SIMULATING FINANCIAL CONTAGION DYNAMICS IN RANDOM INTERBANK NETWORKS

John Leventides
University of Athens, Greece

Kalliopi Loukaki
University of Athens, Greece

Vassilios Papavassiliou
University College Dublin, Ireland
RCEA
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John Leventides\textsuperscript{a,1}, Kalliopi Loukaki\textsuperscript{a,1}, Vassilios Papavassiliou\textsuperscript{b,2,*}

\textsuperscript{a}University of Athens
\textsuperscript{b}University College Dublin

Abstract

The purpose of this study is to assess the resilience of financial systems to exogenous shocks using techniques drawn from the theory of complex networks. We investigate by means of Monte Carlo simulations the fragility of several network topologies using a simple default model of contagion applied on interbank networks of varying sizes. We trigger a series of banking crises by exogenously failing each bank in the system and observe the propagation mechanisms that take effect within the system under different scenarios. Finally, we add to the existing literature by analyzing the interplay of several crucial drivers of interbank contagion, such as network topology, leverage, interconnectedness, heterogeneity and homogeneity across bank sizes and interbank exposures.

Keywords: Interbank contagion, random networks, financial stability, interconnectedness, systemic risk

1. Introduction

The United States subprime mortgage crisis of 2007/8 as well as the European sovereign debt crisis of 2009/10, revealed the weaknesses of financial institutions worldwide and the crucial role financial interconnectedness plays.
in the transmission of financial distress. Although interconnectedness contributes to efficient risk sharing, it may lead to contagious episodes of default following an initial shock in the financial system.

The recent spate of bank failures shed light on the importance of systemic risk for financial stability. The possibility that an event at the company level could trigger severe instability or collapse in the entire economy has raised awareness among policy makers and market regulators of the importance of enhanced transparency and higher standards and efficiency in banking system regulations. According to Pericoli and Sbracia (2003) the banking system of a country can contribute to the transmission of contagion due to moral hazard caused by the presence of institutionally enforced guarantees on deposits and by fluctuations in asset values used as collateral by banks. Thus, enhanced regulations pose significant challenges for the management of systemic risk in today’s integrated global markets.

The complexity of the financial system has led many academics to utilize the network theory to study the effects of interconnectedness and network topology on financial stability. Studying the financial system as a network is one of the methods that have been used to investigate the emergence of systemic risk through the connections of banks. In such network structure, every node represents a bank and the connections between banks are represented by edges. A robust interbank market plays an important role in the stability of the financial system. Through the interbank market, banks which suffer liquidity shortages can borrow from banks with liquidity surpluses. Thus, the interbank market can have a stabilizing effect on the financial system by redistributing funds in an effective way among banks, however, at the same time it can make the system prone to financial contagion through the existing interbank linkages.

For decades, among the various suspects for destabilizing the financial system were large financial institutions whose failure would be disastrous to the greater economic system (too big to fail theory). Such financial institutions must be supported by the government when they face financial distress due to their systemic importance and interconnectedness. However, smaller financial institutions but with lots of connections in the interbank market can have an even larger impact on the financial system if they fail. Higher interconnectedness of the interbank network can reduce the probability of default due to the fact that transmission of a shock can be shared by many counterparties and thus it dissipates faster. On the other hand, when the magnitude of the shock has crossed a critical threshold, due to increased in-
terconnectedness the shock will spread into a large part of the system which can cause a large cascade of defaults. This is the so-called “robust-yet-fragile” property that financial systems exhibit (Acemoglu et al., 2015).

In this paper, we focus our attention on the direct contagion channel and study the effectiveness of various drivers on interbank contagion. The flourishing literature which ensued in recent years has developed theoretical models aimed at addressing the various issues concerning systemic risk. Counterfactual simulations on data have been employed to study interbank contagion under different scenarios related to the topology of the interbank network, the size of interbank exposures and the degree of heterogeneity and interconnectedness within the network. In what follows, we develop a model with banks linked to one another by their interbank claims and investigate by means of Monte Carlo simulations how complexity of an interbank network structure affects interbank contagion under different testable scenarios. Our analysis belongs to the strand of the theoretical literature as we employ computer simulations to construct a large number of bank networks involving entities with interlocking interbank claims/obligations. Using tools from complex network theory, we model how shocks of an initial default may spread from one institution to another (simulated financial networks to study contagion phenomena have also been employed by Nier et al., 2007 and Gai and Kapadia, 2010).

Unlike earlier studies that use particular network structures (Erdős-Rényi network models, scale-free network models and small world network models) to test the resilience of banking systems, we assume that the network structure in our model is arbitrary, that is, the network of interbank claims forms randomly. The assumption of randomness in the network structure has the advantage that our model would contain any possible structure that may emerge in the real world and this is what makes our paper distinct from the earlier literature. We use a direct channel of contagion resulting from the direct interbank linkages among banks which takes effect when an idiosyncratic shock travels through the network of banks and affects the balance sheets of multiple agents.

Our analysis contributes to the literature in a number of ways. First, we study the interplay of several crucial drivers on interbank contagion, such as bank capital ratios, leverage, interconnectedness and heterogeneity across banks’ sizes. Along these lines, we address the following questions: Does heterogeneity, leverage and interconnectedness matter for systemic risk and the propagation of contagion? If so, in what respect? In order to answer
these questions, we build an interbank network model and demonstrate how contagion propagates under various scenarios concerning the degree of the system’s heterogeneity, the balance sheet composition and the level of connectivity among banks\(^1\).

Second, we utilize only two components from a bank’s balance sheet, that is, equity and interbank loans in order to construct a parsimonious regression model. Our regression model is used for testing the impact of crucial drivers recorded in our simulation experiments on interbank contagion. Regression analysis has also been used by Krause and Giasante (2012) to assess the role played by the network’s topological features and balance sheet positions in the transmission of bank failures. The authors utilize a scale-free network model to study interbank contagion with large parameter ranges and many parameters to initialize a balance sheet. However, their model becomes inflexible in operation as it is difficult to compare simulations by varying one of the parameters. On the contrary, our model is easily explainable and reproducible and more amenable to analysis and interpretation.

