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Abstract

This paper analyzes the macroeconomic and distributional effects of declining labor share as observed during the last decades. We use a neoclassical general equilibrium model with two types of households, workers and capitalists, endowed with a CES production function, in which the distributional parameter matches labor share. This implies the existence of a technological nexus between the observed labor share and the distributional parameter of the CES function. We explore that technological nexus and show that both capitalists' and workers' income increase as labor income declines depending on the elasticity of substitution between capital and labor. The effect of labor share changes on income distribution does not depend on the elasticity of substitution, and hence, relative income and relative consumption decrease for workers, increasing inequality. When capital depreciation rate is taken into account, the decline in labor share has a limited impact on the functional distribution of net income.

Keywords: Functional distribution of income; Labor share; Workers; Capitalists.

JEL Classification: E25; J30.

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1. Introduction

During several decades, mainstream economics has considered the labor share of aggregate income as roughly stable over time, a common wisdom that was consolidated by the work of Gollin (2002).[1] Although labor share of income shows some variability along the business cycle, it have been considered fairly stable in the long-run. As a matter of fact, the profession uses a common-wisdom rule in which two-third of aggregate income remunerates the labor factor and one-third the capital factor. However, early empirical evidence showed that labor share increased during the last decades of the nineteenth century and the first half of the twentieth century. Keynes (1939) and Solow (1948) were among the first authors to question the hypothesis of stability of labor share. Later, Kravis (1959) reported that the compensation of employees share increases from 0.55 in 1900 to 0.67 in 1957 in the U.S., a trend associated to the decline of labor in agriculture and the increase in industry, and to the increase in self-employment. Johnson (1954) and Phillips (1960), among others, analyzed how the emergence of owners influenced wage employment in the U.S., highlighting that much of the increase in worker participation in the first half of the 20th century can be attributed to this structural change. Budd (1960) showed that this upward trend in labor share was also observed in the second part of the 19th century, and estimated that labor share rose from about 0.43 in 1870 to around 0.48 in 1910. Kuznets (1966) suggested that demographic changes and urbanization, the rise of the age of entry in the labor market and the incorporation of women to the labor market, were all factors explaining the increase in labor share. Nevertheless, this early empirical evidence has been largely ignored by the literature and the stability of labor share remained a key feature of the aggregate economy, despite some new evidence corroborating early works questioning the constancy in labor share, as Glyn (2011), who observed an upward trend of labor share in 1960s and 1970s among developed countries. However, more recently, a new wave of empirical evidence is challenging again this fact, pointing to a labor share decline in advanced economies. Azmat, Manning and Van Reenen (2011), Elsby, Hobijn and Sahin (2013), Karabarbounis and Neiman, (2014a,b) and Bridgman (2018) provided evidence of falling labor share in developed economies during the last three/four decades.

Several explanations of the observed decline in the labor share during the last decades

[1] The hypothesis of constancy in the functional distribution of income date-backs to Ricardo (1821), who considered this the main issue in Political Economy. Later, a constant labor share was one of the stylized facts enumerated by Kaldor (1957), first established as an empirical regularity by Bowley (1937) for the U.K., and by Johnson (1954) for U.S.

have been proposed in the literature. Karabarbounis and Neiman (2014a) presented evidence that labor share significantly declined since the early 1980s for the majority of OECD countries and industries. They posed a constant elasticity of substitution (CES) production function with elasticity of substitution higher than one, and argued that the well-documented decline in the relative price of investment goods (Gordon, 1990; Cummins and Violante, 2002) induced firms to substitute away from labor to capital. The consequent downsizing in labor demand implied a reduction in the price of labor and hence, in labor share. They concluded that roughly half of the observed decline in labor share can be attributed to this mechanism. Piketty (2014) and Piketty and Zucman (2014) also argued that labor share decline is explained by an increase in the capital-output ratio. Bentolila and Saint-Paul (2003) argued that the evolution of labor share in OECD countries is related not only to the capital-output ratio, but also to the price of imported material and to capital-augmenting technological progress. Azmat et al. (2011) indicate globalization and the consequent outsourcing of labor intensive tasks to low income countries as another factor contributing to the decline in the labor share in developed countries. Autor, Dorn, Katz, Patterson and Van Reenen (2017, 2020) affirm that 10% of the labor share decrease in the U.S. can be explained by few low-labor large market-share superstar firms. Finally, automation (Abdih and Danniger, 2017, Graetz and Michaels, 2018) and the decline in union bargaining power (Fichtenbaum, 2011, Fukao and Perugini, 2020) have been also considered additional factors explaining the observed labor share decline during last decades in advanced economies.

The functional distribution of income is mostly important in the aggregate economy for a number of reasons. First, it measures how total income is distributed between capital and labor. In National Accounts, when measured as aggregate income, GDP is split between the compensation to employees, the return to capital, and taxes. The first term accounts for the contribution of labor rent to GDP, whereas the second one accounts for the contribution of capital. In addition, labor share serves as key variable indicating the level of income inequality in an economy. Piketty (2014) and Adams, Karabarbounis and Neiman (2014) show that the share of aggregate income paid as compensation to labor is frequently used as proxy for income inequality. Because capital is concentrated among high-income agents, the higher the share of capital, the higher the inequality between rich and poor individuals. In fact, data confirm that both across time and countries the higher the share of capital the higher is income inequality in the distribution of personal income (Piketty, 2014). However, Adams et al. (2014) indicate that the relationship between and labor share is not direct, as there are other factors such as the amount of within-labor and within capital income inequality, and the degree to which the highest wage earners are also those earning the highest capital income, that can change the relationship between labor income and income inequality. Finally, labor share is also a key empirical variable to calibrate technological parameters in macro models. Many macro models assume an aggregate production function in which labor share is used to calibrate some technological parameter in the production function. In particular, most macroeconomic models assume a Cobb-Douglas technology, where labor share is considered a deep parameter of the technology, and hence, constant over time. Indeed, the equivalence between the output-labor elasticity parameter of the Cobb-Douglas production function and the labor share has been a key element in the calibration of macroeconomics models.

In this paper, we use the neoclassical growth model as a prism to analyze the effects of declining labor share on other macroeconomic variables. It is worth noting that we are not offering an alternative explanation of why labor share declined over time. In fact, we do not take a neutral stance on the reasons why labor share declines, given that the neoclassical framework embeds a direct positive relationship between labor share and capital accumulation. So, by using this framework we accept the explanation of Karabarbounis and Neiman (2014a) and other authors who related the decline in the labor share with an increase in capital accumulation. We rather take this relationship for granted and then analyze the general equilibrium implications of an exogenous decline in labor share on macro aggregates and, in particular, on income distribution.

The main contribution of this paper is the adoption of a new perspective to the issue, by using a kind of reverse engineering approach, exploiting the technological nexus between the observed labor share and the rest of variables of the economy. We take advantage of the dimensional characteristic of the distribution parameter of the CES production function. An additional equation establishing the relationship between the labor share and the CES distributional parameter is introduced in the model solution, and the corresponding technological parameter value is estimated by internal calibration for a range of values of the labor share. The inclusion of the labor share as target variable making the CES distribution parameter the unknown, is shown to be equivalent to the normalization of a family of CES functions for different elasticities of substitution between capital and labor (de La Granville, 1989, 2016). To perform our analysis, we adopt a neoclassical general equilibrium model with two types of households, workers and capitalists, endowed with a normalized CES production function, in which the distributional parameter matches the change in labor share. In this theoretical framework, a reduction of labor share mechanically triggers a higher capital accumulation which, in turn, pushes upward the level of output. However, two effects counteract this mechanical transmission mechanism. The first one is the elasticity of substitution between capital and labor. We show that the effect of capital accumulation on output can be arbitrarily reduced by increasing the elasticity of substitution. The second one is a general-equilibrium effect operating through the wage rate, which mitigates the negative effect of a reduction in the labor share on labor. When capital increases, it enhances the marginal productivity of labor, thus fading the negative effect of the diminishing distributional parameter on the labor demand. The frictionless model clarifies that this effect emerges in equilibrium even in absence of any nominal or real rigidity on wages. As a result, the equilibrium value of labor is almost invariant in response to reductions in the labor share and, in turn, it does not crow out the positive effect of capital accumulation on output. We show that both capitalists' and workers' income increase as labor share declines. In this model economy, a lower labor share increases the productivity of capital thus provoking a positive income effect for capitalists. This leads to an increase in investment and capital stock, whose magnitude will depend on the elasticity of substitution between capital and labor.

