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PUBLIC DEBT FRONTIER
A TOOLKIT FOR ANALYZING
FISCAL POLICY AND DEBT SUSTAINABILITY

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Public Debt Frontier*

A toolkit for analyzing fiscal policy and debt sustainability

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Abstract

This paper provides a twofold contribution. First, it proposes a synthetic and visual indicator to assess public debt sustainability. This indicator summarizes in a single diagram the linkage between economic activity, government's budget, and the maximum amount of public debt that is sustainable in the long run. The backing theory is a neoclassical growth model augmented with endogenous tax revenues, disaggregated public spending, different production technologies for public and private goods, non-atomistic wage setters in public labor (unions), and a fully specified maturity curve for public bonds. The second contribution of the paper is to develop and present a stand-alone software that analyzes public debt sustainability in response to variations of fiscal policy. This toolkit is useful for managing public debt or to place an additional constraint on government's budget. We provide an example of its usage for the emblematic case of Greece during the last public debt crisis.

JEL Classification: E13, H61, E62, H63, H81, H30.

Keywords: Fiscal policy; Public debt sustainability; Endogenous Tax revenues; DSGE model; Debt-dependent government spending; Python.

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1 Introduction

The recent financial and debt crises brought the issue of excessive public debt and its sustainability again under the spotlight.[1] In a number of EU state members, deteriorating public accounts and the consequent instability of public debt and related assets on international financial markets called for the intervention of the European Commission [EC], European Central Bank [ECB] and International Monetary Fund [IMF] to prevent contagion to public debts and financial assets of other EU members [19]. The set of laws, rules, fiscal measures imposed by these international authorities on public accounts and governments' budgets produced a large debate among economists and policy makers about causes and cures of sovereign debt crises (see [14] for a detailed survey).

In the view of international authorities,[2] the perceived origin of debt crises were: (i) fiscal and current account imbalances due to weak competitiveness of domestic productive system; (ii) financial distress of public accounts either due to irresponsible fiscal discipline (Portugal and Greece), or to obligations towards national banking system (bailouts) in trouble after the 2007-2008 financial crisis (Ireland and Spain). The main guidelines typically indicated for economic recovering were: (1) eliminating economic distortions and liberalizing goods and factors markets to foster the economic activity;[3] (2) undertaking a strict programme of fiscal consolidation to regain sustainability of public debt. In particular, this last condition was to be achieved through a wide-ranging programme of reforms to the pension system − meant to reduce current and future financial disbursements to retiree −, and with restrictive fiscal stance meant to achieve governments' balanced budget or fiscal surplus if needed.

While the intentions of international authorities dictating fiscal austerity are self-explaining, the expected outcome of such policy is less evident. Both the objectives listed above − debt sustainability and fiscal consolidation − crucially depend on the level and expected growth of GDP. Thus, any policy that has a negative impact on GDP, even if effectively improves public accounts, will end up having an indeterminate effect on these objectives. Note that this is the case not only because both objectives are expressed in terms of GDP ratios − public debt over GDP is used to assess debt sustainability and government's budget is expressed in terms of deficit over GDP −, but also because declining economic activity affects negatively tax revenues and therefore on public accounts. Hence, if in the attempt of adjusting fiscal imbalances the government ends up shrinking the economic activity, then it would find himself swimming against a double tide: worsening public accounts and a reduction in the denominator of GDP ratios.

[1] See, among others, [1], [4], [15], [18].

[2] See *The Economic Adjustment Programme for Greece* (2010) for a case study.

[3] These measures include, among others, reforms to productive system, legal system and statute of workers.

This paper is motivated by previous considerations. Which is the impact of fiscal consolidation measures on public accounts, debt sustainability, and government's budget, once accounting for *all* the effects of fiscal policy on macroeconomic variables? We argue that this question can only be answered by using a quantitative model of the aggregate economy in which all the upward and downward forces operating on GDP in reaction to fiscal austerity are properly assessed. To this end, we develop a DSGE model where public spending and taxation affects resources and private decisions and at the same time the economic activity (GDP) affects public accounts and government's budget. In particular, the effect of output on fiscal policy in the model occurs through (i) variations of tax bases and thus tax revenues, (ii) variations of fiscal variables expressed in terms of GDP ratios, i.e., the *snow-ball* effect.[4] The effect of fiscal policy on output occurs instead through the standard public spending channel, through distortionary taxes, and through variations in the amount of public capital and labor services employed as production inputs in private production. At this regard, it is worth noting that government in this model is *OECD-compliant* in the sense that manages all the different chapters of spending and taxes reported in OECD fiscal data. We believe this to be a key feature of a utility-based general equilibrium model because different chapters of spending typically impact on output in different ways, so as different taxes distort either the demand or the supply side of the economy, thus also affecting the economy in very different ways.

In the first part of the paper, we present the model, characterize the general equilibrium solution, and compute a calibrated version of the long run equilibrium (steady state). We show that in the model government budget, the economic activity, and debt dynamics are linked one another, and any change in one of them will affect all of the others. These general-equilibrium linkages constitutes the added value in our analysis with respect to empirical or partial equilibrium analyses (e.g. sustainability equations, fiscal vulnerability, policy criteria like *Maastricht Treaty*) because they allow to account for both the direct and feedback effects of fiscal policy in the calculations of fiscal consolidation. That is, the effect of fiscal tools on macro aggregates and then the impact of changing macro aggregates on public accounts.

The disadvantage of our approach is that the relationships among GDP, public debt and government budget are now expressed in form of cross-equations restrictions to the state-space representation of the DSGE model solution, thus not being explicit analytical forms easy to interpret. In the second part of the paper, we propose an intuitive way to overcome this issue. We show that the salient information contained in the model about the relationships among GDP, debt and government budget can be represented in a single diagram in which two steady-state ratios are depicted: (i) the maximum amount of public debt that is sustainable in the long run given the

[4]Differently from tax revenues, public spending in this model is exogenous. This assumption can be relaxed in future research, but for the interest of present study it seems to fit well in the analysis of fiscal programmes where public spending is typically an exogenous target.

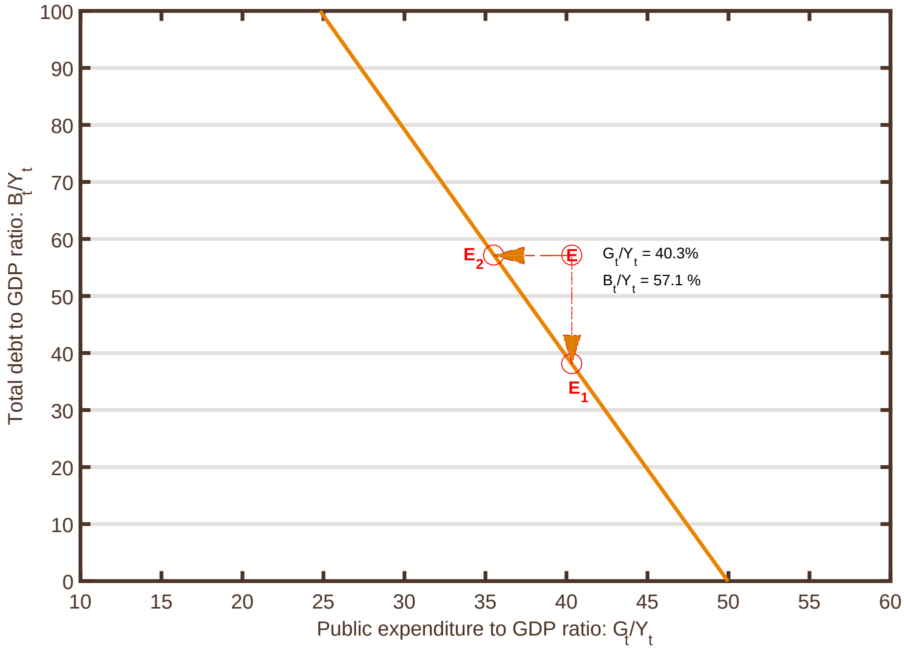

Figure 1: Public Debt Frontier

economic activity, measured in percentages of GDP; (ii) government primary spending in percentages of GDP. Using this diagram, a visual criteria can be applied to assess sustainability of public debt by simply measuring the distance between the actual level of debt and the maximum level of debt that is sustainable in the long run given the level of public spending and the associated GDP in the economy.

To construct the proposed graphical analysis, we perform the following steps. First, we match the characteristics of private sector in the model economy with their empirical counterparts in data, and then we calibrate marginal taxes and composition of public spending in the model using OECD fiscal data. Next, for each possible value of government's primary spending we simulate the model computing GDP and the associated maximum amount of sustainable debt. Intuitively, once the government decides the level of spending, the economy endogenously determines the level of output, which in turn determines tax revenues. Government budget is then closed resulting in a deficit (or surplus) that eventually fixes the new level of debt. At this point, the two ratios mentioned above can be computed and plotted in the diagram, and this is done for each level of primary spending. When joining all of the resulting pair of coordinates we obtain the so-called *public debt frontier*. That is, a line representing the maximum amount of public debt that can be maintained constant over time for a given level of public spending and the associated level of GDP.