Third, unlike most papers in the recent literature (Nier et al., 2007; Gai and Kapadia, 2010; Chinazzi et al., 2015; Amini et al., 2016) we define the term contagion as the situation in which the initial failure of a bank leads to the failure of at least one other bank, while the extent of contagion is measured by the total capital loss in the banking system due to the failure of at least one bank. In other words, we are mostly interested in detecting the magnitude of capital losses in the banking network rather than the number of banks that were adversely affected. Finally, our paper contributes to the existing literature as it examines with the use of a comprehensive network model the knock-on effects an initial default can bring into the interbank network under the assumption of randomness. The assumption that the network of interbank claims and obligations forms randomly, enables us to capture all possible scenarios that may appear in real-world situations. Moreover, we circumvent the problem of data unavailability as real data on interbank exposures are generally only available to central bankers and regulators, thus rendering the empirical analysis of networks problematic. Our

\(^1\)As far as heterogeneity is concerned, a great number of recent empirical studies explore its role along various dimensions, such as network topology (Pegorano, 2012), interbank exposures (Amini et al., 2016), bank sizes and investment opportunities (Iori et al., 2006). Chinazzi et al. (2015) study the role of heterogeneity from a theoretical point of view, using different scenarios concerning bank sizes and interbank exposures.
analysis also differs from networks based on entropy methods which are able to capture unidimensional features of the network structure only, and not dynamic and multifaceted network patterns. Maximum entropy approaches may also not be very reliable in assessing the severity of financial contagion as, depending on the interbank market structure, can lead to undervaluation or overvaluation of the extent of contagion (Mistrulli, 2011).

Our findings show that heterogeneity in bank sizes and interbank exposures matters a great deal in the stability of the financial system, as its absorption capacity is enhanced. Also, the level of interconnectedness hugely impacts on the system’s resilience, especially in smaller and highly interconnected interbank networks. Finally, we provide evidence that highly leveraged banks form the main channel through which financial shocks propagate within the system and such effect is more pronounced in large interbank networks than in smaller ones.

The rest of the paper is organized as follows. Section 2 discusses the related literature on interbank contagion. Section 3 describes our network model of contagion. Section 4 presents and discusses the computer experiments and simulation results. Finally, Section 5 concludes the paper.

2. Related literature

Our study is related to separate strands of the literature on interbank networks. First, it relates to the literature on contagion channels. According to Upper (2011), the channels through which a shock spreads can be broken down into two groups: indirect and direct contagion channels. A direct contagion channel results from the direct interbank linkages among banks and can take effect when an idiosyncratic shock travels through the network. This shock can be due to the inability of some banks to meet their financial obligations or due to interbank exposures that are quite large relative to the lender’s capital. The possibility of the occurrence and transmission of direct contagion depends mainly on the structure and size of the interbank market.

On the other hand, indirect contagion is created by indirect linkages among banks in an interbank network.

Allen and Gale (2000) state that the interbank structure can be complete or incomplete, with contagion being less likely in the case of a complete structure, i.e. the case where the distressed bank has symmetric linkages with all other banks in the banking system. There is also another type of interbank market structure defined as the money centre structure, developed
by Freixas et al. (2000) which implies a symmetrical linkage of a “money
centre” bank to other banks, but without any mutual links among other
(peripheral) banks. In this network structure, the failure of a money centre
bank - the core bank - can cause interbank contagion, while the failure of a
peripheral bank can only affect the neighboring banks.

Allen and Babus (2008) argue that linkages in interbank networks include
identical assets, portfolio returns and overlapping portfolios. If, for exam-
ple, a bank holds identical assets with other banks, the correlation between
their portfolios can cause fire sales in the market during a crisis period, thus
depressing overall prices in the market and inducing significant losses for all
participants. The fear of losses on interbank loans makes banks reluctant to
extent credit and induces them to hoard liquidity.

There are a number of recent studies that have dealt with the issue of
interbank contagion. Memmel and Sachs (2013) simulate interbank conta-
gion effects for the German banking sector and find that bank capital ratios,
the share of interbank assets in the system and the degree of equality in
the distribution of interbank exposures are the most important determinants
for financial stability. Georgescu (2015) compares the contagion potential
of accounting induced regulatory constraints to that of funding constraints
in a bank network and concludes that the interplay between illiquidity and
solvency can lead to bank failures which are manifested by the vulnerable
funding structure of banks during a crisis. Tonzer (2015) examines the rela-
tionship between cross-border bank linkages and financial stability and show
that larger cross-border exposures increase bank risks, however, when bilat-
eral interbank linkages exist there is a shift toward a more stable banking
system. Fink et al. (2016) model contagion in the German interbank market
via the credit quality channel and propose a novel metric which estimates the
potential regulatory capital loss to a banking system due to contagion via
interbank loans. They show that contagion effects can be reduced if banks
alter their lending and borrowing habits in response to policy interventions.

Distinguishing among the various contagion channels is crucial for under-
standing financial contagion and the mechanisms through which it spreads
and evolves. If financial shocks are transmitted via temporary channels,
regulators could use short-run strategies to reduce contagion effects, such
as stricter capital controls. On the other hand, transmission of financial
shocks via more permanent channels would render any short-term measures
inappropriate and would only delay and not prevent contagion (Pericoli and
Sbracia, 2003). Since bailouts are undesirable due to moral hazard consider-
ations, ex-ante measures should be considered in order to limit the possibility of contagion in interbank networks.

Our analysis also relates to the role of heterogeneity in the structure of interbank networks. Iori et al. (2006) use a simulation model of 400 banks comprising the interbank market in which the lending and borrowing functions are endogenously generated. In this model, each bank faces fluctuations in liquid assets and stochastic investment opportunities that mature with delay, creating the risk of liquidity shortages. Banks resort to overnight interbank borrowing only when they face a temporary liquidity shortage. The authors find that the likelihood of contagion is lowered in case the interconnected institutions are homogeneous, i.e. they have similar characteristics such as size or investment opportunities and thus, no institution becomes significant for either borrowing or lending. The authors also suggest, in line with Allen and Gale (2000), that as connectivity increases the system becomes more stable.

Caccioli et al. (2012) study the role of heterogeneity in degree distributions (the number of incoming and outgoing links), balance sheet size and degree correlations between banks. Specifically, they analyze the probability of contagion conditional on (a) the failure of a random bank, (b) the failure of the most connected bank and (c) the default of the biggest bank within the network. They find that networks with heterogeneous degree distributions are shown to be more resilient to contagion triggered by the failure of a random bank, but more fragile with respect to contagion triggered by the failure of highly connected nodes. The authors also provide evidence that when the average degree of connectivity is low, the probability of contagion due to failure of highly connected banks is higher than that due to the failure of large banks. However, when the average degree of connectivity is high, the opposite holds. Since the second scenario seems to be more realistic (networks with high connectivity), having "too big to fail" banks is more effective in eliminating a shock.

Ladley (2013) develops a partial equilibrium model of a closed economy in which heterogeneous banks interact with borrowers and depositors through the interbank market. Banks in the model are subject to regulation and the aim of the model is to qualitatively show how regulation and network structure can constrain or enhance the risk of contagion. The results show that for high levels of connectivity the system is more stable when the shock is small, while the contagion effects are amplified in case of larger initial shocks. Amini et al. (2016) focus on bank heterogeneity in terms of the number of
banks included in the network and the magnitude of their interconnections with other banks. They conclude that the more heterogeneity is introduced, the less resilient the network becomes. Contrary to these findings, the study of Georg and Poschmann (2010) finds no significant evidence that the heterogeneity of the financial system has a negative impact on financial stability.