Finally, because labor share is also a proxy for inequality and the relative welfare of different social groups, we also study the effects of labor share changes on the income distribution. To analyze the overall impact of previous effects on the income distribution, the model accounts for the existence of two types of households. First, *hand-to-mouth* households who do not have access to financial markets, thus obtaining income only from labor rent. Second, standard *capitalist* households who can save and invest in physical capital, thus obtaining income from both capital and labor rents. We find that the effect of labor share changes on the income distribution does not depend on the elasticity of substitution, as the steady state capital-output ratio and wage rate are not affected by how substitutes/complementary labor and capital are. Hence, relative income and relative consumption decrease for workers, increasing inequality. We follow Karabarbounis and Neiman (2014b) and Bridgman (2018) in distinguish between gross and net income. When capital depreciation rate is taken into account, the decline in labor has a limited impact on the functional distribution of net income.

The structure of the rest of the paper is as follows. Section 2 presents a neoclassical general equilibrium model with workers and capitalists. Section 3 describes the calibration and the simulation strategy. Section 4 presents the relationship between labor share, capital accumulation and output. Section 5 studies gross versus net income distribution. Finally, Section 6 presents some concluding remarks.

2. The model

We consider a perfect-foresight economy inhabited by two types of households: workers and capitalists. In particular, we assume a continuum of households indexed $h \in [0, 1]$, in which a fraction ω are hand-to-mouth agents who do not save, thus consuming all of their labor income in each period. These agents are denoted with subscript $i \in [0, \omega]$. The rest of households are rational and forward-looking agents who save and invest in physical capital. They are denoted with subscript $j \in [\omega, 1]$. In the model economy, there is a representative firm producing with a CES production technology and renting labor and capital services in competitive factor markets. It sells its homogeneous product in a perfectly competitive good market. We rely on the representative-firm assumption to match the CES distributional parameter with aggregate data on the Compensation of Employees from National Accounts. We also assume that capitalists provide a fraction of labor employed by the firm to mimic the basic structure of National Accounts regarding the labor share, in which labor income from self-employed workers and entrepreneurs is accounted for in labor and not in capital rent.[2]

2.1. Workers

In each period, households that have no access to financial markets, named workers, consume out all their labor income. The labor supply is endogenous and determined by maximizing the intertemporal utility function defined over consumption, $\{C_{i,t}\}_{t=0}^{\infty}$, and labor, $\{L_{i,t}\}_{t=0}^{\infty}$. The following utility function accommodates these preferences,

$$\sum_{t=0}^{\infty} \beta^t \left[\log C_{i,t} - \phi \frac{L_{i,t}^{1+1/\upsilon}}{1+1/\upsilon} \right] \tag{1}$$

[2]Including a supply of labor from capitalist is consistent with Marshall's argument that income earned by self-employed workers and owners is a compensation for their work, and with National Accounts data that includes these rents in the definition of labor share. This also implies that workers' income is only a fraction of total labor income, as the other fraction is gained by the capitalists. In the literature, it is standard to define capitalists' utility function as a function of consumption, without considering leisure as additional argument. This implies that no labor income is obtained by the capitalists. However, such a modeling strategy is not consistent with the different definitions of labor share used in empirical analyses. We use the terminology *capitalists* instead of entrepreneurs because, as pointed out by Krueger (1999), the labor share infers the division of rents between workers and capitalists.

where β is the discount factor, υ is the Frisch labor elasticity parameter, and $\phi > 0$ is a parameter representing the relative preference for leisure over consumption. The zero-saving assumption implies that their budged constraint is simply

$$C_{i,t} = W_t L_{i,t} \tag{2}$$

where W_t is the wage rate, and the only income is labor income. From first-order conditions we derive the equilibrium condition about workers' labor supply, i.e.,

$$\phi L_t^{1/\upsilon} C_{i,t} = W_t \tag{3}$$

2.2. Capitalists

Capitalists have the same utility function than workers, but they do have access to financial markets. Thus, they can save and invest in physical capital. By working and renting capital to the firms, they get both labor and capital income. Accordingly, their budget constraint is

$$C_{j,t} + I_{j,t} = W_t L_{j,t} + R_t K_{j,t} \tag{4}$$

in which $I_{j,t}$ is investment, $K_{j,t}$ is the capital stock, and R_t is the rental price of capital. As in the standard neoclassical model, capital stock holdings evolve according to the law of motion

$$K_{j,t+1} = (1-\delta) K_{j,t} + I_{j,t} \tag{5}$$

where $0 < \delta < 1$ is the fixed depreciation rate of physical capital.

From first order conditions, we obtain the following equilibrium conditions for capitalists' labor supply and optimal investment choices,

$$\phi L_{j,t}^{1/\upsilon} C_{j,t} = W_t \tag{6}$$

$$\frac{C_{j,t+1}}{C_{j,t}} = \beta \left[R_{t+1} + 1 - \delta\right] \tag{7}$$

2.3. Aggregation

Given the assumption on identical-mass agents, the aggregate value for a generic variable $X_{h,t}$ is

$$X_t = \int_0^1 X_{h,t} dh = \omega X_{i,t} + (1-\omega) X_{j,t} \tag{8}$$

Hence, aggregate variables for consumption, labor, capital stock and investment are given by,

$$C_t = \omega C_{i,t} + (1-\omega) C_{j,t} \tag{9}$$

$$L_t = \omega L_{i,t} + (1-\omega)L_{j,t} \tag{10}$$

$$K_t = (1-\omega)K_{j,t} \tag{11}$$

$$I_t = (1-\omega)I_{j,t} \tag{12}$$

2.4. The firm

We assume a single representative firm who rents capital and employs labor to maximize profits at each period t, taking factor prices as given. As in the standard neoclassical model, profit maximization determines the optimal level of (aggregate) labor and capital services denoted, respectively, L_t and K_t. We assume that the firm is endowed with a CES production function with constant returns to scale,

$$Y_t = [\alpha K_t^\rho + (1-\alpha)L_t^\rho]^{1/\rho} \tag{13}$$

where $-\infty \leq \rho \leq 1$, is a parameter controlling for elasticity of substitution between factors, and $0 \leq \alpha \leq 1$ is the distribution parameter. From the first order conditions of the profit maximization problem we obtain firm's optimal demands for capital and labor, i.e.,

$$R_t = \frac{\alpha Y_t K_t^{\rho-1}}{\alpha K_t^\rho + (1-\alpha)L_t^\rho} = \alpha \left(\frac{Y_t}{K_t}\right)^{1-\rho} \tag{14}$$

$$W_t = \frac{(1-\alpha) Y_t L_t^{\rho-1}}{\alpha K_t^\rho + (1-\alpha)L_t^\rho} = (1-\alpha) \left(\frac{Y_t}{L_t}\right)^{1-\rho} \tag{15}$$

The elasticity of substitution between capital and labor is defined as $\sigma = 1/(1-\rho)$, whereas the distribution parameter has a direct relationship with the functional distribution of income for an elasticity of substitution different from unity, as in this case, is a dimensional parameter and not a share. When the elasticity of substitution approaches one, i.e. $\rho \to 0$, the distribution parameter is equal to the capital share, which is the key property of the Cobb-Douglas production function. When the elasticity of substitution is different from one, there is a relationship between capital share and the distribution parameter α. Our strategy for solving the model exploit that relationship will consist in the consideration of the distribution parameter as an additional endogenous variable to be jointly determined with the rest of endogenous variable at steady state given an exogenous labor share.