In each point of the frontier, tax revenues in equilibrium are large enough to pay for debt service and cover for government's primary spending without generating further deficits, thus implying balanced government budgets throughout the frontier. In other words, economic activity is generating enough fiscal surplus to finance current public expenditures plus debt service with zero deficits, and the government can roll over the existing debt gaining credibility and financial solidity, even in the presence of high levels of debt. Instead, every point at the right of the frontier indicates an equilibrium in which government's financial solvency is at danger and its credibility as borrower cannot last indefinitely. Bad news, lasting crises, delaying recoveries will all configure situations in which rolling over won't be possible, and eventually either default or some bail-out procedure from international authorities should be expected.

Figure 1 provides an example of the frontier and the associated visual criteria mentioned above. In point E the economy is at equilibrium, but the government is running excessive deficit given the stock of existing debt (57.1%). Our analysis suggests that government either reduces primary spending to 35.5%, thus regaining sustainability by itself (point E_2), or asks for some bail-out procedure to reduce debt by 19% (point E_1) without incurring spending reductions (unless otherwise agreed with international authorities providing financial aid). It is worth noting that the indicated reduction in spending (-4.8%) is the sum of two components: (i) the primary reduction needed to bring back total deficit to zero; (ii) the additional reduction needed to cover for the loss of tax revenues generated by the reduction of GDP once public spending diminishes.

The structure of the paper is as follows. Section 2 presents the model (Appendix A characterizes the full system of equilibrium equilibrium). Then, we discuss the application of the model to the Greek economy. The calibration exercise is provided in Section 3 and the main results in Section 4. Finally, Section 5 presents the toolkit and its main features and Section 6 concludes.

2 The model

We develop a general equilibrium model in which government affects private decisions in a number of ways. We consider the role of distortionary taxation, consumption of public goods generating utility, investment in public capital and hiring of public labor enhancing private production possibilities and, finally, issuing of public debt sold to international investors that absorbs domestic resources. In the model economy, firms are represented by a CES production function defined over a labor aggregate nested within a standard Cobb-Douglas. The production of the final output requires four factors: labor services and capital, both private and public. Finally, consumers are modeled in a standard way, but including public goods in the utility function and splitting worked hours between private and public labor. We first describe the behavior of government, then firms, and finally households.

2.1 The Government

Government budget comprises several chapters of spending and different taxes. On the expenditures side, we distinguish four components: public consumption of goods and services; public investment in capital; public wage bill; and transfers. On the fiscal income side, we consider consumption tax, labor income tax, capital income tax and corporate tax, plus revenues from social security.

The budget constraint for government in this economy is defined as follows:

$$G_t + R_t^B B_t + \Delta D_t = T_t + R_t^D D_t + CBT_t + \Delta B_t \tag{1}$$

Equation (1) states that all cash outlays (including transfer payments to households) for non-interest total government spending (G_t), interest payments of total government debt (R_t^B times B_t), and new purchases of financial assets (ΔD_t) must be funded by some combination of tax revenues (T_t), interest earnings on government assets (R_t^D times D_t), transfers from the central bank (CBT_t), and new debt issuance (ΔB_t). For Euro zone countries, transfers from the central bank are zero, and direct purchases of government bonds are precluded by the Treaty (i.e. $CBT_t = 0$). If we denote by B_t the year on year net position of the government, we can also set financial purchases to zero (i.e. $D_t = 0$).

2.1.1 Government spending

Non-interest total government spending is defined as:

$$G_t = C_{g,t} + (1 + \tau_t^{ss}) W_{g,t} L_{g,t} + I_{g,t} + Z_t \tag{2}$$

where $C_{g,t}$ is public consumption of goods and services, $I_{g,t}$ is public investment, $W_{g,t}L_{g,t}$ is the wage bill for public employees, τ_t^{ss} is a social security tax, and Z_t are transfer payments to households, such as welfare, social security or unemployment benefit payments. Public investments accrue into the public structures stock, $K_{g,t}$. We assume the following accumulation process for the public capital:

$$K_{g,t} = (1 - \delta_{K_g}) K_{g,t-1} + I_{g,t} \tag{3}$$

which is analogous to the private capital accumulation process, and where δ_{K_g} is the public physical capital depreciation rate.

We also specify government's spending at the time of calibration. This spending structure implies the selection of i) a certain level of public spending and ii) its distribution among the different components. The level of government spending in the long run, given a certain amount of fiscal revenues, depends on the target levels for the public deficit and public debt. While the Maastricht Treaty establishes limits together with sanctions for deficit and debt sinners, these limits have only been respected to enter into the monetary union, but never after that date. Therefore, we do not consider the Maastricht criteria to be binding for these two variables.

The distribution among the different components of public spending is as follows[5]

$$C_{g,t} = \theta_1 G_t$$

$$I_{g,t} = \theta_2 G_t$$

$$(1 + \tau_t^{ss}) W_{g,t} L_{g,t} = \theta_3 G_t$$

$$Z_t = \theta_4 G_t$$

where $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 1$. We assume that public spending on goods and services are constant proportions of total output and these proportions are kept constant all along the exercise, that is, the government's income and expenditure sides are fully parametric.

2.1.2 Tax revenues

The government obtains resources from the economy by taxing consumption and income from labor, capital and profits, whose effective average tax rates are denoted by τ_t^c, τ_t^l, τ_t^k, τ_t^π, respectively. Additionally, we consider a pay-as-you-go social security system and thus we include the social security tax, τ_t^{ss}. The government budget in each period is given by,

$$\begin{aligned} T_t &= \tau_t^c C_{p,t} + \tau_t^l (W_{p,t} L_{p,t} + W_{g,t} L_{g,t}) + \tau_t^k (R_t - \delta_{K_p}) K_{p,t-1} \\ &\quad + \tau_t^{ss} (W_{p,t} L_{p,t} + W_{g,t} L_{g,t}) + \tau_t^\pi \Pi_t \end{aligned}$$

where $C_{p,t}$ is private consumption, $W_{p,t}$ is private sector wages, $L_{p,t}$ is private labor, R_t is the rental rate of private capital, δ_{K_p} is the depreciation rate of private capital, $K_{p,t}$ is private capital stock, and Π_t are profits to be defined later.

[5] This split of the government expenditures can be thought of as the result of the the policy maker's maximization of preferences of the form $U_g(C_{g,t}, I_{g,t}, W_{g,t}L_{g,t}, Z_t) = \log C_{g,t} + \log I_{g,t} + \log L_{g,t} + \log Z_t$, subject to a budget constraint where the Government can spend G_t.

2.1.3 The government identity

As we previously argued the government budget constraint can be written as:

$$G_t + R_t^B B_t = T_t + B_{t+1} - B_t$$

with the meaning that non financial spending, plus servicing the existing government debt must be financed through taxes plus new debt. Putting together all the elements defined above, the government budget constraint can be written as:

$$\begin{aligned} & C_{g,t} + (1+\tau_t^{ss})W_{g,t}L_{g,t} + I_{g,t} + Z_t + (1+R_t^B)B_t \\ = \; & \tau_t^c C_{p,t} + \tau_t^l (W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) \\ & + \tau_t^k (R_t - \delta_{K_p})K_{p,t-1} + \tau_t^{ss}(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^\pi \Pi_t + B_{t+1} \end{aligned} \tag{4}$$

or, collecting uses and resources:

$$\begin{aligned} & C_{g,t} + W_{g,t}L_{g,t} + I_{g,t} + Z_t + (1+R_t^B)B_t \\ = \; & \tau_t^c C_{p,t} + \tau_t^l (W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^k (R_t - \delta_{K_p})K_{p,t-1} \\ & + \tau_t^{ss} W_{p,t}L_{p,t} + \tau_t^\pi \Pi_t + B_{t+1} \end{aligned} \tag{5}$$

2.2 Labor unions

The public labor market is modeled following the work of [12]. The purpose of the mechanism described in this section is to distort the labor market to prevent wages equalization between the private and the public sector. An analysis of the public labor market among OECD countries show that the public wage bill is a source of major differences among these economies. Our analysis shows that government interventions in the wage setting of public wages can have a significant effect not only on the wage bill, but also in the growth path of the economy affecting the income shares of private inputs, having therefore a long-term effect on the debt budget constraint and the debt limits we want to calculate.

We have chosen a mechanism where the government has preferences over the number of public workers and their pay. To provide an objective function for the government defined over wages and employment, we follow a standard text-book approach (for example see [17])[6] and pose an objective function for the government as the solution of a game between a public sector union that cares about the wages of public-sector employees, $W_{g,t}$, and a government that cares about the level of public employment,

[6] On related grounds [2] and [13] consider the wage bill of the government, employment and wages, separately as arguments of the objective function of the government or the public sector union.

