



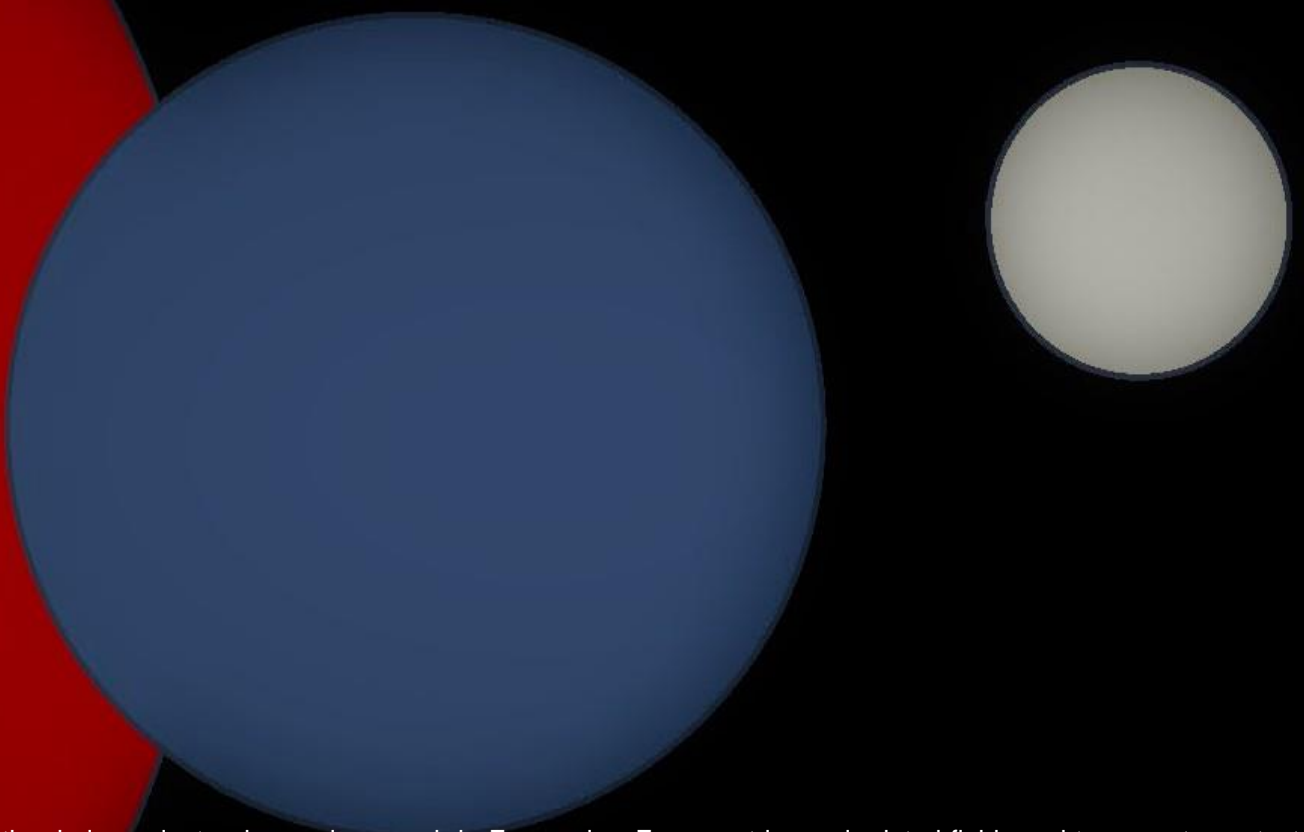
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How would the war and the pandemic affect the stock and cryptocurrency cross-market linkages?

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How would the war and the pandemic affect the stock and cryptocurrency cross-market linkages?

Georgios Bampinas* Theodore Panagiotidis†

Abstract

This paper studies the cross-market linkages between six international stock markets and the two major cryptocurrency markets during the Covid-19 pandemic and the Russian invasion of Ukraine. By employing the local (partial) Gaussian correlation approach, we find that during the Covid-19 pandemic period, both cryptocurrency markets possess limited diversification and safe haven properties, which further diminish during the war. Bootstrap tests for contagion suggest that during the Covid-19 pandemic, the East Asian markets lead the transmission of contagion towards the two cryptocurrency markets. During the Russian invasion, the US stock market emerges as the principal transmitter of contagion. Uncovering the role of pandemic (Infectious Disease EMV Index) and geopolitical risk (GPR index) induced uncertainties, we find that under conditions of high uncertainty and falling prices, the dependency between the US and UK stock markets with both cryptocurrency markets increases considerably. The latter is more profound during the Russian-Ukrainian conflict. Our findings are useful for investors in their search for understanding the differences in asymmetric connectedness between markets during extreme events.

Keywords: Bitcoin, Ethereum, cryptocurrency, stock market, tail dependence, local Gaussian partial correlation, pandemic uncertainty, geopolitical risk uncertainty

JEL Classification: F31; F37; O16; Q40; G11; G12;

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1 Introduction

The Covid-19 pandemic and the subsequent Russian invasion of Ukraine brought about two unprecedented exogenous shocks in financial markets almost worldwide (see [Baker *et al.*, 2020](#)). Shortly after the World Health Organization (WHO) declaration of Covid-19 to be a pandemic, on the 9th of March, the US stock market hit the circuit breaker mechanism, after a sharp S&P500 decline, for the first time since the financial crisis of 2007–08.¹ Concurrently, the European stock markets closed down 11%, while the Asia-Pacific stock markets plunged more than 20% from their highest position at the end of the previous year.² Likewise, on the day of the Russian attack, the S&P 500 index fell by more than 10% from its recent peak, recording its first correction since October 2020, while the European and Asian stock markets dropped on average around 3% to 4% (see, [Boungou and Yatié, 2022](#)).³ During such stressful market situations, the role of cryptocurrencies has been a key talking point either as an alternative investment vehicle to mitigate losses and to hedge against tail-risks or as an indirect monetary tool to avoid the enacting of Russian economic sanctions. Given that digital currencies are a growing asset class, with a total market capitalization of \$980 billion as of September 2022, it is important to understand how this asset class behaves in times of extreme market conditions and adverse events.

This paper addresses this empirical challenge by treating the Covid-19 pandemic and the Russian-Ukraine war as exogenous shocks that allow us to examine whether the cross-market linkages between six international stock markets (Australia, China, Germany, Japan, the UK and US) and two leading cryptocurrency markets (Bitcoin and Ethereum), increase after a shock. We employ the local Gaussian (partial) correlation concept introduced by [Tjøstheim and Hufthammer \(2013\)](#) and extended by [Otneim and Tjøstheim \(2022\)](#), which is a local measure of correlation that accounts for the nonlinear nature of the relationship. The locality of this measure of dependence gives us more flexibility, it is easy to interpret, provides a more complete description of the dependence structure, and allows us to test conditional independence to specific parts of the distribution and to distinguish between positive and

¹<https://www.bloomberg.com/news/articles/2020-03-08/rout-in-u-s-stock-futures-would-trigger-trading-curbs-at-5>

²<https://www.bloomberg.com/news/articles/2020-03-09/perfect-storm-is-plunging-asia-stocks-to-bear-markets-one-by-one>

³<https://www.nytimes.com/2022/03/07/business/stock-market-today.html>

negative conditional dependence, and detect departures from symmetry and linearity.

The recent pandemic of Covid-19 and the consecutive Russian invasion of Ukraine provide us an ideal setting to examine changes between stock and cryptocurrency market linkages for the following reasons: first, both events and their subsequent consequences constitute unexpected shocks to the global stock markets. According to Baker *et al.* (2020), the Covid-19 pandemic has forcefully affected the stock market compared to previous infectious disease outbreaks. Bounou and Yatié (2022) use stock returns from a sample of 94 countries and document a negative relationship between the Ukraine–Russia war and world stock market returns. Second, both shocks are considered exogenous and originated out of public health concerns and geopolitical conflicts that cannot be attributed to economic conditions. Third, as noted in Del Angel *et al.* (2021), the global pandemics have historically had a crucial impact on stock prices. Hudson and Urquhart (2015) and Goel *et al.* (2017) have also highlighted, from a historical perspective mainly focused on World War II, the relationship between wars and stock markets. As such, the recent Covid-19 pandemic and the Russian invasion of Ukraine resulted in a stock market crash almost worldwide. Fourth, the recent pandemic accelerated our advance into a more digital world. During both events, cryptocurrencies have received a great deal of attention from economists and market participants as well as in mainstream media.

An interesting question that arises is whether those cross-market linkages can be attributed to contagion. Contagion is considered to be a main characteristic of financial crisis as the latter spreads from one market to another. Most empirical studies thus far, have mainly focused on financial contagion during the COVID-19 period mostly employing the co-movement and correlation analysis between different types of assets (Corbet *et al.*, 2020a, 2021; Akhtaruzzaman *et al.*, 2021; Goodell and Goutte, 2021; Karamti and Belhassine, 2022). Guo *et al.* (2021a) find that the COVID-19 epidemic increases the number of contagion channels in the international financial system. Guo *et al.* (2021) also analyze the evolution of the contagion dynamics among Bitcoin and other financial markets during the Covid-19 crisis. Le *et al.* (2021) and Yarovaya *et al.* (2021) analyze the spillover effects between various financial assets and show that during the COVID-19 period, the volatility transmission across asset classes was exacerbated.

This study examines the concept of contagion by comparing the local correlation estimations between the tranquil and the turmoil period via a bootstrap test (for an application in financial markets see *Støve et al., 2014; Nguyen et al., 2020; Bampinas et al., 2023*). Regarding the contagion effect we adopt a straightforward framework of *Forbes and Rigobon (2002)*: “contagion is defined as a significant increase in cross-market linkages after a shock”. The transmission of contagion is considered as ‘sequential’ in the sense that an exogenous shock in the stock market is transmitted in the cryptocurrency market.

The consequences of uncertainty for financial markets in times of crisis have been well recognized as far back as the work of *Bloom (2009)*. The rapid spread of COVID-19 pandemic and the subsequent Russian invasion has triggered global uncertainties to a startling extent. Both events induced a significant amount of uncertainty on the equity market (*Christou et al., 2017; Guo et al., 2018; Phan et al., 2018*) as well as on the cryptocurrency markets (*Bouri et al., 2017a; Cheng and Yen, 2020*). Nevertheless, pandemics and wars induced uncertainties may generate different types of contagion compared to financial uncertainty shocks (*Sharif et al., 2020; Yarovaya et al., 2022*). To this end, we distinguish two different types of uncertainty: (i) the pandemic induced uncertainty measured by the Infectious Disease EMV Index of *Baker et al. (2020)* and (ii) the geopolitical risk uncertainty measured by the geopolitical risks (GPR) index constructed by *Caldara and Iacoviello (2022)*, to understand the effects of global tension from the Russian-Ukrainian conflict. By employing a new measure of conditional dependence, the partial local Gaussian correlation (LGPC) approach of *Otneim and Tjøstheim (2022)*, we further shed light on the impact of the recent pandemic and the geopolitical induced uncertainties as drivers of the stock and cryptocurrency market interlinkages.

Our focus would be on the cross-market linkages at the different segments of the distribution as well as the tails, which is crucial for investors who search for alternative assets to offset their (extreme) losses in equities. Therefore, we opt for the local Gaussian partial correlation of *Otneim and Tjøstheim (2022)* through which the conditional dependence between equities and cryptocurrencies across a set of low quantiles is uncovered, such as in situations of financial distress and high uncertainty. The LGPC for this case is a measure of each cryptocurrency market exposure to system wide distress and therefore is akin

to the stress tests performed by individual institutions. We, therefore, provide the hedging and safe-haven properties for cryptocurrency markets when stock markets are crashing, i.e. both markets are in the lower quantiles of the conditional distribution and the shock induced uncertainty (Covid-19 uncertainty or geopolitical risk uncertainty) is at high levels.

Our empirical findings show that the dependence between stock markets and the Bitcoin market has increased in the post- compared to the pre-Covid-19 period. We observe important increases in correlations throughout the conditional distribution, turning from negative to positive, with those linkages being more profound when both markets experience negative returns (left tail). Even stronger cross-market linkages were found between the Ethereum market and the international stock markets during the post-Covid-19 period. The highest increase in dependence is again noted in the left tail of the conditional distribution (market state of crash). This extreme sign change (combined with the resulting positive local correlation estimates between the cryptocurrency markets and each of the rest international stock markets) suggests that the two digital markets may serve as a useful diversification tool only in tranquil times. In turmoil periods, such as the Covid-19 period, these diversification benefits as well as any safe haven properties vanish. The two East Asian stock markets (China and Japan) are found to be the principal transmitters of contagion towards the two leading cryptocurrency markets after the Covid-19 outbreak.

During the Russian invasion of Ukraine, we document a significant increase in dependence between the Bitcoin market and the six equity markets, with the most remarkable increase observed for Australia, Japan and the US. Regarding the Ethereum market, a uniform increase in dependence is also observed for almost all stock market-Ethereum pairs. The increase in dependence observed in a state of market distress (at the left tail of the distribution) questions the diversification and safe-haven benefits of the two cryptocurrency markets with respect to the equity market during the period after the invasion. Moreover, we find strong evidence of contagion between the Bitcoin and the stock market of Australia, Japan and the US. Contagion is also observed between the Ethereum market and Germany, Japan, the UK and the US. We also observe that during the period of the Russian-Ukrainian conflict, the US stock market emerges as the leading market maintaining strong linkages with the two cryptocurrency markets while the China stock market retreats.

