# Redistribution and the Asymmetric Transmission of Aggregate Shocks

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**Abstract**

I study analytically how the redistributive effects of aggregate shocks affect their transmission to macroeconomic variables in a New Keynesian model featuring household heterogeneity and partial insurance against idiosyncratic and aggregate shocks caused by borrowing constraints. I show that output responds more to contractionary aggregate shocks than to expansionary aggregate shocks of equal magnitude. This asymmetry implies that smaller shocks are capable of activating the zero lower bound on nominal interest rates, and that monetary policy should be conducted asymmetrically, with stronger expansionary policy, in order to combat the asymmetric transmission of shocks to output. Quantitatively, the mechanism aligns well with historical evidence on the asymmetric responses of output to monetary policy shocks, and indicates that policy makers should act more aggressively than conventional wisdom might suggest in order to successfully combat the next recession.

# 1 Introduction

When households are heterogeneous and financial markets are incomplete, aggregate shocks affect the distribution of output in addition to its level. In this paper, I study analytically how these redistributive effects feedback into the transmission of aggregate shocks to the level of output. Understanding this feedback effect is vital for policy makers to be able to successfully stabilize the economy in response to business cycle shocks.

The vehicle of my analysis is a heterogeneous agent New Keynesian (HANK) model. In my economy, households have ex-ante heterogeneous labor productivities and profit shares, and are also subject to idiosyncratic labor productivity shocks. I extend the island structure of Heathcote et al. (2014) in order to model financial markets that permit households to achieve partial consumption insurance against idiosyncratic shocks, as in the data (Blundell et al., 2008). On the margin, I assume that consumption insurance is limited by borrowing constraints, reflecting micro foundations such as imperfect commitment (Alvarez and Jermann, 2000). The supply-side of the model is standard: monopolistically competitive firms face price adjustment frictions, and nominal interest rates are set according to a Taylor rule.

I show in closed form that output responds more to contractionary aggregate shocks than to expansionary aggregate shocks of equal magnitude. Therefore, output responses are asymmetric. I show that this mechanism holds for three shocks commonly studied in the New Keynesian literature: monetary policy shocks, cost-push shocks, and TFP shocks.

When output responds to an aggregate shock, the change in aggregate income creates redistribution of income among the heterogeneous households, who then trade in financial markets to insulate their consumption from the shock. However, the binding borrowing constraint prevents some households from increasing their consumption on the margin, and hence reduces aggregate demand. This reduction then amplifies the transmission of contractionary aggregate shocks, but dampens the transmission of expansionary shocks, thus creating asymmetric output responses in equilibrium.

This result has two implications for the design of monetary policy. First, the size of shock at which the zero lower bound on nominal interest rates binds is smaller than in the economy without borrowing constraints. Intuitively, the dampening of expansionary monetary policy implies that a larger nominal interest rate cut is required to overcome a given contractionary shock (increases) to the real interest rate.

Second, policy makers should respond more to falls in output in order to achieve symmetric output responses. I show that an adjusted Taylor type rule that allows for asymmetric responses of interest rates to changes in output attains symmetric output responses.

I then turn to the responses of inflation. In the case of monetary policy shocks, inflation inherits the asymmetry of output, and responds more to contractionary shocks than to expansionary shocks.

This follows from the logic of the New Keynesian Phillips Curve (NKPC) that holds in my economy: inflation is higher when output is higher. Since output responds more to contractionary shocks, so does inflation.

In the cases of cost-push shocks and TFP shocks, inflation exhibits the opposite asymmetry, and responds more to expansionary shocks. This reversal occurs because of two opposing effects that these shocks have on inflation. First, each shock directly affects inflation, where expansionary shocks cause deflation due to lower marginal costs of production. Second, each shock indirectly affects inflation via its effect on output. Crucially, the second channel offsets the first. For example, an expansionary shock directly creates deflationary pressure, which increases aggregate demand, which in turn creates indirect upward pressure on prices. Since the second channel is stronger for contractionary shocks, the overall response of inflation must be larger for expansionary shocks, where the offsetting effect is weaker.

I finish the paper by providing an initial quantitative assessment of the economic mechanism I have analyzed. To do so, I exploit the simple numerical implementation that my model affords. In particular, I show that the range of exposures of household consumption to changes in aggregate consumption is a sufficient statistic for the asymmetry of output responses. This means that I do not need to calibrate other model features such as income processes and the wealth distribution, which greatly simplifies the numerical work.

For concreteness, I focus on the case of monetary policy shocks. Using micro data on household consumption from the Consumer Expenditure Survey (CEX), I regress log household consumption growth on log aggregate consumption growth, where I instrument for aggregate consumption growth using identified monetary policy shocks from Coibon et al. (2017) to avoid contaminating the regression with variation driven by other aggregate shocks. As a simple measure of ex-ante heterogeneity, I group households by the education of the household head, and estimate a different exposure coefficient for each group. I also show that such a grouping will underestimate the true coefficient heterogeneity in the population of households.

I estimate a ratio of the largest coefficient to the smallest of three. Through the lens of my model, this ratio implies that output responds three times more to contractionary monetary policy shocks, than to expansionary monetary policy shocks of equal magnitude. In order to contextualize this finding, I compare it to the direct evidence for asymmetric transmission of monetary policy shocks, using the local projection method of Jorda (2005). I find that contractionary monetary policy shocks are five times more powerful than expansionary shocks. Therefore, the model is capable of accounting for around 60% of the asymmetry.

At the core of my theoretical results is the interaction between redistribution created by aggregate shocks, and incomplete consumption insurance mediated by borrowing constraints. I emphasize that my results do not presume that households live in financial autarky in equilibrium. In contrast, the island structure I adopt allows households to achieve partial insurance against the idiosyncratic shocks and to partially undo the redistribution caused by aggregate shocks. Borrowing constraints merely limit financial trade on the margin, after positive amounts of insurance have been attained.

The generality of my mechanism suggests that it is applicable to any aggregate shock, and implies that recessionary episodes are more severe than expansionary periods of output growth. This accords well with the recent evidence on the negative skewness of output growth, and its tight relationship with financial conditions, as emphasized by Adrian et al. (2019).

**Related Literature** This paper contributes to three strands of the literature.

First, my paper contributes an analytical approach to the study of HANK economies. As noted by the literature, the advantage of closed form analysis is that it helps to shed light on the precise mechanisms at work in HANK economies that are usually prohibitively complex models that must be solved using numerical methods.

The closest existing work to my paper is Bilbiie (2018). Bilbiie analyzes the transmission of monetary policy under an extreme form of ex-ante household heterogeneity, in which one group of households have access to complete insurance markets, while the other group is completely excluded and is forced to live "hand to mouth" (HtM), consuming their whole income in each period. Within this setting, Bilbiie shows that interest rate policy is more powerful when the income share of the HtM households is pro-cyclical. Intuitively, when aggregate income increases in response to an interest rate cut, the HtM households receive disproportionately more of this increase. Since HtM households consume the entire income gain, the indirect general equilibrium feedback from aggregate demand into output is amplified, resulting in a larger overall response of output.

My analysis relaxes two key features of Bilbiie's model. First, I allow all households to attain some level of consumption insurance, thus bringing my model more in line with the data on household consumption smoothing (Blundell et al., 2008; Heathcote et al., 2014). Second, I do not specify exogenously which households exhibit HtM behavior, not the cyclicality of their incomes, but rather let the general equilibrium of the model determine both features of the economy. Relaxing this assumption proves vital for my main result that output responds more to contractionary aggregate shocks. Intuitively, using borrowing constraints to create HtM households ensures that constrained households receive the least of an increase in aggregate income, but are the most exposed to a decline in aggregate income.

In addition to capturing relevant financial frictions such as imperfect commitment, an advantage of my modeling approach is that it is consistent with the empirical evidence for asymmetric MPCs. Recent work by Fuster et al. (2018) and Christelis et al. (2017) find that, across the income distribution, households display high MPCs out of income losses, but low MPCs out of income gains. Binding borrowing constraints are a simple way to rationalize these findings: after an income gain, a household can save and move away from the constraint, and thus exhibits a low MPC. After an income loss, however, the household may become borrowing constrained, and hence be forced to lower her consumption a lot, thus exhibiting high MPC behavior.

In another tractable model, Acharya and Dogra (2018) present an economy in which all households achieve some consumption insurance in equilibrium. However, to retain tractability, they endow households with CARA preferences, rather than the CRRA specification most commonly used in quantitative macro models. In addition, the focus solely on the role of idiosyncratic risk, and abstract entirely from the possibility that households are ex-ante heterogeneous. My analysis complements theirs by focusing on the effects of this important dimension of household heterogeneity, and by developing a tractable economic environment that features CRRA preferences, and a model of consumption insurance that directly builds on the empirical literature (Heathcote et al., 2014).

Second, my analysis offers clean insights on the effects of ex-ante heterogeneity for aggregate shock transmission that should prove useful when calibrating more complex numerical models. For example, the current frontier of these models (Kaplan et al., 2018; McKay et al., 2015) only model household heterogeneity due to idiosyncratic shocks, and abstract from ex-ante heterogeneity and the associated redistributive properties of aggregate shocks. My theoretical and empirical results offer insights into the advantages of including such heterogeneity, and how to use data to discipline the relevant model parameters. In this sense, my results complement those of Patterson (2018), who shows empirically that households with high MPCs are also the most exposed to recessionary declines in aggregate income, and that this correlation amplifies contractionary aggregate shocks. My analysis provide a fully structural foundation for this correlation via borrowing constraints and redistribution, and analyzes both expansionary and contractionary shocks.

In order to quantitatively assess my mechanism, I build on the growing body of empirical work that emphasizes the importance of ex-ante heterogeneity in explaining the incidence of business cycle shocks across households. My model economy can plausibly include the findings by Guvenen et al. (2014, 2016, 2017) that business cycle fluctuations in aggregate income fall mostly on the tails of the income distribution, and uses similar techniques to Parker and Vissing-Jorgensen (2009) and DeGiorgi and Gambetti (2017) to measure the heterogeneous exposures of household consumption to business cycle fluctuations in aggregate consumption.

Finally, my paper relates to the literature on occasionally binding borrowing constraints. In that literature, a representative household faces financial frictions due to borrowing constraints in international financial markets, or collateral constraints in capital or housing markets (for example, Guerrieri and Iacovielli, 2017). Such constraints cause contractionary aggregate shocks to transmit more powerfully than expansionary shocks, as I find here. Relative to this literature, my paper shows how such an asymmetry at the macro level naturally follows from household heterogeneity and market incompleteness at the micro level.

**Outline** The paper proceeds as follows: section 2 describes the economic environment. In section 3, I show how to simplify the equilibrium conditions to a set of four equations, and discuss my key modeling assumptions. Section 4 establishes my main theoretical result on asymmetric output responses to aggregate shocks. Inflation responses are analyzed in section 5. I estimate key model parameters, and compare the implied output response asymmetry to the direct empirical evidence for asymmetry in section 6. Section 7 concludes.

# 2 Environment

I study a New Keynesian economy augmented to allow for a rich description of household heterogeneity and corresponding consumption insurance opportunities. I first describe the the problems that each type of agent solves and then define an equilibrium. I delay the discussion of my key modeling features until after I have derived my analytical framework in section 3.

## 2.1 Households

There is a unit mass of households indexed by $i \in [0,1]$, who each have preferences over infinite sequences of consumption $\{c_{i,t}\}$ given by

$$\mathbb{E}_1 \sum_{t=1}^{\infty} \left(\frac{1}{1+\rho}\right)^{t-1} \frac{c_{i,t}^{1-\sigma}}{1-\sigma}$$

where $\rho > 0$ is a time discount rate, and the expectation is taken over sequences of shocks described below.

Each household receives a unit endowment of labor, that they supply inelastically in every period. When a household supplies her unit of labor, she supplies $\theta_{i,t}$ units of effective labor, where $\theta_{i,t}$ is her labor productivity in period $t$. Effective labor earns the nominal wage $P_t w_t$ where $P_t$ is the nominal price of the final consumption good, and $w_t$ is the real wage. I normalize the mean value of labor productivity across all households to unity in each period, so that $\int_0^1 \theta_{i,t} di = 1$ for all $t$.

Households are subject to two sources of heterogeneity. First, each household has a fixed level of long run labor productivity $\theta_i > 0$ around which $\theta_{i,t}$ fluctuates, and a dividend share $s_i \geq 0$.[1] Let the vector $\upsilon = (\theta, s)$ summarize this source of household heterogeneity, where $\upsilon$ has distribution $F_\upsilon$, with support $\Upsilon \subseteq \mathbb{R}_+^2$. Second, labor productivity $\theta_{i,t}$ is subject to idiosyncratic shocks as described below.

[1]Fixed dividend shares can be interpreted in at least two ways: first, as bonus payments on labor income, and second, as reflecting prohibitively high costs of trading equity at business-cycle frequencies. I note that the structure of financial markets that I introduce below will implicitly allow for some sharing of dividend income risk.

**Island Structure and Idiosyncratic Shocks** Before any markets open, the unit mass of households is partitioned into islands indexed by $\omega \in \Omega$, where each island contains a continuum of households. Each island is defined along two dimensions. First, by an ex-ante unknown sequence of labor productivity shocks $\eta^{\omega} = \{\eta_t^{\omega}\}_{t=1}^{\infty}$ that will apply to all island members. Second by a set $\Upsilon^{\omega} \subseteq \Upsilon$ that determines the possible values of $\upsilon$ that a household located on island $\omega$can draw, where the probability distribution over $\upsilon \in \Upsilon^{\omega}$ is the suitably defined conditional distribution function $F_{\upsilon}^{\omega}$: $F_{\upsilon}^{\omega}(x) = \Pr(\upsilon \leq x | \upsilon \in \Upsilon^{\omega})$.

Households are subject to two types of idiosyncratic shocks to their labor productivity: island-level shocks and individual-level shocks. Formally, for household $i$ located on island $\omega$, her labor productivity evolves according to the process

$$\log \theta_{i,t} = \log \theta_i + \eta_t^{\omega} + \kappa_{i,t} + \epsilon_{i,t}$$

Hence $\theta_{i,t}$ fluctuates around $\theta_i$, with fluctuations due to island-level shocks $\eta_t^{\omega}$, and individual-level shocks $\kappa_{i,t}$ and $\epsilon_{i,t}$. The island-level shocks follow an AR(1) process

$$\eta_{t+1}^{\omega} = \rho_{\eta} \eta_t^{\omega} + \epsilon_{t+1}^{\eta}$$

where the innovation $\epsilon_{t+1}^{\eta}$ is drawn from a distribution $F_{\eta}$ with variance $\sigma_{\eta}^2$ that is common to all islands, and initial values $\eta_1^{\omega}$ are drawn from a distribution $F_{\eta,1}$ that is also common to all islands. The individual-level shock $\kappa_{i,t}$ follows another AR(1) process

$$\kappa_{i,t+1} = \rho_{\kappa} \kappa_{i,t} + \epsilon_{i,t+1}^{\kappa}$$

where the innovation $\epsilon_{i,t+1}^{\kappa}$ is drawn from a distribution $F_{\kappa}$ that is common to all households, and initial values $\kappa_{i,1}$ are drawn from a distribution $F_{\kappa,1}$ that is also common to all households. Finally, the individual-level shocks $\epsilon_{i,t}$ are i.i.d. draws from a distribution $F_{\epsilon}$ that is common to all households.

The persistent-transitory nature of labor productivity shocks follows a long tradition in the macro-labor literature that estimates statistical models for individual labor income. Furthermore, the empirical evidence suggests that the persistent component of shocks to labor income is essentially permanent in nature, so that it is most relevant to consider the case in which $\rho_{\eta}, \rho_{\kappa} \to 1$ (Storesletten et al., 2004; Guvenen et al., 2016). In particular, I will make use of the condition $\rho_{\eta} \to 1$ in my analytical results.[2]

I assume that a Law of Large Numbers can be applied to ensure that individual-level shocks wash out within each island, and island-level shocks wash out across all islands. Aggregate shocks are

[2]Since I ultimately study the responses of aggregate variables around a stationary equilibrium without aggregate shocks, I cannot set $\rho_{\eta} = 1$ since this precludes the existence of such a stationary equilibrium.

introduced below.

**Financial Markets** All assets in the economy are in zero net supply, and each household begins life with zero positions in all assets. Following Heathcote et al. (2014), the structure of financial markets allows for differential insurance opportunities against individual and island-level shocks.

Within an island, households can trade a complete set of Arrow-Debreu securities. Formally, let $\zeta_{t+1}^{\omega} = \left(\upsilon, \kappa_{t+1}, \epsilon_{t+1}, \eta_{t+1}^{\omega}, Z_{t+1}\right)$ index the realizations of shocks in period $t+1$ on island $\omega$, where $\upsilon \in \Upsilon^{\omega}$, and $Z_t$ is a vector of aggregate shocks that will be defined shortly. Note that since $\zeta_{t+1}^{\omega}$ contains individual-level shocks $\kappa_{t+1}$ and $\epsilon_{t+1}$, realizations of $\zeta_{t+1}^{\omega}$ will differ across households on island $\omega$ due to variation in $\kappa_{t+1}$ and $\epsilon_{t+1}$. Given this, let $B_{i,t}\left(\zeta_{t+1}^{\omega}\right)$ denote the quantity of claims to one unit of consumption if $\zeta_{t+1}^{\omega}$ realizes in period $t+1$ purchased in period $t$ by household $i$ on island $\omega$ with current shock $\zeta_{i,t}^{\omega}$. Let $q_t^{\omega}\left(\zeta_{t+1}^{\omega}; \zeta_{i,t}^{\omega}\right)$ denote the price of this claim.

Between islands, financial trade is limited to only a nominal risk-free bond, which is also subject to a no-borrowing constraint. Let $b_{i,t}$ denote the quantity of nominal risk-free bonds purchased by household $i$ in period $t$ where a choice $x_{i,t}$ is understood to be contingent on the realization $\zeta_{i,t}^{\omega}$ for household $i$ on island $\omega$ in period $t$. Each bond earns the gross real interest rate $1 + r_{t+1}$ given by

$$1 + r_t = \frac{1 + \iota_{t-1}}{1 + \pi_t}$$

where $\iota_t$ is the nominal interest rate, and $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ is inflation.

**Household Problem** Household $i$ on island $\omega$ solves

$$\max_{\{c,b,B\}} \mathbb{E}_1 \sum_{t=1}^{\infty} \left(\frac{1}{1+\rho}\right)^{t-1} \frac{c_{i,t}^{1-\sigma}}{1-\sigma}$$

subject to

$$c_{i,t} + b_{i,t} + \int q_t^{\omega}\left(\zeta_{t+1}^{\omega}; \zeta_{i,t}^{\omega}\right) B_{i,t}\left(\zeta_{t+1}^{\omega}\right) d\zeta_{t+1}^{\omega} = w_t \theta_{i,t} + s_i d_t + (1 + r_t)\, b_{i,t-1} + B_{i,t-1}\left(\zeta_{i,t}^{\omega}\right)$$

$$b_{i,t} \geq 0$$

$$\int q_0^{\omega}\left(\zeta_1^{\omega}\right) B_{i,0}\left(\zeta_1^{\omega}\right) d\zeta_1^{\omega} = 0$$

$$b_{i,0} = 0$$

## 2.2 Final Good Firms

A representative competitive final good firm packages the unit mass of intermediate goods indexed by $j \in [0,1]$, using the CES production function

$$Y_t = \left( \int_0^1 y_t(j)^{\frac{\Phi_t - 1}{\Phi_t}} \, dj \right)^{\frac{\Phi t}{\Phi t - 1}}$$

where $\Phi_t > 1$ is the elasticity of substitution across intermediate inputs, and is subject to aggregate shocks. Taking the price of each input and the price of the final good as given, the firm solves

$$\max_{\{y(j)\}} P_t \left( \int_0^1 y_t(j)^{\frac{\Phi_t - 1}{\Phi_t}} \, dj \right)^{\frac{\Phi t}{\Phi_t - 1}} - \int_0^1 p_t(j) \, y_t(j) \, dj$$

Optimization yields a demand function for each intermediate good

$$y_t(j) = \left( \frac{p_t(j)}{P_t} \right)^{-\Phi_t} Y_t$$

and a nominal price index

$$P_t = \left( \int_0^1 p_t(j)^{1-\Phi_t} \, dj \right)^{\frac{1}{1-\Phi_t}}$$

## 2.3 Intermediate Goods Firms

Each intermediate good $j$ is produced by a monopolistically competitive firm employing effective labor $E_t(j)$ and an intermediate input $M_t(j)$ in the production function

$$x_t(j) = A_t M_t(j)^{\alpha} E_t(j)^{1-\alpha}$$

where $A_t$ is aggregate TFP and is subject to aggregate shocks. The intermediate input is itself an aggregate of all intermediate goods,

$$M_t(j) = \left( \int_0^1 m_t(k,j)^{\frac{\Phi_t - 1}{\Phi_t}} \, dk \right)^{\frac{\Phi t}{\Phi t - 1}}$$

so that demand for each intermediate input by firm $j$ satisfies

$$m_t(k,j) = \left( \frac{p_t(k)}{P_t} \right)^{-\Phi_t} M_t(j)$$

Hence, total demand for intermediate $k$ is

$$x_t(k) = \left(\frac{p_t(k)}{P_t}\right)^{-\Phi_t} X_t$$

where

$$X_t = Y_t + \int_0^1 M_t(j)\, dj$$

In each period, firm $j$ solves the cost minimization problem

$$\min_{M_t(j), E_t(j)} P_t M_t(j) + W_t E_t(j)$$

subject to

$$A_t M_t(j)^{\alpha} E_t(j)^{1-\alpha} \geq x_t(j)$$

The FOCs yield the marginal cost of production

$$mc_t = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} w_t^{1-\alpha} \frac{1}{A_t}$$

Each firm faces its own demand curve, and chooses its path of prices to maximize profits subject to quadratic price adjustment costs (Rotemberg, 1982):

$$\max_{p(j)} \mathbb{E}_1 \left[\sum_{t=1}^{\infty} \delta_t \left(p_t(j) x_t(j) - P_t \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} w_t^{1-\alpha} \frac{x_t(j)}{A_t} - \frac{\xi^p}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 P_t X_t\right)\right]$$

subject to

$$x_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\Phi_t} X_t$$

$$p_0(j) = P_0$$

where, for simplicity, I assume that firms discount future profits using a stochastic discount factor driven by aggregate consumption, $\delta_t = \left(\frac{1}{1+\rho}\right)^{t-1}\left(\frac{C_t}{C_1}\right)^{-\sigma}$.[3] I focus on the symmetric equilibrium in which $p_t(j) = P_t$, $x_t(j) = X_t$, $y_t(j) = Y_t$, $M_t(j) = M_t$, and $E_t(j) = E_t$ for all $j \in [0, 1]$. In this case, the aggregate dividend in period $t$ is given by

$$d_t = X_t \left(1 - \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{w_t^{1-\alpha}}{A_t} - \frac{\xi^p}{2}\pi_t^2\right)$$

[3] This choice ensures that my model nests the representative household economy as a special case, as in Bhandari et al. (2018). Furthermore, the choice of stochastic discount factor is innocuous for my theoretical results since I approximate around an equilibrium in which the true stochastic discount factor is constant to first order.