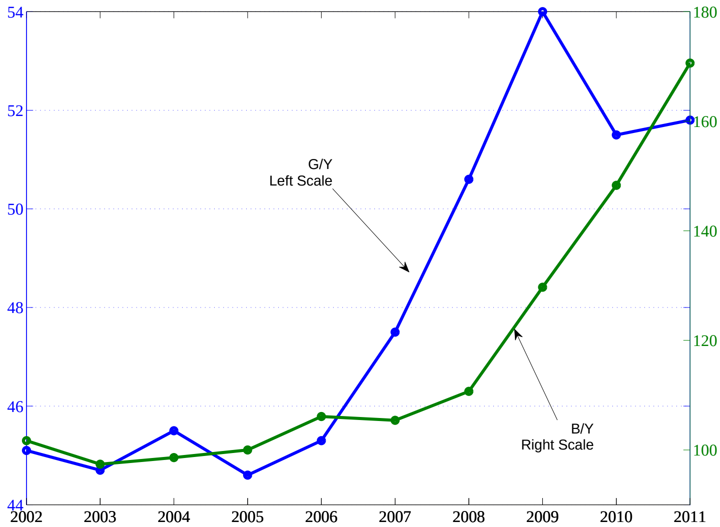

Figure 2: Expenditures and debt 2002-2011. Greece

$L_{g,t}$, given its budget constraint. Thus, the government agrees with the public sector union to maximize the following objective function subject to a budget constraint:

$$\max \left[\omega W_{g,t}^{\rho} + (1-\omega) L_{g,t}^{\rho}\right]^{1/\rho} \tag{6}$$

where ω is the weight given to wages and ρ is a negative parameter indicating the curvature of the trade-off between the elements present in the objective function of the government. If ω is close to zero, then the main goal of the government is to maximize public employment (benevolent government preference), whereas if ω is close to one, the main goal of the government is to maximize public wages (public sector union's preferred option).

Note that expression (6) encompasses the different approaches found in the literature. On the one hand, it takes into account the fact that public employment and wages are determined in an environment different to the private sector. The government itself can increase the number of public employees or can increase public wages subject to the budgetary constraint. On the other hand, it takes into account the fact that labor unions are more important in the public labor sector than in the private sector (see for instance [5]).

As defined previously, the government wage bill is defined as:

$$\theta_3 G_t = (1+\tau_t^{ss}) W_{g,t} L_{g,t} \tag{7}$$

Maximizing the government objective function subject to the government budget constraint is to find critical values for the auxiliary Lagrangian function:

$$\pounds_g(\cdot) = \max \left[\omega W_{g,t}^{\rho} + (1-\omega)L_{g,t}^{\rho}\right]^{1/\rho} + \xi\left(\theta_3 G_t - (1+\tau_t^{ss})W_{g,t}L_{g,t}\right)$$

That provides, upon differentiation, the first order necessary conditions:

$$\begin{aligned}
\frac{\partial \pounds_g(\cdot)}{\partial W_{g,t}} &= \left[\omega W_{g,t}^{\rho} + (1-\omega)L_{g,t}^{\rho}\right]^{1/\rho-1} \omega W_{g,t}^{\rho-1} - \xi(1+\tau_t^{ss})L_{g,t} = 0 \\
\frac{\partial \pounds_g(\cdot)}{\partial L_{g,t}} &= \left[\omega W_{g,t}^{\rho} + (1-\omega)L_{g,t}^{\rho}\right]^{1/\rho-1} (1-\omega)L_{g,t}^{\rho-1} - \xi(1+\tau_t^{ss})W_{g,t} = 0
\end{aligned}$$

Dividing orderly:

$$\omega W_{g,t}^{\rho} = (1-\omega)L_{g,t}^{\rho} \tag{8}$$

Combining this expression with equation (7) we obtain that public wages and employment are equal to:

$$W_{g,t} = \left(\frac{\omega}{1-\omega}\right)^{-1/2\rho}\left[\frac{\theta_3 G_t}{(1+\tau_t^{ss})}\right]^{1/2} \tag{9}$$

$$L_{g,t} = \left(\frac{\omega}{1-\omega}\right)^{1/2\rho}\left[\frac{\theta_3 G_t}{(1+\tau_t^{ss})}\right]^{1/2}, \text{ if } W_{g,t} > W_{p,t} \tag{10}$$

This distribution of the public resources depends on government preferences. However, private and public sectors are competing for the same labor input and as a consequence there is a relationship between public sector and private sector wages inducing a wage premium. The wage premium is implicit in equation (10) and it is part of the solution of the governments problem. This wage premium ensures the government that it's demand for labor will always be satisfied. This relationship will become clearer once we present the household's problem.

2.3 Firms

The problem of the firm is to find optimal values for the utilization of labor and capital in the presence of public inputs. The representative firm operates a CES production function nested within a standard Cobb-Douglas production function, and thus this technology exhibits constant returns to scale. The production of final output, Y, requires labor services, L and capital, K, both private and public. Goods and factors markets are assumed to be perfectly competitive. The firm rents capital and hires labor to maximize period profits, taking factor prices and public labor and capital as given. The technology is given by:

$$Y_t = A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{\alpha_l}{\eta}} \tag{11}$$

where Y_t is aggregate output, A_t is a measure of total-factor productivity that depends on our choice of aggregated production function.

The parameters $0 < \alpha_p < 1$, $0 < \alpha_g < 1$ and $0 < \alpha_l = 1 - \alpha_p - \alpha_g < 1$ are private and public capital share of output and labor respectively, μ $(0 < \mu < 1)$ measures the weight of public employment relative to private employment and $\psi = 1/(1-\eta)$ is a measure of the elasticity of substitution between public and private labor inputs.

If we assume final output to be the unit of account, profits are defined as:

$$\Pi_t = A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]^{\alpha_l/\eta} - (1+\tau_t^{ss}) W_{p,t} L_{p,t} - R_t K_{p,t-1} \tag{12}$$

Under the assumptions that private workers are paid their marginal productivity, we get:

$$(1+\tau_t^{ss}) W_{p,t} = \mu \alpha_l A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]^{(\alpha_l-\eta)/\eta} L_{p,t}^{\eta-1}$$

$$R_t = \alpha_p A_t K_{p,t-1}^{\alpha_p-1} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]^{\alpha_l/\eta}$$

From the above equations, it is found that private factor incomes are:

$$\begin{aligned}(1+\tau_t^{ss}) W_{p,t} L_{p,t} &= \mu \alpha_l A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]^{(\alpha_l-\eta)/\eta} L_{p,t}^{\eta} \\ &= \frac{\mu \alpha_l L_{p,t}^{\eta}}{\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}} Y_t \end{aligned} \tag{13}$$

$$R_t K_{p,t-1} = \alpha_p Y_t \tag{14}$$

The aggregate production function has four productive factors. However, the two public factors have no market price. The government does not usually charge a price that covers the full cost of the services provided with the contribution of public factors. This implies that those rents generated by public factors are not assigned to public factors. As public factors are paid by the government, there is a positive profit, Π_t, which turns out to be:

$$\Pi_t = Y_t - R_t K_{p,t-1} - (1+\tau_t^{ss}) W_{p,t} L_{p,t} > 0$$

Substituting private factor incomes given by expressions (13) and (14) yields:[7]

$$\Pi_t = \left[1 - \alpha_p - \frac{\mu \alpha_l L_{g,t}^{\eta}}{[\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]} \right] Y_t > 0$$

We assume that profits are paid out to households given that they are the owners of the firm.

[7]See appendix A.2 for the derivation of this expression.

2.4 Households

In our model economy, the decisions made by consumers are represented by a stand-in consumer with a period utility where consumption can be decomposed into two components:

$$U(C_t, L_t) = U(C_{p,t}, C_{g,t}, L_t) \tag{15}$$

where $C_{p,t}$ is private consumption and $C_{g,t}$ is consumption of the same private good provided by the government to the consumer. We assume that households obtain utility from the public spending in goods and services. In particular, we assume that:

$$C_t = C_{p,t} + \pi C_{g,t} \quad \text{with } \pi \in (0, 1] \tag{16}$$

Households' preferences are given by the following instantaneous utility function:

$$U(C_t, N_t\overline{H} - L_t) = \gamma \log C_t + (1 - \gamma) \log(N_t\overline{H} - L_t) \tag{17}$$

Leisure is $N_t\overline{H} - L_t$, where $\overline{H}$ is total time endowment and it is calculated as the number of effective hours in the week times the number of weeks in a year times population in the age of taking labor-leisure decisions, N_t, minus the aggregated number of hours worked in a year, L_t. The parameter γ ($0 < \gamma < 1$) is the fraction of private consumption on total private income. Households consume final goods and supply labor to the private and the public sectors,

$$L_t = L_{p,t} + L_{g,t} \tag{18}$$

where L_t is the aggregate level of employment, $L_{p,t}$ is private employment and $L_{g,t}$ is public employment. Public employment is chosen by the government and thus it is exogenous to the households as a quantity constraint. At an aggregate level, the household can only choose the supply of private labor, $L_{p,t} = L_t - L_{g,t}$. Recall that public employment demand is fully covered by the household, provided that $W_{g,t} > W_{p,t}$.