When we consider the role of uncertainty derived from the two specific exogenous shocks, we find that conditional independence is strongly rejected between the two cryptocurrency markets and the two East Asian markets as well as the US market, when pandemic uncertainty is high. Considering the hypothesis of high geopolitical risk, we document increased dependence between all equity-cryptocurrency markets due to the negative developments in Europe (except China). In both turmoil periods, a significant increase in dependence is mainly observed in the parts of the conditional distribution where the stock market returns are negative. In general, stronger dependence in times of financial distress and high uncertainty is observed during the period of Russian-Ukrainian conflict compared to the Covid-19 pandemic period.

The remaining of the paper is organised as follows: Section 2 discusses the relevant literature and our contribution, Section 3 describes the data set, Section 4 presents the empirical methodology and Section 5 demonstrates the empirical results. The last one concludes.

2 Literature review and contribution

Since the seminal paper by Nakamoto (2008), bitcoin has attempted to establish itself as a type of ‘digital gold’ that provides investors with guaranteed scarcity and high mobility. The Covid-19 pandemic and the subsequent Russian-Ukrainian crisis have propagated the reassessment of cryptocurrency markets. While many financial assets were losing value, the pandemic outbreak encouraged investors to review the long-term outlook for cryptocurrencies.⁴ These markets have gained investors’ attention due to the opportunities that their correlation with other financial assets provides and the potential diversification benefits (Corbet *et al.*, 2018; Panagiotidis *et al.*, 2019; Corbet *et al.*, 2020; Charfeddine *et al.*, 2020; Urom *et al.*, 2020). Our study provides the first empirical investigation of the impact of the recent exogenous coronavirus and Russian invasion shocks on the cross-market linkages of equity and cryptocurrency markets. To the best of our knowledge, there are no empirical studies yet exploring the impact of the Ukraine–Russia war between the two markets while evidence on the Covid-19 period remains limited (see, Conlon and McGee, 2020). This paper is an attempt to fill this gap.

⁴<https://www.theguardian.com/technology/2020/nov/17/bitcoin-jumps-to-three-year-high-as-covid-crisis-changes-investor-outlook>

Our work further relates to recent studies for the safe haven properties of cryptocurrencies (Bouri *et al.*, 2017b; Urquhart and Zhang, 2019; Smales, 2019; Shahzad *et al.*, 2019; Kliber *et al.*, 2019; Guesmi *et al.*, 2019). Baur *et al.* (2018) find that Bitcoin is uncorrelated with traditional asset classes in periods of financial turmoil. Corbet *et al.* (2018) provide evidence of the relative isolation of cryptocurrencies from market shocks, including the S&P500 stock market. Charfeddine *et al.* (2020) find that the cross-correlation with conventional assets is rather weak, with the relationship being sensitive to external economic and financial shocks. Borri (2019) concludes that Bitcoin is not exposed to tail-risk to other global assets, such as the US equity market or gold. In this respect, her results indicate that portfolios of cryptocurrencies could offer attractive returns and hedging abilities when included in an investor's portfolio. Bouri *et al.* (2020) reveal a heterogeneous impact of Bitcoin's hedging ability against the downside risk of the US stock sectors, while Bouri *et al.* (2023) demonstrate that Bitcoin prices have significant (in- and out of sample) predictive power for US stock realized volatility.

Similar studies examine the usefulness of cryptocurrency markets as a diversification and a safe haven instrument in extremely stressful periods (Alfaro *et al.*, 2020; Corbet *et al.*, 2020a). Conlon and McGee (2020) investigated the safe haven properties of Bitcoin from a US investor perspective during the Covid-19 bear market and found that Bitcoin was neither a safe haven nor a hedge against the extreme bear market in the S&P500. Moreover, Goodell and Goutte (2020) analyze the Bitcoin reaction to daily Covid-19 world deaths and show that Bitcoin is a safe haven investment. Guo *et al.* (2021) using a dynamic and conditional spillover method show that during the Covid-19 period the contagion effect between Bitcoin and developed markets has been further reinforced. We complement these findings by documenting that during the two recent exogenous shocks (the Covid-19 pandemic and the Russian-Ukrainian conflict), under the circumstances of extreme market distress the diversification and safe-haven properties of cryptocurrency markets diminish to a large extent. This feature is even more pronounced over the recent Russian invasion of Ukraine.

A growing part of the literature deals with how economic policy uncertainty (EPU) affects financial markets (Antonakakis *et al.*, 2013; Dakhlaoui and Aloui, 2016) as well as cryptocurrency markets (Cheng and Yen, 2020; Fang *et al.*, 2019; Yen and Cheng, 2020).

The EPU index is considered as a main determinant of cryptocurrency price dynamics (e.g., [Bouri and Gupta, 2019](#)), while uncertainty risk can be transmitted to Bitcoin market (e.g., [Wang et al., 2019](#)). Some recent studies, analyze the effects of geopolitical risks on Bitcoin returns and volatility ([Aysan et al., 2019](#); [Al Mamun et al., 2020](#)).⁵ We also contribute to this emerging literature on uncovering the stock and cryptocurrency cross-asset linkages by considering the pandemic and geopolitical risk induced uncertainty which are directly connected to the type of exogenous shocks. We provide evidence that under the state of high (pandemic and geopolitical risk) uncertainty the dependence between the stock and cryptocurrency market increases considerably, especially when stock markets are falling. A stronger and wider increase in dependence was also observed during the period of Russian invasion especially for the bitcoin-stock market pairs.

Typically, during a market crash, the larger the shock the more profound the correlations among financial asset prices. Several studies that have considered the measure of correlation between Bitcoin and other asset classes mainly offer strong evidence of a weak correlation (see, [Dyhrberg, 2016](#); [Bouri et al., 2017b](#); [Guesmi et al., 2019](#)). However, the use of linear dependence measures may operate well for approximately bivariate Gaussian variables, but in the presence of nonlinearity they tend to lose power ([Støve et al., 2014](#)). During times of exogenous shocks, there may be periods where values exhibit stronger dependence (positive or negative), while weaker dependence may be observed in other subsets of values. Our work concerns the nonlinear nature of our series and adopts a measure of dependence that is localized and nonlinear. More closely related to our approach are the recent studies for measuring nonparametric dependence based on auto-distance correlation functions, copula models and the cross-quantilogram approach ([Székely and Rizzo, 2009](#); [Zhou, 2012](#); [Oh and Patton, 2017](#); [Han et al., 2016](#)). However, less attention has been given thus far in estimating the strength of cross-dependence, and testing for contagion effects. In this respect, the local Gaussian correlation approach is more straightforward to interpret than auto-distance and copulas, providing a direct measure of both average and upper-lower tail dependence completing the characterization of the cross-asset dependence structure.

⁵[Huang et al. \(2023\)](#) employ TVP-VAR model to examine the time-varying market linkages between Bitcoin and green assets during the COVID-19 pandemic period considering the role of uncertainty related to environmental concerns.

3 Data and descriptive analysis

We collect daily Bitcoin (BTC) and Ethereum (ETH) prices in US dollars from [CryptoCompare.com](https://cryptocompare.com) using an R script to download the data.⁶ The CryptoCompare's index methodology (CCCAGG) calculates the market price of cryptocurrency pairs traded across 230 exchanges by aggregating 24-hour volume-weighted average transaction data for every currency pair (for example, CCCAGG Bitcoin-\$US). Dates with missing values or zero volume values are excluded from our sample.

As a measure of pandemic induced uncertainty, we use the log-level of the Infectious Disease Equity Market Volatility (EMV for short) index obtained from <http://www.policyuncertainty.com/>. This newspaper-based index is updated daily and is constructed in three main stages. In the first stage, terms are specified in four broad sets: economics (E), market (M), volatility (V) and health (ID). In the second stage, daily counts are obtained of newspaper articles that contain at least one term in each of the aforementioned sets across approximately 3,000 US Newspapers. Last, the Infectious Disease EMV Tracker is calculated as the product of the overall EMV tracker value with the share of EMV Articles in which there exists one or more of the following terms: epidemic, pandemic, virus, flu, disease, coronavirus, MERS, SARS, Ebola, H5N1, H1N1 (for more details, see [Baker et al., 2020](#)).

[Caldara and Iacoviello \(2022\)](#) GPR index reflects automated text-search results of the electronic archives of 11 international newspapers. The GPR index is calculated by counting monthly the number of articles related to adverse geopolitical events in each newspaper (as a share of the total number of news articles). The authors define geopolitical risk as every risk related to terrorist attacks or acts, wars, tensions between countries, and every adverse event that affects the peaceful course of international relations. In our study, we consider the logarithmic change in the global GPR index.

Our stock market dataset covers six international equity markets including Australia, China, Germany, Japan, the United Kingdom (UK) and the United States of America (US). Data on the six equity market indices were extracted from the *Wall Street Journal*, major international stock indexes.⁷ We define the daily returns as the logarithmic difference be-

⁶The R package '*cryptor*', provides a basic wrapper around the public API provided by CryptoCompare.

⁷Tickers for the stock markets are abbreviated as: ^DAX for Germany, ^SHCOMP for China, ^SPX for the US, ^XJO for Australia, ^NIK for Japan and ^UKX for the UK.

tween two consecutive closing prices as: $\Delta y_t = \ln(P_t) - \ln(P_{t-1})$. For each stock market weekends and national holidays are excluded from the sample.⁸ To deal with the issue of non-synchronous trading since the six stock markets are closed in different time zones and the cryptocurrency markets operate 24 hours per day, we use the two-day average rolling returns for all variables, in the same spirit of [Forbes and Rigobon \(2002\)](#).

To understand our choice of event windows for the two global shocks under examination, consider [Figure 1](#). [Figure 1](#) depicts the six equity and the two cryptocurrency markets' performance before and after February 24, 2020, as well as during the period of the Russian invasion of Ukraine on February 24, 2022 (dotted lines). February 24, 2020 is the first trading day after the first lockdown in Europe, in Northern Italy. It also marks the start of the 'fever period' noted in [Ramelli and Wagner \(2020\)](#). We thus define the pre-Covid19 period from January 2, 2019 to February 24, 2020 and the post-Covid19 period from February 24, 2020 to November 5, 2020. For the Russian invasion of Ukraine period we define the pre-invasion period from January 1, 2021 to February 23, 2022 and the post-invasion period from February 24, 2022 to July 4, 2022.

[Insert Figure 1 about here.]

Moreover, the operation of the bootstrap test for contagion requires the independence of each variable over time. As noted in [Ghosh *et al.* \(2023\)](#), the return and volatility behavior of cryptocurrencies is similar to that of traditional assets. Therefore, to capture the stylized features of stocks and cryptocurrency markets returns such as fat tails and volatility clustering, we apply a univariate GARCH (1,1) filtering on each series (y_t), with errors following the Student's t distribution (similar to [Støve *et al.*, 2014](#)). For each cleaned series, we apply the following model:

$$\begin{aligned}
 y_t &= \mu + v_t \\
 v_t &= \sigma_t \epsilon_t, \\
 \sigma_t^2 &= \omega + \alpha v_{t-1}^2 + \beta \sigma_{t-1}^2,
 \end{aligned}$$

⁸A battery of unit root and stationarity tests revealed the non-stationarity nature of our time series which became stationary after first differencing. To preserve space unit root tests are not presented here but are available from the authors upon request.

with the usual notation. The standardised residuals are given by $\hat{v}_t = (y_t - \hat{\mu}) / \hat{\sigma}_t^2$ that are used in the subsequent analysis. The diagnostics output for the GARCH filtered series is presented in Table 3 in the Appendix. Both tests for conditional heteroscedasticity (Ljung-Box test and the LM test) signify that the fitted models are satisfactory.

4 Methodology

4.1 Local Gaussian correlation

Tjøstheim and Hufthammer (2013) introduced the local Gaussian correlation approach, where a family of Gaussian bivariate distributions is used to approximate an arbitrary bivariate return distribution. A Gaussian distribution approximates each point of the return distribution. The local correlation is determined as the correlation of the Gaussian distribution in that neighbourhood.

Assume two return series which take the values $\{(X_t, Z_t) \ t = 1, \dots, T\}$. In a neighborhood of each point $y = (x, z)$, a bivariate Gaussian density is fitted as follows,

$$\phi(u, \theta(y)) = \frac{1}{2\pi\sigma_1(y)\sigma_2(y)} \times \exp \left\{ -\frac{1}{2(1-\rho^2(y))} \left[\left(\frac{u_1 - \mu_1(y)}{\sigma_1(y)} \right)^2 + \left(\frac{u_2 - \mu_2(y)}{\sigma_2(y)} \right)^2 - 2\rho(y) \left(\frac{u_1 - \mu_1(y)}{\sigma_1(y)} \right) \left(\frac{u_2 - \mu_2(y)}{\sigma_2(y)} \right) \right] \right\} \quad (1)$$

where $\theta(y) = \phi(\mu_1(y), \mu_2(y), \sigma_1(y), \sigma_2(y), \rho(y))$. $\mu_1(y)$ and $\mu_2(y)$ denote the local means, $\sigma_1(y)$ and $\sigma_2(y)$ the local standard deviations and $\rho(y)$ indicates the local Gaussian correlation at the point $y = (x, z)$. These five parameters are reliant on the location of the point (x, z) , and in this respect $\phi(u, \theta(y))$ may approximate the density function f in neighbourhood of (x, z) . Moving to a different point $y' = (x', z')$ of f a different bivariate Gaussian $\phi(u, \theta(y'))$ is needed to approximate f in that neighbourhood of y' . Therefore, as the location point y varies f may be represented by a family of Gaussian bivariate densities and the local dependence properties may be described by the collection of $\rho(y)$ in each specific neighborhood of y .⁹

⁹A main issue of the local Gaussian correlation estimator relies on the bandwidth choice which is specified by the user. We employ two methods for bandwidth selection, the normal-reference rule-of-thumb (R-package 'MASS') as in Støve *et al.* (2014) and the cross-validation procedure (R-package 'lg') proposed by Lacial and Tjøstheim (2019). We present the bandwidth choice based on the normal-reference rule-of-thumb, since both approaches provide qualitatively similar results,

4.2 A bootstrap contagion test

In the local Gaussian correlation concept the presence of contagion is apparent if the local correlation between two asset classes during the post-event period has increased significantly compared to that before event period (for an application in financial markets see, *Støve et al.*, 2014).