This strategy for solving an augmented model with the distribution parameter as an additional endogenous variable given a exogenous labor share is equivalent to the normalization of the CES production function. As shown by de La Grandville (2016), when production is modeled as a normalized CES function, the distribution parameter must be calibrated as the geometric mean of the capital share and the interest rate, i.e.

$$\alpha = S_{K,0}^{1-\rho} R_0^{\rho} \tag{16}$$

where $S_{K,0}$ and R_0 are, respectively, capital share and interest rate in the normalization point 0. Similarly, the distribution parameter for labor can be defined as,

$$1 - \alpha = S_{L,0}^{1-\rho} W_0^{\rho} \tag{17}$$

where $S_{L,0}$ and W_0 are, respectively, labor share and wage rate in the normalization point 0. This is equivalent to define the distribution parameter as a function of the elasticity of substitution (Klump and de La Granville, 2000), where:

$$\alpha(\sigma) = \frac{S_K K_t^{-\rho}}{S_K K_t^{-\rho} + S_L L_t^{-\rho}} \tag{18}$$

As we will show in the rest of the paper, the relationship between the distribution parameter of the CES production function and the elasticity of substitution between capital and labor will be key for numerically simulate the model and for the results therein.

3. Steady State and Calibration

To perform the quantitative analysis presented in next sections, we solve for the perfect foresight equilibrium of the model and consider the Steady State solution in which all variables are constant over time. For that, we design a reverse engineering approach consisting in calibrating the distributional technological parameter of the aggregate production function, conditioned on a range of exogenous values of the labor share, and using different values of the elasticity of substitution between capital and labor. In particular, we simulate the model for different values of the CES technological parameters, and then we compare the resulting steady state values of endogenous variables. We use three values for the elasticity of substitution between capital and labor: $\{0.9, 1, 1.5\}$. Most macro models assume an elasticity of substitution between labor and capital equal to 1 (Cobb-Douglas production function), and this value has been widely supported in the literature (see Gollin, 2002 for references). Some authors, however, argued that the elasticity of substitution is lower than one, i.e. labor and capital are complements, whereas few papers present evidence in favor of an elasticity of substitution greater than one (Chirinko, 2008). Our sensitivity analysis accounts for all of these options. Next, we consider different values of the labor share, S_L, ranging in the interval [0.5 : 0.8]. The interval includes most of the values used in the theoretical literature and accounts for all the variation in data found by the empirical literature. Note that turning the labor share into a target implies that the distribution parameter of the normalized CES production function must be calibrated accordingly. Because we do not rely on time-varying parameters, we do so by calibrating α jointly with the steady states of the model. In particular, from combining equations (14) and (16), it can be shown that the value of α is given by

$$\begin{aligned} \alpha &= S_K \left(\frac{Y}{K}\right)^{\rho} \qquad \rho \neq 1 \\ \alpha &= S_K \qquad\qquad\quad \rho = 1 \end{aligned} \tag{19}$$

where S_K is the capital share, and we used the restriction: $S_K = 1 - S_L$. The other model parameters are fixed in our analysis and calibrated using standard values of the Real Business Cycle (RBC) literature. We define an annual periodicity in the model and then fix the subjective discount factor, β, equal to 0.97, the depreciation rate of capital, δ, equal to 0.06, and the Frisch elasticity, υ, equal to 0.72. Finally, the willingness-to-work parameter is set to 12.5 to obtain an amount of worked hours of one third of total time endowment when $S_L = 2/3$.

For reader convenience, we report hereafter the system of equilibrium equations used to simulate the model.

$$1 = \beta \left[R + 1 - \delta\right] \tag{20}$$

$$W = \phi L_j^{(1+\upsilon)/\upsilon} C_j \tag{21}$$

$$W = \phi L_i^{(1+\upsilon)/\upsilon} C_i \tag{22}$$

$$C_i = W L_i \tag{23}$$

$$C_j + I_j = W L_j + R K_j \tag{24}$$

$$C = \omega C_i + (1-\omega) C_j \tag{25}$$

$$L = \omega L_i + (1-\omega) L_j \tag{26}$$

$$K = (1-\omega) K_j \tag{27}$$

$$I = (1-\omega) I_j \tag{28}$$

$$\alpha = 1 - S_L \left(\frac{Y}{L}\right)^{\rho} \tag{29}$$

$$R = \alpha \left(\frac{Y}{K} \right)^{1-\rho} \tag{30}$$

$$W = (1 - \alpha) \left(\frac{Y}{L} \right)^{1-\rho} \tag{31}$$

$$Y = C + I \tag{32}$$

$$Y = [\alpha K^{\rho} + (1 - \alpha) L^{\rho}]^{1/\rho} \tag{33}$$

where we drop the time subscripts of variables to denote steady state values. The system of equations (20)-(33) has 14 equations for 14 unknowns including the CES distribution parameter, $(C, I, L, K, C_i, C_j, I_j, K_j, L_i, L_j, Y, W, R, \alpha)$.

Once the above system is solved, we find the corresponding technological distribution parameter for each value of the observed labor share, which depends on the elasticity of substitution between capital and labor, as stated by expression (18). Figure 1 plots the estimated relationship between labor share and the distribution parameter depending on the elasticity of substitution. As S_L diminishes, the distributional parameter α increases. Note that the relationship between variations of S_L and variations of α is proportional only in the model with a Cobb-Douglas production functions (red dashed line in Figure 1). When σ is smaller than one (complement factors), then α varies more than proportionally to variations in S_L. The opposite occurs when σ is greater than 1. For an elasticity of substitution high enough, the distributional parameter remains almost constant for any value of labor share.

[Insert Figure 1 here]

4. Labor share, capital accumulation and output

Recent empirical evidence shows that labor share is declining in a number of developed countries since 1980s (Karabarbounis and Neiman, 2014a,b; Brigman, 2015). We take into account that piece of empirical evidence and simulate the calibrated model to match an exogenously given labor share to assess the macroeconomic consequences of a change in the functional distribution of income. In simulating the model, we also take into account that labor share is not only an indicator of the functional distribution of income and a proxy for inequality and relative welfare of different social groups, but it also related to a fundamental technological parameter in the production function: the distribution parameter of the CES production function. Labor share represents the elasticity of output to labor when the elasticity of substitution between capital and labor is one (a Cobb-Douglas production function), and it is related to the distribution parameter weighting labor when the elasticity of substitution between capital and labor is different from one. For each value of the exogenous labor share in a plausible range we compute the corresponding steady state of the economy using the solution of the model where the parameter α is calculated as an additional endogenous variable depending on the labor share. Given its key role in the results, we also provide a sensitivity analysis on the elasticity of substitution between capital and labor.

As indicated above, our internal calibration method for the distribution parameter of the CES production function is equivalent to the standard normalization of the production function (see de La Grandville, 1989, 2016; Klump and de La Grandville, 2000). For an in-depth description of the implications of the normalization of a CES production function, see Klump, McAdam and Willman (2012). Given that equivalence between our approach and the normalization approach, the simulated model produces a steady state capital-output ratio value, for each exogenous labor share, which does not depend on the elasticity of substitution between capital and labor. Consequently, the steady state consumption-output and investment-output ratios as well as labor supply, for each exogenous value of the labor share, are also independent of the elasticity of substitution. As we will show in the next section, this approach will lead to the important results that the per capita consumption capitalists-workers ratio and the distribution of income between the two social groups (workers and capitalists) does not depend on the elasticity of substitution between capital and labor, as had been considered traditionally from a non-normalized CES production function.