Chinazzi et al. (2015) contribute to the debate on macro-prudential regulation concerning the resiliency of the financial system to exogenous shocks and the conditions that should be met to guarantee a high degree of stability. By forming two distinct models of contagion, a benchmark model and an extended model, the authors explore the interplay between heterogeneity, network structure and balance sheet composition in the transmission of contagion. They argue that heterogeneity in connectivity provides additional resiliency to the system when the initial default is random and also show that ‘too-connected-to-fail’ banks are more dangerous than ‘too-big-to-fail’ ones and should be the primary concern of policy makers since their failure can trigger systemic breakdowns.

Finally, our study is related to a number of empirical papers that have documented various stylized facts in interbank networks. These studies place emphasis on the degree distribution of the nodes that represents the number of incoming and/or outgoing links per node, i.e the number of a bank’s counterparties. Examples of studies that have employed scale-free degree distributions are those of Boss et al. (2004) on the Austrian interbank market, Inaoka et al. (2004) on the Japanese interbank market, Soramäki et al. (2007) on the US Fedwire system, Iori et al. (2008) and Fricke and Lux (2015) on the Italian interbank market and Alves et al. (2013) on the European interbank market for large banks.

Empirical evidence shows that banks tend to be organized in communities and the networks they form tend to be disassortative (Soramäki et al., 2007; Bech and Atalay, 2010). This often reflects the economic rationale that smaller banks, rather than transacting with each other, typically use a small set of money center banks as intermediaries. Also, due to lack of data availability many studies such as Furfine (2003) and Gabrieli (2010) disregard the heterogeneity of interbank relations and focus only on one type of transactions, which according to Bargigli et al. (2015) may provide biased results, and that is the reason recent studies have started focusing on multilayer networks.
3. Network model of contagion

In this section we form a simple network model in which the various financial institutions are randomly linked to one another by their bilateral claims. Our model is tailored to simulate default cascades triggered by an exogenous shock in an interbank network. We first introduce the interbank network model, describe the default cascades initiated by a random negative shock on this network and analyze the parameters that affect interbank contagion.

3.1. The interbank network

We assume that the banking system contains \(i = 1, \ldots, N\) banks. Every bank has its own balance sheet and the accounting equation holds at all times. Total assets are divided in three categories: interbank assets \(A_{i}^{IB}\), other assets \(A_{i}^{OT}\) and cash reserves \(C_{i}\). On the liabilities side of the balance sheet we have included: interbank liabilities \(L_{i}^{IB}\), other liabilities \(L_{i}^{OT}\) and equity capital \(E_{i}\). A schematic overview of the balance sheet is given in Figure 1. Although the proposed balance sheet structure does not capture all elements of a bank balance sheet, it includes all those positions that are relevant to our study.

Let’s consider a finite set \(V = \{v_1, v_2, \ldots, v_n\}\) of unspecified elements and let \(V \times V\) be the set of all ordered pairs \([v_i, v_j]\) of the elements of \(V\), where a relation on the set \(V\) is any subset \(E \subseteq V \times V\). Following Gutman and Polanski (1987), we define a simple interbank network as the pair \(G = (V, E)\), where \(V\) is a finite set of nodes and \(E\) is a symmetric relation on \(V\). We consider the Hilbert space of squared summable functions on the set of nodes \(V\) of the network \(H := l^{2}(V)\), and let \(\{|i> i \in V\}\) be a complete orthonormal basis of \(l^{2}(V)\). Estrada (2011) shows that the adjacency operator of the network acting in \(l^{2}(V)\) is defined as:

\[
(Af)(u) := \sum_{u, v \in E} f(v), f \in H, i \in V
\]  

(1)

We further consider \(A\) as an \(|V| \times |V|\) matrix. For our network \(G = (V, E)\) the entries of the adjacency matrix are defined as:

\[
A_{ij} = \begin{cases} 
1 & \text{if } i, j \in E \\
0 & \text{otherwise} 
\end{cases}
\]
The $u$th row or column of $A$ has $k_u$ entries, where $k_u$ is the degree of the node $u$, which is simply the number of nearest neighbours that $u$ has. Denoting by $1$ a $|V| \times 1$ vector, the column vector of node degrees $\kappa$ is given by:

$$\kappa = (1^T A)^T = A^T 1$$

(2)

We define the **indegree** as the number of links pointing toward a given node, and the **outdegree** as the number of links departing from the corresponding node. Specifically:

$$\kappa^{in} = (1^T A)^T = A^T 1$$

(3)

$$\kappa^{out} = A1$$

(4)

Thus, our interbank network of credit exposures between $n$ banks can be visualized by a graph $G = (V, E)$ where $V$ represents the set of financial institutions - nodes, and $E$ is the set of the edges linking the banks, that is, the set of ordered couples $(i, j) \in V \times V$ indicating the presence of a loan made by bank $i$ to bank $j$. The number of nodes defines the size of the interbank network. Every edge $(i, j)$ is weighted by the face value of the interbank claim and the representation of interbank claims is made by a single weighted $N \times N$ matrix $X$:

$$X = \begin{bmatrix}
0 & \cdots & x_{1j} & \cdots & x_{1N} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
x_{i1} & \cdots & 0 & \cdots & x_{iN} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
x_{N1} & \cdots & x_{Nj} & \cdots & 0
\end{bmatrix}$$

where $x_{ij}$ is the credit exposure of bank $i$ vis-à-vis bank $j$ and $N$ is the number of banks in the network. Interbank assets are represented along the rows while columns represent interbank liabilities. Once $X$ is in place, the interbank entries of each bank are given according to the following rules:
• $A_i = \sum_{j=1}^{N} x_{ij}$ (horizontal summation), where $A_i$ is the total interbank assets of bank $i$

• $L_i = \sum_{j=1}^{N} x_{ij}$ (vertical summation), where $L_i$ is the summation of the total interbank liabilities of bank $j$

One can observe that the diagonal line contains zeros due to the fact that banks do not lend to themselves. In this framework, a random direct network of interbank loans is generated, in which we let vary the outdegree (number of outgoing links) of each node in the system. The outdegree of a bank corresponds to the number of debtors, the indegree corresponds to the number of creditors, while the sum of these two measures gives the degree of each node. The degree of a node is a measure of connectivity which can be both a risk sharing and a risk amplification device. An example of an interbank network, consisting of $N=20$ banks is provided in Figure 2.

