$ if and only if

$$m_0 < \frac{(1+\delta+\delta^2)y + \delta^2(a-1)}{1-\delta^3 a - \delta(a-1)} \tag{16}$$

To prove that $x_0 < x_2$, it suffices to show that for each y that satisfies Assumption (2), inequality (15) implies inequality (16), or

$$\frac{(1+\delta+\delta^2)y + \delta^2(a-1)}{1-\delta^3 a - \delta(a-1)} > \frac{1-\delta^3 a - \delta^2(a-1) - (1+\delta+\delta^2)y}{\delta(a-1)}$$

Rearranging terms, we have

$$(1-\delta^3 a)(1+\delta+\delta^2)(1-\delta a - y) < 0$$

Under Assumption (2), we have $1 > \delta^3 a$. Therefore, it suffices to show that $1-\delta a - y < 0$ when y takes the lowest value that is compatible with Assumption (2), i.e., $y = \frac{1-\delta^3 a^2}{1+\delta(1+\delta)a}$. When $y = \frac{1-\delta^3 a^2}{1+\delta(1+\delta)a}$, $1-\delta a - y < 0$ if and only if

$$-\frac{\delta^2 a(a-1)}{1+\delta(1+\delta)a} < 0$$

Because $a > 1$, the above inequality always holds. Combining all these, we conclude that $x_1 > x_2 > x_0$. ■

B Appendix: Trade of the Durable Good

In this section, we amend our environment in which the durable good cannot be traded. In particular, we now amend the setting so that each period is divided into four subperiods. In the first two subperiods, as before in Section 2, individuals engage in pairwise meetings in which they lend and borrow the production good using the durable good. In the third subperiod, a market for the durable good opens up and each participant can buy the durable good at a price of h_t. In the fourth subperiod, just like in Subperiod 3 in Section 2, all individuals separate and obtain payoffs from the durable good they might hold.

In the durable good market in the third subperiod, individuals can trade the durable good in exchange for a consumption good produced by labor. Everyone has a production technology to produce a unit of the consumption good from each unit of labor. Each individual derives utility of h from consuming h units of the consumption good and disutility of h from h units of labor supply. Recall that utility of y from holding the durable good is obtained at the end of each period (i.e., after the durable good market closes).

In the repo strategy, we now assume that in the first subperiod, a lender requires only one unit of the durable good as collateral. If a borrower loses (resp. a lender gets) the durable good in the second subperiod, he buys (resp. she sells) one unit in the durable good market, by supplying (resp. consuming) h_t units of labor.

In the following proposition, we show that the results of Proposition 2 survive even in this setup. In particular, when the durable good price is $h_t = q_t$ for each $t \in \{0, 1, 2\}$ each individual finds it optimal to follow the repo strategy that we defined in the beginning of Section 4.

Proposition 3. *Under the durable good price $h_t = q_t$ for $t \in \{0, 1, 2\}$, the repo strategy of Section 4 constitutes an equilibrium for the same $m_0 \in (0, 1)$ and $q_2 \in (0, 1)$ as in Proposition 2.*

Proof. To prove the statement, we need to show that each individual has no profitable deviation and that the durable good market clears. First, we show that if the durable good price in period t is q_t, a borrower who does not have the durable good finds it optimal to buy one in the market. Moreover, a lender who has the durable good finds it optimal to sell it.

Consider a borrower with no durable good. If he does not buy the durable good in the market, he receives a payoff of zero forever. In contrast, if he buys one in period t, he receives a payoff of y from the durable good today and a continuation value from future trades from tomorrow onward. Therefore, she receives, in total, $(1 - \delta)y + \delta U_{t+1}$ from purchasing a unit of the durable good. The cost from it is $(1 - \delta)h_t = (1 - \delta)q_t$. From the proof of Lemma 1, we know that when $q_0 = q_1 = 1$ and $q_2 = \frac{(1+\delta+\delta^2)y+\delta(a-1)m_0+\delta^2(a-1)}{1-\delta^3 a}$, we have

$$(1-\delta)y + \delta U_1 > (1-\delta)q_0$$
$$(1-\delta)y + \delta U_2 \geq (1-\delta)q_1$$
$$(1-\delta)y + \delta U_0 = (1-\delta)q_2$$

Therefore, in any period t, a borrower who does not have the durable good chooses to buy one in the market.