Figures 2-5 plot the steady states of the main endogenous variables as function of the labor share. As S_L diminishes, the distributional parameter α increases thus pushing upward capital accumulation (Figure 2). In the neoclassical framework, this mechanical effect increases final output. The effects of a change in labor share are higher as the elasticity of substitution is smaller. When σ is smaller than one (complement factors), then α varies more than proportionally to variations in S_L and the effect on output is amplified. The opposite occurs when σ is greater than 1, thus muting the effect on output. The higher level of capital also raises the marginal productivity of labor. This general equilibrium effect counteracts the direct effect of $(1-\alpha)$ on the wage rate, which is actually increasing in α as shown by Figure 2. Eventually, the positive effect on the wage rate translates proportionally into workers' income equals to worker's consumption (Figure 4). This property is apparent by combining equations (22) and (23). In equilibrium, workers' labor supply is constant and, in particular, it does not depend on the wage rate: $L_i = \phi^{\frac{-\upsilon}{1+\upsilon}}$. As a result, both the income and consumption level of workers increase after a reduction in the labor share. In sum, a decline in labor share provokes a positive wealth effect on workers by increasing wages. This contrast with the common wisdom in which it is generally argued that reductions in labor share automatically imply reductions in workers' income. We show that this statement has some major weaknesses even in a plain-vanilla frictionless neoclassical growth model as the one used here. Indeed, we find that the relationship is rather the opposite, indicating the existence of a negative relationship between labor share and workers labor income.

As expected, the wealth effect on capitalists is also positive. In the model, the increase in their capital income following a reduction in the labor share is paired with an increase in their labor income. In fact, capitalists' labor supply is a negative function of the labor share as expected (Figure 3), given that the negative income effect generated by capital accumulation fully offsets the positive substitution effect of the higher wage. Notwithstanding, the increase in the wage rate counterworks the reduction in the labor supply and, eventually, capitalists' labor income increases (Figure 4). Nevertheless, the main source of the increase in capitalists' income is capital accumulation. As labor share declines, capital stock increases and given that in steady state capital returns are constant, capital income expands. Our simulations show that capitalists' total income and thus consumption is a negative function of labor share. Hence, we conclude that both workers and capitalists are better off after a reduction in the labor share.

[Insert Figure 2 here]

The mechanisms behind these results are the following. In this model economy, a lower labor share increases the productivity of capital resulting in a capital accumulation process until the interest rate returns to its equilibrium. This leads to an increase in investment and capital stock, whose magnitude will depend on the elasticity of substitution between capital and labor, keeping constant the capital-output ratio. The combination of a high level of capital with a constant interest rate in steady state leads to a positive income effect for capitalists. Additionally, the increase in the productivity of capital also contributes to an increase the productivity of labor, resulting in an increase in labor income. This is always true for the case of workers, as their labor supply is fixed. This increase in labor productivity is higher as the elasticity of substitutio is lower.

Figure 3 plots consumption of workers and capitalists. It can be observed that by reducing the labor share increases the consumption of both workers and capitalists. Workers consumption increases, because wage increases. Labor income of capitalists also increases, despite they supply less labor time but a higher wage rate. Both consumption and investment of capitalists increases. In steady state, consumption of capitalists is equal to their total income less capital consumption. Therefore, the capital depreciation rate plays a key role in accounting for the difference between total income and consumption for the capitalists.

[Insert Figure 3 here]

[Insert Figure 4 here]

Our sensitivity analysis on the elasticity of substitution qualifies previous results. On the one end, the effects on output and income of both capitalists and workers depend on the degree of substitutability between capital and labor. When production factors are complements, the impact on output and labor is maximum, but the more increase their substitutability the smaller it gets. In the limit ($\rho \to \infty$) when labor and capital become perfect substitutes, the impact tends to be zero. Yet, it is never negative and thus we can state that the lower the labor share, the better off are all agents in the economy. On the other end, Figure 3 shows that the impact of labor share variations on the aggregate labor supply does not depend on the elasticity of substitution. Again, this result is due to the normalization of the CES production function, which implies that the endogenous calibration of α adjusts in response to a variation of σ for key variables or ratios not to vary. The effects of the necessary normalization of α in the model are apparent by combining equations (29) and (31),

Second, the interest rate in steady state is only determined by the discount factor and the depreciation rate, thus implying that R is invariant to both σ and α. These two features jointly imply that the ratio of capital on output, K/Y, always varies in the same amount after a reduction in the labor share, no matter the value of σ. This point can be directly shown by substituting α in equation (30) using (19), which yields

$$\frac{K}{Y} = \frac{S_K}{R} \tag{34}$$

where S_K is exogenous and $R = 1 - \beta(1 - \delta)$. In a standard model, the equality between interest rate and the marginal product of capital is achieved by variations in labor. Here, that equality is achieved by, not only variations in labor, but mainly by variations in the production distribution parameter depending on the elasticity of substitution. From that expression can be directly shown that the investment-output ratio is also independent on the elasticity of substitution, given that in steady state $I = \delta K$, and hence,

$$\frac{I}{Y} = \frac{\delta S_K}{R} \tag{35}$$

and the same for the consumption-output ratio. By contrast, labor productivity, defined as the output-labor ratio, is a function of σ. Indeed, combining equations (16) and (29), it results that,

$$\frac{Y}{L} = \left(\frac{1 - S_K^{1-\rho} R^{\rho}}{S_L}\right)^{\sigma/(\sigma-1)} \tag{36}$$

Our model predicts that the observed decline in labor share evidenced by a number of authors since 1980s in developed countries should have be followed by a process of capital accumulation, having a positive effect on growth. This is consistent with the empirical evidence reported by Karabarbounis and Neiman (2014a), as they argue that labor share decline is explained by the increase in the capital-output ratio. This is exactly the mechanism of the model with the difference that the underlying cause of this capital accumulation process comes from the change in the distribution parameter of the CES production function. Notice that in our theoretical framework, the interest rate is determined by the discount rate and the physical capital depreciation rates, which are deep parameters and hence, the interest rate is invariant to movements in the capital stock.

In sum, we find that, at an aggregate level, the decline in labor shares is positive for the economy, as this is the result of a process of capital accumulation and increase in the capital-output ratio. In this process, not only gains for capitalists are obtained, but also wages increase, leading to an increase in workers' income and consumption. The macroeconomic consequences of the decline in labor shares are higher as the elasticity of substitution between capital and labor is lower. However, although the size of the cake increases, attention should also be paid to how the portions are distributed.

5. The distribution of income: Gross income versus net income

Next, we focus on the relationship between labor share and the distribution of income between the two types of households. In the literature labor share is also considered a proxy for inequality and relative welfare of different social groups. Although in the previous section we showed that the total size of the cake increases as labor share declines, and that income for both workers and capitalists increases, how the distribution of gains between the two agents and how income inequality reacts to changes in labor share remain key issues. Given the role of capital income in the capitalists's income and capital depreciation, in studying income inequality we must pay attention not only to gross income but also to net income. It is relevant to point out that most of the analyses about labor share and income inequality ignores the distinction between gross and net income, due to capital consumption. Karabarbounis and Neiman (2014b) highlights the importance of the physical capital depreciation rate, often neglected, for the study of income distribution and inequality. Bridgman (2018) shows that depreciation and production taxes are important determinants of the labor share, as they are included in total output, and hence, a fall in labor share may not implies a gaining in capital income. Brigman shows that gross labor share has been falling since 1970s, but that net labor share shows a more stable path. We follow Karabarbounis and Neiman (2014b) and Bridgman (2018) analyses that account for capital depreciation in separating gross and net labor shares. Consistent with those works, we find that when capital depreciation rate is taken into account, the decline in labor share has a limited impact on the functional distribution of net income.