### 3.2. Shock propagation and contagion dynamics

The failure of a bank can affect other banks through their interbank connections. Below, we describe the mechanism through which an initial shock affecting a bank propagates onto its counterparties along the network. Contrary to the recent literature, the term contagion here translates into total capital losses in order to capture the size of contagion and not only the number of the failed banks. The cascade dynamics we use in this study are straightforward to implement and enable us to run a great number of simulations on a variety of different scenarios.

The default procedure starts with an exogenous shock being simulated, typically by setting to zero the equity of one randomly chosen bank $i$ and the cascade of defaults proceeds on a timestep-by-timestep basis, assuming zero recovery for shock transmissions. The zero recovery assumption, which is a realistic one in the short run, is often used in the literature to analyze worst case scenarios and refers to a situation where creditor banks lose all of their interbank assets held against a defaulting bank (Gai and Kapadia, 2010; Chinazzi et al., 2015). A bank’s default implies that it is no longer able to meet its interbank liabilities to its counterparties. Since these liabilities

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2Following Somaräki et al. (2007), connectivity ($p$) can be defined as the unconditional probability that two nodes share a link and equals $p = m/n (n - 1)$ for a directed network, where $n$ is the number of nodes and $m$ the number of links within the network.
constitute other banks’ assets, the banks that get into trouble affect simultaneously their counterparties, leading to write-downs in their balance sheets. The interbank asset loss due to failure of bank \( i \) is subtracted from the bank’s \( j \) capital. Bank \( j \) will fail if its exposure against bank \( i \) exceeds its equity. A second round of bank failure occurs if bank \( j’ \) ’s creditors cannot withstand the losses realized due to its default and eventually, contagion stops if no additional bank goes bankrupt, otherwise a third round of contagion takes place.

An initial shock can be amplified through banks’ interconnections and further transmitted to other institutions, such that the overall effect on the system goes largely beyond the original shock. As Upper and Worms (2004) demonstrate, in response to a liquidity shock banks prefer to withdraw their deposits at other banks instead of liquidating their long-term assets, creating further instability and liquidity dry-ups in the financial system. The mechanics of contagion can be illustrated by a simple example. We assume that we work with an interbank network consisting of \( i = 1, 2, 3, 4 \) four banks, which are equipped with a simple internal structure representing their balance sheet. The balance sheet information for each bank and the interbank relationships among banks can be represented in matrix form. We assume that the banks’ equity is given by a random vector \( b[i], i = 1, ..., 4 \) and their interbank exposure by a squared matrix \( A \) with zeros off the diagonal due to the fact that banks do not lend to themselves. We also assume that the outdegree for each bank is 2, which is set randomly among banks.

\[
b = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \end{bmatrix}, A = \begin{bmatrix} 0 & 20 & 0 & 10 \\ 20 & 0 & 30 & 0 \\ 0 & 30 & 0 & 10 \\ 20 & 0 & 10 & 0 \end{bmatrix}
\]

The above matrix forms a stylized interbank network that allows the representation of a system of interbank claims by a single weighted \( 4 \times 4 \) matrix, in which interbank assets are shown along the rows and interbank liabilities along the columns. The total capital of the system is defined as the summation of vector \( b \) and all magnitudes are expressed in money terms, e.g. euros.

We initially assume that a negative shock wipes out the equity of the first bank - bank 1. Banks that are interconnected with this bank immediately
record losses as bank 1 is unable to repay its liabilities, represented by the summation of the first column of the squared matrix. Note that bank 1 has borrowed the amount of 20 euros from bank 2, denoted by the entry in the first column and second row, and the amount of 20 euros from bank 4 denoted by the corresponding entry in the first column and the fourth row. Thus, both banks 2 and 4 record losses that reduce their equity. The interbank loans of bank 1 that cannot be repaid represent the loss of capital as a percentage of the total capital in the system, while bank’s 1 equity is regarded as the initial loss of capital in the network.

Subsequent to bank’s 1 default, bank 2 becomes insolvent while the amount of 20 euros is subtracted from bank’s 4 equity (40-20=20) making it more vulnerable to subsequent shocks. Thus, the updated vector of equity is now given by vector:

\[
b' = \begin{bmatrix} 0 \\ 0 \\ 30 \\ 20 \end{bmatrix}
\]

The distress caused from the initial default of bank 1 continues to propagate within the interbank network due to banks’ interconnectedness. Bank 2 has borrowed money from bank 1 and bank 3 that cannot be repaid, thus, the amount bank 2 owes to bank 3 has to be subtracted from the equity of bank 3 (30-30=0). This domino effect continues with the default of bank 3, as bank’s 2 default has wiped out its equity. The updated vector of equity is now given by vector:

\[
b'' = \begin{bmatrix} -20 \\ 0 \\ 0 \\ 20 \end{bmatrix}
\]

Bank 3 has borrowed funds from bank 2, which has already gone bankrupt, and from bank 4. The amount borrowed from bank 4 (10 euros) cannot be repaid and has to be subtracted from the updated equity of bank 4 (20-10=10).
A new updated vector of banks’ equity is:

\[
b''' = \begin{bmatrix}
-20 \\
-30 \\
0 \\
10 \\
\end{bmatrix}
\]

The default of bank 1 has caused a total loss of 60 euros or 60 percent of the total capital in the system. The initial loss of capital by defaulting bank 1 is 10 percent of the total capital in the system while the loss of capital at the first stage (interbank loans that cannot be repaid) by defaulting bank 1 as percentage of the system’s total capital is 40 percent. The leverage of the network system defined as total interbank exposure over the total capital in the interbank network, is 1.50 or 150 percent which explains the default cascades in this network \((20 + 10 + 20 + 30 + 30 + 10 + 20 + 10)/100\).

In the above example, banks rely heavily on interbank borrowing which makes the network more vulnerable to a random financial shock. We have described how exactly the default of a single bank can propagate through the interbank network and cause other banks to fail due to contagion effects. The same procedure is repeated for the \(n\) bank in the interbank network which is impacted by the initial random shock.

3.3. Monte Carlo simulations

In this section we apply Monte Carlo simulations in four different stages. In the first stage, we specify the model that will be used. Moreover, we choose the probability distribution specified for the errors. We have selected to work under a student’s \(t\) symmetric distribution. The second stage involves estimating the parameters of interest, i.e. the value of the coefficients in the regression model. In the third stage the test statistics of interest are saved, while in the fourth stage we go back to the first stage and repeat \(N\) times.