Denote $Z_t, t = 1, \dots, T$ as the stock market returns and $X_t, t = 1, \dots, T$ the cryptocurrency market returns. To remove any time and volatility dependence from the data a GARCH(1,1) filter is used to the series. Then, suppose the standardised returns may be written as $d_t = (X_t, Z_t)$, and our sample is divided into two periods: a pre-crisis period (NC) and a post-crisis period (C).¹⁰

Take for example the Covid-19 pandemic period. We define the post-Covid19 period from February 24, 2020 to November 5, 2020. This sample period enables our analysis to capture tranquil as well as turmoil periods around the Covid-19 pandemic. Contagion is evident if the local correlation for the post-Covid-19 period lies above the pre-Covid-19 one. Using a set of fixed gridpoints (x_i, z_i) for $i = 1, \dots, n$ the null and the alternative hypothesis of the contagion test are given as:

$$H_0 : \rho_{NC}(x_i, z_i) = \rho_C(x_i, z_i) \text{ for } i = 1, \dots, n \text{ (no contagion)}$$

$$H_1 : \sum_{i=1}^n (\rho_C(x_i, z_i) - \rho_{NC}(x_i, z_i)) > 0 \text{ (contagion)}$$

The contagion bootstrap test is performed by picking $\{d_1, \dots, d_T\}$ random observations and replacing them in $\{d_1^*, \dots, d_T^*\}$. This resample is then separated in pre- and post-Covid-19 periods (NC and C, respectively) and $\hat{\rho}_{NC}^*(x_i, z_i)$ and $\hat{\rho}_C^*(x_i, z_i)$ is calculated on the values of diagonal grid $(x_i = z_i)$.

Thereafter, the following statistic is calculated:

$$D_1^* = \frac{1}{n} \sum_{i=1}^n [\hat{\rho}_C^*(x_i, x_i) - \hat{\rho}_{NC}^*(x_i, x_i)] w_i(x_i, x_i),$$

where w_i stands for a weight function in order to minimize the distance between the gridpoints and the observations. In the case that a local correlation estimation in a gridpoint is

¹⁰Tests of contagion can be vulnerable to the predefined splits around main events (for a discussion see, *Dungey et al.*, 2005). Thus, in our study regarding the Covid-19 pandemic period, we use two alternative splits when we perform the bootstrap tests for contagion: January 20, when WHO issued the first situation report on the outbreak and March 11, when WHO characterized Covid-19 as a pandemic. Both events give quantitatively similar results and are available from the authors upon request.

far away from any observations, this estimation is avoided. For these resamples we compute D_1^* and construct its distribution. Finally, we use the filtered observations $\{d_1, \dots, d_T\}$ to estimate $\hat{\rho}_{NC}(x_i, x_i)$, $\hat{\rho}_C(x_i, x_i)$ and D_1 . The p -value in terms of the D_1^* distribution is calculated and suggests a rejection of the null hypothesis (H_0) if it lies below a predetermined significance level α .

5 Empirical results

5.1 Cross market dependence between the equity and cryptocurrency markets during the Covid-19 pandemic

Figure 2 illustrates the local Gaussian correlation estimates for the standardized returns of the Bitcoin market and each of the six stock markets, during the Covid-19 pandemic period. For the vast majority of the pairs examined, we observe that the dependence between stock markets and the Bitcoin market has increased in the post-Covid-19 period compared to the pre-Covid-19. The pre-Covid local correlation estimates have either negative or close to zero values for almost all cases. This is in contrast with the findings of [Goodell and Goutte \(2021\)](#), who argue that Bitcoin cryptocurrencies are positively correlated with equities during the pre-Covid-19 period. Post Covid-19, when extreme negative movements are considered (left tail, i.e., Covid-19 period), the same estimates experience a large upward shift, thereby rising to high positive values. This extreme sign change combined with the resulting positive local correlation estimates between the Bitcoin market and each of the rest international stock markets suggests that the Bitcoin market has not served as a diversification tool during the Covid-19 pandemic.

The most remarkable difference in the local correlation estimates between the two periods is observed for the Chinese and Japanese stock markets. For each of the two Asian markets-Bitcoin pair, the correlation has increased from the -0.32 level (in the left tail) to consistently high positive values (absolute difference close to 0.7). The difference in local correlation estimates for Germany and the UK is close to 0.3 in the left tail of the distribution. However, the increase in local correlation estimates for the two Asian markets is observed through the entire distribution. This partly explains the emergence of China and Japan as the primary recipients of investors' flights away from cryptocurrency market. It also points to

the fact that, as the Covid-19 pandemic unfolds, the two Asian stock markets are strongly associated with the Bitcoin market.

[Insert Figures 2 about here.]

The structure of dependence is nonlinear in the post-Covid-19 period and asymmetric for all the stock markets, with a strong positive tail dependence observed in times of extreme market stress (left tail). During the pre-Covid-19 period, the structure of dependence is also non-linear, similar to an inverted U-shape curve, with the most negative values observed in the two tails of the conditional distribution and close to zero values noted in the middle part of the distribution.

The pattern observed above for the local correlation estimates between the Bitcoin market and each of the international stock markets is not confirmed when we consider the Ethereum market. In Figure 3, we observe that during the Covid-19 pandemic for almost all the Ethereum–stock market pairs the entire local correlation curve has moved upwards compared to the pre-Covid-19 period, turning from negative to positive values. When the cases of Japan and China are considered, the local correlation curves post-Covid-19 lie above the pre-Covid-19 ones, across the entire spectrum of the distribution. This finding supports the existence of strong linkages between the Ethereum market and the specific stock markets during the Covid-19 pandemic period. The same holds for Australia and UK, except a region in the right tail and in the middle of the conditional distribution, respectively. During the pre-Covid-19 period, the local Gaussian correlation estimates are close to zero in the middle of conditional distribution and turn negative in the tails. The opposite is observed during the Covid-19 pandemic period, where the highest increase in dependence is noted in the tails of the conditional distribution. Our findings suggest that during tranquil times, Ethereum market may serve as a useful diversification tool for the international stock markets (see for instance, Corbet *et al.*, 2018; Charfeddine *et al.*, 2020), but in turmoil periods, such as the Covid-19 period, the diversification benefits vanish, especially when markets are plunging.

[Insert Figures 3 about here.]

The local Gaussian dependence structure is highly nonlinear and asymmetric in both periods for all the market-pairs. A similar pattern is observed for the Germany, UK and

US-Ethereum pairs, with declining dependence in the right tail of the distribution during the pre-Covid-19 period, and a U-shape dependence curve after the Covid-19 pandemic shock. In the Australian and Asian-Ethereum market pairs, the local correlation curves also seem to have changed shapes, i.e., moving from the left to the right of the conditional distribution, we observe an increasing pattern in the pre-Covid-19 period turning to a declining pattern in the post-Covid-19 period.

Overall, our results show that the degree of dependence between the two cryptocurrency markets and the international equity markets differs across the considered markets and frequencies. During tranquil times, before the Covid-19 outbreak, both cryptocurrency markets may act as safe haven since are negatively correlated with equity markets at the left tail of the conditional distribution. At the same time, Bitcoin and Ethereum may act as diversifiers, as they possess close to zero correlation estimates with the international markets. Due to the weakness of their relationship with established conventional assets, cryptocurrencies appear to have a role in helping investors generate profit by adding a digital asset to a portfolio of conventional assets. Turning to the post-Covid 19 period, we note that the dependence structure is different from the previous ones, with two important observations. First, there is a considerable increase in dependence when markets are in a state of distress, indicating that both cryptocurrency markets cannot act as a diversifier, or even a safe haven during the Covid-19 pandemic, with respect to the respective equity markets. Second, the strongest dependence at this state, is observed with the two Asian markets reaching the 0.4 level on average. Moreover, China has been the first epicenter of the COVID-19 outbreak in January, 2020. The rapid spread of the virus triggered quick reactions from the Chinese government. On 23 January, 2020, the lockdown of the entire city of Wuhan shocked the whole world, later proving to be a very effective policy intervention by the Chinese government. One week later, the WHO declared the outbreak in China to be a public health emergency of international concern (PHEIC). By that time, China had had an outsized presence in East Asian cryptocurrency exchanges, constituting the world's leading crypto market (in terms of crypto activity), capturing close to 31% of all the digital currency transactions in 2020.¹¹. Considering the strong correlation between the East Asian equities and cryptocurrency mar-

¹¹<https://news.bitcoin.com/east-asia-dominates-worlds-onchain-crypto-activity-europe-and-north-america-trail-behind/>

kets during that period of high uncertainty, we may assert that digital assets could not be considered the safest short-term investments in this crisis period.

5.2 Cross market dependence between equity and cryptocurrency markets during the Russian invasion of Ukraine

The nonlinear nature of dependence between the six equity markets and Bitcoin during the Russian invasion of Ukraine is presented in Figure 4. The local Gaussian correlation estimates increased after the post-invasion period for all equity markets, with the most remarkable increase observed for Australia, the UK and the US. For the UK-Bitcoin market pair the difference in local correlation estimates is close to 0.5 at the right tail of the distribution while similar differences are observed for the US and Australia cases at the left shoulders of the distribution. Interestingly, in all cases, there is a surge in the left tail of the local Gaussian correlation curve. More importantly, this implies that when both markets are plummeting the dependence becomes even stronger between the equity and the Bitcoin market, resulting in very limited safe haven benefits. During the period before the invasion, the local Gaussian curve takes values close to zero, especially in the middle regime of the distribution, although an increment in the left tail takes place for Australia and the US.

[Insert Figures 4 about here.]

The cross-market linkages between Ethereum and the six equity markets are demonstrated in Figure 5. For all stock market-Ethereum pairs (except Australia), we observe higher local Gaussian correlation estimates across the whole spectrum of the distribution, during the post-invasion period. Barring the lower-tail (stock markets are in a bearish state), any possibility of diversification benefit or safe haven property seems to have frittered away during the recent episode of Russian invasion. In general, there is consistent evidence across the globe of a lack of diversification and safe haven properties relative to the Ethereum market during the Russian-Ukraine war period.

[Insert Figures 5 about here.]

The Russian-Ukrainian war has brought up crypto's impact on geopolitical issues. At the beginning of the Russian invasion, there was widespread concern about Russian efforts to use crypto markets to raise funds and evade international sanctions. Recent research

also shows that cryptocurrencies are widely used, for illicit financial purposes (Foley *et al.*, 2019). Furthermore, even before the sanctions imposed after the War started, cryptocurrencies were more prevalent in Russia than equity, with up to 12% of Russians owning cryptocurrencies (Khalfaoui *et al.*, 2023). Given the potential benefits and risks associated with crypto—including sanctions evasion, cybercrimes and market volatility—public and private organizations across countries imposed new regulations. For example, the European Union is advancing a new law (known as Markets in Crypto-Assets-MiCA), to create a regulatory regime to improve the cryptocurrencies tracking systems and the legal compliance. The IMF also recently released a guide on the handling of crypto assets for member countries. These new regulations following the increasing global inflation and uncertainty and increasing investors' need for additional liquidity may cause a disassociation between the two markets, especially in the short-run.

For all equity-crypto market pairs the structure of dependence is nonlinear and asymmetric during both periods (pre- and post-invasion). Similar to the Covid-19 period a strong positive tail dependence is observed in times of extreme market stress (left tail). This could serve as an indication of the integration of the equity and the two main cryptocurrency markets that are driven by common adverse shocks. Yet, the plots for the Bitcoin-equity market pairs and to a larger extent for the Ethereum-equity market pairs seem to corroborate that contagion has occurred between the two markets in many cases, an issue that is examined further below.

Moreover, Figures (2-5) also show the sample correlation coefficient point estimates for each stock and cryptocurrency market pair. We observe that these estimates lie to values close to zero for all cases and give an incomplete characterization of the dependence structure between the stock market-Bitcoin pairs compared to the local Gaussian correlation estimates.

5.3 Testing for contagion between the international stocks and cryptocurrency markets

This section examines the hypothesis of contagion transmission from the stock markets to each of the cryptocurrency markets, in the period following the Covid-19 pandemic and the Russian invasion of Ukraine in late February 2020 and 2022, respectively, which marked the beginning of a period of heightened turbulence in the global financial markets. The results

from the bootstrap test for contagion are presented in Table 1 and refer to the transmission (Yes) or no transmission (No) of contagion from the main stock markets (Columns (2)-(7)) to the remaining cryptocurrency markets (Bitcoin and Ethereum).

[Insert Table 1 about here.]