Consider a lender with the durable good. There are several options available to the lender: (i) she can keep the durable good forever; (ii) she can sell it today; (iii) she can keep it today, receive a payoff of y and sell it tomorrow; or (iv) she can keep it for two periods and sell it in period $t+2$ (or in general, she can keep it for N periods and sell it in period $t+N$). Option (i) gives a payoff of y, option (ii) yields a payoff of $(1-\delta)q_t$; option (iii) yields a payoff of $(1-\delta)(y+\delta q_{t+1})$, and option ($iv$) yields a payoff of $(1-\delta)(y+\delta y+\delta^2 q_{t+2})$.

We can show that under the Assumption, a lender with the durable good finds it optimal to sell it within the period in which he receives the durable good for any reason. To check this, first, suppose that $t=0$ or $t=1$. Recall that $q_0 = q_1 = 1 > q_2$. Using the fact that $y < \frac{1-\delta^3-\delta^2(1+\delta)(a-1)-\delta(a-1)m_0}{1+\delta+\delta^2} < 1-\delta < 1$ (which stems from Lemma 1), it can be easily checked that, when $t=0$ or $t=1$, option (ii) strictly dominates the other options. Thus, a lender who receives the durable good in period 0 or in period 1 chooses to sell it immediately within the period.

Now suppose that $t=2$. The payoff from option (i) is less than that from option (iii) because $y = \delta y + (1-\delta)y < \delta(1-\delta) + (1-\delta)y = (1-\delta)(y+\delta)$. The payoff from option (iv) is again less than that from option (iii) because $(1-\delta)(y+\delta(y+\delta)) < (1-\delta)(y+\delta)$. Finally, the payoff from option (ii) is greater than the one from option (iii), and thus, option (ii) is the best if and only if $q_2 \geq y+\delta$. Because $q_2 = \frac{(1+\delta+\delta^2)y+\delta(a-1)m_0+\delta^2(a-1)}{1-\delta^3 a}$, it holds if and only if $y \geq \frac{1+\delta-\delta(1+\delta^2)a-(a-1)m_0}{1+\delta+\delta^2}$. If borrowers have no incentives to deviate in the first and second subperiods, then it must follow that $y \in [\underline{y}_c, \bar{y}_c)$ as we have shown in Lemma 1. Thus, we need to show that

$$\underline{y}_c := \frac{1-\delta^3 a^2 - \delta^2 a(a-1)m_0}{1+\delta(1+\delta)a} \geq \frac{1+\delta-\delta(1+\delta^2)a-(a-1)m_0}{1+\delta+\delta^2}$$
$$\Leftrightarrow -\delta^2 a(a-1+\delta^3 a) - (1-\delta^3 a)(1+\delta a)(a-1)m_0 \leq 0$$

Because Assumption (1) implies that $1 > \delta^3 a$, the last inequality always holds. Therefore, a lender who receives the durable good in period 2 chooses to sell it within the period. Moreover, from Proposition 2, we know that if Assumption (2) holds, then $(1-\delta)q_2 > y$, that is, lenders have no incentives to deviate in the first and second subperiod.

Now, we know that a borrower who does not have the durable good chooses to buy one immediately and a lender who has the durable good chooses to sell it immediately. Using this, we will show that there is no profitable deviation from the repo strategy. First, suppose that a borrower deviates in the second subperiod and does not give the consumption good to the lender. Then, his payoff today increases by $r_t(= q_t)$ from consuming all the consumption good produced. However, such a deviation leads to a loss of the durable good. From above, we know that the borrower chooses to purchase another unit of the durable good in the market immediately within the period. It costs the borrower $h_t = q_t$ to purchase the durable good in the market. Moreover, his future payoffs are unaffected because the borrower starts with the durable good next period. Because the net gain from such a deviation is zero, the borrower finds it optimal to follow the repo strategy.

If a lender deviates in the second subperiod and escapes with the durable good, she loses $r_t(= q_t)$ units of the consumption good. From above, we know that the lender sells the durable good immediately within the period. That yields a benefit of $h_t = q_t$ today and does not affect her future payoffs. Hence, the net gain from the deviation is zero. Therefore, the lender finds it optimal to follow the repo strategy. Therefore, the repo strategy (q_0, q_1, q_2) derived in Proposition 2 constitute an equilibrium under the prices $h_t = q_t$ for $t \in \{0, 1, 2\}$. ■

Under the price of $h_t = q_t$ all individuals are indifferent between participating in the durable good market and refraining from it. Because individuals are indifferent, there are multiple – indeed, a continuum of – equilibria in which some fraction of individuals participate in the durable good market after deviating from the repo strategy, and the rest, as in Sections 3 and 4, follow the repo strategy. In an equilibrium in which a positive measure of individuals participate in the durable good market, borrowers repurchase the durable good from lenders who are different from the ones they traded in pairwise meetings of the first and second subperiods.

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