The budget constraint faced by the stand-in consumer is:

$$\begin{aligned} & (1 + \tau_t^c)C_{p,t} + K_{p,t} - K_{p,t-1} \\ = \; & (1 - \tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + (1 - \tau_t^k)(R_t - \delta)K_{p,t-1} \\ & + Z_t + (1 - \tau_t^\pi)\Pi_t \end{aligned} \tag{19}$$

where $K_{p,t}$ is private capital stock, $W_{p,t}$ is private compensation per employee, $W_{g,t}$ is public compensation per employee, R_t is the rental rate of capital, δ_{K_p} is the capital depreciation rate which is modeled as tax deductible, Z_t are lump sum transfers and entitlements, and Π_t denotes profits from firms, as defined previously. The budget constraint states that consumption and investment in physical capital, cannot exceed the sum of labor and capital rental incomes and profits net of taxes.

Private capital holdings evolve according to:

$$K_{p,t} = (1-\delta_{Kp})K_{p,t-1} + I_{p,t} \tag{20}$$

where $I_{p,t}$ is household's gross investment.

The consumer maximizes the value of her lifetime utility given by:

$$\underset{\{C_t,L_t\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} \beta^t \left[\gamma \log(C_{p,t} + \pi C_{g,t}) + (1-\gamma)\log(N_t\overline{H} - L_{p,t} - L_{g,t})\right] \tag{21}$$

subject to the budget constraint, where (K_{p0}, K_{g0}) and the paths of public employment and taxes are given, and where $\beta \in (0,1)$, is the consumer's discount factor. The Lagrangian auxiliary function is:

$$\begin{aligned}
\pounds_c(\cdot) = & \sum_{t=0}^{\infty} \beta^t \left[\gamma \log(C_{p,t} + \pi C_{g,t}) + (1-\gamma)\log(N_t\overline{H} - L_{p,t} - L_{g,t})\right] \\
& + \lambda_t[(1-\tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + (1-\tau_t^k)(R_t - \delta)K_{p,t-1} + Z_t + (1-\tau_t^\pi)\Pi_t \\
& - (1+\tau_t^c)C_{p,t} - K_{p,t} + K_{p,t-1}]
\end{aligned}$$

The first order conditions for the consumer maximization problem are:

$$\frac{\partial \pounds_c}{\partial C_{p,t}} = \gamma \frac{1}{C_{p,t} + \pi C_{g,t}} - \lambda_t(1+\tau_t^c) = 0 \tag{22}$$

$$\frac{\partial \pounds_c}{\partial L_{p,t}} = -(1-\gamma)\frac{1}{N_t\overline{H} - L_{p,t} - L_{g,t}} + \lambda_t(1-\tau_t^l)W_{p,t} = 0 \tag{23}$$

$$\frac{\partial \pounds_c}{\partial K_{p,t}} = \beta^{t+1}\left[\lambda_{t+1}\left(1 + (1-\tau_{t+1}^k)(R_{t+1} - \delta_{Kp})\right)\right] - \lambda_t\beta^t = 0 \tag{24}$$

plus the budget constraint and a transversality condition stating that the today-value of long distant future values of assets are zero.

This formulation implies that the wage-setting process in the private sector is totally different to that of the public sector. Whereas in the private sector wages are determined in terms of their marginal products, in the public sector a given amount from the government's budget constraint is distributed between public wages and public employment. Note that the above expressions imply that the consumer can only choose the supply of private labor, given that public labor is determined inelastically by the government at a wage that includes a positive premium that guarantees that all public labor demand is covered by the consumer at any market wage $W_{p,t}$.

2.5 International investors

The rest of the world for this economy is modeled as a single international banker whose objective is to maximize the discounted dividend x_t obtained from the asset holdings of government bonds. The discount factor is β, identical to the consumer's discounting parameter. Purchases of government bonds are denoted by b_t. Of course, supply and demand are equal at all times, so $B_t = b_t$.

$$\max_{x_t} \sum_{t=0}^{\infty} \beta^t x_t$$
$$s.t.\ b_{t+1} - b_t + x_t = w^I + R_t^b b_t$$

Where w^I is a constant endowment.

From the above problem we obtain

$$\beta(1 + R_t^b) = 1 \tag{25}$$

Walras's Law is satisfied at all times.[8] From equations (24) and (25) we obtain a non arbitrage steady state condition

$$(1 - \tau^k)(R - \delta_{K_p}) = R^B$$

The net real return to capital has to equate the real return of the government bond, including any risk premium.

3 Calibration

In this section we calibrate the model for the Greek and the German economy to a number of targets. All targets correspond to 2006, just before the crisis. We select these economies as our case study given that they represent a benchmark for studying the causes of a debt crisis, as the former was the first country under the European Monetary Union to lose its triple A rating on government bonds and to adopt a financial program while the latter was put forward as an example of fiscal discipline and a model to follow.

In what follows, we first explain how we calibrate the government parameters, then move on to calibrate the parameters for the rest of the economy, and we explain how we introduce debt maturity into the model.

3.1 Government parameters

The government in our model is defined as a vector of fiscal policy instruments parameters $(\tau^k, \tau^l, \tau^c, \tau^\pi, \tau^{ss}, \theta_1, \theta_2, \theta_3, \theta_4)$, a stock of debt, B, and a fraction of debt that needs to be refinanced every period, N. The first set of parameters are taxes: we pick taxes on capital, labor and consumption directly from OECD, taxes on profits and social security contributions are taken directly from OECD statistics. The second

[8] See Appendix A.1 for a proof.

set of parameters are expenditure shares. We take them directly from the National Accounts. For the Greek economy, public investment represents 10.37% of the expenditure, implying $\theta_2 = 0.1037$, while in the German economy public investment accounts for only 4.32%, implying $\theta_2 = 0.0432$. The expenditure share on public consumption is roughly similar for both countries ($\theta_1 = 0.4467$ for Greece and 0.4024 for Germany). The public wage bill, θ_3, is obtained as the public wage bill over total government expenditures $\theta_3 = (1 + \tau^s)W_g L_g/G$, with values of 0.2441 for Greece and 0.1712 for Germany. Putting together the different fractions of government expenditures we obtain as a residual the value of $\theta_4 = 1 - \theta_1 - \theta_2 - \theta_3$ as total transfers to consumers, which are 0.2055 in Greece and 0.3832 in Germany. Finally, public debt in Greece is equal to $B/Y = 1.10$ and in Germany it is 0.656. All the government parameters are summarized in Table 1.

Last, we use average maturity as in [7] of Greek and German debt at the time of the Great Recession (4 and 6 years, respectively) to pin down the number of periods in which debt payback is followed up. Since each period the government has to refinance a constant fraction $1/N$ of its total debt, the average maturity is equal to

$$\text{maturity} = \frac{1}{N} \times 1 + \frac{1}{N} \times 2 + ... + \frac{1}{N} \times N = \frac{1}{N}\sum_{i}^{N} i = \frac{N+1}{2}.$$

The resulting figures are $N = 7$ for Greece and $N = 11$ Germany.

Table 1: Government parameters

Parameter	Definition	Value	
		Greece	Germany
θ_1	Public consumption/tot. gov't expenditure	0.4467	0.4024
θ_2	Public investment/total gov't expenditure	0.1037	0.0432
θ_3	Public wage bill/tot gov't expenditure	0.2441	0.1712
θ_4	Public transfers/tot. gov't expenditure	0.2055	0.3832
τ^l	Labor income tax rate	0.4100	0.3810
τ^k	Capital income tax rate	0.1640	0.1810
τ^{ss}	Social security contribution	0.3560	0.3390
τ^π	Profit tax rate	0.2500	0.3870
τ^c	Consumption tax rate	0.1480	0.1240
B/Y	Ratio public debt/output	1.1000	0.6560
N	Fraction of debt refinanced every period	7	11

3.2 Technological and preference parameters

The real return of public bonds is $R^B = 0.041$, which corresponds to the interest rate for the Greek and German ten year bonds in 2006. Standard no-arbitrage condition implies that $\beta = 0.9606$ for both economies.

Computing private and public capital depreciation rates is a difficult task, since it involves computing what types of investments are done, and what is the depreciation rate for each of them. Due to its intrinsic difficulty, we use the estimates of [16] for the Spanish economy, which implies that $\delta_{K_p} = 0.08$ and $\delta_{K_g} = 0.04$ for both countries. The depreciation rate for public capital is lower than for private capital given their different composition, since public capital typically contains more infrastructure, which depreciates more slowly. These calculations imply that, in the steady state, the public capital stock represents around 28% of total capital stock, and that total capital stock is 3.26 times total output for Greece, whereas for Germany these figures are 21% and 2.5 respectively.