First, our analysis aims to establish the contagious nature of the international stock markets towards the two major cryptocurrency markets during the main stage of the Covid-19 pandemic. Panel A in Table 1 provides evidence of cross-asset contagion between the two East Asian stock markets and the Bitcoin market. The European markets, Australia, and the US appear unable to generate contagion towards the Bitcoin market after the Covid-19 pandemic shock. Turning to the Ethereum market, contagion effects appear again to stem from the East Asian stock markets. *Corbet et al. (2020)* using intraday data and dynamic correlation analysis also documents contagion effects between Bitcoin and Chinese stock markets in the period after the identification of the Covid-19 pandemic. No evidence of contagion is found for the rest of the markets. In this respect, Asian markets emerge as the principal transmitters of contagion towards the two main cryptocurrency markets. This evidence of contagion is found to be strong in statistical terms since in all cases the null hypothesis of no contagion is rejected at the 5% significance level.

Panel B in Table 1 presents the bootstrap test of contagion between the six equity markets and the two major cryptocurrency markets during the Russian invasion of Ukraine. Considering Bitcoin, we find strong evidence of contagion in the stock markets of Australia, the UK and the US. Contagion is also observed between the Ethereum market and Germany, the UK and the US (marginally for Japan). Interestingly, the contagion phenomena observed between China and the two cryptocurrency markets during the Covid-19 period vanished during the Russian invasion of Ukraine period, while the US and UK stock markets emerge as principal transmitter of contagion in both markets during the period of the Russian-Ukrainian war. This outcome could also be partly attributed to the fact that in late September 2021, the People's Bank of China (PBOC) banned all transactions related to cryptocurrencies. An argument also supported in *Borri (2020)*, who show that domestic regulations affect the trading of cryptocurrencies in the domestic market, and may cause heterogenous spillovers on prices and trading volume across markets and currencies. Moreover, as noted in *Wang and Su*

(2023) the impact of the Russia-Ukrainian conflict deeply affected China, through increased production costs, problems in industrial and supply chains as well as increased instability of financial markets.

5.4 Testing for conditional independence in high levels of uncertainty

In this section, we use the local Gaussian partial correlation approach of Otneim and Tjøstheim (2022) to illustrate the local partial dependence structure and examine for conditional independence between equity and cryptocurrency markets given the higher levels of the pandemic and geopolitical uncertainty of the respective periods. The local Gaussian partial correlation concept and bootstrap test for conditional independence are presented in the Appendix.

Table 2 presents the results from the bootstrap test of conditional independence between the six equity markets and the two cryptocurrency markets during the two different crisis periods. Our main objective is to examine their relationship in times of extreme market conditions. Thus, each equity and cryptocurrency market pair is conditioned to high levels of pandemic (*EMV*, for the Covid-19 period) and geopolitical (*GPR*, for the Russian invasion period) induced uncertainty.

[Insert Table 2 about here.]

Columns (2) and (3) in Table 2, show the results of the conditional independence test for each equity and cryptocurrency market pair during the Covid-19 pandemic period. Regarding the equity-Bitcoin market pairs (upper panel) we observe rejection of the null hypothesis at the 5% significance level for China, Japan and the US. Charfeddine *et al.* (2020) also document different regimes in dependence between the S&P500 and the cryptocurrencies which are related to various financial and economic events. For example, they find a decrease in dependence during the early stages of the Eurozone crisis and an increase in dependence as the crisis started to spread to the other members of the Eurozone. This also reflects the empirical study of Kurka (2019) that spikes in the time-varying spillovers between Bitcoin and traditional asset classes are connected to certain events, such as policy changes, or disruptions to cryptocurrency markets.

In this respect, given a high level of pandemic induced uncertainty, the Bitcoin market possesses significant dependence on the two East Asian markets and the US market. To further scrutinize the structure of the dependence between the equity and bitcoin markets in Figure 6 we present the local partial dependence maps. Each plot illustrates the trivariate local Gaussian partial correlation between Bitcoin and each stock market on the plane defined by high infectious disease EMV levels. The LGPC estimates are positive in all four quadrants for the China and Japan cases as well as for the US but only in the third and fourth quadrant. For China, we observe higher LGPC estimates at the second quadrant (ranging from 0.22 to 0.34). In other words, higher dependence between the Chinese stock market and the Bitcoin market is observed in the state of low stock market returns and high Bitcoin returns. For Japan the LGPC estimates are higher at the second and third quadrant (ranging from 0.17 to 0.35) and for the US at the third quadrant (ranging from 0.06 to 0.3). In general, we observe an increase in dependence mainly at the second and third quadrants in which the stock market returns are negative. Thus, given a high level of pandemic induced uncertainty, we may infer that when the East Asian and the US markets are in a state of distress the cross-market dependence with Bitcoin increases in a significant manner. When the analysis is concentrated in the Ethereum market (lower panel in Table 2) a similar picture emerges with the US, China and Japan to present strong conditional dependence, as well as Australia but to a lesser extend (at the 10% level of significance). The LGPC estimates in Figure 7 show that an increase in dependence is mainly observed in the third quadrant for Japan and the US and in the second quadrant for the case of China. Similar conclusions were reached for the other equity markets. However, in the cases of Australia, Germany, the UK and Japan the LGPC estimates are negative or close to zero in the data rich portions of the sample space.

[Insert Figures 6 and 7 about here.]

Table 2 (columns (3) and (4)) presents the results from the period of the Russian invasion of Ukraine. Estimates from the examination of the conditional independence, reveal that during the Russian-Ukrainian conflict, there is significant dependence between almost all equity market pairs (except China) and the Bitcoin market. The local partial dependence maps for the bitcoin and the six equity markets are given in Figure 8. Overall, we observe an increased dependence between all equity-bitcoin markets due to the negative developments

in Europe. Under the circumstances of high geopolitical risk, the LGPC estimates between the Bitcoin and equity markets are positive and significant providing strong evidence of increased dependence almost globally during the Russian-Ukrainian crisis. Higher LGPC estimates are observed in the second and third quadrants where the stock markets are in a state of distress. This observation holds uniformly for all countries, with the US being the country with the higher LGPC estimates (ranging from 0.35 to 0.61). This points to the existence of strong linkages between the Bitcoin market and the US stock markets during the period of Russian invasion as well as to the degrading role of China during the same period. Similar results in Table 2 are observed for the Ethereum and the equity market pairs with the null hypothesis of independence rejected for the cases of Germany, Japan, the UK and the US (lower panel). Estimates in Figure 9 reveal that there is positive dependence for the cases where the hypothesis of independence has been rejected. Again strong dependence is found on the US stock market followed by the German and UK stock markets.

[Insert Figures 8 and 9 about here.]

This graphical analysis indicates that cryptocurrencies and the considered conventional assets may fall into two different classes of financial assets. Several reasons can explain these high differences in terms of price, returns, and volatility (Garcia and Schweitzer, 2015; Shahzad *et al.*, 2019). A plausible explanation is that the cryptocurrencies market is not mature, compared to the markets of conventional assets. Khalifaoui *et al.* (2023) show that the Russian-Ukraine war attention significantly impact cryptocurrencies, with price falls explained as sell-offs by large holders in seeking liquidity. The cryptocurrency market is also more prone to investors' speculation and speculative bubbles, which create sharp movements in cryptocurrency prices, generating high levels of returns and variability. Moreover, cryptocurrencies are subject to different government regulations, bans, high risks of fraud and forks among many other factors likely to impact their prices (Kristoufek, 2015; Garcia and Schweitzer, 2015; Ciaian *et al.*, 2016; Shahzad *et al.*, 2019). In contrast, conventional assets markets are well established and regulated, less prone to manipulations and asset prices are more linked to the fundamentals of the economy and more likely to reflect the assets' intrinsic value.

6 Conclusion

The first quarter of 2020 was an extraordinary time for the international stock markets. The year started smoothly but it was followed by the fastest collapse ever, caused by the unfolding of an unexpected pandemic. Two years later, on 24 February 2022, Russia officially attacked Ukraine. The ongoing military action further raises concerns about its potential impact on the global economy and in particular on the reaction of global financial markets.

In this paper, we exploit the aforementioned episodes, to examine the cross-market linkages between six international stock market indices (Australia, China, Germany, Japan, the UK and US) and the two major cryptocurrency markets (Bitcoin and Ethereum). Specifically, we are testing for changes in the dependence structure and contagion after the episodes of the Covid-19 pandemic and the Russian invasion of Ukraine. We employ a local Gaussian correlation approach that can describe the dependence structure in every segment of the conditional distribution on the one hand and consider the intrinsic nonlinearity of the relationship on the other. This analysis enabled us to assess the relationship between the markets under extreme conditions.

Our results provide useful information for both portfolio managers and international investors. For example, cross-market dependence analysis results are particularly relevant in terms of diversification gains from a portfolio of the financial markets considered. It also offers further insights into the management of such a portfolio under phases of market distress. We show that Bitcoin and Ethereum markets may serve as a useful diversification tool for the international stock markets in tranquil times. This feature does not hold after the Covid-19 pandemic shock as well as the Russian-Ukrainian war. Dependence between the stock and cryptocurrency markets increases after the shock in both cases (pandemic and war shock), and is higher when stock markets are in a state of distress. Overall, our results are consistent with cryptocurrency investors responding to the recent exogenous shocks by selling in search of additional liquidity, engendering price falls. Additionally, regardless of whether liquidity concerns are driving the fall in cryptocurrency prices, results are useful for investors when making hedging strategies concerning their positions when markets are at a different state (bullish or bearish).

Furthermore, both cryptocurrency markets are not found to be immune to contagion ef-

fects, mainly stemming from the East Asian equity markets during the pandemic period and the US and UK markets during the recent Russian invasion period. In this respect, investors and portfolio managers may construct effective investment strategies and hedges against pandemic and geopolitical risks. In general, the Ethereum market is found to be more vulnerable to contagion effects compared to the Bitcoin market, also illustrating the importance of investor portfolio reallocation to counter different types of risks.

Moreover, the Russian attack has introduced further uncertainty into world stock markets in addition to that related to the COVID-19 pandemic. Since uncertainty is inherently a latent variable, obtaining an appropriate measure for it is not straight-forward. This paper sheds light on the equity-crypto market linkages under conditions of extreme pandemic (for the Covid-19 period) and geopolitical risk (for the Russian invasion period) uncertainty by utilizing the Infectious Disease EMV Index of [Baker *et al.* \(2020\)](#) and the geopolitical risks (GPR) index of [Caldara and Iacoviello \(2022\)](#) into the local Gaussian partial correlation approach. For the two recent episodes, we find that in times of high uncertainty, the hypothesis of conditional independence is strongly rejected. In general, during turmoil times characterized by extremely low stock market returns and high uncertainty the dependence between the equity and cryptocurrency markets is found to be stronger, with this effect being more pronounced during the recent Russian invasion of Ukraine. This issue may suggest that investors should pay a closer look at the asymmetric connectedness between markets, especially when pandemic and geopolitical risk induced uncertainty is at extremely high quantiles, and cryptocurrencies or/and stock markets are at extremely low quantiles (bearish market). Our results can also be useful to investors to make more informed portfolio decisions to control downside risk in the wake of extreme events.

Based on our analysis, investors, portfolio managers and policy makers now have strong empirical evidence that the two major cryptocurrency markets possess some virtues against extreme down movements in the international stock market indexes, but only in normal times. However, stock investors in developed markets have a restricted range of safe-haven assets, given that Bitcoin and Ethereum failed to offer such a property during the two recent worldwide exogenous shocks. Interestingly, the timing of the shocks has coincided with the restrictions taken by the Chinese government against Bitcoin exchanges and trad-

ing activities. Shortly after the Russian invasion of Ukraine national authorities called for a regulatory framework on crypto assets, to prevent Russia from circumventing economic sanctions. However international regulatory bodies have, on the whole, taken very different approaches to regulatory policy for crypto assets. However, it is not clear whether the extent of the different regulatory regimes of the different countries under study has affected our analyses. This would open the door for future research considering the relationship between the two financial markets under different monitoring and regulatory policy approaches.

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Table 1: This table shows p -values from the bootstrap test for contagion considering stock markets as the origin. The null hypothesis indicates no contagion between the stock markets (Australia, China, Germany, Japan, UK and US) and the cryptocurrency markets (Bitcoin and Ethereum). Significance levels at 10%, 5% are denoted by *,**. Yes indicates that the null of no contagion is rejected at 5% level. The bootstrap test for contagion is based on 5000 replications.

Origin:Stock market	Australia	China	Germany	Japan	UK	US
<i>Panel A : Covid – 19 pandemic</i>						
Bitcoin	0.225	0.031**	0.788	0.048**	0.996	0.910
Contagion?	No	Yes	No	Yes	No	No
Ethereum	0.170	0.000**	0.192	0.024**	0.554	0.300
Contagion?	No	Yes	No	Yes	No	No
<i>Panel B : Russian invasion of Ukraine</i>						
Bitcoin	0.000**	0.125	0.368	0.669	0.000**	0.000**
Contagion?	Yes	No	No	No	Yes	Yes
Ethereum	0.480	0.355	0.000**	0.050**	0.038**	0.007**
Contagion?	No	No	Yes	Yes	Yes	Yes

Table 2: Bootstrap test for conditional independence between the equity (R_t) and cryptocurrency (BTC_t and ETH_t for bitcoin and ethereum, respectively) market returns, conditioning on the level of uncertainty (EMV_t and GPR_t). p -values for the null hypothesis are given in columns (2) and (4), respectively. No indicates that the null hypothesis of conditional independence is rejected at 5% level. The bootstrap test for conditional independence is based on 1000 replications.