The quantity \(N\) is the number of replications which should be as large as is feasible. As Monte Carlo is based on random sampling from a given distribution (with results equal to their analytical counterparts asymptotically), setting a small number of replications will yield results that are sensitive to odd combinations of random number draws. Generally speaking, the sampling variation is measured by the standard error estimate, denoted \(S_x = \sqrt{\text{var}(x)/N}\), where \(x\) denotes the value of the parameter of interest and \(\text{var}(x)\) is the variance of the estimates of the quantity of interest over
In order to provide a general assessment of the various parameters that affect financial stability and can trigger contagion in an interbank network, we consider four different scenarios, in line with Chinazzi et al. (2015), where we let vary the degree of heterogeneity in the system, the balance sheet composition and the connectivity among banks. The four scenarios tested are as follows:

- **Scenario 1: Heterogeneous banks with homogeneous exposures.** In this scenario, we construct interbank networks where banks have different equity size and their interbank claims are evenly distributed across the outgoing links.

- **Scenario 2: Heterogeneous banks with heterogeneous exposures.** In this scenario, the interbank networks allow for heterogeneous bank sizes and heterogeneous interbank claims among banks.

- **Scenario 3: Homogeneous banks with heterogeneous exposures.** In this scenario, we construct interbank networks where banks have the same equity size and unevenly distribute their exposures across creditor banks.

- **Scenario 4: Homogeneous banks with homogeneous exposures.** In this last scenario, we construct interbank networks where banks have the same equity size and interbank claims are evenly distributed across creditor banks.

In each case, we allow the connectivity among banks to vary and the number of outgoing links of each bank lies within the range of 2 to 4 links. We examine banking systems consisting of small banks with low, medium and large interbank exposures, as well as systems of large banks with corresponding exposure levels.

We consider a basic model that uses only two components from a bank’s balance sheet, that is, equity and interbank loans - in the words of May and Arinaminpathy (2010) *a caricature for banking ecosystems*. We generate our model in two separate steps. First, we construct a model structure of $N$ nodes representing the banks in our system and randomly assign directed edges to represent lending-borrowing relationships, while in a second step, we assign each node to a stylized balance sheet structure. Once the banking
networks are created, the default propagation dynamics are implemented to examine the effects of an idiosyncratic shock hitting one bank.

The effect of a shock is simulated, typically by setting to zero the equity of the affected bank. We estimate the initial loss of capital by letting the first bank default and subsequently record the loss as percentage of the total capital in the system. Consequently, the defaulted bank will be unable to repay its creditors and the interbank loans that were granted will be written-off, as we have selected to work under a zero recovery assumption. This bad debt will be recorded and expressed as percentage of the total capital in the system. Moreover, the creditors of the defaulted bank will experience a shock on their balance sheets and the recorded losses will be subtracted from their equity.

If at any time the total losses realized by a bank exceed its net worth, the bank is deemed in default and is removed from the network. Note that timesteps are modeled as being discrete and there is the possibility that many banks default simultaneously in each timestep. These shocks propagate to their creditors and take effect in the next timestep. When no further failures are observed, the default procedure terminates and the total losses are recorded and expressed as percentage of the total capital in the system. Figures 1A and 2A in the Appendix formalize the aforementioned mechanism in a pseudocode which simulates the default cascade in the interbank network.

4. Main findings

This section discusses the main findings of this study. Subsection 4.1 describes in full detail the computer experiments conducted while subsection 4.2 discusses the simulation results of all four scenarios considered.

4.1. Computer experiments

Having generated banking systems via a network structure framework and balance sheet allocation, the dynamics of an initial shock affecting a bank within the interbank network can be investigated. Given the complexity of the interbank network outlined above, it is extremely difficult to derive analytical solutions. In order to obtain data to describe the variables that affect contagion, we employ several Monte Carlo simulations. In each realization, we construct an interbank network with \( N \in [20, 50, 80, 100] \) nodes. In a second step, we test the four scenarios mentioned before by varying the equity size of banks and the interbank exposure structure across creditor banks.
For each scenario tested we let the depth of connectivity across banks to vary, such that each bank can have two, three or four outgoing links with other banks. When homogeneity across bank sizes is considered, all banks are assumed to have the same equity size and thus, each bank is endowed with a balance sheet that consists of 100 units of equity. On the other hand, when homogeneity is present with respect to interbank exposures, interbank claims are randomly allocated within the interbank network and are categorized as follows: small loans granted (10 units), medium loans (20 units) and large loans (35 units). With respect to scenarios tested where heterogeneity of bank size is introduced, the amount of equity of each bank is drawn from a uniform distribution in the range: \( b_i \in [0, 100] \), whereas when heterogeneity is introduced with respect to interbank claims, credit is allocated in the following ranges: \( a_{ij} \in [0, 10] \), \( a_{ij} \in [0, 20] \), \( a_{ij} \in [0, 35] \). Then balance sheets are assigned to each node, consistent with each specific scenario tested. We randomly choose a single bank in the system to default due to an exogenous shock and the default cascades proceed sequentially, assuming zero recovery. When no further failures are observed results are recorded before another realization begins. We then impose another shock on the second bank in the network and this procedure continues until all banks in the interbank network are hit by an exogenous shock.

For each scenario tested and for each network size we have nine cases in which we allow the number of outgoing links \( (i = 2, 3, 4) \) and the weight of outgoing links (small, medium and large interbank claims) to vary among banks. Each case gives us 2,000 realizations or, to put it differently, 2,000 banking crises. We deem that 2,000 realizations provide a satisfactory number of runs and robustness to our analysis. Thus, for each scenario tested and each network size we employ \( 2,000 \times 9 = 18,000 \) realizations using the following variables in each realization:

- Total loss of capital due to contagion as percentage of total capital in the system (\( \text{CATEND} \))
- Initial loss of capital by defaulting bank \( i \) as percentage of total capital in the system (\( \text{CATIN1} \)), i.e. bank’s \( i \) depleted equity divided by the equity.

\(^3\)Although those ranges have been selected arbitrarily, they are not sensitive to any regression model employed in the following analysis and thus, our regression results will be unaffected from a qualitative point of view if different ranges are used.
Total equity in the network

- Loss of capital at the first stage (interbank loans that cannot be repaid) by defaulting bank \( i \) as percentage of total capital in the system (CATIN2), i.e. total amount of loans granted to bank \( i \) that cannot be repaid divided by the total equity in the network

- Leverage of the interbank network (LEVIN), i.e. total interbank exposures as measured by the sum of matrix’s \( A \) elements, divided by the total capital in the network

- Number of outgoing links of bank \( i \) (NOUTGOING), i.e. the out-degree of a bank which corresponds to the number of debtors in the network

- Shock propagation variable (COUNT) which measures the number of rounds needed until no further bank defaults

- Variance of capital (equity) (VARCAP) used in those scenarios tested where only heterogeneous bank sizes are considered

- Variance of interbank loans (VARLOANS) used in those scenarios tested where only heterogeneous interbank loan exposures are considered