## 2.4 Monetary Policy

A monetary authority sets the nominal interest rate on the inter-island bond according to the Taylor rule

$$1+\iota_t=\left(1+r^T\right)(1+\pi_t)^{\phi_\pi}\left(\frac{Y_t}{Y^T}\right)^{\phi_y}e^{v_t}$$

where $\phi_\pi > 1$, $\phi_y \geq 0$, and $v_t$ is a monetary policy shock.

In this specification, $r^T$ and $Y^T$ are fixed target levels of the real interest rate, and output.

## 2.5 Market Clearing

In each period, the labor, final good, and asset markets must clear,

$$E_t = 1$$

$$\int_0^1 c_{i,t}di = Y_t\left(1-\frac{\xi^p}{2}\pi_t^2\right)$$

$$\int_0^1 b_{i,t}di = 0$$

$$\int_{I^\omega} B_{i,t}\left(\zeta_{t+1}^\omega\right)di = 0\ \forall \zeta_{t+1}^\omega, \omega$$

where $I^\omega = \{i : i \in \omega\}$ is the set of households located on island $\omega$.

## 2.6 Aggregate Shocks

Aggregate shocks affect aggregate TFP, the elasticity of substitution among intermediate inputs (the source of so called "cost-push" shocks), and the innovations to monetary policy, all of which evolve as AR(1) processes,

$$\log A_{t+1} = \rho_a \log A_t + (1-\rho_a)\log \bar{A} + \epsilon_{t+1}^a$$

$$\log \Phi_{t+1} = \rho_\Phi \log \Phi_t + (1-\rho_\Phi)\log \bar{\Phi} + \epsilon_{t+1}^\Phi$$

$$v_{t+1} = \rho_v v_t + \epsilon_{t+1}^v$$

where $\rho_a, \rho_\Phi, \rho_v \in (0,1)$, and $\left\{\epsilon_t^a, \epsilon_t^\Phi, \epsilon_t^v\right\}_t$ are i.i.d. random variables, each with mean zero and variance $\Sigma^2$. Due to their effects on inflation, and following the New Keynesian convention, I refer to $\epsilon_t^\Phi < 0$ as a positive cost-push shock and $\epsilon_t^\Phi > 0$ as a negative cost-push shock.[4] Given this, we can set $Z_t = (A_t, \Phi_t, v_t)$ as the vector of aggregate shocks.

[4] Intuitively, $\epsilon_t^\Phi > 0$ increases the elasticity of demand faced by each monopolist producer, and hence causes firms to lower their prices.

## 2.7 Equilibrium

**Definition 1.** Given initial conditions $\{F_{\eta,1}, F_{\kappa,1}, P_0\}$, an equilibrium is a sequence of allocations $\left\{\{c_{i,t}, b_{i,t}, B_{i,t}(\cdot)\}_i, Y_t, X_t, M_t, E_t, d_t\right\}_t$ and prices $\{\iota_t, P_t, w_t, \{q_t^{\omega}(\cdot)\}_{\omega}\}_t$ such that

1. $\{c_{i,t}, b_{i,t}, B_{i,t}(\cdot)\}_{i,t}$ solve the household problem taking prices as given.
2. $\{Y_t\}_t$ solve the final good firms' problem taking prices as given.
3. $\{X_t, M_t, E_t, P_t\}_t$ solve the intermediate goods firms' problem.
4. $\{d_t\}_t$ satisfies the dividend equation.
5. $\{\iota_t\}_t$ satisfies the Taylor rule.
6. Markets clear at every time $t \geq 1$.

# 3 A Four Equation New Keynesian Model

In this section, I condense the equilibrium to a set of four equations that fully characterize the responses of aggregate output, inflation, and household consumption to aggregate (and idiosyncratic) shocks. This system is the natural extension of the much-studied three equation New Keynesian model (Gali, 2015), extended to allow for a rich description of household heterogeneity and incomplete financial markets, and allows me to derive closed form expressions for the responses of output and inflation to aggregate shocks.

## 3.1 Stationary Equilibrium

I will derive equations that express the dynamics of output, inflation, and household consumption in terms of deviations around the stationary equilibrium in which there are no aggregate shocks and prices are flexible ($\xi^p = 0$). In this stationary equilibrium, aggregate prices and quantities are constant, but household consumption paths and asset positions are still subject to idiosyncratic shocks.

**Definition 2.** Assume that there are no aggregate shocks and that $\xi^p = 0$. Given an initial price level $P_0$, a stationary equilibrium is a set of allocations $\left\{\{c_{i,t}, b_{i,t}, B_{i,t}(\cdot)\}_{i,t}, Y, X, M, E, d\right\}$ and prices $\left\{r, P_0, w, \{q_t^\omega(\cdot)\}_{\omega,t}\right\}$ such that

1. $\{c_{i,t}, b_{i,t}, B_{i,t}(\cdot)\}_{i,t}$ solve the household problem.
2. $Y$ solves the final good firms' problem.
3. $\{X, M, E, P_0\}$ solve the intermediate goods firms' problem.
4. $d$ satisfies the dividend equation.
5. Markets clear at every time $t \geq 1$.

Since prices are flexible in the stationary equilibrium, monetary policy is neutral. Given this, and to ensure that inflation is zero in the stationary equilibrium (as required by the definition), I set the interest rate and output targets in the Taylor rule to their values attained in the stationary equilibrium: $r^T = r$, and $Y^T = Y$.

The tractability of my framework allows me to explicitly characterize stationary equilibrium, thus establishing its existence and uniqueness. The proofs of this and all other results are contained in the appendix.

**Lemma 1.** *The stationary equilibrium exists, and is unique.*

## 3.2 Characterizing First Order Dynamics

In order to study the responses of endogenous aggregate variables to aggregate shocks, I follow the New Keynesian literature and use first order approximations to express the dynamics of a variable around its value in the stationary equilibrium (Gali, 2015). Therefore, I assume that aggregate shocks are sufficiently small such that this approximation is valid.

Similarly, for tractability at the household level, I assume that the uninsurable idiosyncratic shocks to household consumption captured by $\eta_t^\omega$ are sufficiently small such that a first order approximation around a household's consumption level when $\eta_t^\omega = 0$ is valid.[5]

Furthermore, a robust feature of the empirical literature on idiosyncratic shocks to household income and consumption is that the persistent component of idiosyncratic shocks (both insurable and uninsurable) is essentially permanent in nature. To capture this for uninsurable shocks, I take

[5] Comparing the relevant empirical evidence (e.g. Heathcote et al. (2014) for $\sigma_\eta^2$, and Smets and Wouters (2007) for $\Sigma^2$) suggests that $\sigma_\eta$ is an order of magnitude smaller than $\Sigma$, so that if first order approximations are valid in the aggregate time series, then they are valid in individual time series.

$\rho_\eta \to 1$, which implies that $(1-\rho_\eta)\,\eta_t^\omega$ is an order of magnitude smaller than $\eta_t^\omega$ itself. Since the $\kappa_{i,t}$ shocks are fully insured against, I leave $\rho_\kappa$ unrestricted.

These conditions are summarized in the following assumption.

**Assumption 1.** *Let $\Sigma^2 \to 0$, $\sigma_\eta^2 \to 0$, and $\rho_\eta \to 1$.*

The following proposition contains a set of equations that are necessary and sufficient to characterize the equilibrium dynamics of output and inflation in response to aggregate shocks, expressed in log deviations around the stationary equilibrium in which inflation is zero. The proposition also establishes the dynamics of household consumption, where I express consumption at the island level since the complete markets structure ensures that all households located on a given island achieve the same consumption path in equilibrium. I use the notation $\hat{y}_t = \log Y_t - \log Y$ to denote log deviations of aggregate output from its stationary equilibrium level, and $\hat{c}_{\omega,t} = \log c_{\omega,t} - \log c_\omega$ to denote log deviations of household consumption on island $\omega$ from its level in the stationary equilibrium when $\eta_t^\omega = 0$. Similarly, I define $\hat{\Phi}_t = \log \Phi_t - \log \bar{\Phi}$ and $\hat{a}_t = \log A_t - \log \bar{A}$.

**Proposition 1.** *Under assumption 1, the economy's first order equilibrium dynamics in response to aggregate and idiosyncratic shocks satisfy the system*

$$\iota_t = r + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t$$

$$\pi_t = \varphi_y \hat{y}_t - \varphi_a \hat{a}_t - \varphi_\Phi \hat{\Phi}_t + \frac{1}{1+\rho}\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\hat{c}_{\omega,t} = \beta_\omega^y \hat{y}_t + \beta_\omega^a \hat{a}_t + \beta_\omega^\eta \eta_t^\omega \;\forall \omega$$

$$\min_\omega \left\{\mathbb{E}_t\left[\hat{c}_{\omega,t+1}\right] - \hat{c}_{\omega,t}\right\} = \frac{1}{\sigma}\left(\iota_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \rho\right)$$

*where $\{\beta_\omega^y, \beta_\omega^a, \beta_\omega^\eta\}_\Omega$ and $\varphi_y, \varphi_a, \varphi_\Phi > 0$ depend only on model primitives.*

*$\phi_\pi > 1$ and $\phi_y \geq 0$ are sufficient conditions to ensure that the system has a unique steady state with $\hat{y}_t = 0$ and $\pi_t = 0$.*

The first two equations of proposition 1 are standard features of New Keynesian models. The first equation is the log-linear Taylor rule for the nominal interest rate, which is subject to monetary policy shocks $v_t$.

The second equation is the New Keynesian Phillips Curve (NKPC) linking inflation, output, and aggregate shocks to TFP and the elasticity of substitution parameter $\Phi_t$ (the source of so-called cost-push shocks). Intuitively, if output is higher today, firms face higher marginal costs of production ceteris paribus, and so will optimally choose to raise their prices, leading to inflation. Conversely, a positive TFP shock lowers firms' marginal costs, and thus results in lower optimal prices and deflation. Similarly, a positive shock to $\Phi_t$ lowers firms' price-setting power since the intermediate goods become closer substitutes in final good production. As such, firms are forced to lower their prices closer to marginal cost, which results in deflation. Given this logic, I refer to $\hat{\Phi}_t > 0$ as a negative cost-push shock (since prices fall in response), and $\hat{\Phi}_t < 0$ as a positive cost-push shock (since prices rise in response).

The third equation characterizes the dynamics of per-capita consumption on island $\omega$, and indicates the three sources of variation in equilibrium consumption: changes in output $\hat{y}_t$ caused by an aggregate shock, direct effects of aggregate TFP shocks $\hat{a}_t$, and the direct effect of the island-level component of idiosyncratic income shocks $\eta_t^\omega$. Note that the presence of complete markets on each island implies that each of these channels has the same effect on consumption for every household on each island, so that the coefficients $\{\beta_\omega^y, \beta_\omega^a, \beta_\omega^\eta\}$ do not depend on $i$.

I show in the appendix that $\{\beta_\omega^y, \beta_\omega^a, \beta_\omega^\eta\}$ are expressible in terms of primitive parameters as

$$\beta_\omega^y = \frac{\left(\bar{\Phi} - \alpha\left(\bar{\Phi} - 1\right)\right)}{\alpha} \frac{(1-\alpha)\left(\bar{\Phi} - 1\right)\theta^\omega + s^\omega\left(1 - (1-\alpha)\bar{\Phi}\right)}{(1-\alpha)\left(\bar{\Phi} - 1\right)\theta^\omega + s^\omega}$$

$$\beta_\omega^a = \left(\bar{\Phi} - 1\right) \frac{\bar{\Phi}\left(\frac{1-\alpha}{\alpha}\right)\left(s^\omega - \theta^\omega\right)}{(1-\alpha)\left(\bar{\Phi} - 1\right)\theta^\omega + s^\omega}$$

$$\beta_\omega^\eta = \frac{(1-\alpha)\left(\bar{\Phi} - 1\right)\theta^\omega}{(1-\alpha)\left(\bar{\Phi} - 1\right)\theta^\omega + s^\omega}$$

where $\theta^\omega = \frac{\int_{I^\omega} \theta_i di}{\int_{I^\omega} di}$, $s^\omega = \frac{\int_{I^\omega} s_i di}{\int_{I^\omega} di}$ are the island-level mean labor productivity and dividend share respectively.

Heterogeneity in $\{\beta_\omega^y\}$ captures the redistributive nature of changes in aggregate income. It is straightforward to see that an increase in output causes consumption on island $\omega$ to increase when

$$\beta_\omega^y > 0 \iff \theta^\omega > s^\omega \frac{\bar{\Phi} - \frac{1}{1-\alpha}}{\bar{\Phi} - 1}$$

i.e. when mean labor productivity is sufficiently large relative to the mean dividend share. Intuitively, an increase in output causes wages to rise and dividends to fall ceteris paribus, since firms' labor demand increases. This benefits islands with households who depend on wage income more than non-wage income, i.e. islands on which $\theta^\omega$ is sufficiently large relative to $s^\omega$.

The fact that TFP shocks directly affect household consumption reflects the fact that TFP shocks redistribute income, even if total income is unchanged ($\hat{y}_t = 0$). For example, a positive TFP shock lowers firms' marginal costs since they do not require as much labor to produce the same level of output. As such, the wage share of income falls, and the dividend share rises, holding total income fixed. Given this, it is natural that a positive TFP shock directly causes consumption on island $\omega$ to increase when

$$\beta_\omega^a > 0 \iff s^\omega > \theta^\omega$$

i.e. when the mean dividend share is larger than mean labor productivity, so that households on island $\omega$ are, on average, more dependent on non-wage income than wage income.

Finally, even in the absence of aggregate shocks ($\hat{y}_t = \hat{a}_t = 0$), household consumption varies with the uninsurable component of idiosyncratic income shocks $\eta_t^\omega$. Naturally, a positive island-level shock to labor productivity causes consumption to increase since households on island $\omega$ receive more wage income,

$$\beta_\omega^\eta > 0 \iff \theta^\omega > 0$$

Recall that in equilibrium, households located on the same island will achieve full insurance against idiosyncratic income shocks $\kappa_{i,t}$ and $\epsilon_{i,t}$, so that they do not affect household consumption on island $\omega$.

The forth equation is the consumption Euler equation for my economy, and captures the effect that the borrowing constraint has on inter-island trade in equilibrium. Formally, it says that equilibrium adjustments in the real interest rate are tied to the consumption responses of households with the lowest expected consumption growth.

To understand this condition, note that households' optimal bond positions are shaped by two motives. First, households substitute intertemporally in response to changes in the real interest rate: when the real interest rate increases, household will substitute consumption today in favor of consumption tomorrow. The strength of this motive is governed by the elasticity of intertemporal substitution $\frac{1}{\sigma}$, and is the same for all households.

Second, a household will use the bond in order to smooth her consumption, as captured by her expected growth rate, $\mathbb{E}_t\left[\hat{c}_{\omega,t+1}\right] - \hat{c}_{\omega,t}$. For example, if a household expects her income to fall over time, she will use the bond to save today, at any given real interest rate, in order to insulate her consumption path against her temporal income variation. In contrast to the intertemporal substitution motive, the consumption smoothing motive is in general heterogeneous across households on different islands due to uninsurable idiosyncratic shocks, and the redistributive nature of aggregate shocks.

In equilibrium, these two motives must offset so that the aggregate households bond position is zero. However, the presence of borrowing constraints implies that the bond market will clear when unconstrained households, who have the strongest motive to save on the margin, choose zero bond holdings, and all other households are borrowing constrained. In other words, only the Euler equation for unconstrained households needs to hold in equilibrium.

Since the first saving motive is uniform across households, unconstrained households must have the strongest second motive to save, i.e. they must have the lowest expected consumption growth among all households. Therefore, the equilibrium Euler equation contains the lowest expected consumption growth among all households.

### 3.3 Discussion of Key Model Features

Having exploited the tractability of my framework to condense the equilibrium dynamics into a system of four equations, I now discuss the key features of my model in more detail.

**Island Structure** I build on Heathcote et al. (2014), and use an island structure to retain tractability while allowing for a rich description of household heterogeneity and for partial insurance against idiosyncratic and aggregate shocks. At the individual level, the sharp dichotomy between the insurability of $\kappa_{i,t}$ and $\epsilon_{i,t}$ shocks and the uninsurability of $\eta_t^\omega$ shocks captures the fact that households achieve large amount of insurance against transitory income shocks (Blundell et al., 2008), but only partial insurance against persistent income shocks. As shown in Heathcote et al. (2014), the extent of the partial insurance against persistent income shocks is determined by the relative variances of $\kappa_{i,t}$ shocks and $\eta_t^\omega$ shocks: if $\sigma_\eta^2$ is zero, then there is full insurance against persistent shocks, while if $\sigma_\eta^2 > 0$, then there is some component of the persistent shocks that is uninsurable. Hence overall insurance against persistent income shocks is partial. The empirical exercise in Heathcote et al. (2014) shows how consumption and income data can discipline the extent of partial insurance against idiosyncratic shocks that households achieve in reality.

I extend the island structure by allowing households to achieve partial insurance against their fixed characteristics captured by $\upsilon = (\theta, s)$. In my construction, the extent of partial insurance against $\upsilon$ is determined by the range heterogeneity in $\upsilon$ that exists on an island, as captured by the island-specific support $\Upsilon^\omega$. If $\Upsilon^\omega$ is a singleton set, then all households on island $\omega$ have the same $\upsilon$ and so cannot achieve any insurance against their fixed characteristics using within-island financial markets. At the other extreme, if $\Upsilon^\omega = \Upsilon$ for all $\omega$, then all islands contains the full range of heterogeneity in $\upsilon$, and all households can hence achieve full insurance against their fixed characteristics using only within-island assets.

Heterogeneity in $\upsilon$ implies that households are heterogeneously exposed to aggregate shocks, which therefore have redistributive consequences. Hence, partial insurance against $\upsilon$ translates into partial insurance against these redistributive effects. For example, if $\Upsilon^\omega = \Upsilon$ for all $\omega$, then all households on all islands will achieve full insurance against $\upsilon$, and aggregate shocks will not have redistributive consequences in equilibrium.

In principle, similar techniques to those used in Heathcote et al. (2014) together with data on heterogeneity in long-run values for consumption and income and their co-movements with aggregate consumption and income could be used to infer the extent of insurance against $\upsilon$, and hence the pattern of $\Upsilon^\omega$ across islands. I stress that my theoretical results do not depend on any particular assumptions about $\{\Upsilon^\omega\}_\Omega$. Instead, I highlight how different levels of insurance can affect the transmission of aggregate shocks.

**Borrowing Constraints** The fact that I rule out inter-island trade in equilibrium is consistent with the dichotomy between uninsurable and insurable shocks that I exploit to make my model tractable. Crucially, I do not rely on a “no-trade” result to ensure that households do not want to trade the inter-island bond, but rather explicitly prevent households from doing so using borrowing frictions.

This choice reflects two considerations. First, the presence of business cycle shocks, which are only temporary by construction, makes “no-trade” theorems, which rely on the permanent nature of uninsurable shocks (Heathcote et al., 2014), inapplicable in my setting. Second, the presence of binding borrowing constraints generates a tight link between interest rates and the lowest expected household consumption growth that is a standard feature of asset pricing models with solvency constraints (e.g. Alvarez and Jermann, 2000). As shown by that literature, these constraints can be thought of as a mechanism for decentralizing constrained efficient allocations in the presence of participation constraints when there is a risk that households from a particular island will default on their borrowing from another island.

I also note that the no-borrowing constraint is not the same as exogenously imposing autarky between islands. Instead, the limit on borrowing still requires that prices and quantities adjust in equilibrium so that the bond market clears. In particular, the adjustment must be such that households who wish to save in equilibrium optimally choose zero savings. Therefore, this assumption buys tractability without losing the key transmission mechanism from interest rates to savings choices.[6]

**Irrelevance of wealth as a state variable** An important feature of my model is that household consumption does not depend on household wealth independently of output, TFP shocks, and the uninsurable idiosyncratic shocks to labor productivity. This is in sharp contrast to standard incomplete markets models, in which household choices depend on wealth explicitly, and the wealth distribution must be solved for numerically.

The redundancy of wealth as a state variable follows from the island structure and borrowing constraints that restrict inter-island financial trade. First, the presence of complete markets within an island implies that household allocations can be solved without referring to wealth, as the solution to a planning problem as in Heathcote et al. (2014). Second, the tight borrowing constraints prevent households from making inter-island trades, so that the inter-island wealth distribution remains degenerate at zero. As discussed above, this degeneracy is not as restrictive as it first seems, since households' desires to make inter-island trades depend on the heterogeneity in the supports $\Upsilon^\omega$ across islands $\omega \in \Omega$. For example, if $\Upsilon^\omega = \Upsilon$ for all $\omega \in \Omega$, then the incentive to make inter-island trades would be identical across islands, so that all households would optimally choose a zero bond position in equilibrium, and the borrowing constraints would not bind. Therefore, the borrowing

[6] Werning (2015) uses a similar assumption to analyze how the cyclicality of idiosyncratic risk affects the power of forward guidance.

constraint is best interpreted as constraining financial trade on the margin for a given set of supports $\{\Upsilon^{\omega}\}$.

**First Order Approximations** In order to obtain analytical insights, I assume that the uninsurable idiosyncratic shocks are sufficiently small such that first order approximations are valid. This approach rules out second order phenomena, such as the cyclicality of uninsurable idiosyncratic income risk, that may also affect the responses of aggregate variable to aggregate shocks. For example, analysis by Braun and Nakajima (2011), Werning (2015), and Acharya and Dogra (2018) show how countercyclical idiosyncratic income risk is a force for larger responses of output to both contractionary and expansionary monetary policy shocks.

An important insight of these analyses is that the cyclicality of income risk manifests itself as a source of shocks to the discount rate of households. Intuitively, when output increases and income risk is countercyclical, the strength of the precautionary savings motive decreases, so that households behave as if they are more impatient and have a higher discount rate. Therefore, it is possible to extend my analysis to include this risk channel by specifying a reduced-form relationship between the discount rate at the path of output. However, since the risk channel is symmetric in the sign of the aggregate shock, it does not affect the asymmetry that I am ultimately interested in. Hence I abstract from this extension here.