	Covid-19 pandemic		Russian invasion of Ukraine	
	p-value	Conditional independence?	p-value	Conditional independence?
Hypothesis testing	$H_0 : BTC_t \perp R_t EMV_t$		$H_0 : BTC_t \perp R_t GPR_t$	
Australia	0.240	Yes	0.001**	No
China	0.000**	No	0.122	Yes
Germany	0.062*	Yes	0.024**	No
Japan	0.022**	No	0.044**	No
UK	0.129	Yes	0.032**	No
US	0.045**	No	0.011**	No
Hypothesis testing	$H_0 : ETH_t \perp R_t EMV_t$		$H_0 : ETH_t \perp R_t GPR_t$	
Australia	0.055*	Yes	0.478	Yes
China	0.010**	No	0.398	Yes
Germany	0.363	Yes	0.000**	No
Japan	0.000**	No	0.036**	No
UK	0.102	Yes	0.000**	No
US	0.033**	No	0.000**	No

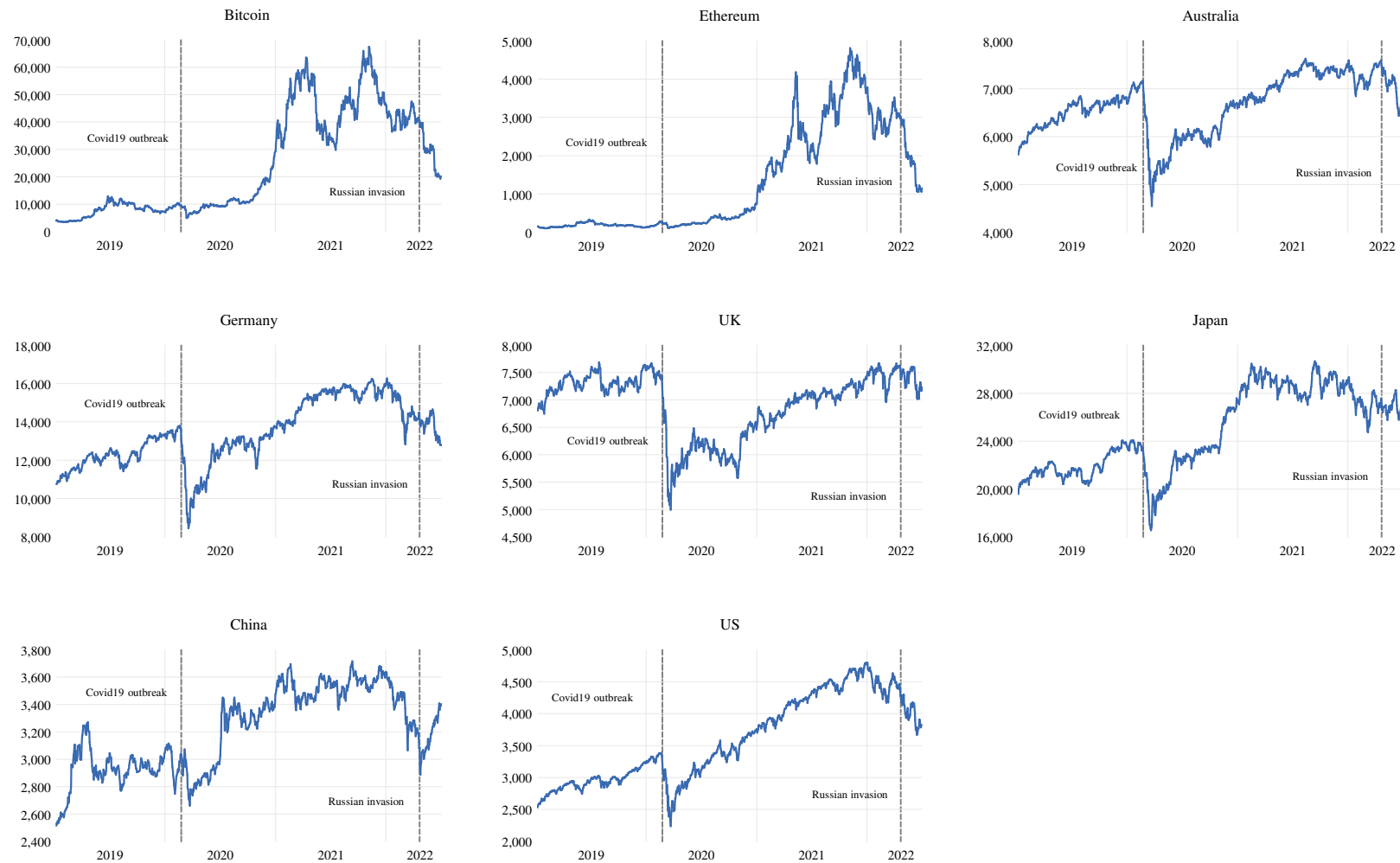


Figure 1: Stock and cryptocurrency market prices. The vertical dotted lines denote the dates of the Covid-19 outbreak and the Russian invasion of Ukraine(24 February 2020 and 2022, respectively).

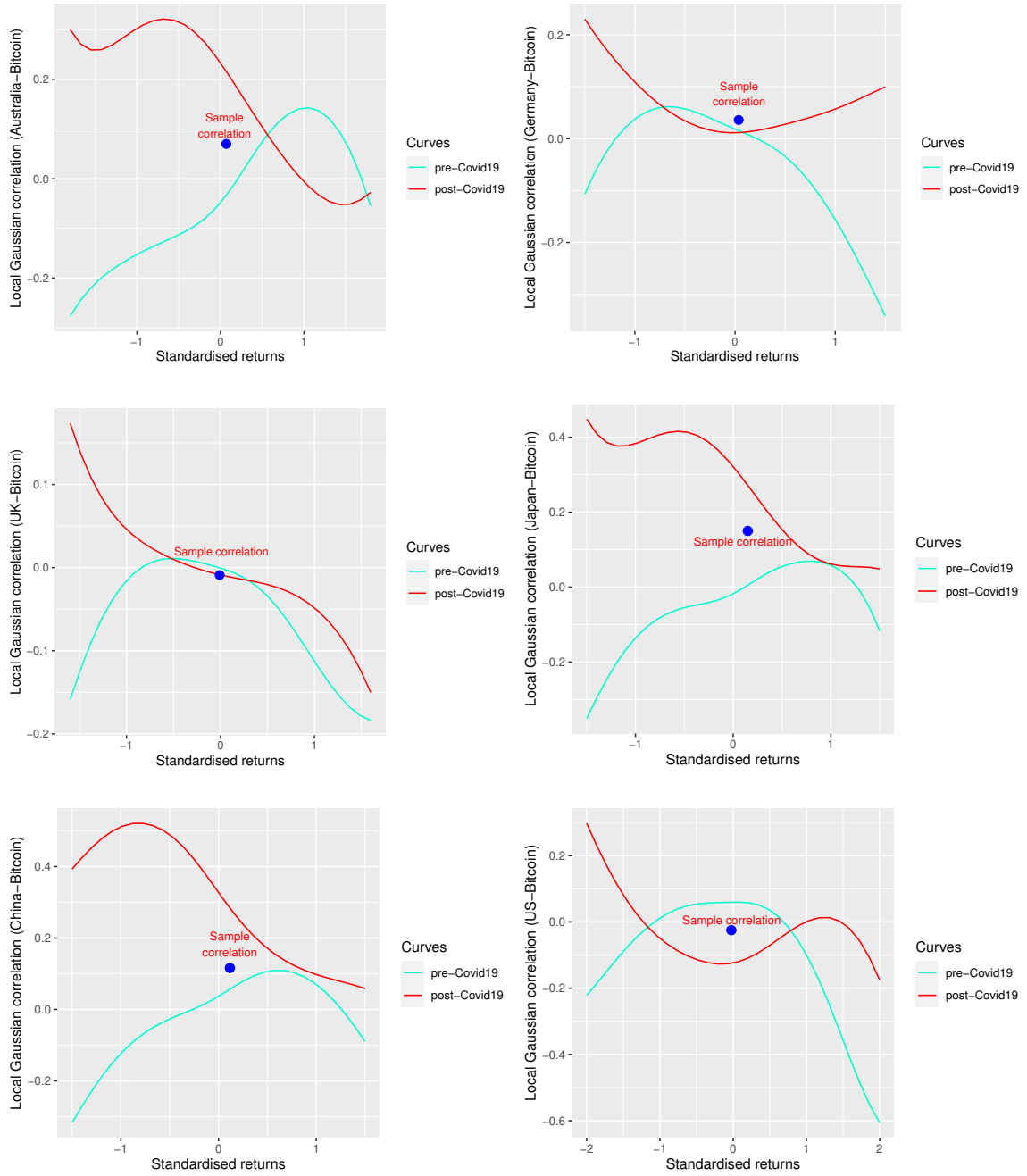


Figure 2: Local Gaussian correlation estimates between the stock markets and the Bitcoin market returns during pre- and post-Covid-19 pandemic period.

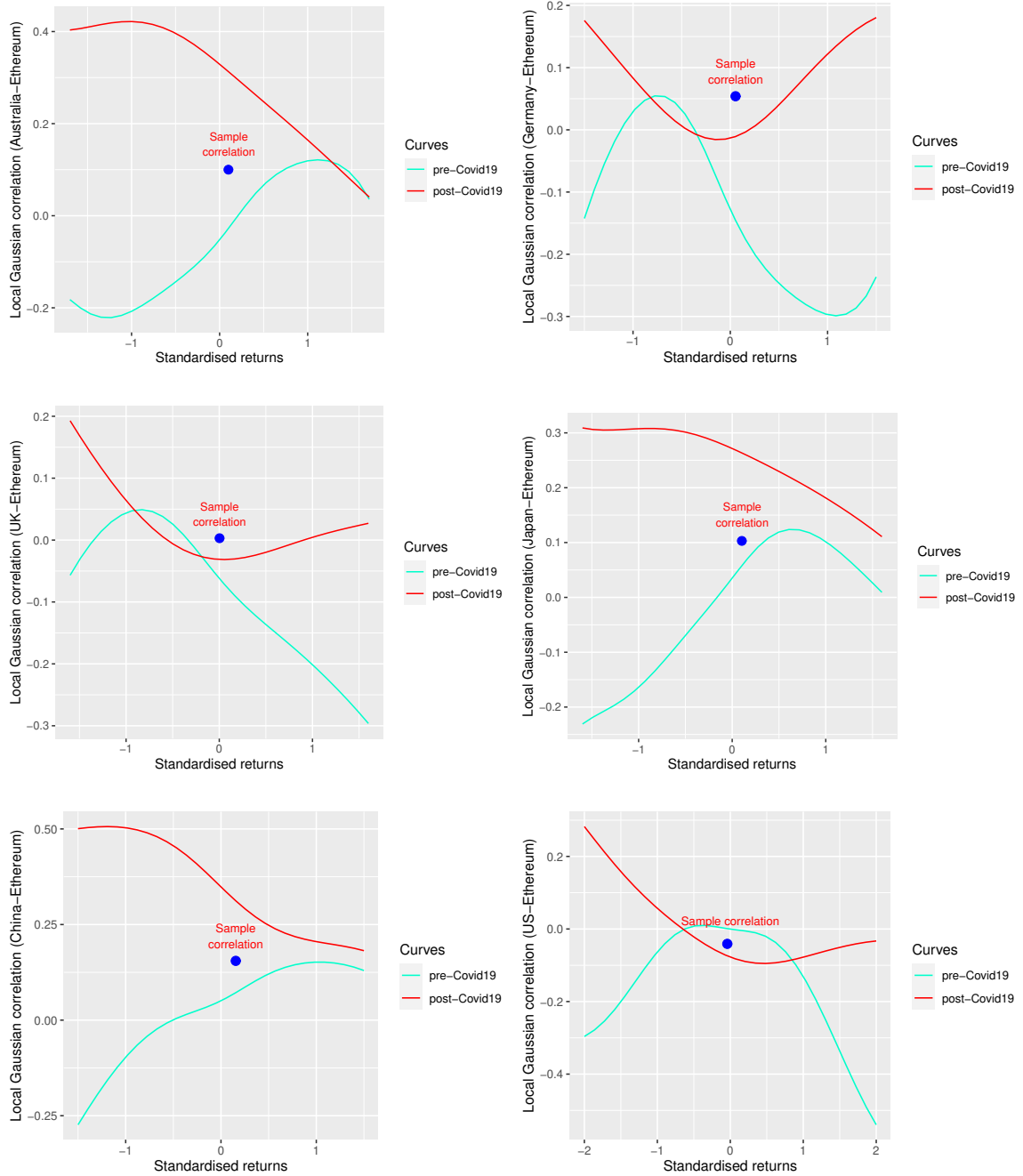


Figure 3: Local Gaussian correlation estimates between the stock markets and the Ethereum market returns during pre- and post- Covid-19 pandemic period