Before studying how the distribution of income between the two different social groups changes, Figure 5 plots per capita consumption for each agent as well as the ratio of per capita consumption of capitalists with respect to workers, deemed to be a proxy of inequality. Some interesting results are observed. First, as expected, steady state per capita consumption of capitalists is higher than that of workers. This is just a consequence derived from the fact that workers have only one source of income (labor), whereas capitalists have two sources (labor and capital). Second, per capita consumption increases as labor share decline for both workers and capitalists, although the magnitude differs. Changes in capitalists' per capita consumption are higher than for workers' per capita consumption, given the different sources of income for these two social groups. Finally, the elasticity of substitution between capital and labor plays a key role in the determination of per capita consumption as labor share declines. As the elasticity of substitution decreases, the higher the gains in per capita consumption for both agents are.

As expected, inequality, measured as the capitalists-workers ratio of per capita consumption, increases as labor share declines. This effect is observed in spite that capital consumption by capitalists also increases. This ratio is independent on the elasticity of substitution between capital and labor. Importantly, these results suggest that a declining labor share does not necessarily imply declining living standards for workers. Even if average labor income grows less than capital income, workers may still be better off to the extent the decline in labor share is produced together with an increase in workers' labor income. Nevertheless, it remains true that differences in welfare across earning groups are increasing. This is consistent with the capital deepening hypothesis.

The total consumption-output ratio is given by:

$$\frac{C}{Y} = 1 - \frac{\delta S_K}{R} \tag{37}$$

where total consumption is defined as $C = \omega C_i + (1-\omega)C_j$, and where per capita consumption is $c_i = C_i/\omega$, $c_j = C_j/(1-\omega)$, and $c = c_i + c_j$. From expressions (22) and (21), the capitalists-workers per capita consumption ratio is,

$$\frac{c_j}{c_i} = \frac{\omega}{1-\omega}\left(\frac{L_i}{L_j}\right)^{\frac{1+\upsilon}{\upsilon}} \tag{38}$$

where L_i is a constant by construction and L_j does not depend on the elasticity of substitution by normalization.

[Insert Figure 5 here]

Finally, we calculate income shares for workers and capitalists. As a fraction of capitalists' income comes from labor, this is not exactly equivalent to the functional distribution of income, and represents in a clearer way the distribution of income between the two social groups. A first important result that emerges is that the distribution of income between workers and capitalists does not depend on the elasticity of substitution. From first order conditions, labor and capital shares defined as S_L and S_K, respectively, are given by,

$$S_K = \frac{R_t K_t}{Y_t} = \alpha\left(\frac{Y_t}{K_t}\right)^{-\rho} \tag{39}$$

$$S_L = \frac{W_t L_t}{Y_t} = (1-\alpha)\left(\frac{Y_t}{L_t}\right)^{-\rho} \tag{40}$$

Apparently, labor and capital shares depend on the elasticity of substitution between capital and labor. However, this is just an artifice as the parameter α is not independent from the capital share. As shown by Klump et al. (2012) without normalization the CES technological parameters have no economic interpretation, as the distribution parameter is not independent on the elasticity of substitution, and neglecting the normalization of the CES production function will produce misleading results. In particular, when the elasticity of substitution is different from one, there is a relationship between capital share and the distribution parameter α, that must be taken into account. This relationship between the parameter α and ρ, can be defined as:

$$\alpha = 1 - S_L\left(\frac{Y}{L}\right)^{\rho} \tag{41}$$

From equations (31) and (17) results that labor income share is not a function of the elasticity of substitution,

$$\frac{WL}{Y} = S_L \tag{42}$$

as S_L is exogenous. Equation (34) also clarifies that parameter ρ is not a determinant of capitalists' capital income share, given that $K = (1-\omega)K_j$, and thus

$$\frac{K_j R}{Y} = \frac{S_K}{1-\omega} \tag{43}$$

These results, in turn, imply that total income share of capitalists does not depend on ρ. As a result, also workers' labor income share is not affected by variations of σ.

[Insert Figure 6 here]

Figure 6 plots the workers and capitalists income share for gross and net capital incomes, where differences provoked by capital depreciation are clearly observed. For gross income, the workers' share is always below the labor share, as another fraction is total labor income is got by capitalists. Calculating total income using gross capital income results that when labor share is 0.8, the workers income represents a 65.83% of total income. This proportion is lower than the labor share because in our model also a fraction of labor income is rented by capitalists. The rest 34.17% corresponds to capitalists income which includes both labor and capital incomes. When labor share is 0.5, workers income share declines to 43.15% of total income, whereas capitalists gross income is 56.85%. This means that the difference of workers income share in the considered range is around 22 percentage points. However, when we account for the depreciation of physical capital, the functional distribution of income changes dramatically. When labor share is 0.8 (the maximum value of the range), the workers income share is 76.89% of total income. However, as we reduce the labor income share, this fractions only changes slightly. For a labor share of 0.5 (the minimum value of the range), we obtain that workers income share is 68.05%.

These quantitative results evidence how capital depreciation is an important variable in determining how net income is distributed between workers and capitalists. For a high labor share, say close to 0.8, we find that the proportion of net income received by workers is lower. This is explained by a lower capital stock and hence, a low depreciation. However, as labor share decline, this induces an accumulation of capital and hence, more depreciation. This favor the re-distribution of net income toward workers. As a consequence, the workers' income share increases above the labor share.

The implications for income distribution of the physical capital depreciation rate are important. Several authors argue that the physical capital depreciation rate is increasing, as the new equipment assets incorporated through investment are characterized by a high depreciation rate. This is the case of Information and Communication Technologies (ICTs). This implies that capital deepening is not a sufficient condition for inequality to increases, as that implies a higher proportion of capital income that cannot be devoted to consumption due to capital consumption.

Three main results are derived from previous analysis. First, the income distributional effects of the labor share are invariant to the elasticity of substitution and therefore income distribution is uniquely identified for any value of the elasticity of substitution between capital and labor. This contrast with the literature, where the elasticity of substitution had been considered as a fundamental parameter in setting income distribution in a context of capital deepening. Here, we demonstrate that this statement is incorrect and a direct consequence of the lack of normalization of the CES production function. Second, only for very high values of labor share, workers' income share of net income is lower than the observed labor share. For values of labor share in the empirical observed rage, workers' income share of net income is higher than the labor share calculated over gross income. Third, whereas labor share decline is a serious issue when looking at gross income distribution, is not a big problem when net income is considered. The resulting difference in the range 0.5-0.8 is 68.05-76.89, that is, only a difference of 9 percentage points for a range of values of 30 percentage points (approximately only a 1/3 of the difference).