# 4 Output Responses to Aggregate Shocks

In this section, I obtain analytical expressions for the responses of output to aggregate shocks, and show that output responds more to contractionary aggregate shocks than to expansionary aggregate shocks of equal magnitude. For expositional clarity, I begin with monetary policy shocks, and then consider cost-push and TFP shocks. I focus on output responses in this section, and consider the corresponding inflation responses in section 5.

The equilibrium system derived in proposition 1 is inherently non-linear due to the presence of potentially binding borrowing constraints. As such, closed form expressions for impulse responses are unattainable. Therefore, in order to make progress analytically, I study the responses of output to so called "MIT" shocks. Such shocks are one time, zero-probability events, after which the economy transitions deterministically back to the steady state of the system.

In my economy, incomplete risk sharing results in heterogeneous exposures of household consumption to fluctuations in output, as summarized by the heterogeneity in the $\{\beta^{y}_{\omega}\}$ coefficients. It turns out that this heterogeneity has implications for both the existence and uniqueness of responses that I am interested in. Therefore, I now state a condition that ensures that all responses exist and are unique. I delay the interpretation of this condition until after I have studied the responses themselves.

**Assumption 2.** *Define* $\underline{\beta}^y = \min_\omega \{\beta_\omega^y\}$, *and assume that* $\underline{\beta}^y + \frac{1}{\sigma}(\phi_\pi \varphi_y + \phi_y) > 0$.

Unless otherwise stated, I impose assumptions 1 and 2 from now on.

## 4.1 Asymmetric Output Responses to Monetary Policy Shocks

Suppose the economy is initially in the steady state (stationary equilibrium). Consider the response of output to a one time, zero probability, monetary policy shock of the form $v_t = \rho_v^{t-1} v_1$.

**Proposition 2.** *The first order transitional dynamics of output in response to a monetary policy shock are given by*

$$\hat{y}_t = \begin{cases} -\frac{1}{(1-\rho_v)\underline{\beta}^y + \frac{1}{\sigma}\left((\phi_\pi - \rho_v)\frac{1+\rho}{1+\rho-\rho_v}\varphi_y + \phi_y\right)} \frac{1}{\sigma} \rho_v^{t-1} v_1 & \textit{if} \quad v_1 > 0 \\ \\ -\frac{1}{(1-\rho_v)\bar{\beta}^y + \frac{1}{\sigma}\left((\phi_\pi - \rho_v)\frac{1+\rho}{1+\rho-\rho_v}\varphi_y + \phi_y\right)} \frac{1}{\sigma} \rho_v^{t-1} v_1 & \textit{if} \quad v_1 < 0 \end{cases}$$

*where*

$$\bar{\beta}^y = \max_\omega \{\beta_\omega^y\}$$
$$\underline{\beta}^y = \min_\omega \{\beta_\omega^y\}$$

$\bar{\beta}^y > \underline{\beta}^y$ implies that output responds more to a positive (contractionary) monetary policy shock than to a negative (expansionary) monetary policy shocks of equal magnitude.

The asymmetry follows from the interaction of the redistributive effects of the monetary policy shock, and the borrowing constraints that limit inter-island asset trade in equilibrium. This interaction is captured by the third and forth equations of proposition 1.

Consider first an expansionary monetary policy shock, that causes output to increase in equilibrium. Because households are ex-ante heterogeneous, the increase in aggregate income is not spread equally among the population of households, so that households whose incomes increase the least will try to borrow from other households. However, such households become borrowing constrained, preventing financial trades from occurring, so that household consumption paths inherit the redistributive consequences of the aggregate shock. This redistribution is captured by the heterogeneity in the $\{\beta_\omega^y\}$ coefficients in the third equation of proposition 1. Furthermore, borrowing constrained households cannot increase their consumption, dampening the overall increase in aggregate demand, and hence dampening the equilibrium response of output to the shock.

In response to a contractionary monetary policy shock, output falls in equilibrium. In this case, heterogeneity implies that households whose incomes fall the most will try to borrow from other households. Borrowing constraints again cause consumption to inherit the redistribution of income caused by the aggregate shock, and imply that constrained households cut their consumption causing a fall in aggregate demand that amplifies the overall decline in output.

It is striking that uninsurable idiosyncratic shocks do not appear in the expressions for the output responses. To understand their absence, consider the forth equation of proposition 1, which states that Euler equation only holds for households with the lowest expected equilibrium consumption growth. To first order, the contribution of idiosyncratic shocks to a household's expected consumption growth is given by

$$\beta_\omega^\eta \left(\mathbb{E}_t \left[\eta_{t+1}^\omega\right] - \eta_t^\omega\right) = \beta_\omega^\eta \left(1 - \rho_\eta\right) \eta_t^\omega \approx 0$$

where the approximation follows from the fact that $\rho_\eta \to 1$ under assumption 1. In other words, when the uninsurable idiosyncratic shocks that transmit to consumption are very persistent (as is the case empirically), their first order contribution to expected consumption growth is zero. Intuitively, households do not expect their consumption to change due to uninsurable idiosyncratic shocks, and so do not need to trade the inter-island bond in response to such shocks. The fact that permanent uninsurable idiosyncratic shocks do not induce inter-island trade is at the core of the tractability obtained in Heathcote et al. (2014).

In addition to idiosyncratic shocks, household consumption is subject to aggregate shocks via their effects of aggregate income $\hat{y}_t$. Furthermore, the heterogeneity captured by the $\{\beta_\omega^y\}_\Omega$ coefficients implies that households' consumption paths are heterogeneously exposed to changes in aggregate income.

Therefore, in equilibrium, a household's expected consumption growth is determined solely by her consumption exposure to the change in output. The first order contribution of changes in aggregate income to a household's expected consumption growth is given by

$$\beta_\omega^y \left[\mathbb{E}_t \left[\hat{y}_{t+1}\right] - \hat{y}_t\right] = -\left(1 - \rho_v\right) \beta_\omega^y \hat{y}_t$$

where the equality follows from the fact that $\mathbb{E}_t \left[\hat{y}_{t+1}\right] = \rho_v \hat{y}_t$ along the transition path after the MIT shock has hit the economy. Therefore, the identity of the household with the lowest expected consumption growth depends on whether the aggregate shock is expansionary ($\hat{y}_t > 0$) or contractionary ($\hat{y}_t < 0$).

Consider first an expansionary monetary policy shock ($v_1 < 0$) that raises output, so that $\hat{y}_t > 0$. In this case, the households with the lowest expected consumption growth must have the largest consumption exposure coefficient, $\bar{\beta}^y = \max_\omega \{\beta_\omega^y\}$, and hence experience the largest increase in consumption among all households. Since these households are unconstrained in equilibrium, their consumption increase must be proportional to the decline in the real interest rate caused by the monetary shock, as stated by the Euler equation. Since the largest increase in household consumption is proportional to the decline in the real interest rate, the equilibrium increase in output must be less than proportional to the decline in the real interest rate.

Now consider a contractionary monetary policy shock ($v_1 > 0$) that lowers output, so that $\hat{y}_t < 0$. In this case, the households with the lowest expected consumption growth must have the smallest consumption exposure coefficient, $\underline{\beta}^y = \min_\omega \{\beta^y_\omega\}$, and hence experience the smallest decrease in consumption among all households. Since these households are unconstrained in equilibrium, their consumption decrease must be proportional to the increase in the real interest rate caused by the monetary shock, as stated by the Euler equation. Since the smallest decrease in household consumption is proportional to the increase in the real interest rate, the equilibrium decrease in output must be more than proportional to the increase in the real interest rate. Hence, output responds more to contractionary monetary shocks than to expansionary monetary shocks of equal magnitude.

## 4.2 Asymmetric Output Responses to Cost-Push Shocks

Cost-push shocks are a source of variation in inflation that is exogenous to changes in real activity, and play an important role in explaining economic data using medium-scale New Keynesian DSGE models (Smets and Wouters, 2007). In this section, I study their transmission in my economy, and highlight the similarities and differences to the transmission of monetary policy shocks.

Consider the response of output to a one time, zero probability, cost-push shock of the form $\hat{\Phi}_t = \rho_\Phi^{t-1}\hat{\Phi}_1$.

**Proposition 3.** *The first order transitional dynamics of output in response to a cost-push shock are given by*

$$\hat{y}_t = \begin{cases} \frac{(\phi_\pi - \rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_\Phi}{(1-\rho_\Phi)\bar{\beta}^y + \frac{1}{\sigma}\left((\phi_\pi-\rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_y + \phi_y\right)}\frac{1}{\sigma}\rho_\Phi^{t-1}\hat{\Phi}_1 & \textit{if} \quad \hat{\Phi}_1 > 0 \\ \\ \frac{(\phi_\pi - \rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_\Phi}{(1-\rho_\Phi)\underline{\beta}^y + \frac{1}{\sigma}\left((\phi_\pi-\rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_y + \phi_y\right)}\frac{1}{\sigma}\rho_\Phi^{t-1}\hat{\Phi}_1 & \textit{if} \quad \hat{\Phi}_1 < 0 \end{cases}$$

*where*

$$\bar{\beta}^y = \max_\omega \{\beta^y_\omega\}$$

$$\underline{\beta}^y = \min_\omega \{\beta^y_\omega\}$$

The asymmetry of cost-push shocks follows from the fact that a positive cost-push ($\hat{\Phi}_1 < 0$) shock creates inflation which causes interest rates to rise via the Taylor rule, while a negative cost-push shock ($\hat{\Phi}_1 > 0$) causes interest rates to fall. These interest rate movements then initiate the same mechanism as in the monetary policy shock case, causing output to respond more to the contractionary movement than to the expansionary movement.

### 4.3 Asymmetric Output Responses to TFP Shocks

In this section, I analyze the responses of output to TFP shocks. The key insight relative to monetary and cost-push shocks is that changes in the level of technology have distributional consequences which feed into the general equilibrium responses of aggregate variables when households are heterogeneous and financial markets are incomplete.

Consider the response of output to a one time, zero probability, TFP shock of the form $\hat{a}_t = \rho_a^{t-1}\hat{a}_1$.

**Proposition 4.** *The first order transitional dynamics of output in response to a TFP shock are given by*

$$\hat{y}_t = \begin{cases} c_y^+ \rho_a^{t-1}\hat{a}_1 & \textit{if} \quad \hat{a}_1 > 0 \\ \\ c_y^- \rho_a^{t-1}\hat{a}_1 & \textit{if} \quad \hat{a}_1 < 0 \end{cases}$$

*where $c_y^- > c_y^+$, and $c_y^- \geq 0$. $c_y^+$ and $c_y^-$ are the unique solutions to the non-linear equations*

$$\max_{\omega} \left\{ (\rho_a - 1)\left(\beta_\omega^y c_y^- + \beta_\omega^a\right) \right\} = \frac{1}{\sigma}\left( (\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y \right) c_y^- - \frac{1}{\sigma}(\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_a$$

$$\min_{\omega} \left\{ (\rho_a - 1)\left(\beta_\omega^y c_y^+ + \beta_\omega^a\right) \right\} = \frac{1}{\sigma}\left( (\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y \right) c_y^+ - \frac{1}{\sigma}(\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_a$$

The fact that $c_y^- > c_y^+$ implies that output responds more to contractionary TFP shocks than to expansionary TFP shocks of equal magnitude. Therefore the output responses exhibit the same asymmetry as in the monetary shock and cost-push shock cases.

The response of output to a TFP shock is determined by two forces. First, TFP shocks affect firms' marginal costs and hence their optimal prices, thereby affecting inflation, and interest rates via the Taylor rule. Second, the change in marginal costs caused by a TFP shock endogenously changes the shares of total income that flow to labor in the form of wages, and non-labor in the form of dividends. The presence of the second channel makes explicit solutions for $c_y^+$ and $c_y^-$ unattainable.

Consider a negative TFP shock, $\hat{a}_1 < 0$. The first channel is as follows: a decrease in productivity increases firms' marginal costs of production, which causes them to raise their prices, creating inflation. This inflation causes interest rates to rise via the Taylor rule, which causes output to decrease via the same mechanism as in the monetary shock case. The asymmetry built into that mechanism implies that the decrease in output will be relatively large.

The second channel reinforces the fall in output. The increase in marginal costs endogenously redistributes income towards wages from dividends, and so causes households who depend mainly on labor income to save in equilibrium. This is captured by the $\beta_\omega^a$ term inside the $\min$ operator of the $c_y^-$ equation: households with smaller $\beta_\omega^a$ coefficients have smaller dividend shares, and hence benefit more from the negative TFP shock.

In isolation, the endogenous redistribution caused by the TFP shock is a force for output to fall in equilibrium. Intuitively, saver households' current consumption must be consistent with the fact that interest rates are unchanged. Therefore, the increase in consumption due to the negative TFP shock must be offset by a decline in consumption due to a drop in aggregate income, i.e. output. Therefore, both channels force output to drop in response to the negative TFP shock.

When the TFP shock is positive, $\hat{a}_1 > 0$, the first channel is as follows: an increase in productivity lowers firms' marginal costs of production, which causes them to lower their prices, creating deflation. This deflation causes interest rates to fall via the Taylor rule, which causes output to increase via the same mechanism as in the monetary shock case. The asymmetry built into that mechanism implies that the increase in output will be relatively small.

The second channel now works in the opposite direction to the first. The drop in marginal costs endogenously redistributes income from wages to dividends, and so causes households who depend mainly on non-labor income to become savers in equilibrium. This is captured by the $\beta_\omega^a$ term inside the $\max$ operator of the $c_y^+$ equation: households with larger $\beta_\omega^a$ coefficients have larger dividend shares, and hence benefit more directly from the positive TFP shock.

In isolation, this channel is a force for output to fall in equilibrium since the equilibrium consumption response of saver households must be consistent with the fact that real interest rates are unchanged. Therefore, the increase in consumption of savers due to the positive TFP shock must be offset by a decline in consumption due to a drop in aggregate income, i.e. output. Therefore, the second channel offsets the first, and results in a smaller increase in output in response to the positive TFP shock, thus establishing the asymmetry of responses.

In extreme cases, it is possible for the second channel to dominate the first, so that output actually falls in response to a positive TFP shock, and a technology improvement has contractionary effects.[7] To see when this is a plausible outcome, solve the equation for $c_y^+$ to get

$$c_y^+ = \frac{\frac{1}{\sigma}\left(\phi_\pi - \rho_a\right)\frac{1+\rho}{1+\rho-\rho_a}\varphi_a - \left(1-\rho_a\right)\beta_{\omega^+}^a}{\left(1-\rho_a\right)\beta_{\omega^+}^y + \frac{1}{\sigma}\left(\left(\phi_\pi - \rho_a\right)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right)}$$

where $\omega^+ = \arg\max_\omega \left\{\beta_\omega^y c_y^+ + \beta_\omega^a\right\}$. This expression shows that $c_y^+$ is negative when $\frac{1}{\sigma}\frac{\phi_\pi-\rho_a}{1-\rho_a}\frac{1+\rho}{1+\rho-\rho_a}\varphi_a < \beta_{\omega^+}^a$. This inequality holds when a few households receive the bulk of the increase in dividends that

[7] The fact that both channels go in the same direction for negative TFP shocks rules this possibility out when $\hat{a}_1 < 0$. Hence $c_y^- \geq 0$, as per proposition 4.

results from a positive TFP shock, so that $\beta^a_{\omega+}$ is large and positive, and when the response of the monetary authority to the deflation created by the positive TFP shock is sufficiently weak, so that $\phi_\pi$ is small (and positive). In this case, the fall in aggregate demand that results from these households choosing to save their additional income can overturn the increase in output stemming from the monetary response to the improvement in productivity. This possibility is particularly pronounced when the transmission from TFP shocks into inflation is weak, so that $\varphi_a > 0$ is small, or when the TFP shock is not very persistent, so that $\rho_a \to 0$.

Previous work by Basu et al. (2006) has argued that technology improvements do indeed have contractionary short-run effects in the data. The mechanism I have highlighted here therefore offers a theoretical justification for such empirical findings, that complement the mechanisms outlined in that paper. In addition, it links the contractionary nature of positive TFP shocks the extent of inequality in the economy, since technology improvements are more likely to contractionary when there exists a few households who receive the lion's share of the increase in dividends that results from the shock.

### 4.4 Lessons for Monetary Policy Design

At the heart of the output asymmetry mechanism is the fact that borrowing constraints cause interest cuts to transmit less powerfully than interest rate hikes. This fact has two implications for the design of monetary policy in response to aggregate shocks.

**Binding Zero Lower Bound** Consider an aggregate shock that, ceteris paribus, causes the real interest rate to rise. The response of monetary policy is to cut the nominal interest rate so that the real interest rate does not rise as much in equilibrium. However, the strong transmission of increases in the real interest rate implies that the size of the nominal interest rate cut called for by the Taylor rule is larger for a given size of aggregate shock. Therefore, the size of shock at which the nominal rate response implied by the Taylor rule becomes negative, and the zero lower bound binds, is smaller in the presence of binding borrowing constraints.

Formally, consider the economy's response to cost-push shocks in the presence of a nominal interest rate rule adjusted for a zero lower bound constraint,

$$\iota_t = \max\left\{r + \phi_\pi \pi_t + \phi_y \hat{y}_t, 0\right\}$$

Using the equilibrium responses of output and inflation to a cost push shock that raises the real interest rate directly, $\Phi_1 > 0$, we can derive the equilibrium response of nominal interest rates, and hence compute the shock size at which the zero lower bound constraint binds.

**Proposition 5.** *Consider a one time, zero probability cost-push shock with zero persistence. The zero lower bound constraint binds if and only if*

$$\hat{\Phi}_1 > \hat{\Phi}_{ZLB} = \frac{\rho + (\phi_\pi \varphi_y + \phi_y) \frac{1}{\bar{\beta}^y} \frac{1}{\sigma} \rho}{\phi_\pi \varphi_\Phi}$$

*where $\hat{\Phi}_{ZLB}$ is decreasing in $\bar{\beta}^y$.*

Intuitively, when $\bar{\beta}^y$ is larger, the transmission of expansionary monetary policy is weaker, and larger nominal interest rate cuts are required to overcome a given increase in the real interest rate caused by an aggregate shock. Therefore, smaller shocks will cause the zero lower bound to bind than in the economy without borrowing constraints.

This result suggests that borrowing constraints make it more likely that the zero lower bound impedes the ability of monetary policy to stabilize the economy in response to aggregate shocks.

**Restoring Symmetric Output Responses** The results above also highlight that a linear Taylor rule is insufficient to ensure symmetric transmission of both interest rate cuts and hikes.

From the perspective of policy makers, it is therefore useful to understand how to overcome this asymmetry of monetary policy transmission. As a useful benchmark, I consider the case of cost-push shocks, and consider the following question: how much more do interest rates need to respond to falls in output relative to increases in output in order to achieve symmetric equilibrium responses to cost-push shocks?

As a simple departure from the standard Taylor rule, I consider a piece-wise rule,

$$\iota_t = r + \phi_\pi \pi_t + \phi_y^+ \max\{\hat{y}_t, 0\} + \phi_y^- \min\{\hat{y}_t, 0\}$$

that allows for differential responses of monetary policy to increases and decreases in output.

Using this interest rate rule for monetary policy, the following result shows how to determine $\phi_y^-$ relative to $\phi_y^+$ to ensure symmetric responses of output to cost-push shocks.

**Proposition 6.** *Given $\phi_y^+ > 0$, setting*

$$\phi_y^- = \sigma (1 - \rho_\Phi) \left(\bar{\beta}^y - \underline{\beta}^y\right) + \phi_y^+$$

*ensures that output has symmetric first order transitional dynamics in response to cost-push shocks.*

The proposition shows that in order to restore symmetry, expansionary monetary policy must be stronger than contractionary monetary policy in order to overcome the asymmetric transmission caused by borrowing constraints. Furthermore, the size of the gap between $\phi_y^-$ and $\phi_y^+$ is determined by the range of consumption exposure coefficients $\bar{\beta}^y - \underline{\beta}^y$, which is in turn dependent on the extent of uninsurable consumption risk present in the economy. The larger is this gap, the larger is the asymmetry of monetary policy transmission, and hence the larger that interest cuts need to be relative to interest rate hikes to have the same size effect on output.

## 4.5 Discussion of Mechanism

Having established my main results, I now discuss the economic mechanism in the context of the existing literature. I focus on the role of permanent uninsurable heterogeneity, marginal propensities to consume, and the concept of the wealthy hand to mouth.

**The Role of Uninsurable Permanent Heterogeneity** Recall that the coefficients $\{\beta^y_\omega, \beta^a_\omega, \beta^\eta_\omega\}_\Omega$ capture the effects of uninsurable ex-ante heterogeneity in my economy. In particular, heterogeneity in $\{\beta^y_\omega, \beta^a_\omega\}_\Omega$ summarizes how aggregate shocks redistribute household consumption along the equilibrium response path. In the knife edge case in which $\beta^y_\omega \equiv \beta^y$ and $\beta^a_\omega = \beta^a$ for all islands, there is no uninsurable heterogeneity and aggregate shocks do not have redistributive consequences in equilibrium. As a result, the key mechanism breaks down, and there is no asymmetry of output responses. Intuitively, the absence of redistribution implies that households have uniform motives to trade the inter-island bond, and therefore must all choose a zero, unconstrained bond position in equilibrium. Hence, borrowing constraints do not play a role in determining the equilibrium responses of output to monetary policy shocks, which are hence symmetric to first order.

This result explains why other papers that study HANK-style economies do not report asymmetric responses of output to monetary policy shocks, or indeed any aggregate shocks (e.g. Kaplan et al., 2017). The standard approach of merging a Huggett (1993) or Aiyagari (1994) type model of heterogeneous households with a New Keynesian supply side implicitly imposes that all households are identical ex-ante. Therefore, the key heterogeneity required to generate asymmetric output responses is ruled by by construction. In contrast, recent work by Patterson (2018) explicitly allows for permanent heterogeneity across households, which generates amplification of contractionary aggregate shocks using a similar mechanism to the one that I study in generality here.

Crucially, this type of fixed heterogeneity across households seems important empirically. For example, using high quality administrative data for the US, Guvenen et al. (2016, 2017) document that households' incomes processes both exhibit permanent heterogeneity in the mean level of income, and are differentially exposed to aggregate fluctuations, as captured by heterogeneity in $\upsilon_i$ across households in my model. On the consumption side, Parker and Vissing-Jorgensen (2009) and DeGiorgi and Gambetti (2017) show that households' consumption paths are also differentially correlated with changes in output, which provides some direct evidence of heterogeneity in the $\{\beta^y_\omega\}$ coefficients, as required for asymmetric responses.