We use OECD data series on public sector labor and wages. Public and private compensation of employees and public and private employment are taken from OECD Economic Outlook database December 2007 Issue, for the period 1960-2006. The public wage bill is calculated as total final public compensation of employees. In 2006, public employment over private employment is 24.0% for Greece and 13.04% for Germany. The other target is the wage premium, W_g/W_p, which is 1.4935 for Greece and 1.1999 for Germany. Simultaneously, we observe from the same database the ratios of public labor to private labor L_g/L_p which is 0.24 for Greece and 0.13 for Germany. These figures imply that both public employment and public wages are higher in Greece than in Germany. These figures are consistent with the ratio of public wage bill over total government spending for each economy. Since workers are paid their marginal product, we obtain that the ratio of public wages to private wages is

$$\frac{W_{g,t}}{W_{p,t}} = \frac{1-\mu}{\mu} \frac{L_{g,t}^{\eta-1}}{L_{p,t}^{\eta-1}}. \tag{26}$$

The estimation we follow is closely related to [12], which implies $\eta = 0.4326$ and $\mu = 0.6008$ for Greece, and $\eta = 0.5762$ and $\mu = 0.6640$ for Germany.

We move on to compute factor shares in the production function. We use a standard[9] no-arbitrage condition for capital and bonds to find that $R_B = (R_p - \delta_p) \times (1 - \tau^k)$, where R_p is the return on private capital investment. Given that $R_p = \alpha_p K_p / Y$, we find that α_p is 0.3005 in Greece and 0.2556 in Germany. We use total compensation of employees over GDP to compute α_l, given that in the model $\alpha_l = (W_p L_p + W_g L_g)(1 + \tau^{SS})/Y$. We can write this expression as a function of three previous targets as $\alpha_l = ((W_p/W_g)(L_p/L_g) + 1)\theta_3 G/Y$. We find that this number is equal to 0.3327 in Greece and 0.6026 in Germany for a G/Y of 0.36 for Greece and 0.48 for Germany. Finally, α_g is found as the residual so the sum of shares equals 1 in each country.

Finally, we calibrate A to normalize output in the economy to 100. To this end, we

[9]In our case this condition comes from equations (24) and (25)

use expression

$$A = \frac{Y}{K_p^{\alpha_p} K_g^{\alpha_g} [\mu L_p^{\eta} + (1-\mu) L_g^{\eta}]^{\frac{\alpha_l}{\eta}}},$$

evaluated at Y=100 for the two countries. Similarly, we set γ in order for the labor supply equation to generate observed labor force participation, $L/H = 0.5750$ in Greece and $L/H = 0.7017$ in Germany, using equation

$$\gamma = \frac{C_p + C_g}{C_p + C_g + (H - L_p - L_g) W_p \frac{1-\tau_l}{1+\tau_k}},$$

which implies $\gamma = 0.8956$ in Greece and $\gamma = 0.8792$ in Germany. All the parameters for the economy are reported in Table 2.

Table 2: Calibration of the economy

Parameter	**Definition**	**Value**	
		Greece	Germany
β	Discount factor	0.9606	0.9606
δ_{Kp}	Private capital depreciation rate	0.0800	0.0800
δ_{Kg}	Public capital depreciation rate	0.0400	0.0400
η	Public-Private labor elasticity of substitution	0.4326	0.5762
μ	Private employment weight	0.6008	0.6640
α_p	Private capital income share	0.3005	0.2556
α_l	Labor share	0.3327	0.6026
α_g	Public capital technical parameter	0.3668	0.1418
A	TFP	1.2015	1.6422
γ	Consumption-leisure preferences	0.8956	0.8792

Greek figures for taxes, fiscal revenues, total government spending and its distribution are not so different from the figures for the rest of countries in the euro area. The tax menu is very similar to countries such as Germany. Fiscal revenues (including social security contributions) to GDP ratio for Greece is in the line of the rest euro area countries and even higher than countries like Ireland. Furthermore, government spending to GDP ratio was about 45% for Greece compared to the 47% for Germany or 53% for France, and public to private labor ratio is around 24% for Greece compared to about 32% for France.

3.3 Equilibrium and public debt frontier

With the model economy calibrated to replicate the size of the government for the period 2002-2006 we proceed to define a steady state where the economy can roll over the existing debt as follows.

Definition of steady state with rolling over: An equilibrium for this economy is a vector of prices $(W_g^*, W_p^*, R_p^*, R_g^*, R^B)$, a vector of input quantities $(L_g^*, L_p^*, K_g^*, K_p^*)$, and a vector of private consumption and investment (C_p^*, I_p^*) such that for a given fiscal policy summarized by a collection of taxes $(\tau_c, \tau_l, \tau_k, \tau_{ss}, \tau_\pi)$ and expenditure proportions $(\theta_1, \theta_2, \theta_3, \theta_4)$, induces a vector of public consumption, investment, transfers, and debt services $(C_g^*, I_g^*, Z^*, R^B B^*)$, such that the optimization problems of the household, the firm, and the government are satisfied in a way that the resources constraints are satisfied and all markets clear.

This steady state induces a level of welfare for the consumer given by

$$U^* = \frac{1}{1-\beta} U(C_p^*, C_g^*, L^*) \tag{27}$$

We can compute one steady state with rolling over for every ratio G/Y and build what we call the "debt frontier", defined as the sustainable debt limit for each level of public expenditure: figure 3 shows those debt limits that are consistent with each ratio G/Y. Sustainable debt limit here stands for a level where fiscal income is sufficient to cover current government expenditures and the service of debt. This notion of sustainable debt limit coincides formally with the steady state level of debt (with constant bond yields) for the model we have presented.

From the model we obtain a numerical representation of the trade-off between public debt long-run sustainable limit and government size measured as the total government spending to GDP ratio. A larger government size, given a constant level of public revenues, corresponds to a lower long-run sustainable level of public debt. The debt frontier is the relationship between public expenditure to GDP ratio, G/Y, and total debt to GDP ratio, B/Y, implied by the government budget constraint. Above the curve, we have all pairs where given the ratio G/Y, the amount of endogenous fiscal revenues are not enough to cover the services of total debt, $R^B B$. Below the curve, we have all data pairs where fiscal revenues suffice to cover the given G/Y ratio and services the outstanding debt. Figure 3, shows that the ratios of public expenditures and total debt where very far from the debt limit, calculated with the real return of bonds set at 1% for the period 2002-2006. Figure 3, also plots the actual values of G/Y and B/Y ratios for the period 2002-2006, shown in figure 2 as time series. These ratios, remained almost constant for the period 2002-2006 at a value of total public spending/GDP of 45% and a public debt/GDP of around 100%. The intuition behind this result is simple. In our model, public debt is modeled as if bond markets were infinitely liquid and thus, any maturing bond can always be rolled over at the given rate in the steady state. In this context, the long term sustainable amount of debt depends on both public revenues and expenditures and on the public bond interest rate. The sustainable debt limit is increasing in public revenues and decreasing in public expenditure and bond interest rate. A negative shock to output will reduce