Finally, our analysis provides a contribution to the recent debate on the consequences of falling labor share on income inequality [Piketty (2014)]. Piketty argued that the capital share of income will rise as the capital-output ratio increases. Some authors criticized this reasoning arguing that the analysis is true only if the elasticity of substitution between capital and labor greater than one. They argued that the empirical evidence shows otherwise. That is, the elasticity of substitution is typically lower than one (Chirinko, 2008), or close to one but no evidence supports the hypothesis of an elasticity of substitution greater than one. The elasticity of substitution between capital and labor can be defined as

$$\sigma = \frac{d\log(L/K)}{d\log(Y_L/Y_K)} \tag{44}$$

where Y_L is the marginal productivity of labor and Y_K the marginal productivity of capital. From that, it is derived that the elasticity of the capital share, defined as $Y_K K/Y$, with respect to the capital-output ratio, is given by,

$$\frac{d\log(Y_K/Y)}{d\log(K/Y)} = 1 - \frac{1}{\sigma} \tag{45}$$

where capital share increases as K/Y increases only if $\sigma > 1$. If instead $\sigma < 1$, Piketty's argument reverses. In this paper, we show that equation (45) holds in the neoclassical growth framework only if the distribution parameter of the CES production function (or Cobb-Douglas) is treated as an exogenous non-dimensional parameter. When instead we modeled that parameter as endogenous depending on the observed capital share and the elasticity of substitution, then the capital-output ratio is invariant to the elasticity of substitution. In this framework, changes in the elasticity of the capital share with respect to the capital-output ratio do not depend on the elasticity of substitution, and the change is capital share in steady state is simply equal to the change in the capital-output ratio, that is,

$$\frac{d\log(Y_K/Y)}{d\log(K/Y)} = 1 \tag{46}$$

given that in the steady state interest rate is a constant. This result supports the validity of Piketty's as general argument, as the positive relationship between capital share and the capital-output ratio is proven true for any value of the elasticity of substitution between capital and labor. This result also illustrates the importance of using a normalization when working with CES production functions with different elasticities of substitution between capital and labor.

6. Concluding remarks

This paper investigates the implications of labor share for output and income distribution. The paper uses a reverse engineering approach where a model with workers and capitalists is solved for an exogenous range of values of the labor share. The key issue in this approach is that labor share is related to the distribution technological parameter of a CES production function, and this distribution parameter is calibrated internally depending on the labor share. This approach is equivalent to the normalization of a family of CES functions for a range of values of the elasticity of substitution between capital and labor.

As expected in a neoclassical model, we find that output increases as labor share declines, being higher the effect the lower the elasticity of substitution between capital and labor is. The direct effect of a decline in labor share consists in an increases in capital investment due to higher returns. This investment process leads to an expansion in output depending on the elasticity of substitution and in the capital-output ratio. We find that income distribution is not affected by the elasticity of substitution.

The paper opens a number of interesting questions. First, the observed decline in labor share, from an aggregate point of view, is a good news for the economy, not only for the capitalists but also for workers. Labor share declines not only increases the total size of the cake but also the size of the portion for the two groups of agents. We find that the decline in labor shares is a factor contributing to expanding income for all the agents in the economy, even if some group has no capital. However, that effect is asymmetric, increasing inequality. Second, implications for inequality are completely different whether income share is calculated using gross or net income. This is an important issue, as depreciation rate does not remain constant over time, but depends on the new types of capital assets incorporated to the economy through investment. If the functional distribution of income is calculated using net values, labor share decline has little effect on increasing inequality between workers and capitalists. Finally, this paper shows that when using a CES production function, the issue of dimensional constants, not present in the cases of a Cobb-Douglas or Leontief technologies, must be treated with care, especially when assessing the consequences of the elasticity of substitution between capital and labor on income inequality.

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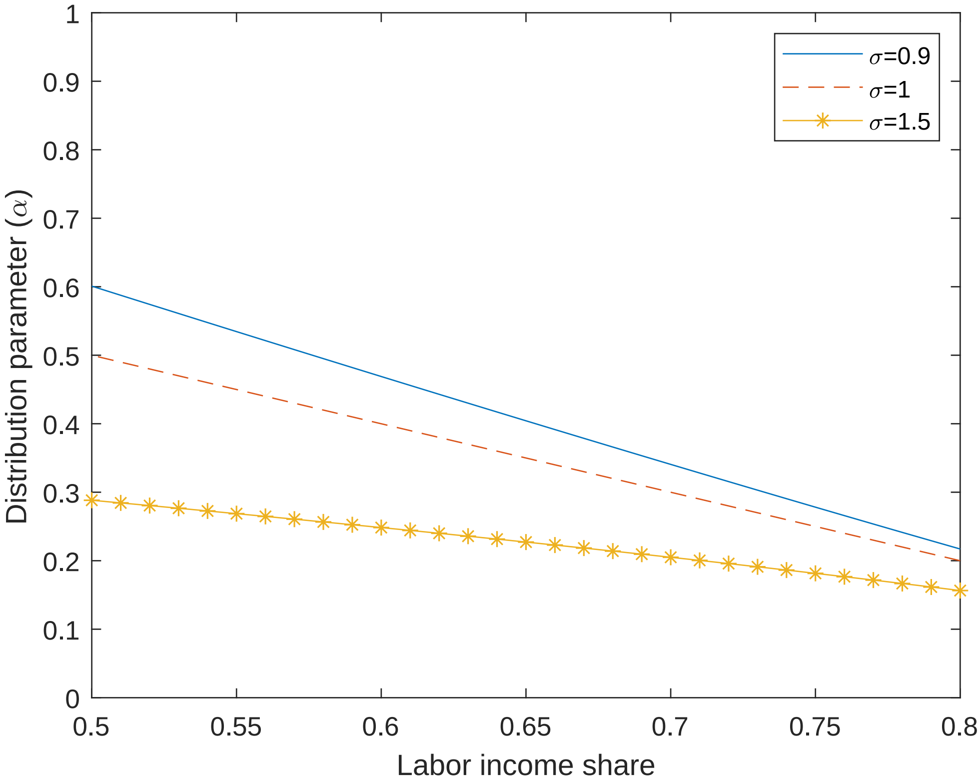

Figure 1: Steady state relationship between labor share and the technological distribution parameter of the CES production function.