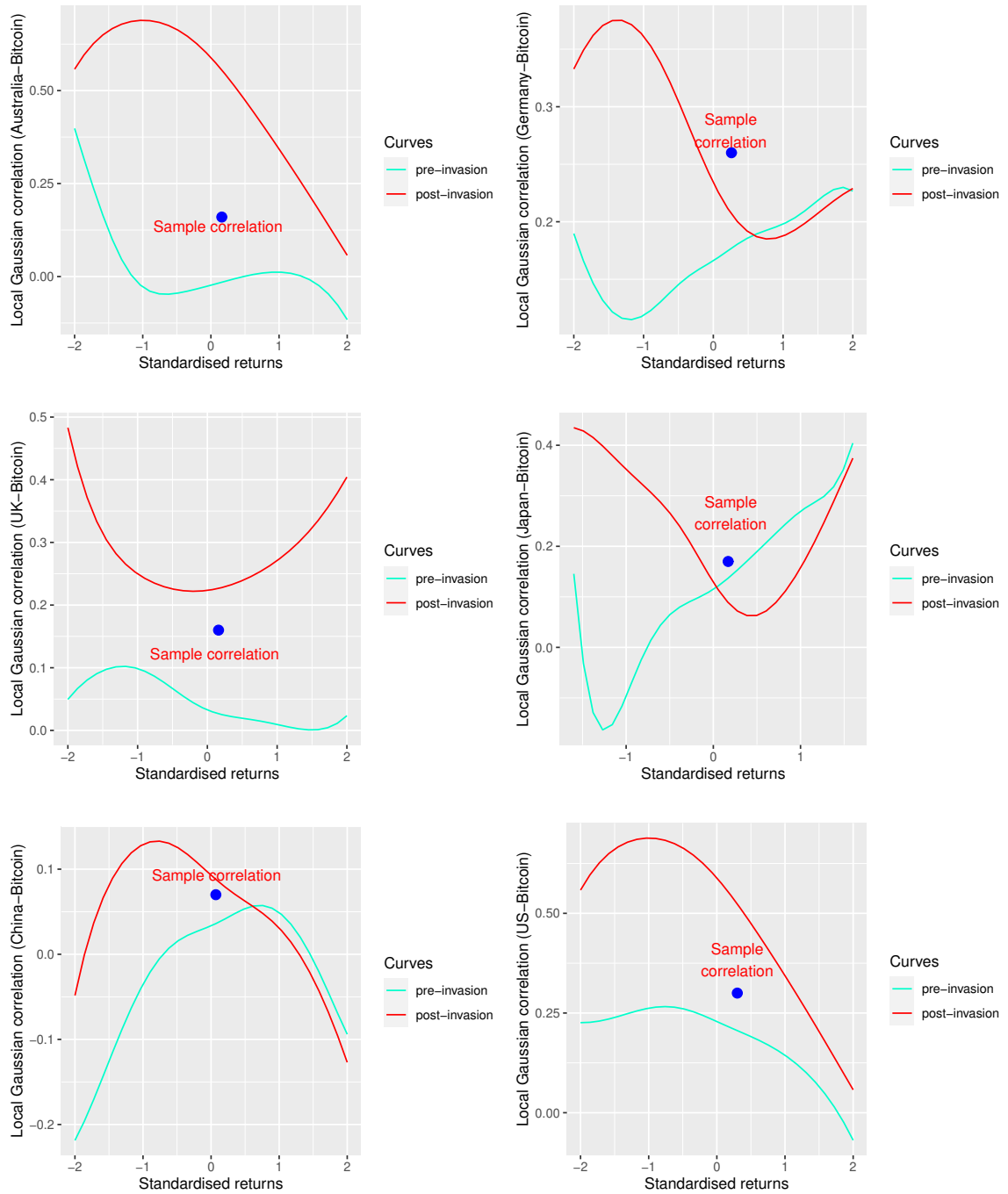


Figure 4: Local Gaussian correlation estimates between the stock markets and the Bitcoin market returns during pre- and post- Russian invasion of Ukraine.

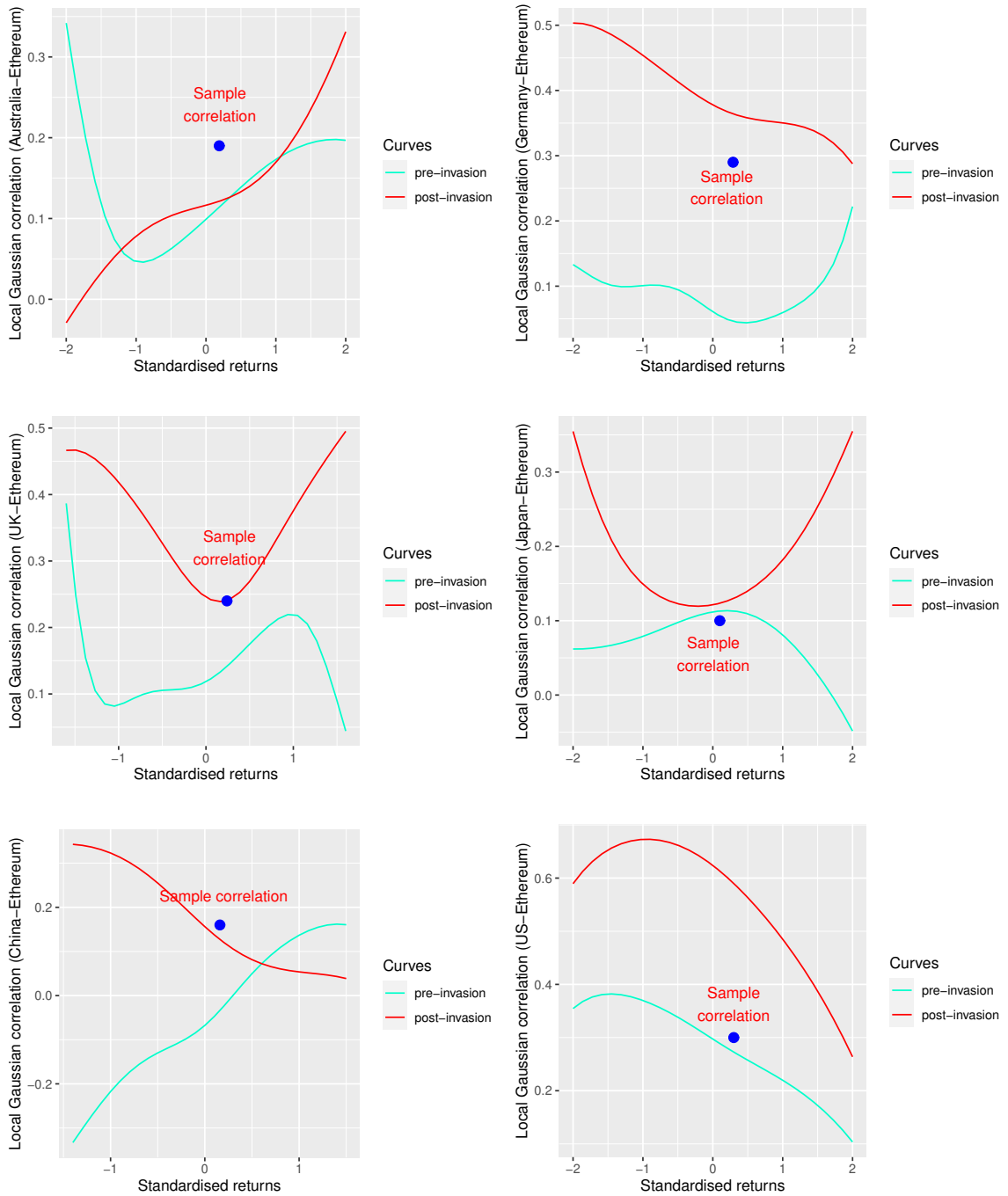
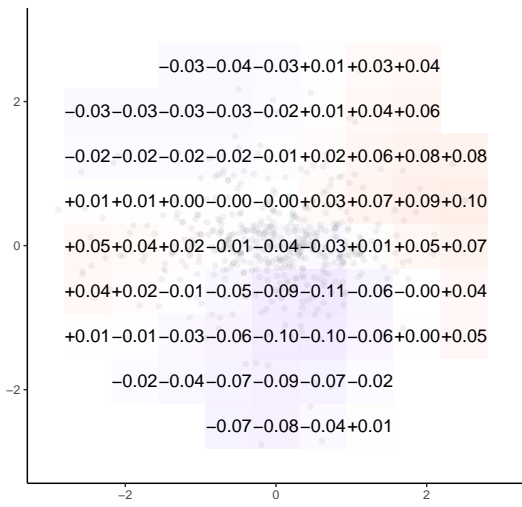
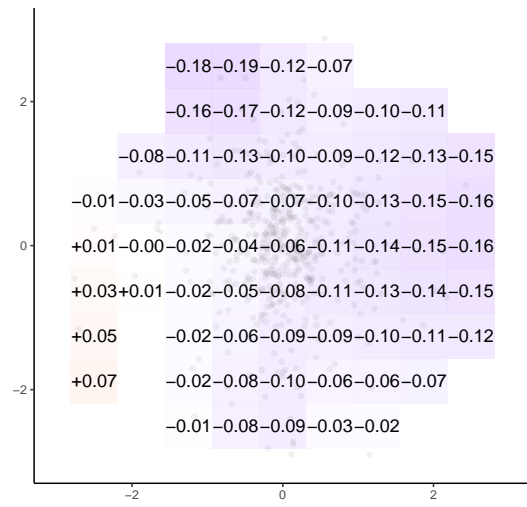


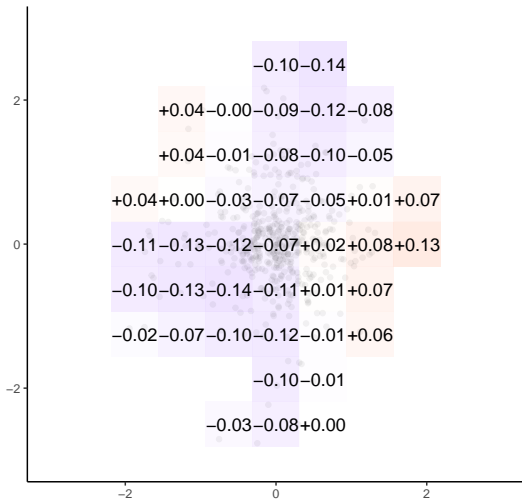
Figure 5: Local Gaussian correlation estimates between the stock markets and the Ethereum market returns during pre- and post- Russian invasion of Ukraine.



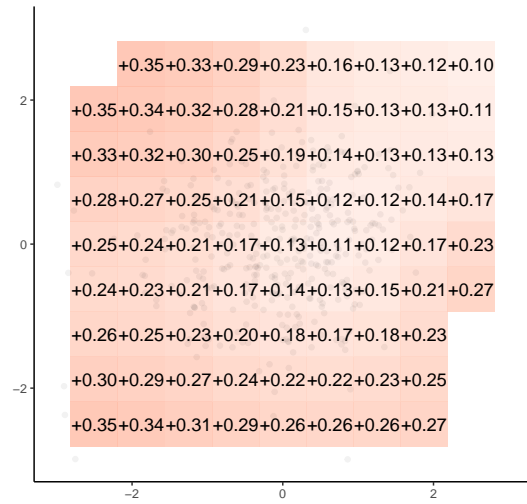
(a) Australia



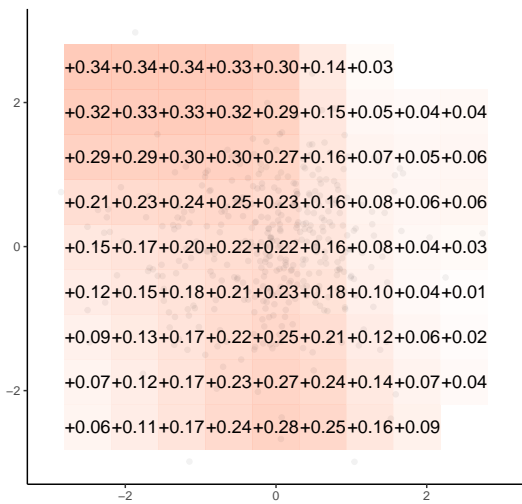
(b) Germany



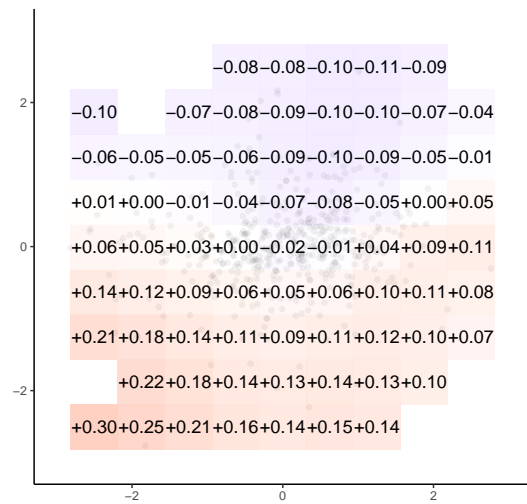
(c) UK



(d) Japan

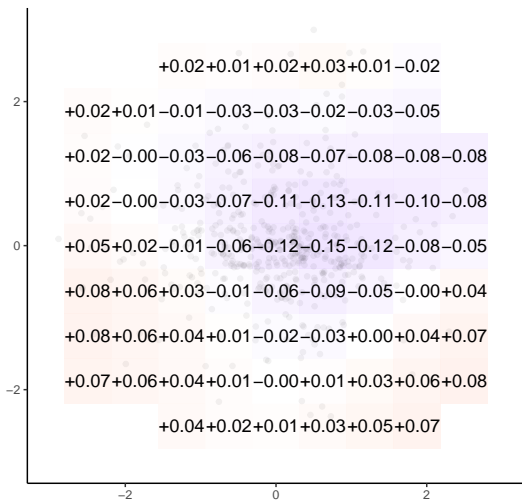


(e) China

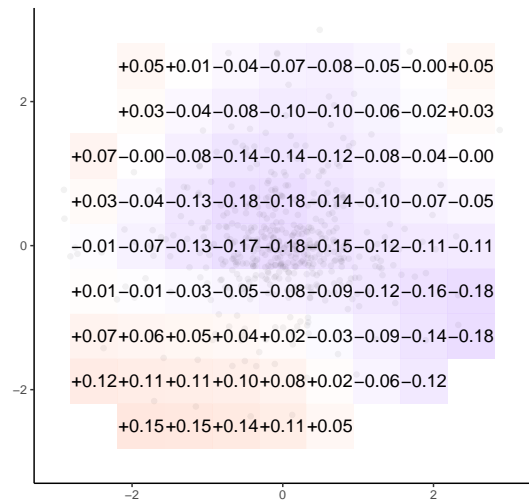


(f) US

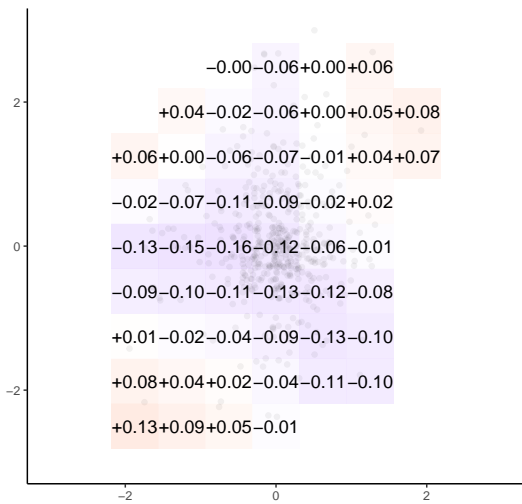
Figure 6: Local partial dependence map of Bitcoin vs. Stock market returns on the plane defined by high infectious disease EMV levels ($EMV_t = 3$)