Our selection of variables is motivated by economic intuition and by the findings of previous studies on the dynamics of systemic risks (Nier et al., 2007). In order to study the effect the aforementioned variables have on contagion risk, we estimate the following ordinary least squares (OLS) models:

\[
CATEND = \beta_1 CATIN1 + \beta_2 CATIN2 + \beta_3 LEVIN + \beta_4 NOUTGOING + \beta_5 COUNT + \beta_6 VARCAP
\]  

(5)

\[
CATEND = \beta_1 CATIN1 + \beta_2 CATIN2 + \beta_3 LEVIN + \beta_4 NOUTGOING + \beta_5 COUNT + \beta_6 VARCAP + \beta_7 VARLOANS
\]  

(6)
The model described in equation (5) is applied to scenarios involving heterogeneous bank sizes with homogeneous exposures in the network structure, equation (6) refers to a situation where emphasis is placed on heterogeneous interbank loan exposures combined with heterogeneous bank sizes, equation (7) takes into account homogeneous banks with heterogeneous exposures while equation (8) considers only homogeneous bank sizes and interbank claims. The variable CATIN1 has been omitted from equations (7)-(8) due to the fact that banks in the interbank system are homogeneous, i.e. we keep constant the equity of each bank and thus CATIN1 remains stable during our simulation runs.

### 4.2. Simulation results

In this section, we discuss the regression results of all four scenarios. Since our variables are measured on different scales, we cannot directly infer which independent variable has the largest effect on the dependent variable. In order to circumvent this problem we standardize our series to have zero mean and unit variance. Table 1 presents the regression results of the first scenario using the OLS model described in equation (5), where heterogeneous banks distribute evenly their interbank claims across the outgoing links of a network consisting of \( N = 20, 50, 80, 100 \) banks. All regressor coefficients are found to be statistically significant in all cases regardless of the size of the network. \( R \)-squared coefficients take on large values ranging from 72 to 76 percent and highlight the ability of our selected variables to explain financial distress in interbank networks.

The variable CATIN1 captures the initial effect defaulting bank \( i \) exerts on the network, whereas the magnitude of interconnectedness across the banks that comprise the interbank network is measured through parameter CATIN2. Financial shocks will propagate into the defaulting bank’s counter-
parties along the network, erode their capital and make them more vulnerable to subsequent shocks. The magnitude of the positive relationship between CATIN2 and CATEND - the dependent variable - seems to decrease as the size of the interbank network increases. This finding implies that as we move from smaller to larger network settings, systemic risk and the likelihood of contagion declines. Figure 3 visually illustrates the extent of contagion as a function of the percentage loss of capital due to bank’s i default. It is shown that as the network size increases capital losses decline, confirming the findings from the regression model.

As expected, we also find that there is a positive relationship between the leverage of the network and the capital losses due to contagion. This result is in line with the findings of Nier et al. (2007) who provide evidence that systemic risk increases when system-wide leverage increases. Highly leveraged banks in the interbank network are clearly more exposed to the risk of default on interbank loans. The greater the size of default on debt is, the larger the losses are that banks transmit to their neighbors, other things being equal. Thus, highly leveraged banks contribute more to systemic risk as they become a vehicle for transmitting shocks within the network. Moreover, it is shown that the magnitude of the positive relationship between the network’s leverage and contagion risk increases as we move from smaller to larger interbank networks (illustrated in Figure 4).

Our results also suggest that connectivity, expressed in our experiments as the outdegree of the first bank that defaults, has a negative effect on interbank contagion. The fact that we have allowed connectivity in the network to vary, has provided additional resilience to it. Interestingly, the magnitude of connectivity decreases as the size of the network increases. In relatively small interbank networks, a high level of connectivity will allow an efficient absorption of shocks, whereas in larger networks the increased connectivity will spread the shock throughout the system, potentially leading to market-wide collapses. Our regression analysis also shows that the COUNT variable which measures the number of rounds until no further bank defaults, has a positive impact on interbank contagion and this relationship becomes more statistically significant as the size of the network increases.

Heterogeneity expressed as the variance of capital exhibits a negative and statistically significant relationship with interbank contagion, showing that size heterogeneity can have positive effects on the stability of an interbank network. An interbank network consisting of banks of various sizes can more easily withstand a negative shock, therefore no institution becomes significant
for either borrowing or lending. Furthermore, in such network both smaller and larger banks can act as shock absorbers when an initial shock hits the banking system, making contagion a less likely phenomenon. This finding is in line with the results of Iori et al. (2006) concerning bank size heterogeneity.

Table 2 presents the regression results of the second scenario using the model described in equation (6), where banking institutions with heterogeneous bank sizes are linked to one another via heterogeneous interbank claims. The regressor coefficients are statistically significant in almost all cases and the $R^2$-squared values are quite high and lie in the vicinity of 78 to 86 percent, highlighting the good explanatory power of the model. CATIN1 impacts in a statistically significant manner the dependent variable in all network segments and the magnitude of standardized coefficients exceeds the corresponding magnitude of those derived from the first scenario. In other words, an initial shock from defaulting bank $i$ will dissipate more easily and will not spillover in the network as intensively as in the first scenario. Again, CATIN2 has a large positive impact on contagion risk, however, its magnitude fades away as we move from smaller to larger networks - in the last case of $N=100$ banks it becomes statistically insignificant. It should also be highlighted that the CATIN2 coefficients are smaller than those recorded in the first scenario when it comes to small and medium-sized networks, while the reverse holds for larger interbank markets. An initial shock following the default of bank $i$ does not seem to contribute much to a banking crisis scenario within small and medium-sized networks and the size of total capital losses is smaller than that documented in the first scenario. Figure 5 depicts the extent of contagion as a function of the percentage loss of capital due to default of the first bank and confirms the results recorded in Table 2.

The results also show that there still exists a positive relationship between leverage and contagion, however, the coefficient estimates are much smaller than those recorded in the previous scenario. Moreover, the magnitude of the leverage coefficients increases as the number of banks in the interbank network increases, with the only exception being the 50 bank network segment which follows an autonomous path and is inversely related to contagion (although statistically insignificant). Results on connectivity are qualitatively similar to those of the first scenario, showing that connectivity negatively impacts contagion risk especially in larger interbank networks. The number of rounds until no further bank defaults positively impacts contagion risk and contributes the most to total capital losses in the banking system when large interbank networks are formed.
Under this scenario, the heterogeneity allowed on both bank sizes and interbank exposures has had a great impact on the resilience of the network system. Heterogeneity impacts negatively on interbank contagion although its intensity decreases as the size of the network increases. Moreover, we provide evidence that heterogeneity of bank size contributes more to the resilience of the interbank network than heterogeneity of interbank exposures. The heterogeneity of interbank exposures acts as a diversification tool and contributes to a smaller extent to an unfolding crisis compared to the scenario where homogeneous banks are interconnected via heterogeneous exposures (shown in Table 3).