**Relation to Marginal Propensities to Consume (MPCs)** An important insight from the existing HANK literature is that a non-negligible fraction of households are constrained in equilibrium, and hence are "off their Euler equation". These households act in a "hand to mouth" (HtM) fashion, consuming most, if not all, of their income in each period, and exhibit large marginal propensities to consume (MPCs) as a result. Furthermore, analysis by Bilbiie (2017) and Acharya and Dogra (2018) shows that the cyclicality of the income of these HtM households affects the strength of the transmission of monetary policy: when HtM households' income share is pro-cyclical, interest rate policy is more powerful than in an economy without HtM households. Intuitively, when aggregate income increases in response to an interest rate cut, the HtM households receive disproportionately more of this increase. Since HtM households consume the entire income gain, the indirect general equilibrium feedback from aggregate demand into output is amplified, resulting in a larger overall response of output.

My economy also features households who are constrained in equilibrium. Indeed, the presence of borrowing constraints that inhibit inter-island trade achieves exactly this. However, in contrast to the aforementioned literature, my economy does not feature a group of households who can be permanently labeled at HtM. Instead, which households become constrained in equilibrium depends crucially on the equilibrium response of aggregate income to the aggregate shock that hits the economy.

When an expansionary shock hits the economy, aggregate income increases in equilibrium. As described above, when aggregate income rises, constrained households are those whose incomes are the least pro-cyclical among all households. In other words, constrained households receive disproportionately less of the overall income gain, which dampens the general equilibrium feedback from aggregate demand into output, and thus results in a smaller overall output response.

In contrast, when aggregate income falls in response to a contractionary aggregate shock, constrained households' incomes must be the most pro-cyclical among all households. In this case, constrained households are disproportionately exposed to the fall in income, which amplifies the general equilibrium feedback effect, and results in a larger overall output response.

I endogenize the cyclicality of constrained households' incomes by using borrowing constraints as my key source of market incompleteness. In addition to capturing relevant financial frictions mentioned above, an advantage of my modeling approach is that it is consistent with the empirical evidence for asymmetric MPCs. Recent work by Fuster et al. (2018) and Christelis et al. (2017) find that, across the income distribution, households display high MPCs out of income losses, but low MPCs out of income gains. Binding borrowing constraints are a simple way to rationalize these findings: after an income gain, a household can save and move away from the constraint, and thus exhibits a low MPC. After an income loss, however, the household may become borrowing constrained, and hence be forced to lower her consumption a lot, thus exhibiting high MPC behavior.

**The Wealthy Hand to Mouth** Kaplan et al. (2014) stress that high MPC and HtM behavior is not confined to households who are both income and wealth poor, but is also a feature of household behavior further up the income and wealth distributions. I argue that my model is both consistent with this fact, and clarifies the conditions under which it is poor households or wealthy households who exhibit high MPC behavior.

My mechanism emphasizes that different households are borrowing constrained when different aggregate shocks hit the economy. Households whose consumption paths are highly correlated with aggregate consumption are constrained when output falls in response to a contractionary shock, while households with low correlation consumption paths are constrained when output increases in response to an expansionary shock.

In the empirical analysis, I find that households with more education have consumption paths that are more positively correlated with aggregate consumption. Since education is likely itself highly correlated with measures of income and wealth (DeGiorgi and Gambetti (2017) provide evidence of this correlation in the Consumer Expenditure Survey), the theoretical analysis suggests that households with higher incomes and wealth levels becomes constrained when output falls in response to a contractionary shock, and thus exhibit behavior in line with the "wealth hand-to-mouth" described by Kaplan et al. (2014).

This feature also highlights that binding borrowing constraints to do preclude the existence of positive wealth holding in my economy. As described above, each island features a non-degenerate wealth distribution that results from households achieving complete insurance against transitory idiosyncratic shocks and partial insurance against persistent idiosyncratic shocks and aggregate shocks.

## 4.6 Existence and Uniqueness of Responses

An intuitive description of how to solve for the responses of output to aggregate shocks is as follows: when an aggregate shock hits the economy, it directly affects the demand for bonds of saver households. For example, an expansionary monetary policy shock lowers the real interest rate and hence lowers their demand for bonds on the margin. Since all other households are borrowing constrained, the equilibrium output response must be such that it has an equal and opposite effect on the demand for bonds of saver households in order to clear the bond market.

In order to find the equilibrium response of output $\hat{y}_t$ to an arbitrary transitory aggregate shock, consider the two effects that a small increase in $\hat{y}_t$ has on saver households' demand for bonds. First, it increases the income of saver households by an amount $\beta^y_{\omega^s}$, where $\omega^s$ is an island on which saver households are located. Ceteris paribus, this increase in income creates an increase in bond demand of $\beta^y_{\omega^s}$. Second, a small increase in output causes an endogenous increase in inflation of $\varphi_y$. The monetary authority responds to the increases in output and inflation by raising the real interest rate by an amount specified by the Taylor rule coefficients, $\phi_\pi \varphi_y + \phi_y > 0$. By the logic of the Euler equation, this interest rate hike causes saver households to increase their bond demand by $\frac{1}{\sigma}(\phi_\pi \varphi_y + \phi_y)$.

Now consider assumption 2, which states that

$$\underline{\beta}^y + \frac{1}{\sigma}(\phi_\pi \varphi_y + \phi_y) > 0$$

Since $\beta^y_{\omega^s} \geq \underline{\beta}^y$ by definition, assumption 2 guarantees that

$$\beta^y_{\omega^s} + \frac{1}{\sigma}(\phi_\pi \varphi_y + \phi_y) > 0$$

which implies that a small increase in output will always cause saver households to increase their demand for bonds. This strictly monotonic relationship between output and the demand for bonds of saver households is sufficient to ensure that, for any given change in bond demand caused by an arbitrary aggregate shock, there exists a unique output adjustment that will exactly offset it, thus establishing the existence and uniqueness of output responses to arbitrary aggregate shocks.

In my empirical analysis, I find that $\underline{\beta}^y > 0$, which guarantees that assumption 2 is satisfied. Nonetheless, it is instructive to consider what is possible were the assumption to fail. In this case, the relationship between output and the demand for bonds of saver households need not be strictly monotonic, which opens up the possibility that there are either multiple output adjustments that exactly offset the effect of the aggregate shock on bond demand, or none at all. Therefore, assumption 2 is a useful condition to ensure both the existence and uniqueness of the output responses under investigation.

# 5 Inflation Responses to Aggregate Shocks

Having studied the responses of output to aggregate shocks, I now turn to the responses of inflation. I maintain assumptions 1, and 2 throughout.

## 5.1 Inflation Responses to Monetary Policy Shocks

The responses of inflation to monetary policy shocks are simple to derive given the output responses.

**Proposition 7.** *The first order transitional dynamics of inflation in response to a monetary policy shock are given by*

$$\pi_t = \begin{cases} -\dfrac{\frac{1+\rho}{1+\rho-\rho_v}\varphi_y}{(1-\rho_v)\underline{\beta}^y+\frac{1}{\sigma}\left((\phi_\pi-\rho_v)\frac{1+\rho}{1+\rho-\rho_v}\varphi_y+\phi_y\right)}\frac{1}{\sigma}\rho_v^{t-1}v_1 & \textit{if} \quad v_1>0 \\ \\ -\dfrac{\frac{1+\rho}{1+\rho-\rho_v}\varphi_y}{(1-\rho_v)\bar{\beta}^y+\frac{1}{\sigma}\left((\phi_\pi-\rho_v)\frac{1+\rho}{1+\rho-\rho_v}\varphi_y+\phi_y\right)}\frac{1}{\sigma}\rho_v^{t-1}v_1 & \textit{if} \quad v_1<0 \end{cases}$$

*where*

$$\bar{\beta}^y = \max_\omega \{\beta^y_\omega\}$$
$$\underline{\beta}^y = \min_\omega \{\beta^y_\omega\}$$

The proposition shows that inflation inherits the asymmetry of output. This occurs because the response of inflation to monetary shocks is entirely determined by the response of output via the logic of the NKPC: higher output implies higher marginal costs which causes firms to increase their prices, thus raising inflation. Therefore, inflation moves in the same direction as output in response to monetary shocks. Since output responds more to contractionary monetary shocks than to expansionary shocks, so does inflation.

## 5.2 Inflation Responses to Cost-Push Shocks

**Proposition 8.** *The first order transitional dynamics of inflation in response to a cost-push shock are given by*

$$\pi_t = \begin{cases} -\dfrac{\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_\Phi\left((1-\rho_\Phi)\bar{\beta}^y+\frac{1}{\sigma}\phi_y\right)}{(1-\rho_\Phi)\bar{\beta}^y+\frac{1}{\sigma}\left((\phi_\pi-\rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_y+\phi_y\right)}\rho_\Phi^{t-1}\hat{\Phi}_1 & \textit{if} \quad \hat{\Phi}_1>0 \\ \\ -\dfrac{\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_\Phi\left((1-\rho_\Phi)\underline{\beta}^y+\frac{1}{\sigma}\phi_y\right)}{(1-\rho_\Phi)\underline{\beta}^y+\frac{1}{\sigma}\left((\phi_\pi-\rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_y+\phi_y\right)}\rho_\Phi^{t-1}\hat{\Phi}_1 & \textit{if} \quad \hat{\Phi}_1<0 \end{cases}$$

*where*

$$\bar{\beta}^y = \max_\omega \{\beta^y_\omega\}$$
$$\underline{\beta}^y = \min_\omega \{\beta^y_\omega\}$$

In contrast to monetary policy shock transmission, the asymmetry of the inflation responses is the opposite to that of output: inflation responds more to cost-push shocks that increase output. This occurs because the responses of inflation are determined by two forces. First, cost-push shocks directly affect firms' marginal costs and so directly affect inflation (positive cost-push shocks increase marginal costs and inflation). Second, the equilibrium response of output affects inflation via the NKPC as in the monetary shock case. Crucially, the second effect pushes inflation in the opposite direction to the first: a positive cost-push shock directly drives inflation up, which raises interest rates causing output to fall, which creates an offsetting downward force on inflation. The strength of this offsetting force inherits the asymmetry of the output responses, so that inflation responds more overall when output responds less and the offsetting force is weaker. Therefore inflation responds with the opposite asymmetry to output.

### 5.3 Inflation Responses to TFP Shocks

**Proposition 9.** *The first order transitional dynamics of inflation in response to a TFP shock are given by*

$$\pi_t = \begin{cases} c_\pi^+ \epsilon_t^a & \text{if} \quad \epsilon_t^a > 0 \\ \\ c_\pi^- \epsilon_t^a & \text{if} \quad \epsilon_t^a < 0 \end{cases}$$

*where*

$$c_\pi^+ = \frac{1+\rho}{1+\rho-\rho_a}\left(\varphi_y c_y^+ - \varphi_a\right)$$

$$c_\pi^- = \frac{1+\rho}{1+\rho-\rho_a}\left(\varphi_y c_y - \varphi_a\right)$$

*and* $c_\pi^- > c_\pi^+$.

Similar to the case of cost-push shocks, inflation exhibits the opposite asymmetry to output, and responds more to positive TFP shocks than to negative TFP shocks. As in the cost-push shock case, the reversal occurs because the inflation response is the sum of a symmetric direct effect, and an asymmetric indirect effect stemming from the response of output. For example, a positive TFP shock lowers firms' marginal costs, which causes them to lower their prices, creating deflation. This causes the monetary authority to lower rates, which in turn causes output to rise. The rise in output then offsets the initial fall in inflation. Since the offsetting force will be stronger for negative TFP shocks than for positive TFP shocks, inflation will respond more overall to positive TFP shocks.

## 6 Quantitative Exercise

My theoretical analysis has highlighted the role that heterogeneous exposures of household consumption to changes in aggregate income play in generating asymmetric responses of output to aggregate shocks. In this section, I use data on household consumption patterns in the US to directly estimate these exposures, and then use these estimates to compute the asymmetry of the responses of output to monetary policy shocks implied by the model.

I find significant heterogeneity in consumption exposures, which I argue imply that output responses to contractionary monetary policy shocks are at least three times as large as the responses to expansionary monetary policy shocks. This level of asymmetry accounts for around 60% of the asymmetry found accords well with the direct evidence for asymmetric responses that I describe at the end of this section.

Given the pre-existing evidence for asymmetric responses of output to monetary policy shocks, I focus on this aggregate shock in my quantitative analysis. However, the methods I describe are directly applicable to other aggregate shocks.

## 6.1 Quantifying the Asymmetry

In order to assess the quantitative magnitude of the output response asymmetry to a monetary policy shock, I define the ratio of the contractionary response to the expansionary response,

$$\mathcal{R} = \frac{\left(1-\rho_v\right)\bar{\beta}^y + \frac{1}{\sigma}\left(\frac{1+\rho}{1+\rho-\rho_v}\varphi_y\left(\phi_\pi-\rho_v\right)+\phi_y\right)}{\left(1-\rho_v\right)\underline{\beta}^y + \frac{1}{\sigma}\left(\frac{1+\rho}{1+\rho-\rho_v}\varphi_y\left(\phi_\pi-\rho_v\right)+\phi_y\right)}$$

The key parameters for quantifying the asymmetry are $\bar{\beta}^y$ and $\underline{\beta}^y$, which measure the highest and lowest equilibrium sensitivities of household consumption to changes in output. In particular, when $\frac{1}{\sigma}\left(\frac{1+\rho}{1+\rho-\rho_v}\varphi_y\left(\phi_\pi-\rho_v\right)+\phi_y\right)=0$, the response ratio is simply the ratio of the consumption sensitivities, $\mathcal{R}=\frac{\bar{\beta}^y}{\underline{\beta}^y}$.

The parameters $\bar{\beta}^y$ and $\underline{\beta}^y$ are "sufficient statistics" for computing the output response asymmetry (Chetty, 2009).[8] In other words, to compute $\mathcal{R}$, I only need to know the values of $\bar{\beta}^y$ and $\underline{\beta}^y$, and do not need quantitative information on the underlying structural mechanism that generates them. In my setting, this means that I do not need to know quantitative details concerning the structure of financial markets or, in particular, empirical features of the underlying equilibrium wealth distribution. This is in sharp contrast to quantitative HANK models, such as Kaplan et al. (2018), which require detailed knowledge of income processes, financial market structures, technology, and preferences in order to obtain numerical results.

The simplicity of my model's numerical implementation follows from the financial markets structure I adopt. The island construction ensures that household consumption dynamics are consistent

[8]Sufficient statistics approaches have recently become popular in macroeconomics. See, for example, Auclert and Rognlie (2017).

with the empirical evidence on the transmission of idiosyncratic income shocks, and allows for heterogeneous exposures to aggregate shocks. These exposures are simple to estimate using micro data on household consumption. Estimates of $\bar{\beta}^y$ and $\underline{\beta}^y$ can then be plugged into $\mathcal{R}$ to immediately quantify the asymmetry.

## 6.2 Data

**Household Consumption** I use the Consumer Expenditure surveys (CEX) from 1996 to 2008 to measure consumption of non-durables and services at the household level.[9] In order to ensure the consistency of consumption measurements between the CEX and the aggregate data in the NIPA tables, I sum across the relevant categories of expenditure in the CEX, and define non-durable and services consumption as total expenditures on food, services, heating fuel, public and private transport, personal care, and clothing and footwear.[10] I deflate nominal expenditures using the personal consumption expenditure price deflator.

Each household reports their consumption four times at three month intervals. From these reports, I compute three quarterly growth rates of log consumption for each household. Since different households are interviewed each month, I have quarterly growth rates of household consumption, available at a monthly frequency.

I restrict the sample to urban households, not in student status, where the household head is of working age (25-64), and only consider households who respond to all four interview waves. In order to remove consumption variation caused by factors outside of my model, I first regress log real consumption on a polynomial in age of the household head, family size, and number of children under the age of eighteen, and use the residuals from this regression as my measures of household consumption.

**Output** As my measure of output, I use quarterly growth rates of per-capita personal consumption expenditures of non-durable goods and services (at a monthly frequency), taken from the NIPA tables, deflated using the personal consumption expenditure price deflator.

My choice of growth in per-capita personal consumption expenditures as the right-hand side variable reflects two considerations. First, the theoretical models I have studied in this paper have all abstracted from capital investment and government spending, so that aggregate consumption is the theoretically consistent measure of total output. Second, unlike measures of GDP, personal consumption expenditures are available at a monthly frequency, which enables me to exploit all of the variation in the micro-data and to maintain a reasonable sample size.

[9] The only alternative to the CEX would be to use imputed consumption series in the PSID, as in Blundell et al. (2008). However, the data is only available at an annual frequency, which smooths out much of the business cycle frequency variation I am most interested in measuring.

[10] My results are robust to variations in this definition.

**Monetary Policy Shocks** In order to extract the variation in $\Delta \log Y_t$ driven by monetary policy shocks, I follow Coibon et al. (2017), who use the methods introduced by Romer and Romer (2004) to identify innovations to monetary policy that are orthogonal to economic conditions. Formally, the authors run the regression

$$\Delta FFR_t = x_t'\Gamma + \epsilon_t^v$$

where $\Delta FFR_t$ is the change in the federal funds rate from period $t-1$ to $t$, and $x_t$ is a vector of controls that contains forecasts of GDP growth, inflation, and the unemployment rate taken from the Greenbooks at each Federal Open Market Committee meeting. The residuals from this regression, $\{\hat{\epsilon}_t^v\}$, are then taken as the series of monetary policy shocks, with the interpretation that $\hat{\epsilon}_t^v > 0$ is a contractionary shock, and $\hat{\epsilon}_t^v < 0$ is an expansionary shock.

Using this method, Coibon et al. (2017) generate a series of monetary policy shocks at a monthly frequency from 1969 to 2008, which I plot in figure 1. The shocks are evenly spread over positive and negative values, and are very volatile during the Volcker disinflation period in the early 1980s.

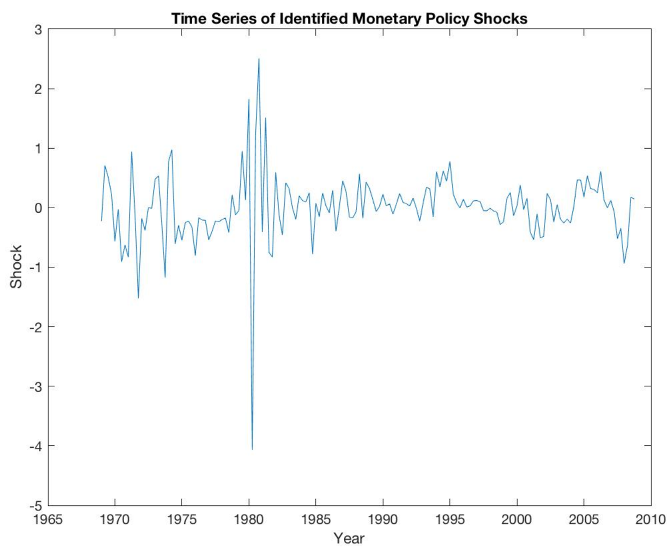


Figure 1: Identified Monetary Policy Shocks from Coibon et al. (2017). The authors run the regression $\Delta FFR = x_t'\gamma + \epsilon_t$ where $\Delta FFR_t$ is the change in the federal funds rate from period $t-1$ to $t$, and $x_t$ is a vector of controls that contains forecasts of GDP growth, inflation, and the unemployment rate taken from the Greenbooks at each Federal Open Market Committee meeting. The residuals from this regression, $\{\hat{\epsilon}_t\}$, are then taken as the series of monetary policy shocks, with the t interpretation that $\hat{\epsilon}_t > 0$ is a contractionary shock, and $\hat{\epsilon}_t < 0$ is an expansionary shock.

## 6.3 Estimation Procedure

Let $\{c_{i,t}\}$ and $\{Y_t\}$ be data on household consumption and output respectively. The equation describing the dynamics of household consumption in proposition 1 suggests that we can recover estimates of the $\{\beta_\omega^y\}$ coefficients by considering the following pooled OLS regression

$$\Delta \log c_{i,t} = \sum_{\omega\in\Omega} \mathbf{1}\{i\in\omega\}\,\alpha_\omega + \sum_{\omega\in\Omega} \mathbf{1}\{i\in\omega\}\,\beta_\omega^y \Delta\log Y_t + u_{i,t}$$

where $u_{i,t}$ contains idiosyncratic shocks to household consumption, and measurement error in the household consumption data.

As it stands, this regression features a number of exogeneity and feasibility concerns, which I now describe how to tackle.

**Variation in Output** In order to measure the sensitivities of household consumption to changes in output driven by monetary policy shocks, the variation in $\Delta\log Y_t$ must be due to monetary policy shocks only. However, the variation in raw output data is driven by multiple aggregate shocks hitting the economy simultaneously in each period. Running the above regression would therefore result in estimates of $\{\beta_\omega^y\}$ that measure the sensitivity of household consumption to changes in output driven by multiple shocks, and would not correspond to the theoretical parameters $\{\beta_\omega^y\}$.

In order to alleviate this issue, I first project the output data onto a set of identified, lagged monetary policy shocks (described in more detail below), $Z_t = \left(\epsilon_{t-1}^v, \ldots, \epsilon_{t-L}^v\right)$,

$$\Delta\log Y_t = \alpha_y + Z_t'\gamma + e_t$$

so that the fitted values $\left\{\Delta \hat{\log} Y_t\right\}$ capture the variation in $\Delta\log Y_t$ driven by monetary policy shocks only. This specification is consistent with the structural vector-autoregression paradigm, in which aggregate variables are expressible as a moving average of the (infinite) history of structural shocks (see Barnichon and Matthes (2016) for a review and extension of this approach to the non-linear case).[11] I then use these fitted values in my main regression specification, so that $\{\beta_\omega^y\}$ correctly identify the sensitivity of household consumption to changes in output driven by monetary policy shocks only.[12]

**Identification of Island Groupings** The ideal regression requires us to group households by their island, which acts as a latent grouping variable. While methods have recently been developed that go some way to dealing with this issue, implementing such a procedure here is beyond the scope of the paper.[13] Instead, I describe a simple solution to this problem that results in a lower

[11] My theoretical results suggest that $\Delta\log Y_t$ should depend non-linearly on the history of monetary policy shocks. However, for the purposes of extracting the variation in $\Delta\log Y_t$ driven by monetary policy shocks, I abstract from this complication. I investigate non-linear responses in section 6.7.

[12] Intuitively, this process amounts to Two-Stage-Least-Squares (2SLS) estimation, where the first stage extracts the variation in $\Delta\log Y_t$ due to monetary policy shocks only, and the second stage estimates the household sensitivity parameters using this variation alone.

[13] For example, Bonhomme et al. (2017) develop OLS-type estimators that allow for discrete unobserved heterogeneity in the underlying population.

bound estimate of the true extent of heterogeneity in the $\{\beta^y_\omega\}$ coefficients.