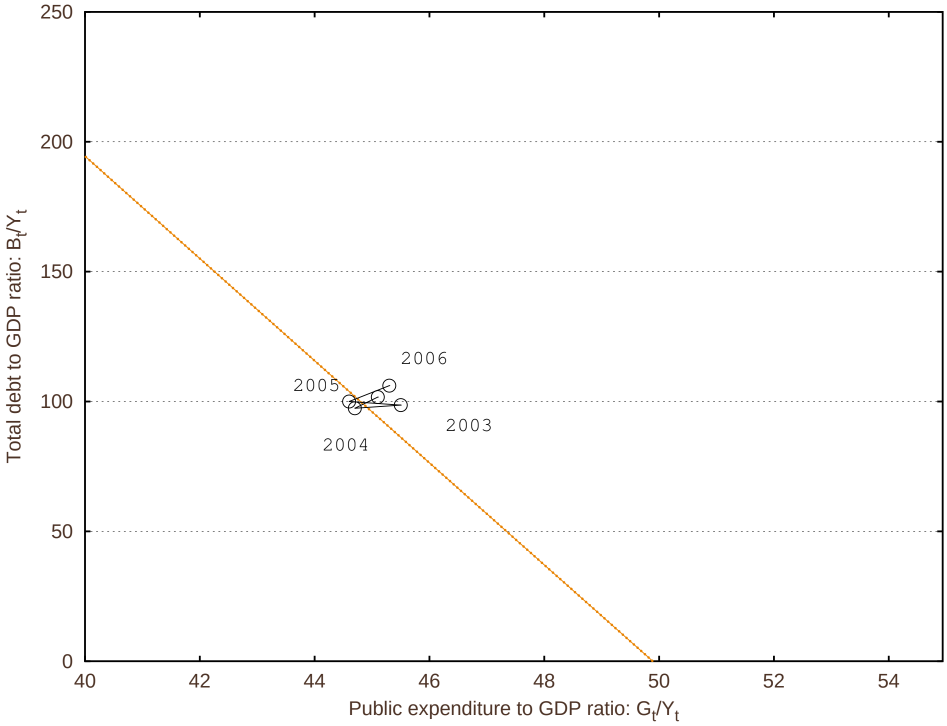

Figure 3: The debt frontier 2002-2006. Greece

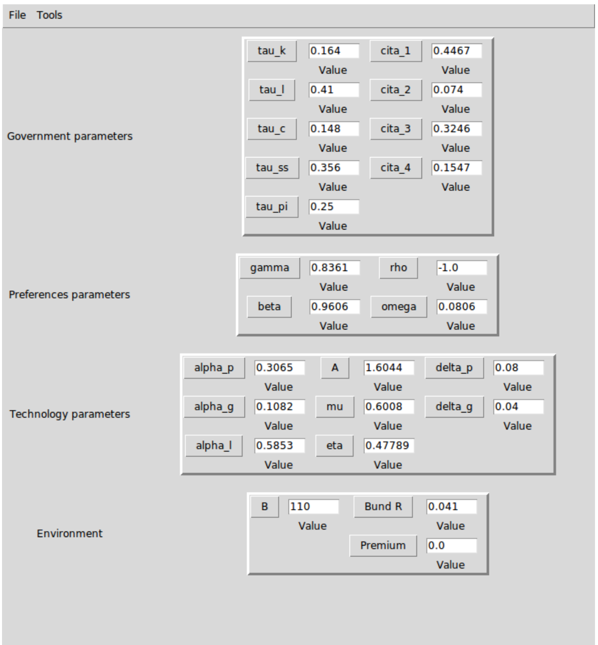

Figure 4: Main window of the application

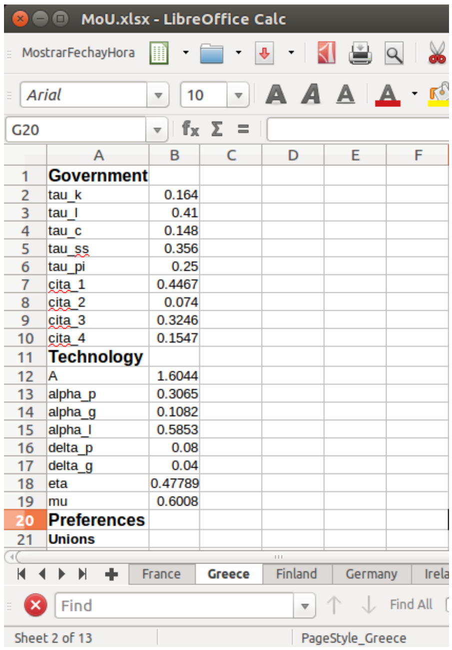

Figure 5: File MoU.xslx

both the public income/output ratio and the public expenditure/output ratio, driving the economy toward the long-run unsustainable debt area on one hand, and reducing the long-run sustainable amount of debt on the other hand.

4 A Python toolkit for policy evaluation

In this section, we present a software written in Python in which researchers and policy makers can freely calibrate and experiment changes in government's behavior to assess the consequences on debt sustainability.

The main window of the application is in Figure 4, and contains the pre-loaded parameter values of a country. This country can be any of the OECD data set used to feed the application. The country that is currently pre-loaded is Greece.

Figure 4, shows four panels with data. Each of the panels collects the parameter values of each of the four institutional sectors of our economy: the government, the household, the technology (represented by the firm and the labor union), and the external sector, characterized by an international interest rate, a country specific risk premium and the current debt to GDP ratio that is steadily rolled over period by period.

From the top menu of the application we can select the sub menu **File**, where a drop-down menu unfolds, showing a set of options depicted in figure 6. There, we

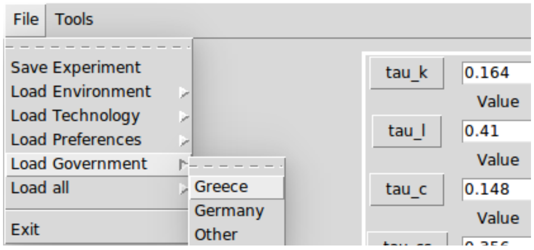

Figure 6: Menu File

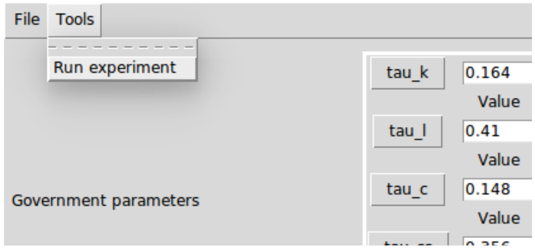

Figure 7: Menu Tools

can select the parameter set for the government, for the technology, the consumer and the environment described by the foreign sector. It is also possible to download the parameter set of a whole country. It is therefore possible to create mixed economies, with, for example, the Italian technology, the German government, the Japanese debt and the American consumer.

Once the economy has been selected, from the **Tools** menu, we can execute the experiment we want to run. Figure 7 displays the singleton drop-down menu that allows the user to run the experiment once the parameter set has been set to the desired mixture of economies.

Once the option **Run experiment** is chosen, a window pops-up in the screen of the computer. It contains the 'debt frontier' computed with the model presented in the paper, and with the parameter selection obtained from the Libre Office Calc[10] file from where the python interpreter reads the stored data. Figure 9 displays the calculated frontier superimposed to data consisting of trios $[(G/Y, B/Y), year]$

4.1 Example 1

We want to know if Greece could have improved its fiscal financial position had it assumed in advance the German tax code. To run this experiment we first execute the application as a python executable. From the top menu, choose **File→ Load all→ Greece** as shown in Figure 10 to fill all parameters with the values taken from the file **MoU.xlsx** shown in Figure 5. Once the data is loaded into the application execute **Tools→ Run experiment** and Figure 9 pops-up. It contains two elements: the 'debt frontier' and a collection of points in the space $(G/Y, B/Y)$ together with a year associated to each point. Note that the line passes through the cluster of points marked 2002-2006. Those are the years used for the calibration of the 'debt frontier' and that is the reason why the line passes trough those points. Once the crisis hit Greece, the negative shock on Y, given G swings the ratio G/Y to the right. This

[10]LibreOffice Calc is a free software spread sheet included in the LibreOffice suit. It was forked, developed and maintained by The Document Foundation, and released under the GNU Lesser General Public License

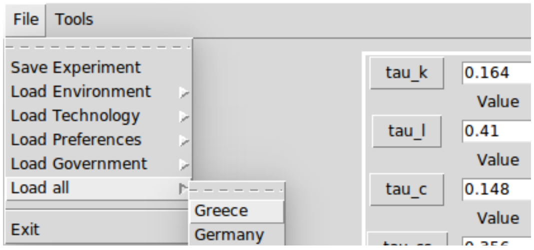

Figure 8: Load Greek parameters

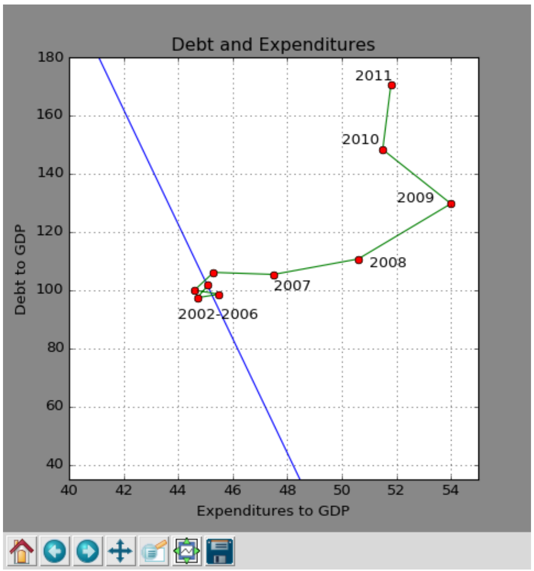

Figure 9: Pop-up window with the debt frontier

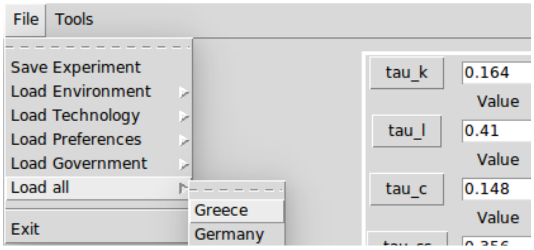

Figure 10: Load Greek parameters

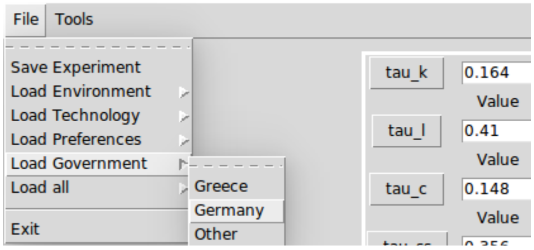

Figure 11: Load German fiscal parameters

movement makes the current situation unsustainable in the long run.