Table 3 depicts the results of the third scenario as described in equation (7). In this scenario, we construct network systems where banks have the same equity size and unevenly distribute their exposures across creditor banks. We note that an initial shock fades away with the failure of the first bank and does not spillover to other banks within the network. This is mainly due to our choice of parameters regarding the equity of each bank, the links among banks and the interbank claims among creditor banks. In order to observe default cascades we relax our initial assumptions and lower the equity of each bank in the network system. Specifically, each bank is now endowed with a balance sheet that consists of 25 units of equity and interbank claims among creditor banks are distributed in the following ranges: \( a_{ij} \in [0, 10], a_{ij} \in [0, 20], a_{ij} \in [0, 35] \).

Similar to the previous scenarios, the regressor coefficients are statistically significant in most cases and the \( R \)-squared values are still large, in fact the largest of all three scenarios tested. Variable CATIN2 has a highly significant positive impact on systemic risk that fades away as the network system gets larger. The same observation holds for the level of connectivity in the banking system i.e. a strong negative impact that dissipates as \( N \) increases. The leverage of the system has a positive impact on systemic risk and its magnitude increases as the size of the network increases. The standardized coefficients are much larger than those reported in the second scenario, implying that highly leveraged banks are less capable of absorbing negative shocks, something that can amplify the initial impact of a shock that is transmitted to neighbor banks via interlinkages. Figures 7 and 8 illustrate the third scenario as a function of the percentage loss of capital due to default of the first bank in the network and as a function of leverage in the banking system, respectively.

As in the previous cases, we find the number of rounds until no further
bank defaults to affect contagion risk positively and statistically significantly, and such impact is magnified in relatively larger interbank networks. We also note that the standardized coefficients are more statistically significant than those reported in the first and second scenario. The heterogeneity of interbank exposures plays a significant role in the stability of the financial network and its impact declines with the number of banks included in the network, and such impact is stronger than that found in heterogeneous bank network settings.

Finally, Table 4 depicts the results of the fourth scenario as described in equation (8). In this scenario, we construct network systems where banks have the same equity size and interbank claims are evenly distributed across creditor banks. We acknowledge the fact that this scenario is a bit unrealistic as banks in real-world interbank networks do not have the same equity size and do not necessarily distribute their interbank claims evenly across their creditors. However, by varying the topology of the interbank market and the degree of heterogeneity of the system we are in a position to effectively examine the effect of different calibrations on systemic risk. Thus, although this scenario can be regarded as a special case with magnifying effects, it provides useful insights on interbank market resiliency during periods of stress.

The variable CATIN2 has a strong positive impact on systemic risk that dissipates as the network system gets larger. Simulations show that this scenario yields qualitatively similar results with the previous three scenarios in relation to the leverage of the network, that is, leverage positively and significantly affects contagion risk and such effect becomes stronger progressively when the number of constituent banks in the network increases. Figure 9 illustrates that the more leveraged a banking system is, the less resilient it becomes once a random shock hits. For instance, for the less leveraged network systems (0.5 percent - 1.5 percent) and as the number of banks increases the total loss of capital due to contagion as percentage of total capital in the system drops to nearly 0 percent.

Figure 10 illustrates that the extent of contagion as a function of the percentage loss of capital in the network is magnified in this last scenario as capital losses exceed those documented in the previous scenarios. Connectivity impacts negatively on interbank contagion and follows a similar pattern to that of previous scenarios and dissipates as the number of banks in the network increases, although at a much slower rate than in previous cases. Finally, the number of rounds until no further bank defaults affects contagion risk in a statistically significant manner especially when large interbank
networks are considered. The main intuition behind these results is that higher interconnectedness of a homogeneous interbank network can reduce the probability of contagion in case the first bank defaulting is less leveraged, as the shock will be absorbed by many counterparties and will dissipate at a faster rate. However, if the first bank defaulting is highly leveraged, the shock absorption capacity of the network will decrease and default cascades will prevail.

5. Conclusions and implications for future work

This paper investigates how complexity of an interbank network structure affects interbank contagion. In particular, we explore the interplay between heterogeneity, network structure and balance sheet composition in the spreading of contagion using four basic scenarios. Our findings clearly indicate that heterogeneity plays a significant role in the stability of the financial system. In our numerical simulations, we observe that when heterogeneity is introduced with respect to the size of each bank, the system’s shock absorption capacity is enhanced. An interbank network consisting of banks of different sizes can more easily withstand a random shock, making contagion a less likely phenomenon. Furthermore, when we allow for the presence of heterogeneous interbank exposures in our model, we observe additional resilience to the interbank network as an initial shock dissipates more easily than in the case of homogeneous interbank claims.

We also find that the likelihood of contagion declines as we move from smaller to larger network settings. As far as connectivity is concerned, our analysis reveals that interconnectedness has a large impact on the resilience of the interbank network. Financial shocks will be absorbed more efficiently in relatively small and highly interconnected interbank networks, whereas in larger systems increased connectivity will spread the shock into a large part of the system causing a cascade of defaults. Highly leveraged banks are more exposed to default risk and thus contribute more to systemic risk, especially to that of large interbank networks.

Avenues for future research can include the study of non-performing loans (NPLs) in relation to contagion risk in a unified framework. A second objective within this setting would be to test how asset devaluations and haircuts depicted on bank balance sheets can affect interbank contagion. Under such setting, various weaknesses of network systems can be identified and additionally, the role systemic banks play in causing market-wide effects can be
further explored. This becomes extremely relevant to the case of the European sovereign debt crisis, whose aftermath is still fresh in the financial system.

6. References

Table 1: OLS regression analysis for Scenario 1 (Heterogeneous banks with homogeneous exposures). The table presents the regression results for Scenario 1. The dependent variable is CATEND measured as the total loss of capital due to contagion as percentage of total capital in the network. Explanatory variables are CATIN1, CATIN2, LEVIN, NOUTGOING, COUNT and VARCAP. Each cell displays the OLS standardized coefficients along with the corresponding $t$-statistics (shown in parentheses). The sample comprises of 18,000 realizations (simulated banking crises). ** and *** denote significance at the 5 and 1 percent level, respectively.