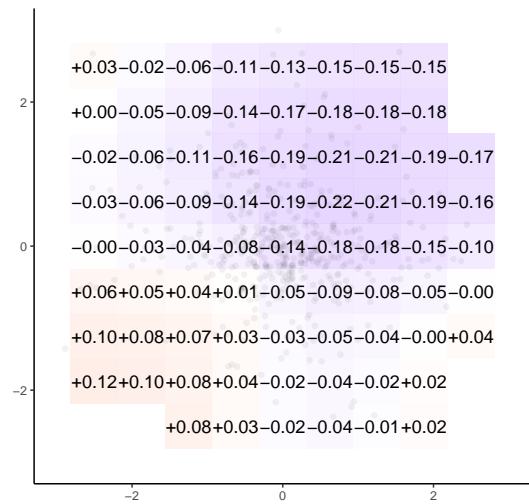
(a) Australia



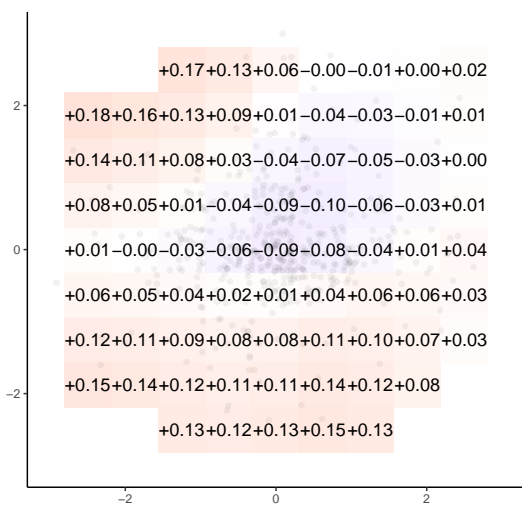
(b) Germany



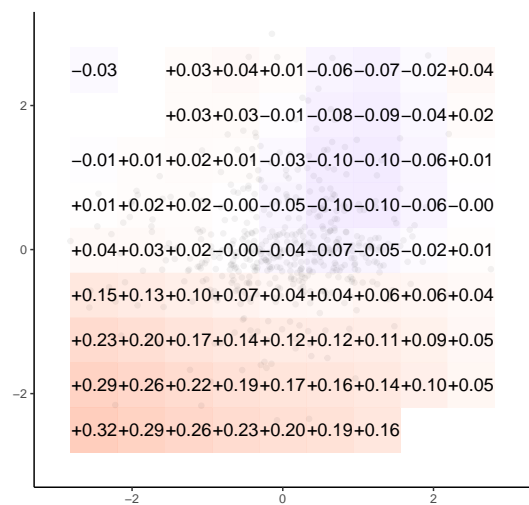
(c) UK



(d) Japan

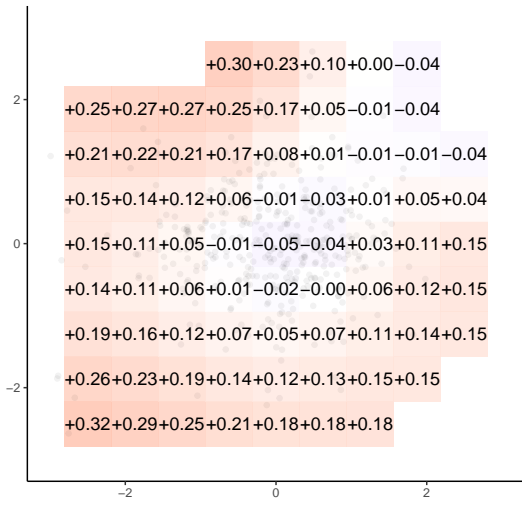


(e) China

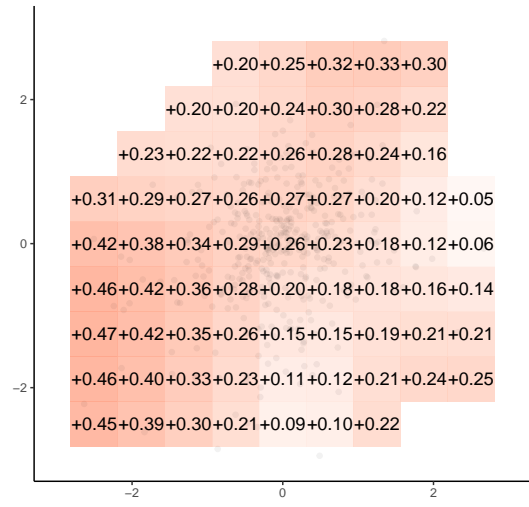


(f) US

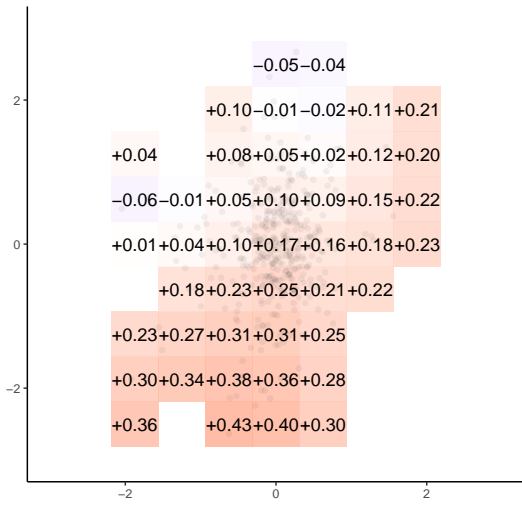
Figure 7: Local partial dependence map of Ethereum vs. Stock market returns on the plane defined by high infectious disease EMV levels ($EMV_t = 3$)



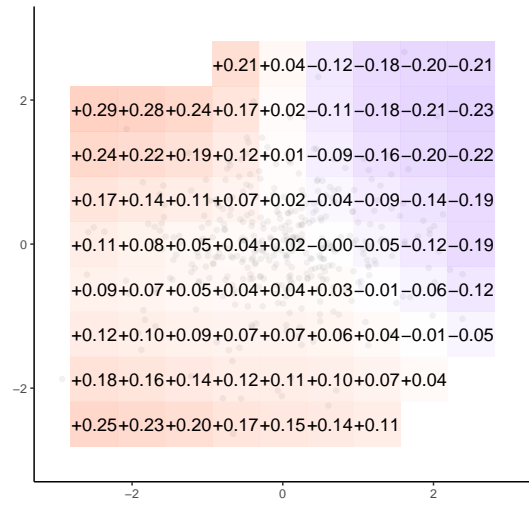
(a) Australia



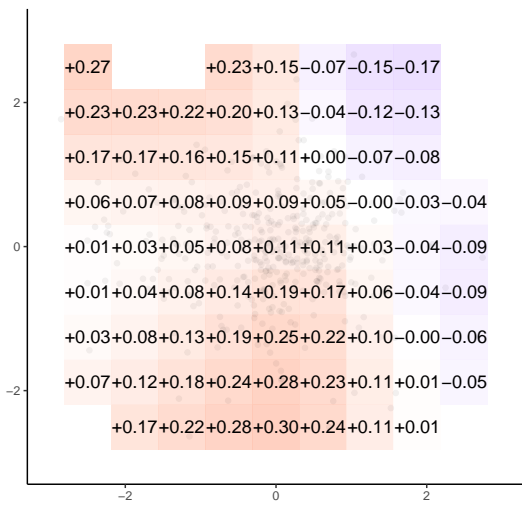
(b) Germany



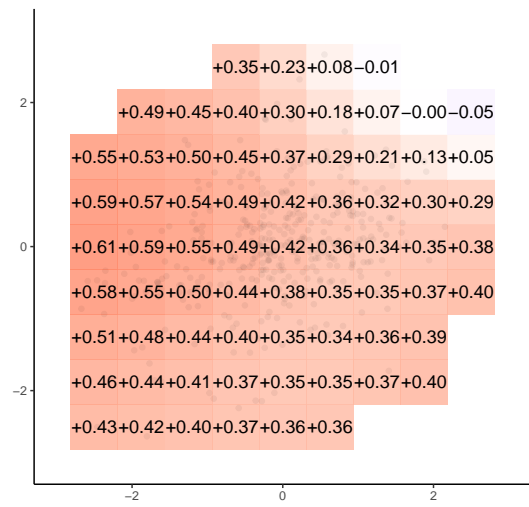
(c) UK



(d) Japan

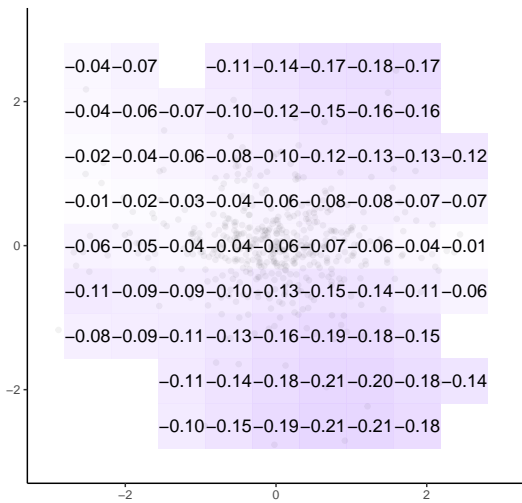


(e) China

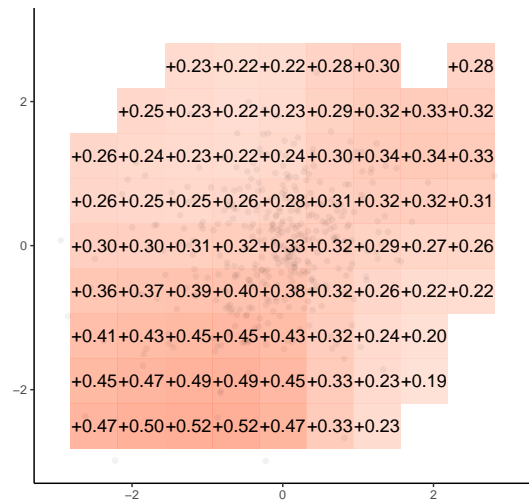


(f) US

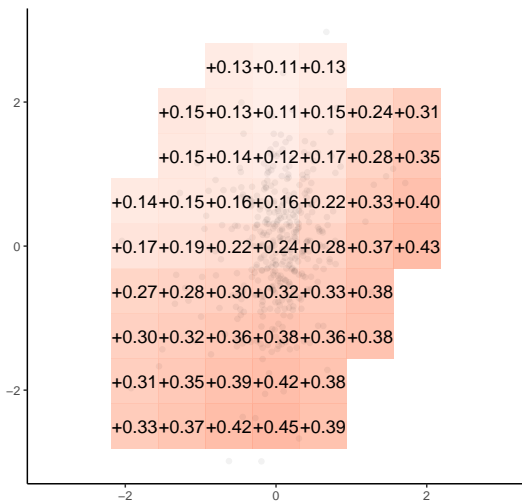
Figure 8: Local partial dependence map of Bitcoin vs. Stock market returns on the plane defined by high geopolitical risk uncertainty levels ($GPR_t = 5$)



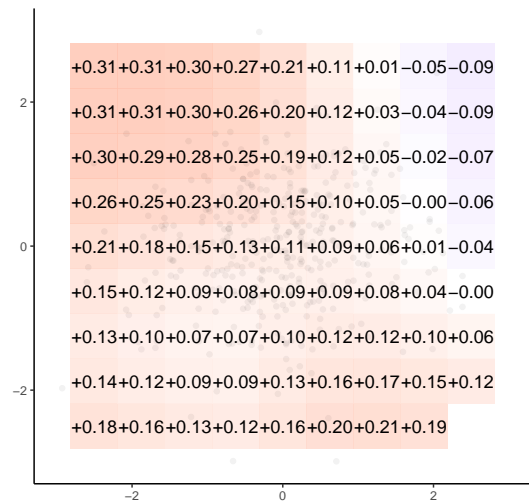
(a) Australia



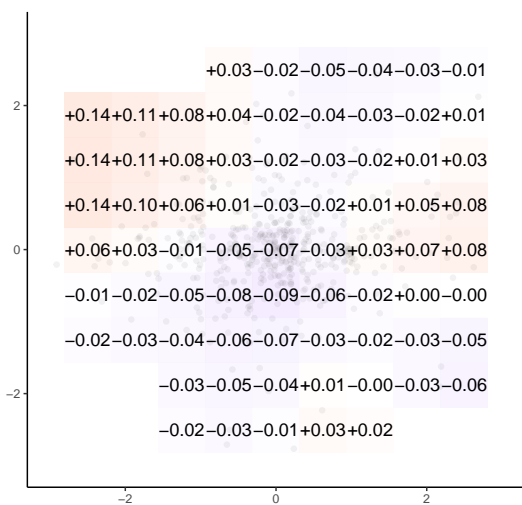
(b) Germany



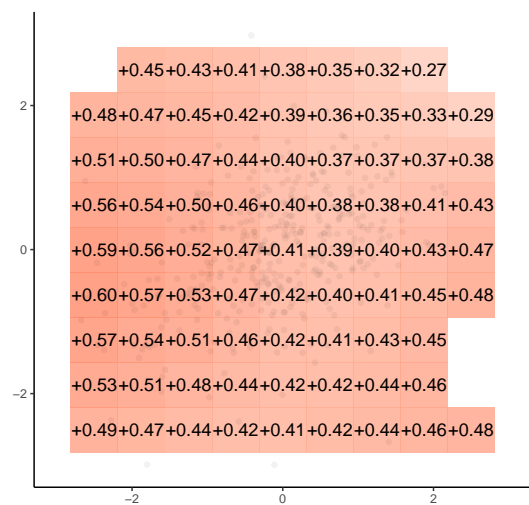
(c) UK



(d) Japan



(e) China



(f) US

Figure 9: Local partial dependence map of Ethereum vs. Stock market returns on the plane defined by high geopolitical risk uncertainty levels ($GPR_t = 5$)

Appendix

Table 3: The upper panel of the table shows the Ljung–Box statistics for tests of lack of correlation of squared standardised residuals for lags $p= 1, 5, 15$ and 20 , denoted as $Q^2(p)$. The lower panel of the table illustrates the p -value of ARCH LM test that tests the null hypothesis of no ARCH effects in standardised residuals (see, Engle, 1982).