To improve Greece's financial situation, the German tax code could be proposed. To check how the German tax code affects the long term fiscal position for Greece, we next get back to the main window of the application and select **File→ Load Government→ Germany** as show in Figure 11

Once the German tax code is loaded into the, otherwise Greek economy, we can run again the experiment with **Tools→ Run experiment** to get a new line obtained from a Greek economy that has German taxes. Figure 12 shows that Greece with the German tax code would have been worse off than it was when the crisis hit.

Other experiments are also interesting. For example, Germany could have had Greek taxes, and we could check that for Germany that would have pose any financial problem. At the same time, Greece with German parameters in the technology would have produced a very solvent country, whereas the converse is not true: Germany would

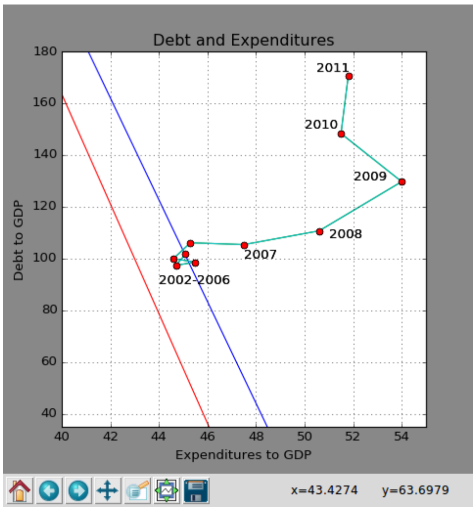

Figure 12: E1, Greece with German taxes

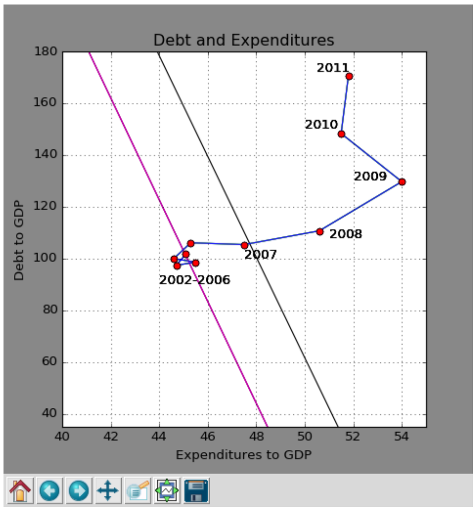

Figure 13: E2, Greek reform

have defaulted its own debt operating the Greek technology.

4.2 Example 2

Policies oriented to increase productivity, together with a fiscal package that includes increases in VAT, labor taxes and corporate taxes, plus a re-structuring of public expenditures increasing public investment, at the expense of transfers, can be effective to solve a debt crisis. The proposed combination of increasing by 10% the following vector of policy instruments $(\tau_k, \tau_l, \tau_\pi, \theta_3)$ would depress output by -2.37%, it would depress private consumption and public consumption by -2.30% and -2.37% respectively, and it would depress total investment by -4.98%, but it would rise the debt ceiling by 24.47%. as shown in Figure 13

4.3 Example 3

We complete our analysis with a variation of the yield. Figure 14 shows how the frontier moves inwards as a consequence of an increase in the yield of the Greek bond. We represent the frontier for a 4%, 5% and a 7% yields, re-calibrating the other parameters values of the model economy to the new interest rate. Notice that the effective spreads of the Greek bond with respect to the German Bund were much larger. Since the very beginning of the negotiations of the details of the rescue package for Greece by April 2010, the spreads skyrocketed due to a number of reasons. One of those reasons is discussed in [6]. They argue that the seniority of the new bonds issued to finance the rescue program would disincentive other private investors from buying Greek bonds. However, we agree with [3] in saying that the rescue package was an effective mechanism

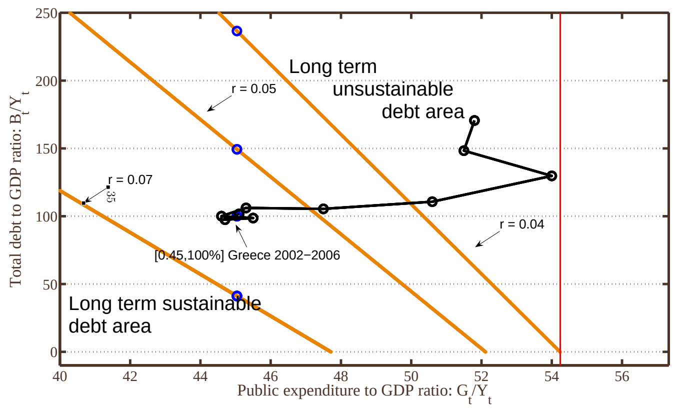

Figure 14: The frontier at different real rates

to provide liquidity to the Greek State at a controlled yield. Figure 14 shows that the fiscal ratios displayed by the Greek economy prior to the crisis were sustainable at the yield of 5%, that is, the real return of the rescue package bond was consistent with a long-term sustainability of the Greek State prior to the unfolding of events that drove Greece to the current crisis. Nevertheless, the pre-crisis figures were unsustainable at the yield of 7% as shown in Figure 14

5 Final remarks

This paper develops a DSGE model in which the government is fully characterized in both income and spending sides. This allows to relate detailed fiscal policy to the evolution of public debt and to compare model predictions about debt dynamics with the concept of sustainable debt. Allowing for different taxes and chapters of spending in a general equilibrium model ultimately provides a model of competing taxes and spending, showing which fiscal policies are more costly in terms of debt sustainability and which in terms of welfare for households.

The paper also develops a toolkit to implement previous analysis on a quantitative basis. This toolkit works as a stand alone software written in Python, which has the advantage that can be both directly used in PC (operating Windows or GNU/Linux) or Mac and embedded in apps thus ready to be executed on smart phones and tablets.

To test our theory and the associated toolkit, we calibrate the model to the Greek economy and show the impact of different fiscal policy on the sustainability of public debt in Greece. In the paper, we only focus on long run implications of fiscal policy on debt sustainability, but in principle our framework allows to include in the analysis the short run implications of fiscal policy on debt dynamics if the researcher wishes so. For now, we leave the short run perspective for future research.

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Appendix A.1: Walras' Law

Take the budget constraint faced by the consumer:

$$\begin{aligned}
& (1+\tau_t^c)C_{p,t} + K_{p,t} - K_{p,t-1} \\
= \; & (1-\tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + (1-\tau_t^k)(R_t-\delta)K_{p,t-1} + Z_t + \Pi_t
\end{aligned}$$

And substitute the value of

$$Z_t = G_t - C_{g,t} - (1+\tau_t^{ss})W_{g,t}L_{g,t} - I_{g,t}$$

to obtain:

$$
\begin{aligned}
& (1+\tau_t^c)C_{p,t} + I_{gt} + K_{p,t} - K_{p,t-1} \\
= \; & (1-\tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + (1-\tau_t^k)(R_{p,t}-\delta)K_{p,t-1} \\
& + G_t - C_{g,t} - (1+\tau_t^{ss})W_{g,t}L_{g,t} + \Pi_t
\end{aligned}
$$

Or,

$$
\begin{aligned}
& C_{pt} + C_{g,t} + I_{gt} + I_{pt} \\
= \; & -\tau_t^c C_{p,t} + (1-\tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + R_{p,t}K_{p,t-1} - \tau_t^k \left(R_{p,t} - \delta_{K_p}\right) K_{p,t-1} \\
& + G_t - (1+\tau_t^{ss})W_{g,t}L_{g,t} + \Pi_t
\end{aligned}
$$

But, the government identity establishes the following relation:

$$
(1+R_t^B)B_t - B_{t+1} = T_t - G_t
$$

Direct substitution yields

$$
\begin{aligned}
& C_{pt} + C_{g,t} + I_{gt} + I_{pt} - T_t - B_{t+1} + (1+R_t^B)B_t \\
= \; & -\tau_t^c C_{p,t} + (1-\tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + R_{p,t}K_{p,t-1} - \tau_t^k \left(R_t - \delta_{K_p}\right) K_{p,t-1} \\
& -(1+\tau_t^{ss})W_{g,t}L_{g,t} + \Pi_t
\end{aligned}
$$

Government fiscal income is given by:

$$
\begin{aligned}
T_t = \; & \tau_t^c C_{p,t} + \tau_t^l(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^k(R_{p,t} - \delta_{K_p})K_{p,t-1} \\
& + \tau_t^{ss}(W_{p,t}L_{p,t} + W_{g,t}L_{g,t})
\end{aligned}
$$

Substitution and elimination drives to:

$$
\begin{aligned}
& C_{pt} + C_{g,t} + I_{gt} + I_{pt} - B_{t+1} + (1+R_t^B)B_t \\
= \; & W_{p,t}L_{p,t} + R_t K_{p,t-1} + \Pi_t + \tau_t^{ss}W_{p,t}L_{p,t}
\end{aligned}
$$

From the definition of profits we find that,

$$
\Pi_t = Y_t - (1+\tau_t^{ss})W_{p,t}L_{p,t} - R_{p,t}K_{p,t}
$$

Substitution yields:

$$
C_{pt} + C_{g,t} + I_{gt} + I_{pt} = Y_t + B_{t+1} - (1+R_t^B)B_t
$$

Which implies that all uses come from all available resources from an open economy. Therefore, Walras' Law is satisfied at all times.