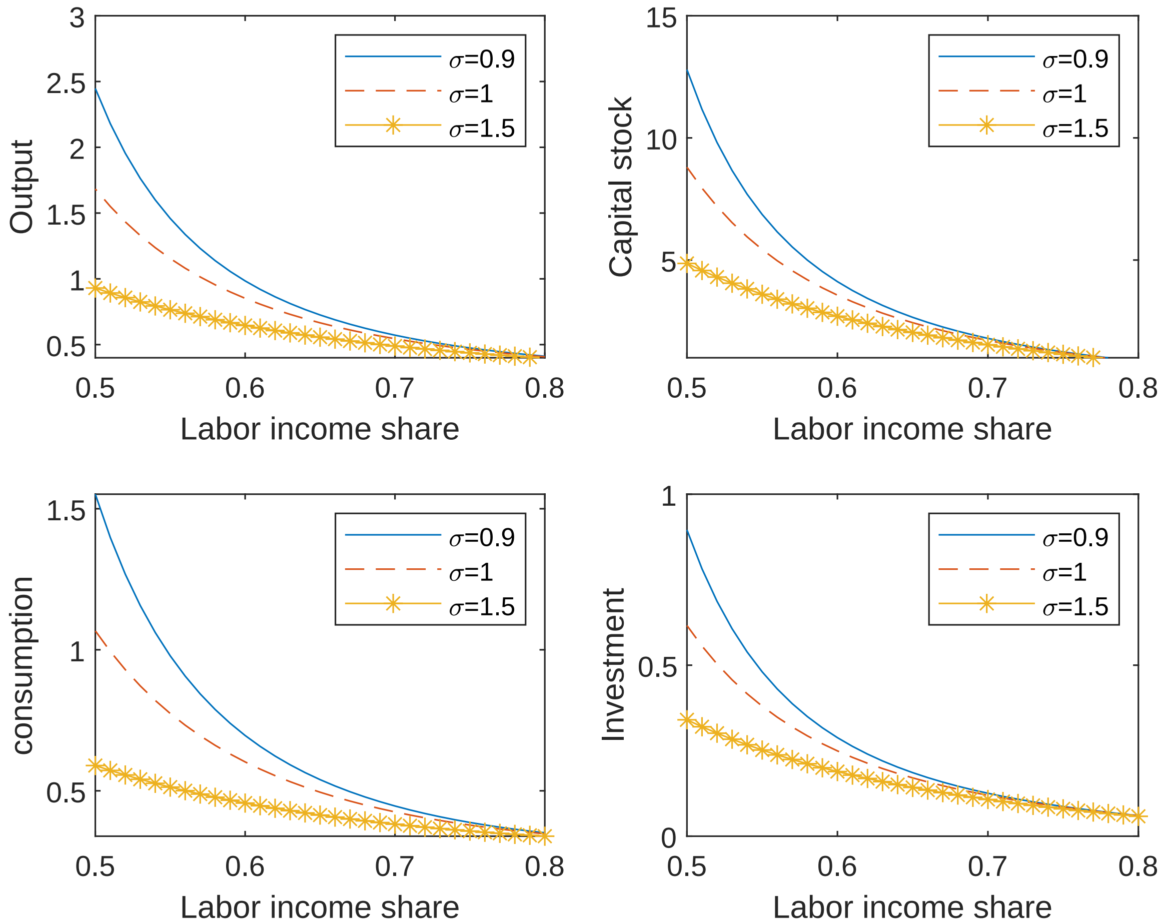

Figure 2: Steady state output, investment, capital and labor as a function of the labor share and the elasticity of substitution between capital and labor.

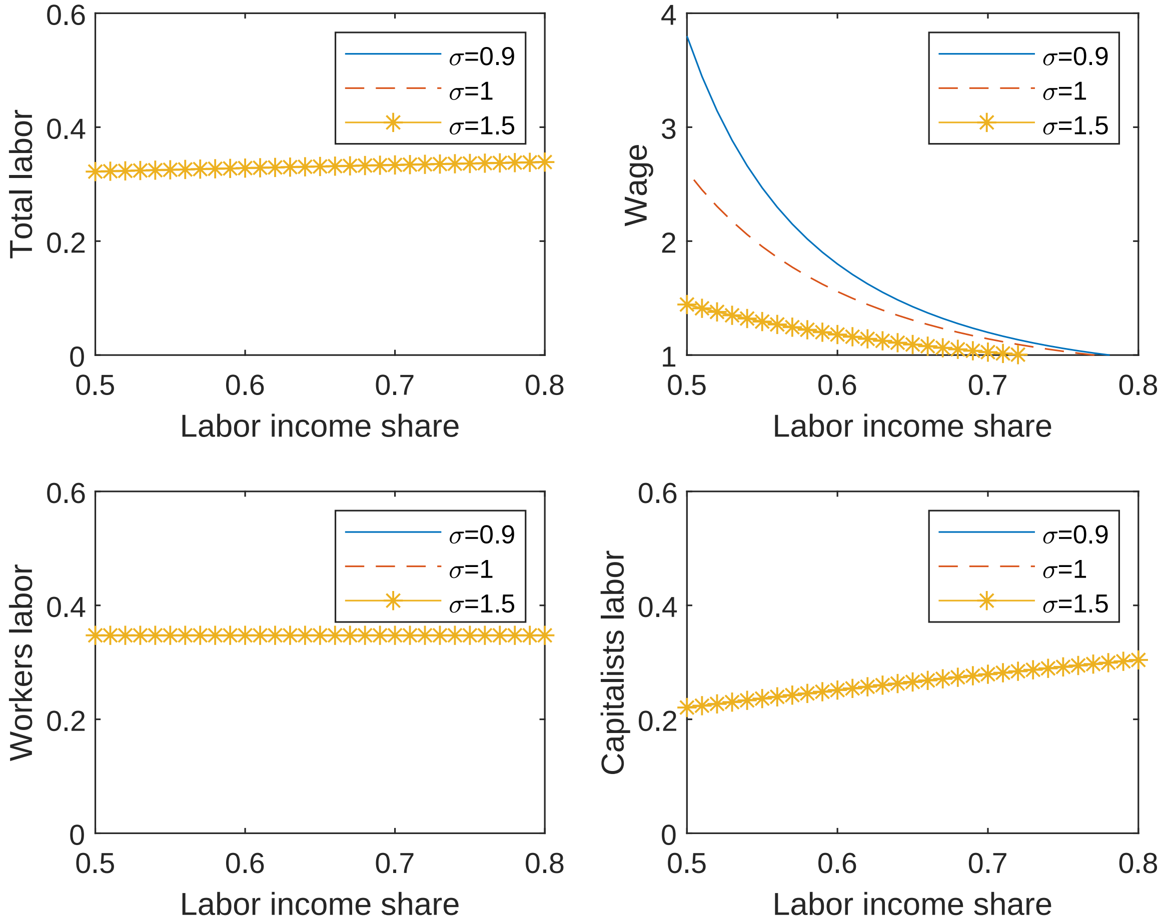

Figure 3: Steady state labor and wage as a function of the labor share and the elasticity of substitution between capital and labor.

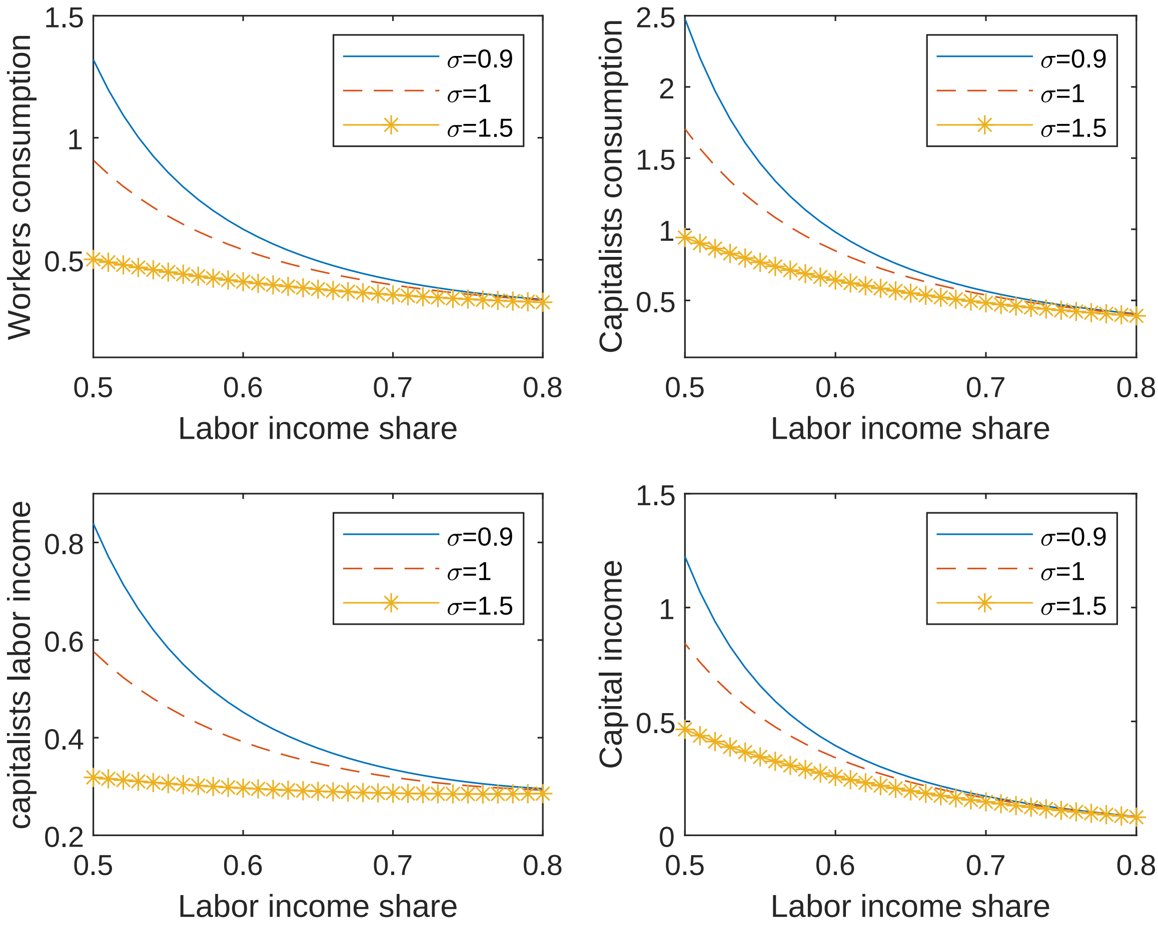

Figure 4: Steady state consumption and income as a function of the labor share and the elasticity of substitution between capital and labor..