Formally, let $\mathcal{G}$ be a surjective function that maps household $i$ in period $t$, i.e. the household-period tuple $(i,t)$, into a finite set of groups $\{1,2,...,G\}$. $\mathcal{G}$ represents an arbitrary group formation process, and nests fixed group assignment as a special case, $\mathcal{G}(i,t)$ fixed for all $t$.

If the island groupings were observable, we could use such data to construct the correct $\mathcal{G}$ function. Since such data is unavailable, I instead postulate that households are grouped according to some observable characteristic.

Given a choice of $\mathcal{G}$ function, consider the pooled OLS regression for a group $g \in \{1,2,...,G\}$,

$$\Delta \log c_{i,t} = \sum_{g=1}^{G} \mathbf{1}\{(i,t) \in g\}\,\alpha_g + \sum_{g=1}^{G} \mathbf{1}\{(i,t) \in g\}\,\beta^y_g \Delta \hat{\log} Y_t + e_{i,t}$$

where the pooling occurs over the set $\{(i,t) : \mathcal{G}(i,t) = g\}$ of household-periods assigned to group $g$. Estimating this regression for each group implies that the key parameters for quantifying the asymmetry can be estimated as $\bar{\beta}^y = \max_g \left\{\hat{\beta}^y_g\right\}$ and $\underline{\beta}^y = \min_g \left\{\hat{\beta}^y_g\right\}$.

When the $\mathcal{G}$ function assigns each household $i$ to a fixed group over time, the implied asymmetry parameters will always be weakly bounded by the true asymmetry parameters $\max_i \{\beta_i\}$ and $\min_i \{\beta_i\}$. Therefore, pooled OLS using fixed group assignments will always weakly underestimate the true asymmetry.

**Proposition 10.** *Suppose the model for household consumption growth is given by*

$$\Delta \log c_{i,t} = \sum_{\omega \in \Omega} \mathbf{1}\{i \in \omega\}\,\alpha_\omega + \sum_{\omega \in \Omega} \mathbf{1}\{i \in \omega\}\,\beta^y_\omega \Delta \hat{\log} Y_t + u_{i,t}$$

*If $\mathcal{G}$ does not depend on $t$ for all $i$, then the asymmetry parameters implied by the pooled OLS regressions*

$$\Delta \log c_{i,t} = \sum_{g=1}^{G} \mathbf{1}\{i \in g\}\,\alpha_g + \sum_{g=1}^{G} \mathbf{1}\{i \in g\}\,\beta^y_g \Delta \hat{\log} Y_t + e_{i,t}$$

*are weakly bounded by $\max_\omega \{\beta^y_\omega\}$ and $\min_\omega \{\beta^y_\omega\}$, i.e.*

$$\max_g \left\{ \textit{plim}_{T\to\infty} \hat{\beta}^y_g \right\} \leq \max_\omega \{\beta^y_\omega\}$$

$$\min_g \left\{ \textit{plim}_{T\to\infty} \hat{\beta}^y_g \right\} \geq \min_\omega \{\beta^y_\omega\}$$

Intuitively, when group assignments are fixed over time, the estimated consumption exposure of a group $g$ is a convex combination of the consumption exposures of each household in that group.

Therefore, each group's consumption exposure is weakly smaller than the largest household exposure, and weakly larger than the smallest household exposure. This immediately says that the asymmetry implied by the estimates must be bounded by the true asymmetry at the household level.

When $\mathcal{G}$ assigns households to different groups over time, it is difficult to say whether the implied asymmetry from pooled OLS over- or underestimates the true asymmetry. As an extreme example, suppose that $\beta_i = 1$ for all $i$ (so that the true asymmetry is nil) and consider the following assignment process for a fixed group $g$. When $\Delta \hat{\log} Y_t > 0$, assign households with the highest consumption growths to group $g$. When $\Delta \hat{\log} Y_t < 0$, assign households with the lowest consumption growths to group $g$. Such a process will result in an estimate of $\hat{\beta}_g$ much larger than 1, due to the selection bias created by the assignment mechanism's dependence on idiosyncratic shocks, and will therefore overestimate the true asymmetry. Furthermore, the opposite assignment process will clearly result in an underestimate of the true asymmetry.

In light of this discussion, I choose as a benchmark, an assignment mechanism that is fixed over time, so that the estimated asymmetry is known to be a lower bound on the true asymmetry (in the limit $T \to \infty$). In the CEX data, the best candidate for this is the level of education of the household head.[14] Over the year long cycle during which the household reports consumption, the education level of the household head is fixed and is certainly exogenous to changes in output over the same period. Therefore, I sort households into five groups based on the education level of the household head: less than high school, high school, some college, full college, and beyond college (advanced degree).

**Measurement Error and Short Panels** As mentioned, $u_{i,t}$ captures both idiosyncratic shocks to consumption and measurement error at the household level (Aguiar and Bils, 2015),. While the idiosyncratic shock component is uncorrelated with $\Delta \hat{\log} Y_t$ by definition, the measurement error component may not be, and so could induce bias into the estimates of consumption sensitivities. However, as long as measurement error is independent across households, and each group $g$ consists of a sufficiently large number of households in each period, applying a cross-sectional Law of Large Numbers implies that the composite measurement error term is approximately zero in every period for a given group, and is therefore uncorrelated with $\Delta \hat{\log} Y_t$. I assume that this condition holds in my analysis.

I also note that the grouping of households, either into hidden islands or into exogenous groups, effectively creates synthetic panels of household consumption data. This helps to alleviate the short-panel nature of the CEX data, and is common among analyses that use CEX data to analyze trends

[14]The very short panel nature of the CEX data implies that other potential fixed attributes such as permanent income are difficult to plausibly compute. Education is of course likely to be correlated with this and other fixed attributes.

and fluctuations in household consumption (see, for example, Parker and Vissing-Jorgensen (2009), Primiceri and van Rens (2009), and De Giorgi and Gambetti (2017)).

## 6.4 Baseline Regression Specification

In sum, I estimate the following two stage model:

First, estimate

$$\Delta \log Y_t = \alpha_y + Z_t' \gamma + e_t$$

to get fitted values $\left\{ \Delta \hat{\log} Y_t \right\}$. Then, use these fitted values to estimate the model

$$\Delta \log c_{i,t} = \sum_{g=1}^{G} \mathbf{1} \{ i \in g \} \alpha_g + \sum_{g=1}^{G} \mathbf{1} \{ i \in g \} \beta_g^y \Delta \hat{\log} Y_t + e_{i,t}$$

where the groups $g \in \{1, ..., G\}$ consist of five education levels: less than high school, high school, some college, full college, and beyond college (advanced degree). In practice, I use a two-stage least squares estimation procedure rather than running separate regressions for each stage.

## 6.5 Results

In my baseline results, I project $\Delta \log Y_t$ onto a vector of the ninety six most recent identified monetary policy shocks $Z_t = \left( \hat{\epsilon}_{t-1}^v, ..., \hat{\epsilon}_{t-96}^v \right)$. This allows the effects of monetary policy shocks to persist for up to eight years. Since the empirically relevant range of monetary policy shock persistence is two to three years (Gertler and Karadi, 2015; Christiano et al., 2005), the choice of $L = 96$ is a reasonable approximation of the history of shocks that matter for variation in output growth. In appendix B, I show that my results are robust to variations in the lag length $L$. All regressions are weighted using the CEX survey weights provided in the data sets.

Table 1 shows the estimated coefficient $\hat{\beta}_g^y$ for each education group, together with its standard error, which I cluster at the household level, and total sample size. The estimated coefficients are strong increasing with respect to education. A 1% increase in the growth of aggregate consumption caused by monetary policy shocks is associated with a 3.58% increase in the consumption growth of households with an advanced degree, but a 1.23% increase in the consumption growth of households with only a high-school diploma.

| | Less than High School | High School | Some College | Full College | Advanced Degree |
|---|---|---|---|---|---|
| $\hat{\beta}^y_g$ | 1.28 | 1.23 | 1.71 | 2.86 | 3.58 |
| s.e. | 1.23 | 0.79 | 0.71 | 0.77 | 1.12 |
| $n$ | 9,621 | 21,396 | 27,025 | 19,206 | 11,022 |

Table 1: Estimated $\left\{\hat{\beta}^y_g\right\}$ exposure coefficients across household groups with different education levels using monthly data over the period 1996-2008. Standard errors are clustered at the household level.

These results imply an estimate for the sensitivity ratio of $\frac{\bar{\beta}^y}{\underline{\beta}^y} \approx 2.9$. Therefore, the most sensitive households are approximately three times as sensitive to changes in aggregate consumption than the least sensitive households.

This finding is in line with previous studies of heterogeneous consumption sensitivities. For example, Parker and Vissing-Jorgensen (2009) group households in period $t$ by their consumption level in period $t-1$, and find a sensitivity ratio of 5. While this estimate is larger than the lower bound of 2.9, the grouping strategy fails the conditions in proposition 10 so that it likely yields a biased estimate the true sensitivity ratio.

The slight "U-shaped" pattern of sensitivities is also consistent with the evidence on heterogeneous income sensitivities. For example, Guvenen et al. (2017) run a similar regression using worker level income data and unconditional variation in GDP growth across percentiles of the permanent income distribution, and find a "U-shaped" pattern of sensitivities such that the highest and lowest permanent income workers are the most sensitive to unconditional changes in GDP growth (see figure 4). This finding supports the theory that borrowing constraints cause household consumption to inherit the sensitivity of household income to changes in output.

## 6.6 Quantitative Assessment

Given estimates for $\bar{\beta}^y$ and $\underline{\beta}^y$, the other key parameters in $\mathcal{R}$ are the slope of the NKPC, $\varphi_y$, and the coefficient in output in the Taylor rule, $\phi_y$. I set $\phi_y = 0$, which is in line with the existing literature that uses calibrated Taylor rules. In order to set $\varphi_y$, I appeal to the empirical evidence from the literatures on inflation forecasting and estimation of the NKPC.

Both of these literatures suggest that $\varphi_y$ is very small. The forecasting literature suggests that $\varphi_y = 0$ is very plausible (Atkeson and Ohanian, 2001), while the estimation literature tends to find $\varphi_y$ around 0.05, but with a decent dose of uncertainty (Schorfheide, 2008). Therefore, as a convenient benchmark, I set $\varphi_y = 0$.

When $\varphi_y = 0$, the asymmetry ratio is $\mathcal{R} = 2.9$. Therefore, the output response to a contractionary monetary policy shock is three times as large as the output response to an expansionary monetary policy shock of equal magnitude. I compare this asymmetry to the macro evidence for asymmetry in the next section.

For completeness, figure 2 plots $\mathcal{R}$ as a function of $\varphi_y$ using a standard calibration of the other parameters.[15] The ratio declines as $\kappa_y$ increases, but remains above 2.3 throughout the range, which covers the most plausible values of $\varphi_y$ away from zero.

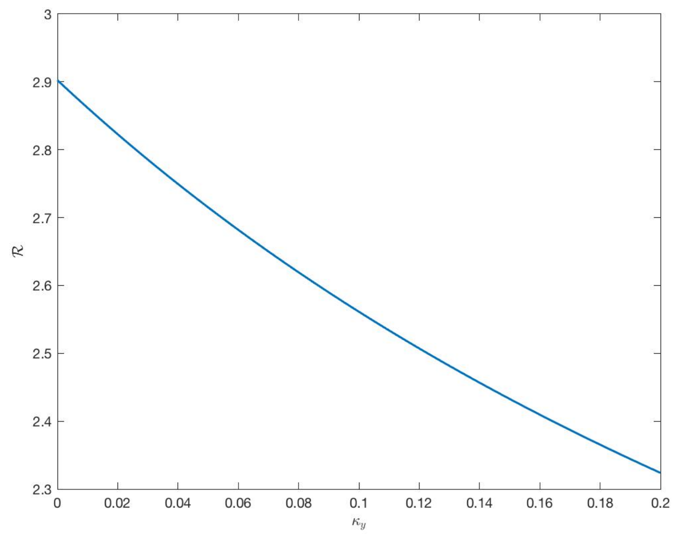


Figure 2: Asymmetry ratio $\mathcal{R}$ as a function of $\varphi_y$ when $\rho_v = 0.6$, $\phi_\pi = 1.25$, $\phi_y = 0$, $\sigma = 1.5$, and $\rho = 0.5\%$.

Intuitively, when output increases after an expansionary shock, $\varphi_y > 0$ implies that inflation also increases. Higher inflation causes high nominal rates via the Taylor rule, which offsets some of the initial expansionary shock. The same logic implies that $\varphi_y > 0$ causes deflation to offset the contractionary shock. Since the initial output response is larger for a contractionary shock, the offsetting force is larger too, which shrinks the overall asymmetry.

### 6.7 Empirical Evidence of Monetary Policy Asymmetry

The micro evidence on heterogeneous consumption sensitivities implies that contractionary monetary policy shocks are three times more powerful than expansionary monetary policy shocks. In this section, I show that this result is in line with the macro-econometric evidence for asymmetric

[15] I set $\rho_v = 0.6$ to reflect the quarterly persistence of monetary policy shocks estimated in the data (Christiano et al. (2005), Gertler and Karadi (2015)). I set $\phi_\pi = 1.25$ and $\phi_y = 0$, which is a commonly used specification for the Taylor rule, and set $\frac{1}{\sigma} = 0.67$ in line with estimates for the EIS (Vissing-Jorgensen, 2002). Finally, I set $\delta = 0.995$, which is consistent with an annual real interest rate of 2%.

monetary policy transmission. Specifically, I use local projection methods (Jorda, 2005) to demonstrate that contractionary monetary policy shocks are approximately four times more powerful than expansionary shocks.[16]

**Empirical Specification** I follow Jorda (2005), and estimate the impulse response of output to monetary policy shocks using local projection methods. Formally, I estimate the specification

$$y_{t+h} = \alpha^h + \beta^{h,+} \max\left\{\hat{\epsilon}_t^v, 0\right\} + \beta^{h,-} \min\left\{\hat{\epsilon}_t^v, 0\right\} + \sum_{l=0}^{L} \gamma_{y,l}^h y_{t-l} + \sum_{l=1}^{L} \gamma_{FFR,l}^h FFR_{t-l} + u_{t+h}^h$$

for horizons $h = 1, ..., H$. Here, $\{y_t\}$ is linearly de-trended output (in logs), $\{\hat{\epsilon}_t^v\}$ is the series of identified monetary policy shocks, and $\{FFR_t\}$ is the federal funds rate. The estimated coefficients $\left\{\hat{\beta}^{h,+}\right\}_1^H$ and $\left\{\hat{\beta}^{h,-}\right\}_1^H$ are the impulse responses of $y$ to positive and negative shocks of unit size respectively.

I use quarterly frequency data over the period 1969 - 2008. In order to be consistent with the microdata evidence, I use per-capita aggregate consumption of non-durables and services as my measure of output. I set $L = 1$, and note that the inclusion of contemporaneous aggregate consumption as a regressor is consistent with the convention that monetary policy shocks only affect measures of aggregate demand with a 1 period delay (Christiano et al., 1999). Finally, I estimate the system of equations over $h = 1, ..., H$ jointly, and compute Driscoll-Kraay (1998) standard errors that are robust to arbitrary serial and cross-sectional correlation across time and horizons.

**Results** Figure 3 plots the estimated impulse responses of output to contractionary (positive) and expansionary (negative) monetary policy shocks of 1% size over fifteen quarters. The dashed lines are 90% confidence intervals. For ease of comparison, I have multiplied the expansionary response by -1. Both impulse responses exhibit the "U-shape" that is a common feature of output responses to monetary policy shocks (Christiano et al., 1999).[17]

[16] The literature on asymmetric monetary policy goes back to at least Cover (1992) and DeLong and Summers (1988), who both find contractionary shocks are more powerful than expansionary shocks. More recently, Angrist et al. (2013), and Barnichon and Matthes (2016), introduce novel methodologies to measure asymmetric effects, and also find that contractionary monetary policy shocks are more powerful than expansionary shocks.

[17] Since my simple model does not contain ingredients such as consumption habits or investment frictions that are typically found in medium-scale DSGE models, it cannot generate the "hump-shaped" impulse responses found in the data.

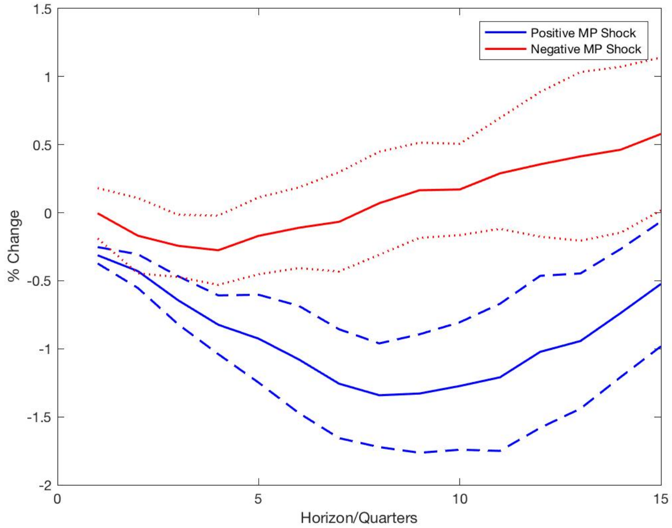


Figure 3: Impulse responses of aggregate consumption (from NIPA) estimated using local projection methods. The dashed lines are 90% confidence intervals computed using Driscoll-Kraay standard errors.

The contractionary shock generates a maximum response that is approximately four times as large as the maximum response to an expansionary shock. A simple t-test for point-wise equality of the responses confirms that the difference in responses is statistically significant over horizons of one to fours years (see figure 5 in the appendix).

The asymmetry is robust to alternative regressions specifications and sample restrictions. See the appendix for details.

As a simple metric of comparison, I compare the ratio of the maximum responses in the data to the ratio of responses in the model, $\mathcal{R}$. According to this metric, the asymmetry estimated in the macro data is reasonably consistent with the asymmetry implied by the micro-data. The fact that the sensitivity ratio implied by the micro data is a lower bound implies that a quantitative version of model can explain at least 60% of the asymmetry found in the macro data, and could plausibly explain much more if we can estimate the true exposure ratio at a more granular level of household heterogeneity than education.

# 7 Conclusion

When output falls in response to a contractionary aggregate shock, the decrease in consumption of unconstrained households is necessarily the smallest among all households. Therefore, the fall in output is greater than the response of unconstrained households alone. In contrast, when output increases in response to an expansionary aggregate shock, the increase in consumption of unconstrained households is necessarily the largest among all households. Therefore, the increase in output is smaller than the response of unconstrained households alone. Hence, output responds more to contractionary aggregate shocks than to expansionary shocks of equal magnitude.

The micro-data suggests that the largest sensitivity of household consumption to changes in output is approximately three times the size of the smallest sensitivity. When inflation is unresponsive to changes in output, output should respond three times as much to contractionary monetary policy shocks than to expansionary monetary policy shocks. This quantitative result can therefore explain at least 60% of the asymmetry found in the macro data directly.

My analysis provides evidence against linearizing DSGE models that feature household heterogeneity and incomplete markets, and suggests that care should be taken when selecting sufficient statistics to summarize the heterogeneity in a simple manner. By construction, linearization rules out asymmetric responses to aggregate shocks, even if the underlying non-linear model admits them by design, as my analysis suggests. Similarly, the evidence that MPCs are asymmetric implies that they are not sufficient statistics for household consumption behavior. In contrast, my analysis highlights alternative sufficient statistics that are estimable from similar data sources, and summarize key aspects of household heterogeneity while still allowing for non-linear responses of output to aggregate shocks.

Finally, it is natural to conjecture that the mechanism in this paper applies to any aggregate shock. It would therefore be interesting to investigate the asymmetric transmission of other aggregate shocks, and to see how well the model does at explaining the asymmetry.

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# A Additional Figures

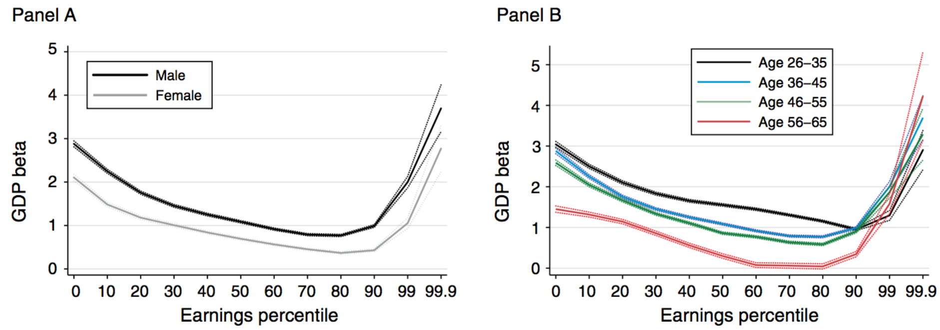


FIGURE 1. GDP BETA AT AGE 36–45 BY GENDER AND FOR MALES BY AGE GROUP

*Notes:* Earnings percentiles (conditional on gender and age group) are based on average real earnings from year $t-6$ to $t-2$, while real earnings growth used to estimate GDP beta is from year $t-1$ to $t$. The dotted lines represent the 95 percent confidence interval.

Figure 4: Reproduction of Figure 1 from Guvenen et al. (2017), which plots the $\beta_g$ coefficients from the pooled OLS regression $\Delta \log y_{i,t} = \alpha_g + \beta_g \Delta \log Y_t + u_{i,t}$, where $y_{i,t}$ is worker $i$'s income in period $t$ as reported on her W-2 form, $Y_t$ is GDP, and the groups $\{g\}$ are the gender-specific (Panel A) or age-specific (Panel B) percentiles of the permanent income distribution computed using incomes in periods $t-6$ to $t-2$.

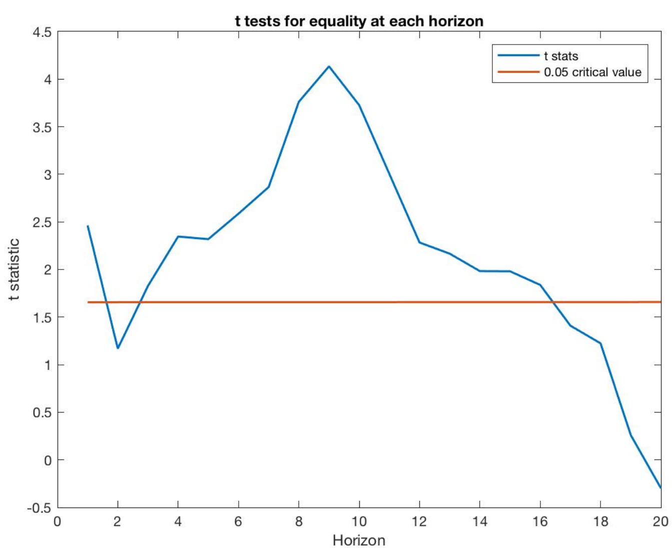


Figure 5: T statistics for testing $\hat{\beta}^{h,+} = \hat{\beta}^{h,-}$ at each horizon $h$.