Appendix A.2: Positive profits

In a private economy where the government supply capital and labor with market pricing, the firm would have a profit function as:

$$\bar{\Pi}_t = Y_t - (1+\tau_t^{ss})(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) - R_{p,t}(K_{p,t-1} + K_{g,t-1})$$

where

$$Y_t = A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{\alpha_l}{\eta}}$$

Under the assumptions that private factors are paid their marginal productivity, we get:

$$(1+\tau_t^{ss})W_{p,t} = \mu\alpha_l A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{(\alpha_l-\eta)}{\eta}} L_{p,t}^{\eta-1} \tag{A.2.1}$$

$$(1+\tau_t^{ss})W_{g,t} = (1-\mu)\alpha_l A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{(\alpha_l-\eta)}{\eta}} L_{g,t}^{\eta-1} \tag{A.2.2}$$

$$R_{p,t} = \alpha_p A_t K_{p,t-1}^{\alpha_p-1} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{\alpha_l}{\eta}}$$

$$R_{g,t} = \alpha_g A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g-1} [\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{\alpha_l}{\eta}}$$

Division of equation (A.2.1) by (A.2.2) yields equation (26) of Section 3. From the above equations we can obtain all income shares as:

$$\begin{aligned}(1+\tau_t^{ss})W_{p,t}L_{p,t} &= \mu\alpha_l A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{(\alpha_l-\eta)}{\eta}} L_{p,t}^{\eta} \\ &= \frac{\mu\alpha_l L_{p,t}^{\eta}}{\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}} Y_t\end{aligned}$$

$$\begin{aligned}(1+\tau_t^{ss})W_{g,t}L_{g,t} &= (1-\mu)(1-\alpha_p-\alpha_g) A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{(\alpha_l-\eta)}{\eta}} L_{g,t}^{\eta} \\ &= \frac{(1-\mu)\alpha_l L_{g,t}^{\eta}}{\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}} Y_t\end{aligned}$$

$$R_{p,t}K_{p,t-1} = \alpha_p Y_t$$

and

$$R_{g,t}K_{g,t-1} = \alpha_g Y_t$$

Profits are zero because of the homogeneity of the production function:

$$\begin{aligned}
\bar{\Pi}_t &= Y_t - \frac{\mu \alpha_l L_{p,t}^{\eta}}{\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}} Y_t - \frac{(1-\mu)\alpha_l L_{p,t}^{\eta}}{\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}} Y - \alpha_p Y_t - \alpha_g Y_t, \\
\bar{\Pi}_t &= Y_t \left(1 - \alpha_l - \alpha_p - \alpha_g\right) = 0
\end{aligned}$$

If, on the contrary, the government pays public factor through taxes as it is assumed in the paper, then there are positive profits which can be calculated as the difference between total output and the rents paid to the private factors:

$$\Pi_t = Y_t - R_{p,t}K_{p,t-1} - (1+\tau_t^{ss})W_{p,t}L_{p,t} > 0$$

Substituting private factor incomes yields:

$$\Pi_t = \left[1 - \alpha_p - \frac{\mu(1-\alpha_p-\alpha_g)L_{g,t}^{\eta}}{[\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]}\right] Y_t > 0$$

Appendix A.3: Equilibrium conditions

The collection of the model's first order conditions, market clearing and resource constraints are:

$$\frac{\gamma}{C_{p,t} + \pi C_{g,t}} - \lambda_t (1+\tau_t^c) = 0 \tag{A.3.1a}$$

$$\frac{1-\gamma}{N_t \overline{H} - L_{p,t} - L_{g,t}} - \lambda_t (1-\tau_t^l) W_{p,t} = 0 \tag{A.3.1b}$$

$$\beta \left[\lambda_{t+1} \left(1 + (1-\tau_{t+1}^k)(R_{t+1} - \delta_{Kp})\right)\right] - \lambda_t = 0 \tag{A.3.2}$$

$$\lambda_{t-1} - \beta \lambda_t (1 + R_t^B) = 0 \tag{A.3.3}$$

$$Y_t - A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]^{\frac{\alpha_l}{\eta}} = 0 \tag{A.3.4}$$

$$R_{p,t} - \alpha_p A_t K_{p,t-1}^{\alpha_p - 1} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]^{\frac{\alpha_l}{\eta}} = 0 \tag{A.3.5}$$

$$(1+\tau_t^{ss})W_{p,t} - \mu\alpha_l A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]^{\frac{(\alpha_l - \eta)}{\eta}} L_{p,t}^{\eta - 1} = 0 \tag{A.3.6}$$

$$\Pi_t - \left[\alpha_g + \frac{(1-\mu)\alpha_l L_{g,t}^{\eta}}{[\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]}\right] Y_t = 0 \tag{A.3.7}$$

$$K_{p,t} - ((1-\delta_{Kp})K_{p,t-1} + I_{p,t}) = 0 \tag{A.3.8}$$

$$K_{g,t} - \big((1-\delta_{K_g})K_{g,t-1} + I_{g,t}\big) = 0 \tag{A.3.9}$$

$$G_t - (C_{g,t} + (1+\tau_t^{ss})W_{g,t}L_{g,t} + I_{g,t} + Z_t) = 0 \tag{A.3.10}$$

$$C_{g,t} - \theta_1 G_t = 0 \tag{A.3.11}$$

$$I_{g,t} - \theta_2 G_t = 0 \tag{A.3.12}$$

$$(1+\tau_t^{ss})W_{g,t}L_{g,t} - \theta_3 G_t = 0 \tag{A.3.13}$$

$$Z_t - \theta_4 G_t = 0 \tag{A.3.14}$$

$$W_{g,t} - \left(\frac{\omega}{1-\omega}\right)^{-1/2\theta} \left[\frac{\theta_3 G_t}{(1+\tau_t^{ss})}\right]^{1/2} = 0 \tag{A.3.15}$$

$$L_{g,t} - \left(\frac{\omega}{1-\omega}\right)^{1/2\theta} \left[\frac{\theta_3 G_t}{(1+\tau_t^{ss})}\right]^{1/2} = 0 \tag{A.3.16}$$

$$T_t - \left(\begin{array}{c} \tau_t^c C_{p,t} + \tau_t^l (W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^k (R_t - \delta_{K_p}) K_{p,t-1} \\ +\tau_t^{ss}(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^k R_t^B B_t(z) + \tau_t^{\pi}\Pi_t \end{array}\right) = 0 \tag{A.3.17}$$

$$G_t + (1+R_t^B)B_t(z) - (T_t + B_{t+1}(z)) = 0 \tag{A.3.18}$$

$$L_t - L_{p,t} - L_{g,t} = 0 \tag{A.3.19}$$

This set of conditions fully characterizes a unique solution for any given policy vector. The set of equations of the model is completed with the budget constraint of the consumer and the following transversality conditions:

$$\lim_{t\to\infty} \beta^t \lambda_t K_t = 0$$

$$\lim_{t\to\infty}(1+R_t^B)^{-t}B_t(z)=0$$

The first transversality condition means that the present value of future capital, K_t, must be going to zero. The second transversality condition requires a zero limit of future government debt discounted at the bond rate.

Appendix B: Data Sources

The frequency of the data is annual for the period 2002-2011. The model is calibrated using data for the sub-period 2002-2006, which is selected as the steady state for our model economy. GDP, government expenditure, public debt, private consumption, private investment, public investment and public consumption are taken from the OECD Statistics data base and Eurostat. Data on capital stock are taken from the EU-KLEMS database.

Public and private compensation of employees and public and private employment are taken from OECD Economic Outlook database December 2007 Issue, for the period 1960-2006. Public wage bill is calculated as total final public compensation of employees.

Finally, real return of Greek bond corresponds to the 10 year bond yield are taken from Bloomberg database, and the average maturity of debt can be seen in http://www.pdma.gr/index.php/en/public-debt-strategy/public-debt/historical-characteristics/weighted-average-cost-maturity-of-annual-funding