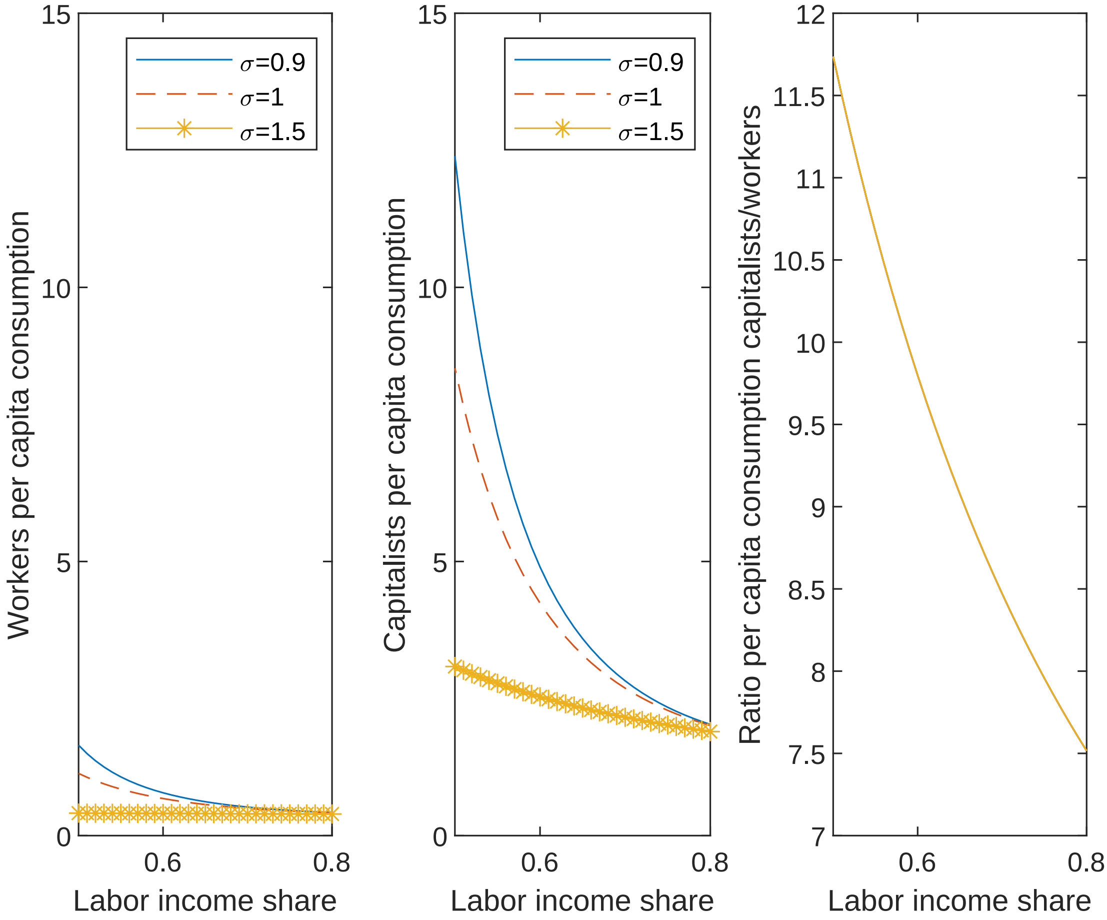

Figure 5: Workers-capitalists consumption rate.

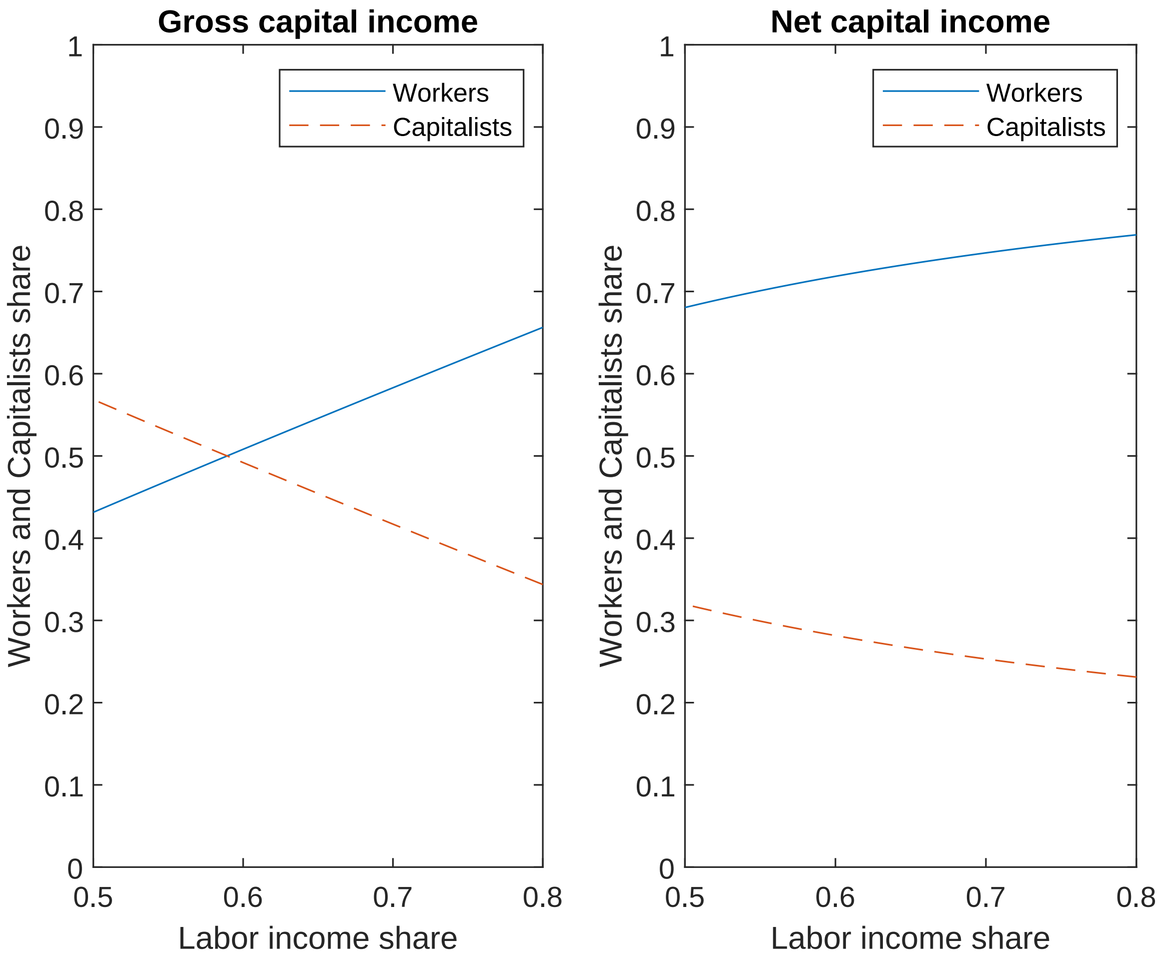

Figure 6: Workers and capitalists gross and net income share.