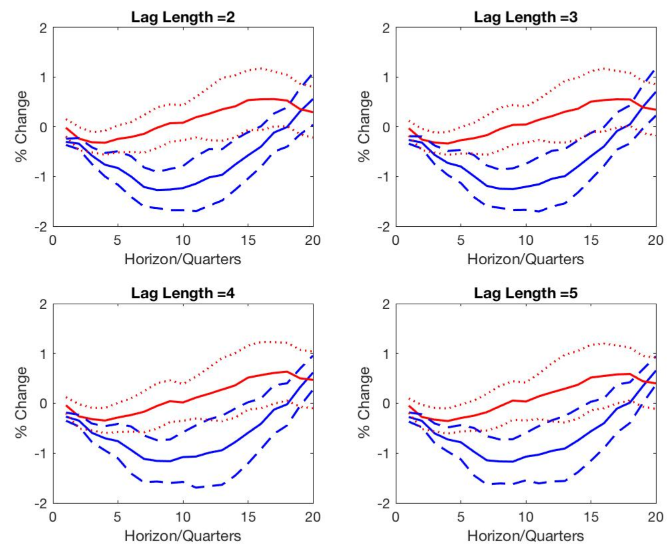


Figure 6: Impulse responses of real GDP (from NIPA) estimated using local projection methods and different lag structures.

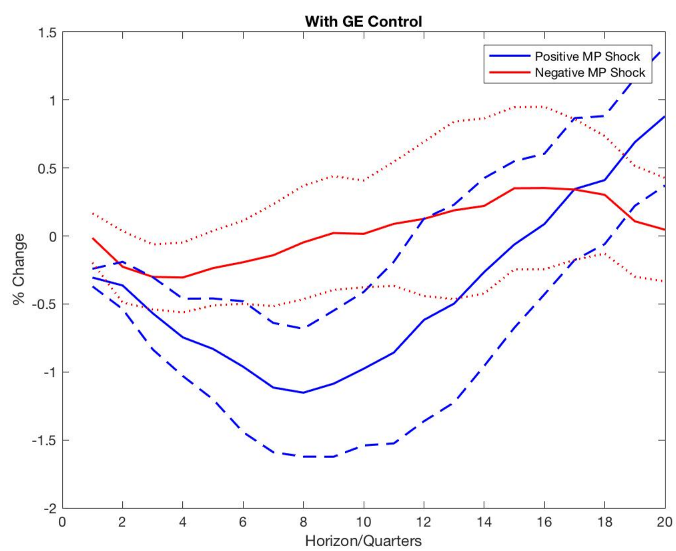


Figure 7: Impulse responses of real GDP (from NIPA) estimated using local projection methods with inflation as an additional control variable.

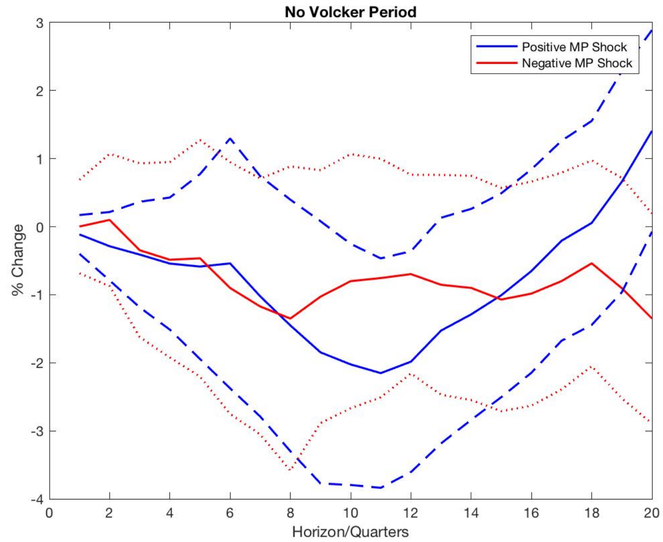


Figure 8: Impulse responses of real GDP (from NIPA) estimated using local projection methods, using only the post-Volcker sample.

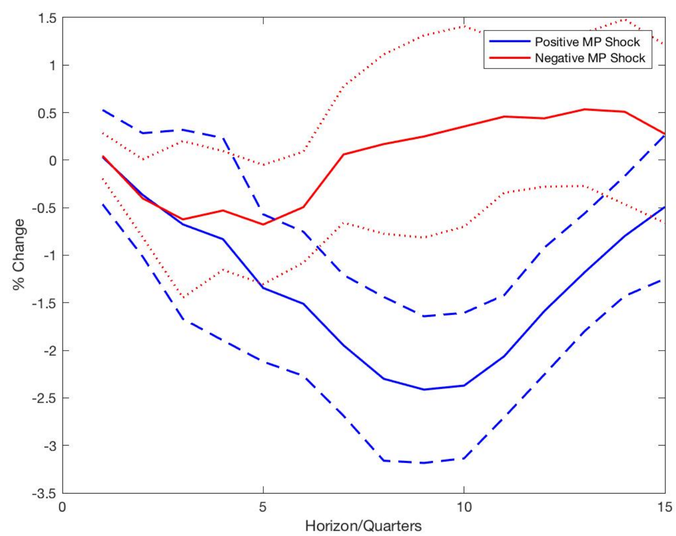


Figure 9: Impulse responses of real aggregate consumption of non-durables and services (from NIPA) estimated using local projection methods.

# B Robustness for CEX Regressions

| $\hat{\beta}_g$ | $L = 84$ | $\mathbf{L = 96}$ | $L = 108$ | $L = 120$ | $L = 132$ |
|---|---|---|---|---|---|
| Less than High School | 0.791 | 1.272 | 1.230 | 1.157 | 1.202 |
| | (1.332) | (1.227) | (1.206) | (1.185) | (1.180) |
| High School | 0.315 | 1.233 | 1.544** | 1.405* | 1.269* |
| | (0.856) | (0.785) | (0.761) | (0.747) | (0.744) |
| Some College | 1.057 | 1.711** | 1.991*** | 2.027*** | 2.070*** |
| | (0.774) | (0.713) | (0.684) | (0.669) | (0.659) |
| College | 2.822*** | 2.856*** | 2.803*** | 2.842*** | 2.648*** |
| | (0.838) | (0.774) | (0.753) | (0.743) | (0.740) |
| Advanced Degree | 4.176*** | 3.579*** | 3.623*** | 3.523*** | 3.628*** |
| | (1.240) | (1.123) | (1.078) | (1.066) | (1.066) |

Robust standard errors clustered at household level in parentheses
*** p<0.01, ** p<0.05, * p<0.1

# C Robustness for Local Projections

Here, I show that the asymmetric responses of output to monetary policy shocks are robust to regression specifications with different lag and control variable structures, sample restrictions that exclude the Volcker disinflation period, and when I change the dependent variable to GDP. All figures are in the appendix.

My baseline choice of $L = 1$ is optimal according to the Bayesian Information Criterion (BIC) given by

$$T \log \left(RSS/T\right) + k \log T$$

where $RSS$ is the residual sum of squares from the regressions and $T$ is the sample length. I also consider the Akaike Information Criterion (AIC), which is given by

$$T \log \left(RSS/T\right) + 2k$$

and also suggests an optimal choice of $L = 1$. Furthermore, figure B plots the impulse responses for $L \in \{2, 3, 4, 5\}$, and shows that the asymmetry is similar to the baseline specification in all cases.

The baseline regression includes aggregate demand and the federal funds rate as control variables. However, most New Keynesian models imply that inflation is also determined as part of the equilibrium system, and so affects the path of aggregate demand. To this end, figure 7 plots the impulse responses with inflation (measured by the Personal Consumption Expenditure deflator) as an additional control variable that follows the same lag structure as aggregate demand. The asymmetry is essentially unchanged.

It is well known that the Volcker disinflation period in the early 1980s resulted in volatile monetary policy, as exhibited by the large shocks in figure 1. While these shocks provide useful variation in the explanatory variable, it is useful to check that they are not the driving force behind the result. Therefore, in figure 8 I plot the impulse responses from the baseline regression having restricted the sample to 1985Q1 onwards, thus dropping the entire Volcker episode. While the smaller sample results in much wider confidence intervals, the asymmetry is still clear to see, with contractionary shocks having twice the effect of expansionary shocks. Note that in this case, the micro evidence can explain all of the asymmetry.

Finally, I run the baseline regression with real GDP as the dependent variable instead of aggregate consumption. Figure 9 plots the impulse responses, which exhibit similar levels of asymmetry, although they are slightly more noisily estimated.

# D Proofs

**Proof of Lemma 1** I begin by characterizing the solution to the household problem. Since households within an island can trade a full of Arrow securities, and cannot trade between islands in equilibrium, we can solve for consumption and the implied security prices using a sequence of static planning problems, as in Heathcote et al. (2014). Let $I^{\omega} = \{i : i \in \omega\}$ denote the set of households located on island $\omega$. In period $t$, the island-level planner solves the static problem

$$\max \int_{I^{\omega}} \frac{c_{i,t}^{1-\sigma}}{1-\sigma} di$$

subject to

$$\int_{I^{\omega}} \left(w_t \theta_i \exp\left(\eta_t^{\omega}\right) \exp\left(\kappa_{i,t} + \epsilon_{i,t}\right) + s_i d_t - c_{i,t}\right) di \geq 0$$

Attaching multiplier $\lambda^{\omega}$ to the constraint, the FOCs yield

$$c_{i,t} = \left(\lambda^{\omega}\right)^{-\frac{1}{\sigma}}$$

and

$$\int_{I^{\omega}} \left(w_t \theta_i \exp\left(\eta_t^{\omega}\right) \exp\left(\kappa_{i,t} + \epsilon_{i,t}\right) + s_i d_t - c_{i,t}\right) di = 0$$

so that

$$\left(\lambda^{\omega}\right)^{-\frac{1}{\sigma}} \int_{I^{\omega}} di = w_t \exp\left(\eta_t^{\omega}\right) \int_{I^{\omega}} \theta_i di \int_{I^{\omega}} \exp\left(\kappa_{i,t} + \epsilon_{i,t}\right) di + d_t \int_{I^{\omega}} s_i di$$

By group construction, $\int_{I^{\omega}} \exp\left(\kappa_{i,t} + \epsilon_{i,t}\right) di = 1$ so that household consumption on island $\omega$ is given by

$$c_{i,t} = w_t \exp\left(\eta_t^{\omega}\right) \theta^{\omega} + s^{\omega} d_t$$

for all $i \in I^{\omega}$, where

$$\theta^{\omega} = \frac{\int_{I^{\omega}} \theta_i di}{\int_{I^{\omega}} di}$$

$$s^{\omega} = \frac{\int_{I^{\omega}} s_i di}{\int_{I^{\omega}} di}$$

In order to decentralize this planning solution, we need to verify prices for the Arrow securities, and back out the implied asset positions of households. We can use the household FOCs to back out the implied Arrow security prices. Attaching multiplier $\lambda\left(\zeta_{i,t}\right)$ to the flow budget constraint in the household problem yields the FOCs

$$\left(\frac{1}{1+\rho}\right)^{t-1} c_{i,t}^{-\sigma} = \lambda\left(\zeta_{i,t}^{\omega}\right)$$

$$q_t^{\omega}\left(\zeta_{t+1}^{\omega}; \zeta_{i,t}^{\omega}\right) = \frac{\lambda\left(\zeta_{t+1}^{\omega}\right)}{\lambda\left(\zeta_{i,t}^{\omega}\right)} \Pr\left(\zeta_{t+1}^{\omega} | \zeta_{i,t}^{\omega}\right)$$

so that

$$q_t^{\omega}\left(\zeta_{t+1}^{\omega}; \zeta_{i,t}^{\omega}\right) = \frac{1}{1+\rho} \left(\frac{c_{t+1}}{c_{i,t}}\right)^{-\sigma} \Pr\left(\zeta_{t+1}^{\omega} | \zeta_{i,t}^{\omega}\right)$$

Now use

$$c_{i,t} = w_t \exp\left(\eta_t^{\omega}\right) \theta^{\omega} + s^{\omega} d_t$$

to get

$$q_t^\omega\left(\zeta_{t+1}^\omega;\zeta_{i,t}^\omega\right)=\frac{1}{1+\rho}\left(\frac{w_{t+1}\exp\left(\eta_{t+1}^\omega\right)\theta^\omega+s^\omega d_{t+1}}{w_t\exp\left(\eta_t^\omega\right)\theta^\omega+s^\omega d_t}\right)^{-\sigma}\Pr\left(\zeta_{t+1}^\omega|\zeta_{i,t}^\omega\right)$$

Now we can back out the asset positions $\left\{B_{i,t}\left(\zeta_{i,t+1}^\omega\right)\right\}$ that implement the consumption path chosen by the planner. Substituting consumption into the flow budget constraint and setting $b_{i,t}=0$ yields an expression for realized wealth at node $\zeta_{i,t}^\omega$,

$$B_{i,t-1}\left(\zeta_{i,t}^\omega\right)=w_t\exp\left(\eta_t^\omega\right)\left(\theta^\omega-\theta_i\exp\left(\kappa_{i,t}+\epsilon_{i,t}\right)\right)+\left(s^\omega-s_i\right)d_t+\int q_t^\omega\left(\zeta_{t+1}^\omega;\zeta_{i,t}^\omega\right)B_{i,t}\left(\zeta_{t+1}^\omega\right)d\zeta_{t+1}^\omega$$

Define

$$T_{i,t}\left(\zeta_{i,t}^\omega\right)=w_t\exp\left(\eta_t^\omega\right)\left(\theta^\omega-\theta_i\exp\left(\kappa_{i,t}+\epsilon_{i,t}\right)\right)+\left(s^\omega-s_i\right)d_t$$

Using the expressions for the prices $\left\{q_t^\omega\left(\zeta_{t+1}^\omega\right)\right\}$, this can be solved recursively to yield

$$B_{i,t}\left(\zeta_{i,t+1}^\omega\right)=T_{i,t+1}\left(\zeta_{i,t+1}^\omega\right)+\sum_{j=1}^{\infty}\int\left(\frac{1}{1+\rho}\right)^j\left(\frac{c_{i,t+1+j}}{c_{i,t+1}}\right)^{-\sigma}T_{t+1+j}\left(\zeta_{t+1+j}^\omega\right)\Pr\left(\zeta_{t+1+j}^\omega|\zeta_{i,t+1}^\omega\right)d\zeta_{t+1+j}^\omega$$

$$B_{i,0}\left(\zeta_{i,1}^\omega\right)=T_{i,1}\left(\zeta_{i,1}^\omega\right)+\sum_{j=1}^{\infty}\int\left(\frac{1}{1+\rho}\right)^j\left(\frac{w_{1+j}\exp\left(\eta_{1+j}^\omega\right)\theta^\omega+s^\omega d_{1+j}}{w_1\exp\left(\eta_1^\omega\right)\theta^\omega+s^\omega d_1}\right)^{-\sigma}T_{1+j}\left(\zeta_{t+j}^\omega\right)\Pr\left(\zeta_{1+j}^\omega|\zeta_{i,1}^\omega\right)d\zeta_{1+j}^\omega$$

which ensure that household budget constraints are satisfied as required.

We can now characterize the aggregate quantities and prices in the stationary equilibrium.

In the stationary equilibrium, island level per-capita consumption is given by

$$c_{\omega,t}=w\exp\left(\eta_t^\omega\right)\theta^\omega+s^\omega d$$

with security prices

$$q_t^\omega\left(\zeta_{i,t+1}^\omega\right)=\frac{1}{1+\rho}\left(\frac{w\exp\left(\eta_{t+1}^\omega\right)\theta^\omega+s^\omega d}{w\exp\left(\eta_t^\omega\right)\theta^\omega+s^\omega d}\right)^{-\sigma}\Pr\left(\zeta_{i,t+1}^\omega\right)$$

The FOC of the intermediate goods producer implies that the wage is given by

$$w=\left(\alpha^\alpha\left(1-\alpha\right)^{1-\alpha}\bar{A}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{1}{1-\alpha}}$$

so that dividends satisfy

$$d=X\left(1-\frac{1}{\alpha^\alpha\left(1-\alpha\right)^{1-\alpha}}\frac{w^{1-\alpha}}{\bar{A}}\right)$$

From the cost-minimization problem, we have

$$w^{1-\alpha}=\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\bar{A}^{-\frac{1-\alpha}{\alpha}}X^{\frac{1-\alpha}{\alpha}}$$

so that

$$d=X-\frac{1}{\alpha}\bar{A}^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}$$

and

$$Y=w+d=X-\bar{A}^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}$$

Finally, combining

$$w=\left(\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\bar{A}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{1}{1-\alpha}}$$

and

$$w^{1-\alpha}=\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\bar{A}^{-\frac{1-\alpha}{\alpha}}X^{\frac{1-\alpha}{\alpha}}$$

yields

$$X=\left(\alpha\bar{A}^{\frac{1}{\alpha}}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{\alpha}{1-\alpha}}$$

$$Y=\left(\alpha\bar{A}^{\frac{1}{\alpha}}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\alpha\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)>0$$

$$d=\left(\alpha\bar{A}^{\frac{1}{\alpha}}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\alpha\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)-\left(\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\bar{A}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{1}{1-\alpha}}$$

Hence we have

$$M=\frac{\left(\alpha\bar{A}^{\frac{1}{\alpha}}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\bar{A}}\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\bar{A}\frac{\bar{\Phi}-1}{\bar{\Phi}}$$

$$E=\frac{1-\alpha}{\alpha}\frac{\left(\alpha\bar{A}^{\frac{1}{\alpha}}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\bar{A}}\left(\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\bar{A}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{-\frac{\alpha}{1-\alpha}}=1$$

$$c_{\omega,t}=\left(\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\bar{A}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{1}{1-\alpha}}\exp\left(\eta_t^{\omega}\right)\theta^{\omega}+s^{\omega}\left(\left(\alpha\bar{A}^{\frac{1}{\alpha}}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\alpha\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)-\left(\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\bar{A}\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{1}{1-\alpha}}\right)$$

Finally, the real interest rate on the inter-island bond is such that the Euler equation holds with weak inequality,

$$c_{\omega,t}^{-\sigma}\geq\frac{1+r}{1+\rho}\mathbb{E}_t\left[c_{\omega,t+1}^{-\sigma}\right]$$

In equilibrium, the Euler equation of households with the strong saving motive will hold with equality so that

$$1=\frac{1+r}{1+\rho}\max_{\omega}\frac{1}{c_{\omega,t}^{-\sigma}}\mathbb{E}_t\left[c_{\omega,t+1}^{-\sigma}\right]$$

which yields

$$1+r=\frac{1+\rho}{\max_{\omega}\mathbb{E}_t\left[\left(\frac{w\exp\left(\eta_{t+1}^{\omega}\right)\theta^{\omega}+s^{\omega}d}{w\exp\left(\eta_t^{\omega}\right)\theta^{\omega}+s^{\omega}d}\right)^{-\sigma}\right]}$$

thus establishing the existence of a unique stationary equilibrium. □

**Proof of Proposition 1** The first equation is derived by taking logs of the Taylor rule, and using the approximation $\log(1+x)\approx x$.

To derive the linearized NKPC, start with the FOC of firm $j$,

$$\left(1-\Phi_t\right)p_t\left(j\right)^{-\Phi_t}P_t^{\Phi_t}X_t+\Phi_tP_t^{1+\Phi_t}\frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}w_t^{1-\alpha}\frac{1}{A_t}p_t\left(j\right)^{-\Phi_t-1}X_t-\xi^p\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}-1\right)\frac{1}{p_{t-1}\left(j\right)}P_tX_t+$$

$$+\frac{1}{1+\rho}\xi^p\mathbb{E}_t\left[\left(\frac{p_{t+1}\left(j\right)}{p_t\left(j\right)}-1\right)\frac{p_{t+1}\left(j\right)}{p_t\left(j\right)^2}P_{t+1}X_{t+1}\right]=0$$

Impose symmetry: $p_t(j) = P_t$, $x_t(j) = X_t$, to get

$$(1-\Phi_t) + \Phi_t \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} w_t^{1-\alpha} \frac{1}{A_t} - \xi^p \pi_t (1+\pi_t) + \frac{1}{1+\rho} \xi^p \mathbb{E}_t \left[ \pi_{t+1} (1+\pi_{t+1}) \frac{P_{t+1} X_{t+1}}{P_t X_t} \right] = 0$$

Recall from the cost-minimization that

$$E_t(j) = \frac{1-\alpha}{\alpha} \frac{x_t(j)}{\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} A_t w_t^\alpha}$$

so that imposing $E_t(j) = 1$ yields

$$w_t = \frac{1-\alpha}{\alpha} A_t^{-\frac{1}{\alpha}} X_t^{\frac{1}{\alpha}}$$

and

$$w_t^{1-\alpha} = \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} A_t^{-\frac{1-\alpha}{\alpha}} X_t^{\frac{1-\alpha}{\alpha}}$$

Hence

$$M_t(j) = \frac{x_t(j)}{\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} A_t w_t^{\alpha-1}}$$

becomes

$$M_t = A_t^{-\frac{1}{\alpha}} X_t^{\frac{1}{\alpha}}$$

so that all firms demand the same composite intermediate good, which also equals total demand for composite intermediate goods. Therefore,

$$Y_t = X_t - A_t^{-\frac{1}{\alpha}} X_t^{\frac{1}{\alpha}}$$

Using

$$w_t^{1-\alpha} = \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} A_t^{-\frac{1-\alpha}{\alpha}} X_t^{\frac{1-\alpha}{\alpha}}$$

the NKPC becomes

$$\pi_t (1+\pi_t) = \frac{1-\Phi_t}{\xi^p} + \frac{\Phi_t}{\xi^p} \frac{1}{\alpha} A_t^{-\frac{1}{\alpha}} X_t^{\frac{1-\alpha}{\alpha}} + \frac{1}{1+\rho} \mathbb{E}_t \left[ \pi_{t+1} (1+\pi_{t+1}) \frac{P_{t+1} X_{t+1}}{P_t X_t} \right]$$

which has log linearization steps

$$\frac{\Phi_t}{\xi^p} \frac{1}{\alpha} A_t^{-\frac{1}{\alpha}} X_t^{\frac{1-\alpha}{\alpha}} \approx \frac{\bar\Phi}{\xi^p} \frac{1}{\alpha} \bar A^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} + \frac{1}{\xi^p} \frac{1}{\alpha} \bar A^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} \left(\Phi_t - \bar\Phi\right) - \frac{1}{\alpha} \frac{\bar\Phi}{\xi^p} \frac{1}{\alpha} \bar A^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} \hat a_t + \frac{1-\alpha}{\alpha} \frac{\bar\Phi}{\xi^p} \frac{1}{\alpha} \bar A^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} \hat x_t$$

$$\pi_t = \frac{\bar\Phi}{\xi^p} \left( \frac{1}{\alpha} \bar A^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} - 1 \right) \left( \log \Phi_t - \log \bar\Phi \right) - \frac{\bar\Phi}{\xi^p} \frac{1}{\alpha} \frac{1}{\alpha} \bar A^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} \hat a_t + \frac{\bar\Phi}{\xi^p} \frac{1-\alpha}{\alpha} \frac{1}{\alpha} \bar A^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} \hat x_t + \delta \mathbb{E}_t [\pi_{t+1}]$$

where

$$\frac{1}{\alpha} \bar A^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} = \frac{\bar\Phi - 1}{\bar\Phi}$$

so that

$$\pi_t = \frac{\bar\Phi - 1}{\xi^p} \frac{1-\alpha}{\alpha} \hat x_t - \frac{1}{\xi^p} \hat\Phi_t - \frac{\bar\Phi - 1}{\xi^p} \frac{1}{\alpha} \hat a_t + \frac{1}{1+\rho} \mathbb{E}_t [\pi_{t+1}]$$