	$Q^2(1)$ [<i>p-value</i>]	$Q^2(5)$ [<i>p-value</i>]	$Q^2(10)$ [<i>p-value</i>]	$Q^2(15)$ [<i>p-value</i>]	$Q^2(20)$ [<i>p-value</i>]
Australia	1.519 [0.234]	1.749 [0.883]	4.350 [0.930]	12.555 [0.434]	21.082 [0.160]
China	2.985 [0.945]	3.240 [0.663]	4.703 [0.910]	10.566 [0.311]	13.023 [0.578]
Germany	4.088 [0.465]	5.417 [0.367]	9.054 [0.527]	9.294 [0.730]	10.587 [0.811]
Japan	6.725 [0.541]	9.657 [0.086]	11.738 [0.303]	7.634 [0.396]	11.070 [0.697]
UK	3.744 [0.364]	8.145 [0.148]	4.163 [0.166]	4.931 [0.474]	10.959 [0.121]
US	2.855 [0.325]	2.325 [0.802]	3.118 [0.978]	6.677 [0.621]	8.526 [0.596]
Bitcoin	4.764 [0.124]	1.077 [0.184]	2.953 [0.982]	18.228 [0.369]	18.279 [0.765]
Ethereum	4.805 [0.198]	1.582 [0.133]	2.607 [0.989]	19.045 [0.136]	19.510 [0.687]
	<i>ARCH</i> (1)	<i>ARCH</i> (5)	<i>ARCH</i> (10)	<i>ARCH</i> (15)	<i>ARCH</i> (20)
Australia	0.138	0.931	0.538	0.859	0.996
China	0.204	0.662	0.924	0.537	0.644
Germany	0.628	0.366	0.492	0.464	0.986
Japan	0.845	0.168	0.399	0.748	0.889
UK	0.463	0.176	0.178	0.554	0.212
US	0.648	0.809	0.978	0.724	0.707
Bitcoin	0.303	0.964	0.986	0.814	0.148
Ethereum	0.232	0.904	0.979	0.951	0.667

The local Gaussian partial correlation and the conditional independence test of **Otneim and Tjøstheim (2022)**

Assume that $\mathbf{Y} = (Y_1, \dots, Y_p)$ is a random vector and (Y_1, Y_2, Y_3) denotes a partition of vectors of \mathbf{Y} with dimensions d_1, d_2, d_3 respectively. In this case, $\mathbf{Y}^{(1)} = (\mathbf{Y}_1, \mathbf{Y}_2) = (Y_1, \dots, Y_{d_1+d_2})$ incorporates the first $d_1 + d_2$ components in \mathbf{Y} and $\mathbf{Y}^{(2)} = \mathbf{Y}_3 = (Y_{d_1+d_2}, \dots, Y_{d_1+d_2+d_3})$ contains the remaining d_3 variables ($p = d_1 + d_2 + d_3$). Partitioning the the mean vector μ and covariance matrix Σ of \mathbf{Y} :

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}, \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \Sigma_{21} & \Sigma_{22} & \Sigma_{23} \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{pmatrix} \quad (2)$$

where Σ_{ij} is the covariance matrix of $(\mathbf{Y}_i, \mathbf{Y}_j)$ $i, j = 1, 2, 3$. When $\mathbf{Y}^{(2)} = \mathbf{Y}_3$ we may separate between two concepts of correlation, the *partial* and the *conditional* correlation, which may coincide in the Gaussian distribution as well as in several other joint distributions.

Based on the partial correlation when defining the local Gaussian partial correlation (LGPC), the partial variance-covariance matrix of $\mathbf{Y}^{(1)} = (\mathbf{Y}_1, \mathbf{Y}_2)$ given $\mathbf{Y}^{(2)} = \mathbf{Y}_3$ can be written as:

$$\Sigma_{12|3} = \Sigma^{11} - \Sigma^{12} (\Sigma^{22})^{-1} \Sigma^{21}, \quad (3)$$

where $\Sigma_{12|3}$ denotes the covariance matrix in the conditional (Gaussian) distribution of $\mathbf{Y}^{(1)}$ given \mathbf{Y}_3 , under the assumption that \mathbf{Y} is jointly normal. Next, we may define the partial correlation matrix between \mathbf{Y}_1 and \mathbf{Y}_2 given \mathbf{Y}_3 , as:

$$\mathbf{R}_{12|3} = \mathbf{D}^{-1/2} \Sigma_{12|3} \mathbf{D}^{-1/2} \quad (4)$$

where $\mathbf{D} = \text{diag}(\Sigma_{12|3})$. Similarly, we identify the partial correlation matrix shown in (3) with the correlation matrix in the conditional (Gaussian) distribution of $\mathbf{Y}^{(1)}$ given $\mathbf{Y}^{(2)}$ in the case that \mathbf{Y} is jointly normal. Equations (8) and (9) may serve as the primary point the local partial correlation definition.

Assuming that the components of \mathbf{Y} are continuous, given a point y we approximate the joint density function f_Y in a neighborhood of y by a multivariate Gaussian density $\phi(y, \nu)$, which at the point $y = \nu$ is

$$\phi(y) = \frac{1}{(2\pi)^{p/2} |\Sigma(y)|^{1/2}} \exp \left\{ -1/2 (\mathbf{y} - \mu(\mathbf{y}))^T \Sigma^{-1}(\mathbf{y}) (\mathbf{y} - \mu(\mathbf{y})) \right\}, \quad (5)$$

where $\mathbf{y} = (y_1, \dots, y_p)$, $\mu(\mathbf{y}) = \{\mu_j(y)\}$ and $\Sigma(\mathbf{y}) = \{\sigma_{jk}(y)\}$ for $j, k = 1, \dots, p$. Moving to a different point, say x , there exists an another Gaussian approximation $\phi(x, \nu)$. In this case, f_Y is approximated by a family of multivariate Gaussian densities given by a set of smooth parameter functions $\{\mu(\mathbf{y}), \Sigma(\mathbf{y})\}$. Assuming that f_Y is a Gaussian density the parameter functions converge to constants corresponding to the true parameter values, conceding that $\phi(\mathbf{y}) = f_Y(\mathbf{y})$.

Otneim and Tjøstheim (2017, 2018) show that the local parameter functions estimation $\{\mu(\mathbf{y}), \Sigma(\mathbf{y})\}$ becomes straightforward when transforming each Y_j to a standard normal variable $Z_j = \Phi^{-1}(U_j)$, where U_j is a uniform variable $U_j = F_j(Y_j)$ with F_j denoting the cumulative distribution function of Y_j . On the occasion that we define the random vector Z as transformation of Y to marginal standard normality:

$$Z = (\Phi^{-1}F_1(Y_1), \Phi^{-1}F_2(Y_2), \dots, \Phi^{-1}F_p(Y_p))$$

This conversion enables us to further simplify the local Gaussian approximation (11) by writing the density f_Z of \mathbf{Z} at the point $v = z$ as,

$$f_z(\mathbf{z}) = \phi(\mathbf{z}, R(\mathbf{z})) = \frac{1}{|2\pi\mathbf{R}(\mathbf{z})|^{1/2}} \exp\{-1/2\mathbf{z}^T\mathbf{R}^{-1}(\mathbf{z})\mathbf{z}\} \quad (6)$$

in which we may derive fixed local means and standard deviations $\mu_j(\mathbf{z}) = 0$ and $\sigma_j^2(\mathbf{z}) = 1$, $j = 1, \dots, p$, and where $\mathbf{R}(\mathbf{z}) = \{\rho_{jk}(\mathbf{z})\}$ is the local correlation matrix. We will refer to Z and its probability function F_Z as being on the z -scale.

Denoting the partitioning of \mathbf{Z} , as $(\mathbf{Z}^{(1)}, \mathbf{Z}^{(2)}) = (\mathbf{Z}_1, \mathbf{Z}_2, \mathbf{Z}_3)$, a clarification of the local partial covariance matrix of $\mathbf{Z}^{(1)} | \mathbf{Z}^{(2)}$ is the local version of eq. (9):

$$\Sigma_{12|3}(\mathbf{z}) = \mathbf{R}^{11}(\mathbf{z}^{(1)}) - \mathbf{R}^{12}(\mathbf{z})(\mathbf{R}^{22}(\mathbf{z}^{(2)}))^{-1}\mathbf{R}^{21}(\mathbf{z}), \quad (7)$$

In the case that $d_1 = d_2 = 1$, then $\Sigma_{12|3}(\mathbf{z})$ is a 2 X 2 matrix, and we define the local Gaussian partial correlation $\alpha(\mathbf{z})$ between the two variables in $\mathbf{Z}^{(1)} = (Z_1, Z_2)$ given $\mathbf{Z}^{(2)} = \mathbf{Z}_3$ likewise the ordinary (global) partial correlation provided by eq. (3):

$$\alpha(\mathbf{z}) = \mathbf{R}_{12|3}(\mathbf{z}) = \frac{\{\Sigma_{12|3}(\mathbf{z})\}_{12}}{\{\Sigma_{12|3}(\mathbf{z})\}_{11}^{1/2}\{\Sigma_{12|3}(\mathbf{z})\}_{22}^{1/2}}, \text{ which in the case that } \mathbf{Z}^{(2)} = \mathbf{Z}_3 \text{ is}$$

scalar reduces to

$$\alpha(\mathbf{z}) = \rho_{12|3}(z_1, z_2 | z_3) = \frac{\rho_{12}(z_1, z_2) - \rho_{13}(z_1, z_3)\rho_{23}(z_2, z_3)}{\sqrt{1 - \rho_{13}^2(z_1, z_3)}\sqrt{1 - \rho_{23}^2(z_2, z_3)}}$$

The local Gaussian partial correlation between Y_1 and Y_2 given $Y^{(2)} = Y_3$, on the z -scale is estimated by

$$\hat{\alpha}(\mathbf{z}) = \mathbf{R}_{12|3}(\mathbf{z}) = \frac{\{\Sigma_{12|3}(\mathbf{z})\}_{12}}{\{\Sigma_{12|3}(\mathbf{z})\}_{11}^{1/2}\{\Sigma_{12|3}(\mathbf{z})\}_{22}^{1/2}}, \quad (8)$$

a corresponding value of $\hat{\alpha}(\mathbf{y})$ at the point $\mathbf{y} = \hat{F}(\Phi(z))$ is obtained by replacing in the above equation $\mathbf{z} = \Phi^{-1}(\hat{F}(\mathbf{y}))$.

Otneim and Tjøstheim (2022) also construct a test for conditional independence based on the local Gaussian partial correlation approach. A test statistic for testing the null hypothesis of $H_0 : Y_1 \perp Y_2 | \mathbf{Y}_3$ or, equivalently, a test statistic in terms of the marginally Gaussian pseudo observations is:

$$H_0 : Z_1 \perp Z_2 | \mathbf{Z}_3 \quad (9)$$

by aggregating our local measure of dependence over the sample space of \mathbf{Y} (or \mathbf{Z}). A analogous test statistic on the z -scale is:

$$T_{n,b} = \int_S h(\hat{\alpha}_b(\mathbf{z})) dF_n(\mathbf{z})$$

where $h(\cdot)$ is an even and non-negative real-valued function in most standard applications and $S \subseteq \mathbb{R}^p$ denotes an integration area that can be altered in order to test specific portions of the sample space. Under the laws of large numbers and regularity conditions, $T_{n,b}$ converges in probability towards its population value,

$$T = \int_S h(\alpha(\mathbf{z})) dF_n(\mathbf{z})$$

Departures from conditional independence lead to large values of $T_{n,b}$. In other words, if $T_{n,b}$ is larger than a critical value, this leads to the rejection of the null hypothesis. For technical details, interested readers can refer to [Otneim and Tjøstheim \(2022\)](#), section 5.