To replace $\hat x_t$ by $\hat y_t$, use

$$Y_t = X_t - A_t^{-\frac{1}{\alpha}} X_t^{\frac{1}{\alpha}}$$

which has log-linearization

$$\hat{x}_t = \frac{X - A^{-\frac{1}{\alpha}} X^{\frac{1}{\alpha}}}{X - A^{-\frac{1}{\alpha}} \frac{1}{\alpha} X^{\frac{1}{\alpha}}} \hat{y}_t - \frac{\frac{1}{\alpha} A^{-\frac{1}{\alpha}} X^{\frac{1}{\alpha}}}{X - A^{-\frac{1}{\alpha}} \frac{1}{\alpha} X^{\frac{1}{\alpha}}} \hat{a}_t$$

Substitution into the NKPC yields

$$\pi_t = \frac{\bar{\Phi}}{\xi^p} \left( \frac{1}{\alpha} \bar{A}^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} - 1 \right) \hat{\Phi}_t - \frac{\bar{\Phi}}{\xi^p} \frac{1}{\alpha} \frac{1}{\alpha} \bar{A}^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} \hat{a}_t +$$

$$+ \frac{\bar{\Phi}}{\xi^p} \frac{1-\alpha}{\alpha} \frac{1}{\alpha} \bar{A}^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} \left( \frac{X - A^{-\frac{1}{\alpha}} X^{\frac{1}{\alpha}}}{X - A^{-\frac{1}{\alpha}} \frac{1}{\alpha} X^{\frac{1}{\alpha}}} \hat{y}_t - \frac{\frac{1}{\alpha} A^{-\frac{1}{\alpha}} X^{\frac{1}{\alpha}}}{X - A^{-\frac{1}{\alpha}} \frac{1}{\alpha} X^{\frac{1}{\alpha}}} \hat{a}_t \right) + \frac{1}{1+\rho} \mathbb{E}_t \left[ \pi_{t+1} \right]$$

$$\pi_t = \frac{\bar{\Phi}}{\xi^p} \frac{1-\alpha}{\alpha} \frac{1}{\alpha} \bar{A}^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} \frac{X - A^{-\frac{1}{\alpha}} X^{\frac{1}{\alpha}}}{X - A^{-\frac{1}{\alpha}} \frac{1}{\alpha} X^{\frac{1}{\alpha}}} \hat{y}_t + \frac{\bar{\Phi}}{\xi^p} \left( \frac{1}{\alpha} \bar{A}^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} - 1 \right) \hat{\Phi}_t -$$

$$- \left( \frac{\bar{\Phi}}{\xi^p} \frac{1}{\alpha} \frac{1}{\alpha} \bar{A}^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} + \frac{\bar{\Phi}}{\xi^p} \frac{1-\alpha}{\alpha} \frac{1}{\alpha} \bar{A}^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} \frac{\frac{1}{\alpha} A^{-\frac{1}{\alpha}} X^{\frac{1}{\alpha}}}{X - A^{-\frac{1}{\alpha}} \frac{1}{\alpha} X^{\frac{1}{\alpha}}} \right) \hat{a}_t + \frac{1}{1+\rho} \mathbb{E}_t \left[ \pi_{t+1} \right]$$

where using

$$\frac{1}{\alpha} \bar{A}^{-\frac{1}{\alpha}} X^{\frac{1-\alpha}{\alpha}} = \frac{\bar{\Phi} - 1}{\bar{\Phi}}$$

$$X = \bar{A}^{\frac{1}{1-\alpha}} \left( \alpha \frac{\bar{\Phi} - 1}{\bar{\Phi}} \right)^{\frac{\alpha}{1-\alpha}}$$

$$X^{\frac{1}{\alpha}} = \bar{A}^{\frac{1}{\alpha(1-\alpha)}} \left( \alpha \frac{\bar{\Phi} - 1}{\bar{\Phi}} \right)^{\frac{1}{1-\alpha}}$$

yields

$$\pi_t = \frac{\bar{\Phi} - 1}{\xi^p} \frac{1-\alpha}{\alpha} \left( \bar{\Phi} - \alpha \left( \bar{\Phi} - 1 \right) \right) \hat{y}_t - \frac{\bar{\Phi} - 1}{\xi^p} \frac{1}{\alpha} \left( \alpha + (1-\alpha) \bar{\Phi} \right) \hat{a}_t - \frac{1}{\xi^p} \hat{\Phi}_t + \frac{1}{1+\rho} \mathbb{E}_t \left[ \pi_{t+1} \right]$$

Hence the NKPC is given by

$$\pi_t = \varphi_y \hat{y}_t - \varphi_a \hat{a}_t - \varphi_\Phi \hat{\Phi}_t + \frac{1}{1+\rho} \mathbb{E}_t \left[ \pi_{t+1} \right]$$

where

$$\varphi_y = \frac{\bar{\Phi} - 1}{\xi^p} \frac{1-\alpha}{\alpha} \left( \bar{\Phi} - \alpha \left( \bar{\Phi} - 1 \right) \right)$$

$$\varphi_a = \frac{\bar{\Phi} - 1}{\xi^p} \frac{1}{\alpha} \left( \alpha + (1-\alpha) \bar{\Phi} \right)$$

$$\varphi_\Phi = \frac{1}{\xi^p}$$

In equilibrium, we can express consumption of household $i$ located on island $\omega$ as

$$c_{\omega,t} = w_t \exp\left( \eta_t^\omega \right) \theta^\omega + s^\omega d_t$$

where

$$w_t^{1-\alpha} = \left( \frac{1-\alpha}{\alpha} \right)^{1-\alpha} A_t^{-\frac{1-\alpha}{\alpha}} X_t^{\frac{1-\alpha}{\alpha}}$$

$$d_t = X_t - \left( \frac{1}{\alpha} \right) A_t^{-\frac{1}{\alpha}} X_t^{\frac{1}{\alpha}} - \frac{\xi^p}{2} \pi_t^2 X_t$$

so that

$$c_{\omega,t}=\left(\frac{1-\alpha}{\alpha}\theta^{\omega}\exp\left(\eta_t^{\omega}\right)-\frac{1}{\alpha}s^{\omega}\right)A_t^{-\frac{1}{\alpha}}X_t^{\frac{1}{\alpha}}+s^{\omega}X_t-s^{\omega}\frac{\xi^p}{2}\pi_t^2X_t$$

This expression for equilibrium consumption can be log linearized in terms of $\hat{x}_t$, $\hat{a}_t$, and $\eta_t^{\omega}$,

$$c_{\omega,t}=\beta_{\omega}^{x}\hat{x}_t+\beta_{\omega}^{a,x}\hat{a}_t+\beta_{\omega}^{\eta}\eta_t^{\omega}$$

where

$$\beta_{\omega}^{x}=\frac{\left(\left(\frac{1-\alpha}{\alpha}\right)\theta^{\omega}-\left(\frac{1}{\alpha}\right)s^{\omega}\right)\bar{A}^{-\frac{1}{\alpha}}\frac{1}{\alpha}X^{\frac{1}{\alpha}}+s^{\omega}X}{\left(\left(\frac{1-\alpha}{\alpha}\right)\theta^{\omega}-\left(\frac{1}{\alpha}\right)s^{\omega}\right)\bar{A}^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}+s^{\omega}X}$$

$$\beta_{\omega}^{a,x}=-\frac{\left(\left(\frac{1-\alpha}{\alpha}\right)\theta^{\omega}-\left(\frac{1}{\alpha}\right)s^{\omega}\right)X^{\frac{1}{\alpha}}\frac{1}{\alpha}\bar{A}^{-\frac{1}{\alpha}}}{\left(\left(\frac{1-\alpha}{\alpha}\right)\theta^{\omega}-\left(\frac{1}{\alpha}\right)s^{\omega}\right)\bar{A}^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}+s^{\omega}X}$$

$$\beta_{\omega}^{\eta}=\frac{\left(\frac{1-\alpha}{\alpha}\right)\theta^{\omega}\bar{A}^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}}{\left(\left(\frac{1-\alpha}{\alpha}\right)\theta^{\omega}-\left(\frac{1}{\alpha}\right)s^{\omega}\right)\bar{A}^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}+s^{\omega}X}$$

Using

$$\frac{1}{\alpha}\bar{A}^{-\frac{1}{\alpha}}X^{\frac{1-\alpha}{\alpha}}=\frac{\bar{\Phi}-1}{\bar{\Phi}}$$

$$X=\bar{A}^{\frac{1}{1-\alpha}}\left(\alpha\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\bar{A}^{-\frac{1}{\alpha}}\frac{1}{\alpha}X^{\frac{1}{\alpha}}=\bar{A}^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{\bar{\Phi}-1}{\bar{\Phi}}\right)^{\frac{1}{1-\alpha}}$$

we get

$$\beta_{\omega}^{x}=\frac{\left(\frac{1-\alpha}{\alpha}\theta^{\omega}-\frac{1}{\alpha}s^{\omega}\right)\frac{\bar{\Phi}-1}{\bar{\Phi}}+s^{\omega}}{\left(\frac{1-\alpha}{\alpha}\theta^{\omega}-\frac{1}{\alpha}s^{\omega}\right)\frac{\bar{\Phi}-1}{\bar{\Phi}}\alpha+s^{\omega}}$$

$$\beta_{\omega}^{a,x}=-\frac{\left(\frac{1-\alpha}{\alpha}\theta^{\omega}-\frac{1}{\alpha}s^{\omega}\right)\frac{\bar{\Phi}-1}{\bar{\Phi}}}{\left(\frac{1-\alpha}{\alpha}\theta^{\omega}-\frac{1}{\alpha}s^{\omega}\right)\frac{\bar{\Phi}-1}{\bar{\Phi}}\alpha+s^{\omega}}$$

$$\beta_{\omega}^{\eta}=\frac{\left(1-\alpha\right)\theta^{\omega}\frac{\bar{\Phi}-1}{\bar{\Phi}}}{\left(\frac{1-\alpha}{\alpha}\theta^{\omega}-\frac{1}{\alpha}s^{\omega}\right)\frac{\bar{\Phi}-1}{\bar{\Phi}}\alpha+s^{\omega}}$$

Then log linearize

$$Y_t=X_t-A_t^{-\frac{1}{\alpha}}X_t^{\frac{1}{\alpha}}$$

to obtain

$$\hat{y}_t=\frac{X-A^{-\frac{1}{\alpha}}\frac{1}{\alpha}X^{\frac{1}{\alpha}}}{X-A^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}}\hat{x}_t+\frac{\frac{1}{\alpha}A^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}}{X-A^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}}\hat{a}_t$$

which can be replaced in the previous linearization,

$$\hat{x}_t=\frac{X-A^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}}{X-A^{-\frac{1}{\alpha}}\frac{1}{\alpha}X^{\frac{1}{\alpha}}}\hat{y}_t-\frac{\frac{1}{\alpha}A^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}}{X-A^{-\frac{1}{\alpha}}\frac{1}{\alpha}X^{\frac{1}{\alpha}}}\hat{a}_t$$

to obtain

$$\hat{c}_{\omega,t}=\beta_{\omega}^{y}\hat{y}_t+\beta_{\omega}^{a}\hat{a}_t+\beta_{\omega}^{\eta}\eta_t$$

where

$$\beta_{\omega}^{y}=\beta_{\omega}^{x}\frac{X-A^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}}{X-A^{-\frac{1}{\alpha}}\frac{1}{\alpha}X^{\frac{1}{\alpha}}}=\frac{1}{\alpha}\frac{\left(1-\alpha\right)\left(\bar{\Phi}-1\right)\theta^{\omega}+s^{\omega}\left(1-\left(1-\alpha\right)\bar{\Phi}\right)}{\left(1-\alpha\right)\left(\bar{\Phi}-1\right)\theta^{\omega}+s^{\omega}}\left(\bar{\Phi}-\alpha\left(\bar{\Phi}-1\right)\right)=$$

$$= \frac{1}{\alpha}\left(1-\frac{s^{\omega}\left(1-\alpha\right)\bar{\Phi}}{\left(1-\alpha\right)\left(\bar{\Phi}-1\right)\theta^{\omega}+s^{\omega}}\right)\left(\bar{\Phi}-\alpha\left(\bar{\Phi}-1\right)\right)$$

$$\beta_{\omega}^{a} = \beta_{\omega}^{a,x}-\beta_{\omega}^{x}\frac{\frac{1}{\alpha}A^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}}{X-A^{-\frac{1}{\alpha}}\frac{1}{\alpha}X^{\frac{1}{\alpha}}} = \left(\bar{\Phi}-1\right)\frac{\bar{\Phi}\left(\frac{1-\alpha}{\alpha}\right)\left(s^{\omega}-\theta^{\omega}\right)}{\left(1-\alpha\right)\left(\bar{\Phi}-1\right)\theta^{\omega}+s^{\omega}}$$

$$\beta_{\omega}^{\eta} = \frac{\left(\frac{1-\alpha}{\alpha}\right)\theta^{\omega}\bar{A}^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}}{\left(\left(\frac{1-\alpha}{\alpha}\right)\theta^{\omega}-\left(\frac{1}{\alpha}\right)s^{\omega}\right)\bar{A}^{-\frac{1}{\alpha}}X^{\frac{1}{\alpha}}+s^{\omega}X} = \frac{\left(1-\alpha\right)\left(\bar{\Phi}-1\right)\theta^{\omega}}{\left(1-\alpha\right)\left(\bar{\Phi}-1\right)\theta^{\omega}+s^{\omega}}$$

Euler equation: To derive this equation, note that the Euler equation for household $i$ is given by

$$c_{i,t}^{-\sigma} \geq \frac{1}{1+\rho}\mathbb{E}_t\left[c_{i,t+1}^{-\sigma}\left(1+r_{t+1}\right)\right]$$

where the inequality is strict if the borrowing constraint binds.

Therefore, the Euler equation features a "distortion" only if household $i$ would like to borrow in equilibrium,

$$c_{i,t}^{-\sigma} > \frac{1}{1+\rho}\mathbb{E}_t\left[c_{i,t+1}^{-\sigma}\left(1+r_{t+1}\right)\right] \Rightarrow b_{i,t}=0$$

Hence, in equilibrium, there exists a household $i^*\left(t\right)$ such that

$$1 = \frac{1}{1+\rho}\mathbb{E}_t\left[\left(1+r_{t+1}\right)\left(\frac{c_{i^*(t),t+1}}{c_{i^*(t),t}}\right)^{-\sigma}\right] \geq \delta\mathbb{E}_t\left[\left(1+r_{t+1}\right)\left(\frac{c_{i,t+1}}{c_{i,t}}\right)^{-\sigma}\right]$$

for all $i \neq i^*\left(t\right)$. Hence $i^*\left(t\right)$ satisfies

$$i^*\left(t\right) \in \arg\max_{i}\mathbb{E}_t\left[\left(1+r_{t+1}\right)\left(\frac{c_{i,t+1}}{c_{i,t}}\right)^{-\sigma}\right]$$

The aggregate Euler equation is therefore given by

$$1 = \frac{1}{1+\rho}\max_{\omega}\mathbb{E}_t\left[\left(1+r_{t+1}\right)\left(\frac{c_{\omega,t+1}}{c_{\omega,t}}\right)^{-\sigma}\right]$$

Taking logs and using the first-order approximation $\log\mathbb{E}_t\left[x_{t+1}\right] \approx \mathbb{E}_t\left[\log x_{t+1}\right]$ yields

$$0 = \log\frac{1}{1+\rho}+\max_{\omega}\left\{\mathbb{E}_t\left[\log\left(1+r_{t+1}\right)\right]-\sigma\mathbb{E}_t\left[\log\left(\frac{c_{\omega,t+1}}{c_{\omega,t}}\right)\right]\right\}$$

Using the approximation $\log\left(1+r\right) \approx r$ together with the definition of the real interest rate simplifies the equation to

$$0 = \iota_t-\mathbb{E}_t\left[\pi_{t+1}\right]-\rho-\sigma\min_{\omega}\left\{\mathbb{E}_t\left[\log c_{\omega,t+1}\right]-\log c_{\omega,t}\right\}$$

where I have also used the fact that

$$\max_{\omega}\left\{-X_{\omega}\right\} = -\min_{\omega}\left\{X_{\omega}\right\}$$

Using the log linearization $\hat{c}_{\omega,t}$ to obtain

$$\min_{\omega}\left\{\mathbb{E}_t\left[\hat{c}_{\omega,t+1}\right]-\hat{c}_{\omega,t}\right\} = \frac{1}{\sigma}\left(\iota_t-\mathbb{E}_t\left[\pi_{t+1}\right]-\rho\right)$$

thus completing the derivation of the four equation system.

In the absence of aggregate shocks, the steady state of the system satisfies

$$\iota = r + \phi_\pi \pi + \phi_y \hat{y}$$

$$\pi = \varphi_y \hat{y} + \frac{1}{1+\rho}\pi$$

$$\hat{c}_{\omega,t} = \beta^y_\omega \hat{y} + \beta^\eta_\omega \eta^\omega_t \ \forall \omega$$

$$\min_\omega \left\{ \mathbb{E}_t \left[ \hat{c}_{\omega,t+1} \right] - \hat{c}_{\omega,t} \right\} = \frac{1}{\sigma} \left( \iota - \pi - \rho \right)$$

Substitution yields

$$\min_\omega \left\{ \beta^\eta_\omega \left( \mathbb{E}_t \eta^\omega_{t+1} - \eta^\omega_t \right) \right\} = \frac{1}{\sigma} \left( r - \rho \right) + \frac{1}{\sigma} \left( \phi_\pi \frac{1+\rho}{\rho} \varphi_y + \phi_y - \frac{1+\rho}{\rho} \varphi_y \right) \hat{y}$$

where

$$\min_\omega \left\{ \beta^\eta_\omega \left( \mathbb{E}_t \eta^\omega_{t+1} - \eta^\omega_t \right) \right\} = \frac{1}{\sigma} \left( r - \rho \right)$$

from the stationary equilibrium. Hence the economy is described by the single equation

$$0 = \frac{1}{\sigma} \frac{1+\rho}{\rho} \left( \left( \phi_\pi - 1 \right) \varphi_y + \frac{\rho}{1+\rho} \phi_y \right) \hat{y}$$

so that given $\phi_\pi, \phi_y \geq 0$,

$$\left( \phi_\pi - 1 \right) \varphi_y + \frac{\rho}{1+\rho} \phi_y > 0$$

is sufficient for a unique steady state $\hat{y} = 0$ and $\pi = 0$. $\square$

**Proof of Propositions 2 and 7** Recall the system

$$\iota_t = r + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t$$

$$\pi_t = \varphi_y \hat{y}_t + \frac{1}{1+\rho} \mathbb{E}_t \left[ \pi_{t+1} \right]$$

$$\hat{c}_{\omega,t} = \beta^y_\omega \hat{y}_t + \beta^\eta_\omega \eta^\omega_t \ \forall \omega$$

$$\min_\omega \left\{ \mathbb{E}_t \left[ \hat{c}_{\omega,t+1} \right] - \hat{c}_{\omega,t} \right\} = \frac{1}{\sigma} \left( \iota_t - \mathbb{E}_t \left[ \pi_{t+1} \right] - \rho \right)$$

where I have set $\hat{a}_t = \hat{\Phi}_t = 0$ by construction. Consider an MIT shock $v_t = \rho_v^{t-1} v_1$, and simplify the system to

$$\pi_t = \varphi_y \hat{y}_t + \frac{1}{1+\rho} \mathbb{E}_t \left[ \pi_{t+1} \right]$$

$$\min_\omega \left\{ \beta^y_\omega \left( \mathbb{E}_t \left[ \hat{y}_{t+1} \right] - \hat{y}_t \right) \right\} = \frac{1}{\sigma} \left( \phi_\pi \pi_t + \phi_y \hat{y}_t + \rho_v^{t-1} v_1 - \mathbb{E}_t \left[ \pi_{t+1} \right] \right)$$

Suppose $v_1 > 0$, and guess $\hat{y}_t = c_y^+ \rho_v^{t-1} v_1$ and $\pi_t = c_\pi^+ \rho_v^{t-1} v_1$, so that

$$c_\pi^+ = \frac{1+\rho}{1+\rho-\rho_v} \varphi_y c_y^+$$

$$\left( 1 - \rho_v \right) \min_\omega \left\{ -\beta^y_\omega c_y^+ \right\} = \frac{1}{\sigma} \left( \left( \phi_\pi - \rho_v \right) \frac{1+\rho}{1+\rho-\rho_v} \varphi_y + \phi_y \right) c_y^+ + \frac{1}{\sigma}$$

and assumption 2 implies that $c_y^+ < 0$. Hence

$$c_y^+ = -\frac{1}{(1-\rho_v)\min_\omega\{\beta_\omega^y\} + \frac{1}{\sigma}\left((\phi_\pi - \rho_v)\frac{1+\rho}{1+\rho-\rho_v}\varphi_y + \phi_y\right)}\frac{1}{\sigma}$$

Now suppose $v_1 < 0$, and guess $\hat{y}_t = c_y^- \rho_v^{t-1} v_1$ and $\pi_t = c_\pi^- \rho_v^{t-1} v_1$, so that

$$c_\pi^- = \frac{1+\rho}{1+\rho-\rho_v}\varphi_y c_y^-$$

$$-(1-\rho_v)\min_\omega\{\beta_\omega^y c_y^-\} = \frac{1}{\sigma}\left((\phi_\pi - \rho_v)\frac{1+\rho}{1+\rho-\rho_v}\varphi_y + \phi_y\right)c_y^- + \frac{1}{\sigma}$$

and assumption 2 implies that $c_y^- < 0$. Hence

$$c_y^- = -\frac{1}{(1-\rho_v)\max_\omega\{\beta_\omega^y\} + \frac{1}{\sigma}\left((\phi_\pi - \rho_v)\frac{1+\rho}{1+\rho-\rho_v}\varphi_y + \phi_y\right)}\frac{1}{\sigma}$$

as required. $\square$

**Proof of Propositions 3 and 8** Recall the system

$$\iota_t = r + \phi_\pi \pi_t + \phi_y \hat{y}_t$$

$$\pi_t = \varphi_y \hat{y}_t - \varphi_\Phi \hat{\Phi}_t + \frac{1}{1+\rho}\mathbb{E}_t[\pi_{t+1}]$$

$$\hat{c}_{\omega,t} = \beta_\omega^y \hat{y}_t + \beta_\omega^\eta \eta_t^\omega \ \forall\omega$$

$$\min_\omega\{\mathbb{E}_t[\hat{c}_{\omega,t+1}] - \hat{c}_{\omega,t}\} = \frac{1}{\sigma}(\iota_t - \mathbb{E}_t[\pi_{t+1}] - \rho)$$

where I have set $\hat{a}_t = v_t = 0$ by construction. Consider an MIT shock $\hat{\Phi}_t = \rho_\Phi^{t-1}\hat{\Phi}_1$, and simplify the system to

$$\pi_t = \varphi_y \hat{y}_t - \varphi_\Phi \rho_\Phi^{t-1}\hat{\Phi}_1 + \frac{1}{1+\rho}\mathbb{E}_t[\pi_{t+1}]$$

$$\min_\omega\{\mathbb{E}_t[\beta_\omega^y \hat{y}_{t+1}] - \beta_\omega^y \hat{y}_t\} = \frac{1}{\sigma}(\phi_\pi \pi_t + \phi_y \hat{y}_t - \mathbb{E}_t[\pi_{t+1}])$$

Suppose $\hat{\Phi}_1 > 0$, and guess $\hat{y}_t = c_y^+ \rho_\Phi^{t-1}\hat{\Phi}_1$, $\pi_t = c_\pi^+ \rho_\Phi^{t-1}\hat{\Phi}_1$, so that

$$c_\pi^+ = \frac{1+\rho}{1+\rho-\rho_\Phi}(\varphi_y c_y^+ - \varphi_\Phi)$$

$$(1-\rho_\Phi)\min_\omega\{-\beta_\omega^y c_y^+\} = \frac{1}{\sigma}\left((\phi_\pi - \rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_y + \phi_y\right)c_y^+ - \frac{1}{\sigma}(\phi_\pi - \rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_\Phi$$

and assumption 2 implies that $c_y^+ > 0$. Hence

$$c_y^+ = \frac{(\phi_\pi - \rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_\Phi}{(1-\rho_\Phi)\max_\omega\{\beta_\omega^y\} + \frac{1}{\sigma}\left((\phi_\pi - \rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_y + \phi_y\right)}\frac{1}{\sigma} > 0$$

Now suppose $\hat{\Phi}_1 < 0$, and guess $\hat{y}_t = c_y^- \rho_\Phi^{t-1} \hat{\Phi}_1$, $\pi_t = c_\pi^- \rho_\Phi^{t-1} \hat{\Phi}_1$, so that

$$c_\pi^- = \frac{1+\rho}{1+\rho-\rho_\Phi} \left( \varphi_y c_y^- - \varphi_\Phi \right)$$

$$-\left(1-\rho_\Phi\right) \min_\omega \left\{ \beta_\omega^y c_y^- \right\} = \frac{1}{\sigma} \left( \left( \phi_\pi - \rho_\Phi \right) \frac{1+\rho}{1+\rho-\rho_\Phi} \varphi_y + \phi_y \right) c_y^- - \frac{1}{\sigma} \left( \phi_\pi - \rho_\Phi \right) \frac{1+\rho}{1+\rho-\rho_\Phi} \varphi_\Phi$$

and assumption 2 implies that $c_y^- > 0$. Hence

$$c_y^- = \frac{\left( \phi_\pi - \rho_\Phi \right) \frac{1+\rho}{1+\rho-\rho_\Phi} \varphi_\Phi}{\left(1-\rho_\Phi\right) \min_\omega \left\{ \beta_\omega^y \right\} + \frac{1}{\sigma} \left( \left( \phi_\pi - \rho_\Phi \right) \frac{1+\rho}{1+\rho-\rho_\Phi} \varphi_y + \phi_y \right)} \frac{1}{\sigma} > 0$$

as required. □

**Proof of Propositions 4 and 9** Recall the system

$$\iota_t = r + \phi_\pi \pi_t + \phi_y \hat{y}_t$$

$$\pi_t = \varphi_y \hat{y}_t - \varphi_a \rho_a^{t-1} \hat{a}_1 + \frac{1}{1+\rho} \mathbb{E}_t \left[ \pi_{t+1} \right]$$

$$\hat{c}_{\omega,t} = \beta_\omega^y \hat{y}_t + \beta_\omega^a \rho_a^{t-1} \hat{a}_1 + \beta_\omega^\eta \eta_t^\omega \ \forall \omega$$

$$\min_\omega \left\{ \mathbb{E}_t \left[ \hat{c}_{\omega,t+1} \right] - \hat{c}_{\omega,t} \right\} = \frac{1}{\sigma} \left( \iota_t - \mathbb{E}_t \left[ \pi_{t+1} \right] - \rho \right)$$

where I have set $v_t = \hat{\Phi}_t = 0$ by construction. Simplification yields

$$\pi_t = \varphi_y \hat{y}_t - \varphi_a \rho_a^{t-1} \hat{a}_1 + \frac{1}{1+\rho} \mathbb{E}_t \left[ \pi_{t+1} \right]$$

$$\min_\omega \left\{ \mathbb{E}_t \left[ \beta_\omega^y \hat{y}_{t+1} + \beta_\omega^a \rho_a^t \hat{a}_1 \right] - \beta_\omega^y \hat{y}_t - \beta_\omega^a \rho_a^{t-1} \hat{a}_1 \right\} = \frac{1}{\sigma} \left( \phi_\pi \pi_t + \phi_y \hat{y}_t - \mathbb{E}_t \left[ \pi_{t+1} \right] \right)$$

Suppose $\hat{a}_1 > 0$, and guess $\hat{y}_t = c_y^+ \rho_a^{t-1} \hat{a}_1$, $\pi_t = c_\pi^+ \rho_a^{t-1} \hat{a}_1$, so that

$$c_\pi^+ = \frac{1+\rho}{1+\rho-\rho_a} \left( \varphi_y c_y^+ - \varphi_a \right)$$

$$\min_\omega \left\{ \left( \rho_a - 1 \right) \left( \beta_\omega^y c_y^+ + \beta_\omega^a \right) \right\} = \frac{1}{\sigma} \left( \left( \phi_\pi - \rho_a \right) \frac{1+\rho}{1+\rho-\rho_a} \varphi_y + \phi_y \right) c_y^+ - \frac{1}{\sigma} \left( \phi_\pi - \rho_a \right) \frac{1+\rho}{1+\rho-\rho_a} \varphi_a$$

To establish uniqueness, define

$$f^+\left(c_y\right) = \min_\omega \left\{ \left( \rho_a - 1 \right) \left( \beta_\omega^y c_y + \beta_\omega^a \right) \right\} - \frac{1}{\sigma} \left( \left( \phi_\pi - \rho_a \right) \frac{1+\rho}{1+\rho-\rho_a} \varphi_y + \phi_y \right) c_y + \frac{1}{\sigma} \left( \phi_\pi - \rho_a \right) \frac{1+\rho}{1+\rho-\rho_a} \varphi_a$$

so that for $c_{y,1} < c_{y,2}$, we have

$$f^+\left(c_{y,2}\right) - f^+\left(c_{y,1}\right) = \min_\omega \left\{ \left( \rho_a - 1 \right) \left( \beta_\omega^y c_{y,2} + \beta_\omega^a \right) \right\} - \min_\omega \left\{ \left( \rho_a - 1 \right) \left( \beta_\omega^y c_{y,1} + \beta_\omega^a \right) \right\} + \frac{1}{\sigma} \left( \left( \phi_\pi - \rho_a \right) \frac{1+\rho}{1+\rho-\rho_a} \varphi_y + \phi_y \right) \left( c_{y,1} - c_y \right.$$

so that

$$f^+\left(c_{y,2}\right) - f^+\left(c_{y,1}\right) \leq \left( \rho_a - 1 \right) \left( \beta_{\omega_1}^y c_{y,2} + \beta_{\omega_1}^a \right) - \left( \rho_a - 1 \right) \left( \beta_{\omega_1}^y c_{y,1} + \beta_{\omega_1}^a \right) + \frac{1}{\sigma} \left( \left( \phi_\pi - \rho_a \right) \frac{1+\rho}{1+\rho-\rho_a} \varphi_y + \phi_y \right) \left( c_{y,1} - c_{y,2} \right)$$

where $\omega_1 \in \arg\min_\omega \{(\rho_a - 1)(\beta^y_\omega c_{y,1} + \beta^a_\omega)\}$. Hence

$$f^+(c_{y,2}) - f^+(c_{y,1}) \leq \left((1-\rho_a)\beta^y_{\omega_1} + \frac{1}{\sigma}\left((\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right)\right)(c_{y,1} - c_{y,2}) < 0$$

where the second strict inequality follows from assumption 2, $\rho_a \in [0,1)$, and $c_{y,1} < c_{y,2}$. Therefore $f^+$ is strictly decreasing, and hence has a unique solution to $f^+(c_y) = 0$, which yields the coefficient $c^+_y$.

Now suppose $\hat{a}_1 < 0$, and guess $\hat{y}_t = c^-_y \rho_a^{t-1}\hat{a}_1$, $\pi_t = c^-_\pi \rho_a^{t-1}\hat{a}_1$, so that

$$c^-_\pi = \frac{1+\rho}{1+\rho-\rho_a}\left(\varphi_y c^-_y - \varphi_a\right)$$

$$\max\left\{(\rho_a - 1)\left(\beta^y_\omega c^-_y + \beta^a_\omega\right)\right\} = \frac{1}{\sigma}\left((\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right)c^-_y - \frac{1}{\sigma}(\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_a$$

To establish uniqueness, define

$$f^-(c_y) = \max_\omega\{(\rho_a - 1)(\beta^y_\omega c_y + \beta^a_\omega)\} - \frac{1}{\sigma}\left((\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right)c_y + \frac{1}{\sigma}(\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_a$$

so that for $c_{y,1} < c_{y,2}$, we have

$$f^-(c_{y,2}) - f^-(c_{y,1}) = \max_\omega\{(\rho_a - 1)(\beta^y_\omega c_{y,2} + \beta^a_\omega)\} - \max_\omega\{(\rho_a - 1)(\beta^y_\omega c_{y,1} + \beta^a_\omega)\} + \frac{1}{\sigma}\left((\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right)(c_{y,1} - c$$

so that

$$f^-(c_{y,2}) - f^-(c_{y,1}) \leq (\rho_a - 1)\left(\beta^y_{\omega_2} c_{y,2} + \beta^a_{\omega_2}\right) - (\rho_a - 1)\left(\beta^y_{\omega_2} c_{y,1} + \beta^a_{\omega_2}\right) + \frac{1}{\sigma}\left((\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right)(c_{y,1} - c_{y,2})$$

where $\omega_2 \arg\max_\omega \{(\rho_a - 1)(\beta^y_\omega c_{y,2} + \beta^a_\omega)\}$. Hence

$$f^-(c_{y,2}) - f^-(c_{y,1}) \leq \left((1-\rho_a)\beta^y_{\omega_2} + \frac{1}{\sigma}\left((\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right)\right)(c_{y,1} - c_{y,2}) < 0$$

where the second strict inequality follows from assumption 2, $\rho_a \in [0,1)$, and $c_{y,1} < c_{y,2}$. Therefore $f^-$ is strictly decreasing, and hence has a unique solution to $f^-(c_y) = 0$, which yields the coefficient $c^-_y$.

To show $c^-_y \geq 0$, suppose $c^-_y < 0$. Then,

$$\max_\omega\left\{(\rho_a - 1)\left(\beta^y_\omega c^-_y + \beta^a_\omega\right)\right\} - \frac{1}{\sigma}\left((\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right)c^-_y = -\frac{1}{\sigma}(\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_a$$

implies that $\min_\omega\left\{\beta^y_\omega c^-_y + \beta^a_\omega\right\} > 0$. Now recall the expressions for $\beta^y_\omega$ and $\beta^a_\omega$,

$$\beta^y_\omega = \frac{\left(\bar{\Phi} - \alpha\left(\bar{\Phi} - 1\right)\right)}{\alpha}\frac{(1-\alpha)\left(\bar{\Phi} - 1\right)\theta^\omega + s^\omega\left(1 - (1-\alpha)\bar{\Phi}\right)}{(1-\alpha)\left(\bar{\Phi} - 1\right)\theta^\omega + s^\omega}$$

$$\beta^a_\omega = \left(\bar{\Phi} - 1\right)\frac{\bar{\Phi}\left(\frac{1-\alpha}{\alpha}\right)(s^\omega - \theta^\omega)}{(1-\alpha)\left(\bar{\Phi} - 1\right)\theta^\omega + s^\omega}$$

Consider an island $\omega'$ with $\beta^y_{\omega'} \geq 1$ so that $\theta^\omega \geq s^\omega$, which implies that $\beta^a_{\omega'} \leq 0$, and $\beta^y_{\omega'} c^-_y + \beta^a_{\omega'} \leq 0$, which contradicts that the min is positive. Therefore $c^-_y \geq 0$.

To show that $c_y^- > c_y^+$, consider

$$f^+\left(c_y^+\right) - f^-\left(c_y^-\right) = 0$$

which is equivalent to

$$\min_\omega \left\{(\rho_a - 1)\left(\beta_\omega^y c_y^+ + \beta_\omega^a\right)\right\} - \max\left\{(\rho_a - 1)\left(\beta_\omega^y c_y^- + \beta_\omega^a\right)\right\} + \frac{1}{\sigma}\left((\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right)\left(c_y^- - c_y^+\right) = 0$$

Note that

$$\min_\omega \left\{(\rho_a - 1)\left(\beta_\omega^y c_y^+ + \beta_\omega^a\right)\right\} - \max\left\{(\rho_a - 1)\left(\beta_\omega^y c_y^- + \beta_\omega^a\right)\right\} \leq (1-\rho_a)\,\beta_{\omega^-}^y\left(c_y^- - c_y^+\right)$$

where $\omega^- \in \arg\max_\omega \left\{(\rho_a - 1)\left(\beta_\omega^y c_y^- + \beta_\omega^a\right)\right\}$. Hence we have

$$0 \leq \left((1-\rho_a)\,\beta_{\omega^-}^y + \frac{1}{\sigma}\left((\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right)\right)\left(c_y^- - c_y^+\right)$$

so that $c_y^- \geq c_y^+$ since $(1-\rho_a)\,\beta_{\omega^-}^y + \frac{1}{\sigma}\left((\phi_\pi - \rho_a)\frac{1+\rho}{1+\rho-\rho_a}\varphi_y + \phi_y\right) > 0$ by assumption 2. The inequality is strict whenever $c_y^+ \neq c_y^-$. $\square$

**Proof of Proposition 5** Consider the four equation model extended to allow for a zero lower bound on nominal interest rates,

$$\iota_t = \max\left\{r + \phi_\pi \pi_t + \phi_y \hat{y}_t + \upsilon_t, 0\right\}$$

$$\pi_t = \varphi_y \hat{y}_t - \varphi_a \hat{a}_t - \varphi_\Phi \hat{\Phi}_t + \frac{1}{1+\rho}\mathbb{E}_t\left[\pi_{t+1}\right]$$

$$\hat{c}_{\omega,t} = \beta_\omega^y \hat{y}_t + \beta_\omega^a \hat{a}_t + \beta_\omega^\eta \eta_t^\omega \;\forall\omega$$

$$\min_\omega\left\{\mathbb{E}_t\left[\hat{c}_{\omega,t+1}\right] - \hat{c}_{\omega,t}\right\} = \frac{1}{\sigma}\left(\iota_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \rho\right)$$

Consider the transitional dynamics of output and inflation in response to a one time, zero probability cost-push shock with zero persistence, and suppose that the zero lower bound binds.

Solving the system under the assumption $\iota_1 = 0$ yields

$$\hat{y}_1 = \frac{1}{\bar{\beta}^y}\frac{1}{\sigma}\rho$$

$$\pi_1 = \varphi_y \frac{1}{\bar{\beta}^y}\frac{1}{\sigma}\rho - \varphi_\Phi \hat{\Phi}_1$$

Given these responses, we can derive the bound on $\hat{\Phi}_1$ to ensure that $\iota_1 = 0$,

$$\rho + \phi_\pi\left(\varphi_y \frac{1}{\bar{\beta}^y}\frac{1}{\sigma}\rho - \varphi_\Phi \hat{\Phi}_1\right) + \phi_y \frac{1}{\bar{\beta}^y}\frac{1}{\sigma}\rho < 0 \iff \hat{\Phi}_1 > \hat{\Phi}_{ZLB} = \frac{\rho + (\phi_\pi \varphi_y + \phi_y)\frac{1}{\bar{\beta}^y}\frac{1}{\sigma}\rho}{\phi_\pi \varphi_\Phi}$$

where the bound is decreasing in $\bar{\beta}^y$.

**Proof of Proposition 6** In the case of cost-push shocks, symmetry requires

$$(1-\rho_\Phi)\,\underline{\beta}^y + \frac{1}{\sigma}\left((\phi_\pi - \rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_y + \phi_y^-\right) = (1-\rho_\Phi)\,\bar{\beta}^y + \frac{1}{\sigma}\left((\phi_\pi - \rho_\Phi)\frac{1+\rho}{1+\rho-\rho_\Phi}\varphi_y + \phi_y^+\right)$$

so that

$$\phi_y^- = \sigma\left(1-\rho_\Phi\right)\left(\bar{\beta}^y - \underline{\beta}^y\right) + \phi_y^+$$

as in the proposition. □

**Proof of Proposition 10** Pooled OLS estimation for group $g$ yields

$$\hat{\beta}_g = \frac{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right) \Delta \log c_{i,t}}{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

where $\bar{y} = \frac{1}{T}\sum \Delta \hat{\log} Y_t$, and the summation over $i$ is read as "sum over all households $i$ such that $\mathcal{G}(i,t) = g$". Substituting in the true model for household consumption yields

$$\hat{\beta}_g = \frac{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)\left(\alpha_i + \beta_i \Delta \hat{\log} Y_t + u_{i,t}\right)}{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

$$\hat{\beta}_g = \frac{\sum_t \sum_i \left(\alpha_i \Delta \hat{\log} Y_t + \beta_i \left(\Delta \hat{\log} Y_t\right)^2 + u_{i,t}\Delta \hat{\log} Y_t - \bar{y}\alpha_i - \beta_i \bar{y} \Delta \hat{\log} Y_t - \bar{y} u_{i,t}\right)}{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

$$\hat{\beta}_g = \frac{\sum_t \Delta \hat{\log} Y_t \sum_i \alpha_i + \sum_t \left(\Delta \hat{\log} Y_t\right)^2 \sum_i \beta_i + \sum_t \Delta \hat{\log} Y_t \sum_i u_{i,t}}{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2} -$$

$$- \frac{\bar{y}\sum_t \sum_i \alpha_i + \bar{y}\sum_t \Delta \hat{\log} Y_t \sum_i \beta_i + \bar{y}\sum_t \sum_i u_{i,t}}{\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

$$\hat{\beta}_g = \frac{\sum_t \Delta \hat{\log} Y_t \alpha_{g,t} + \sum_t \left(\Delta \hat{\log} Y_t\right)^2 \beta_{g,t} + \sum_t \Delta \hat{\log} Y_t u_{g,t} - \bar{y}\sum_t \alpha_{g,t} - \bar{y}\sum_t \Delta \hat{\log} Y_t \beta_{g,t} - \bar{y}\sum_t u_{g,t}}{\frac{1}{n}\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

where

$$\alpha_{g,t} = \frac{1}{n_{g,t}} \sum_{i \in g,t} \alpha_i$$

$$\beta_{g,t} = \frac{1}{n_{g,t}} \sum_{i \in g,t} \beta_i$$

$$u_{g,t} = \frac{1}{n_{g,t}} \sum_{i \in g,t} u_{i,t}$$

are parameter means over households in group $g$ in period $t$. Note that $\beta_{g,t} \in (\min_i \beta_i, \max_i \beta_i)$ by definition.

Continuing,

$$\hat{\beta}_g = \frac{\frac{1}{T}\sum_t \left(\Delta \hat{\log} Y_t\right)\left(\Delta \hat{\log} Y_t\right)\beta_{g,t} - \bar{y}\frac{1}{T}\sum_t \Delta \hat{\log} Y_t \beta_{g,t} + \frac{1}{T}\sum_t \Delta \hat{\log} Y_t \alpha_{g,t} - \bar{y}\frac{1}{T}\sum_t \alpha_{g,t}}{\frac{1}{n_{g,t}}\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2} +$$

$$+\frac{\frac{1}{T}\sum_t \Delta \hat{\log} Y_t u_{g,t} - \bar{y}\frac{1}{T}\sum_t u_{g,t}}{\frac{1}{n_{g,t}}\sum_t \sum_i \left(\Delta \hat{\log} Y_t - \bar{y}\right)^2}$$

Hence as $T \to \infty$, we can apply a suitable Law of Large Numbers (e.g. Proposition 7.5 in Hamilton (1994)) to obtain

$$\hat{\beta}_g \to^p \frac{Cov\left(\Delta \hat{\log} Y_t, \Delta \hat{\log} Y_t \beta_{g,t}\right) + Cov\left(\Delta \hat{\log} Y_t, \alpha_{g,t} + u_{g,t}\right)}{V\left[\Delta \hat{\log} Y_t\right]}$$

where, if $\mathcal{G}$ does not alter group assignments over time (i.e. is exogenous to changes in $\Delta \hat{\log} Y_t$), then

$$Cov\left(\Delta \hat{\log} Y_t, \alpha_{g,t} + u_{g,t}\right) = 0$$

and

$$Cov\left(\Delta \hat{\log} Y_t, \Delta \hat{\log} Y_t \beta_{g,t}\right) = V\left[\Delta \hat{\log} Y_t\right] \frac{1}{n} \sum_{i \in g} \beta_i$$

so that

$$\hat{\beta}_g \to^p \frac{1}{n} \sum_{i \in g} \beta_i \in \left(\min_i \{\beta_i\}, \min_i \{\beta_i\}\right)$$

as required. □