<table>
<thead>
<tr>
<th></th>
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<th>$N=50$</th>
<th>$N=80$</th>
<th>$N=100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CATIN1</td>
<td>0.080 (23.029)***</td>
<td>0.030 (7.671)***</td>
<td>0.024 (6.245)***</td>
<td>0.017 (4.591)***</td>
</tr>
<tr>
<td>CATIN2</td>
<td>0.208 (24.266)***</td>
<td>0.088 (9.191)***</td>
<td>0.020 (2.109)**</td>
<td>0.023 (2.568)**</td>
</tr>
<tr>
<td>LEVIN</td>
<td>0.078 (13.376)***</td>
<td>0.131 (20.473)***</td>
<td>0.127 (19.898)***</td>
<td>0.160 (25.829)***</td>
</tr>
<tr>
<td>NOUTGOING</td>
<td>-0.147 (-25.061)***</td>
<td>-0.085 (-12.220)***</td>
<td>-0.023 (-3.212)***</td>
<td>-0.013 (-1.963)**</td>
</tr>
<tr>
<td>COUNT</td>
<td>0.721 (152.451)***</td>
<td>0.734 (147.324)***</td>
<td>0.762 (155.048)***</td>
<td>0.749 (158.296)***</td>
</tr>
<tr>
<td>VARCAP</td>
<td>-0.103 (-45.749)***</td>
<td>-0.063 (-39.920)***</td>
<td>-0.048 (-38.314)***</td>
<td>-0.042 (-39.818)***</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.760</td>
<td>0.717</td>
<td>0.716</td>
<td>0.745</td>
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</table>
Table 2: OLS regression analysis for Scenario 2 (Heterogeneous banks with heterogeneous exposures). The table presents the regression results for Scenario 2. The dependent variable is CATEND measured as the total loss of capital due to contagion as percentage of total capital in the network. Explanatory variables are CATIN1, CATIN2, LEVIN, NOUTGOING, COUNT, VARCAP and VARLOANS. Each cell displays the OLS standardized coefficients along with the corresponding \( t \)-statistics (shown in parentheses). The sample comprises of 18,000 realizations (simulated banking crises). *** denotes significance at the 1 percent level.

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<tr>
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<th>( N=80 )</th>
<th>( N=100 )</th>
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<tr>
<td>CATIN1</td>
<td>0.670 (223.788)***</td>
<td>0.727 (271.376)***</td>
<td>0.640 (187.087)***</td>
<td>0.592 (177.760)***</td>
</tr>
<tr>
<td>CATIN2</td>
<td>0.121 (20.243)***</td>
<td>0.085 (15.635)***</td>
<td>0.064 (9.387)***</td>
<td>0.003 (0.524)</td>
</tr>
<tr>
<td>LEVIN</td>
<td>0.014 (2.616)***</td>
<td>-0.004 (-0.878)</td>
<td>0.068 (9.952)***</td>
<td>0.027 (4.186)***</td>
</tr>
<tr>
<td>NOUTGOING</td>
<td>-0.119 (-25.986)***</td>
<td>-0.081 (-18.756)***</td>
<td>-0.089 (-15.751)***</td>
<td>-0.060 (-11.101)***</td>
</tr>
<tr>
<td>COUNT</td>
<td>0.530 (145.530)***</td>
<td>0.541 (171.862)***</td>
<td>0.579 (147.590)***</td>
<td>0.674 (178.780)***</td>
</tr>
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<td>VARCAP</td>
<td>-0.101 (-53.995)***</td>
<td>-0.068 (-61.788)***</td>
<td>-0.061 (-50.322)***</td>
<td>-0.050 (-54.927)***</td>
</tr>
<tr>
<td>VARLOANS</td>
<td>-0.057 (-10.622)***</td>
<td>-0.023 (-4.303)***</td>
<td>-0.080 (-11.667)***</td>
<td>-0.034 (-5.396)***</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.823</td>
<td>0.865</td>
<td>0.782</td>
<td>0.796</td>
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</table>
Table 3: OLS regression analysis for Scenario 3 (Homogeneous banks with heterogeneous exposures). The table presents the regression results for Scenario 3. The dependent variable is CATEND measured as the total loss of capital due to contagion as percentage of total capital in the network. Explanatory variables are CATIN2, LEVIN, NOUTGOING, COUNT and VARLOANS. Each cell displays the OLS standardized coefficients along with the corresponding $t$-statistics (shown in parentheses). The sample comprises of 18,000 realizations (simulated banking crises). ** and *** denote significance at the 5 and 1 percent level, respectively.

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<tbody>
<tr>
<td>CATIN2</td>
<td>0.238</td>
<td>0.166</td>
<td>0.124</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>(51.837)***</td>
<td>(31.641)***</td>
<td>(21.978)***</td>
<td>(15.126)***</td>
</tr>
<tr>
<td>LEVIN</td>
<td>0.053</td>
<td>0.080</td>
<td>0.084</td>
<td>0.089</td>
</tr>
<tr>
<td></td>
<td>(11.723)***</td>
<td>(15.699)***</td>
<td>(15.145)***</td>
<td>(15.489)***</td>
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<tr>
<td>NOUTGOING</td>
<td>-0.186</td>
<td>-0.162</td>
<td>-0.147</td>
<td>-0.133</td>
</tr>
<tr>
<td></td>
<td>(-61.481)***</td>
<td>(-44.323)***</td>
<td>(-37.350)***</td>
<td>(-33.313)***</td>
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<tr>
<td>COUNT</td>
<td>0.875</td>
<td>0.901</td>
<td>0.911</td>
<td>0.927</td>
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<tr>
<td></td>
<td>(258.811)***</td>
<td>(247.480)***</td>
<td>(232.873)***</td>
<td>(241.892)***</td>
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<tr>
<td>VARLOANS</td>
<td>-0.199</td>
<td>-0.229</td>
<td>-0.235</td>
<td>-0.232</td>
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<tr>
<td></td>
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<td>(-41.527)***</td>
<td>(-39.356)***</td>
<td>(-37.024)***</td>
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<tr>
<td>R-squared</td>
<td>0.887</td>
<td>0.856</td>
<td>0.834</td>
<td>0.835</td>
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Table 4: OLS regression analysis for Scenario 4 (Homogeneous banks with homogeneous exposures). The table presents the regression results for Scenario 4. The dependent variable is CATEND measured as the total loss of capital due to contagion as percentage of total capital in the network. Explanatory variables are CATIN2, LEVIN, NOUTGOING and COUNT. Each cell displays the OLS standardized coefficients along with the corresponding $t$-statistics (shown in parentheses). The sample comprises of 18,000 realizations (simulated banking crises). *** denotes significance at the 1 percent level.

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<tr>
<td>CATIN2</td>
<td>0.513</td>
<td>0.484</td>
<td>0.471</td>
<td>0.465</td>
</tr>
<tr>
<td></td>
<td>(136.409)***</td>
<td>(122.675)***</td>
<td>(122.258)***</td>
<td>(121.843)***</td>
</tr>
<tr>
<td>LEVIN</td>
<td>0.006</td>
<td>0.021</td>
<td>0.027</td>
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<td></td>
<td>(2.250)***</td>
<td>(8.188)***</td>
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<td>(10.425)***</td>
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<tr>
<td>NOUTGOING</td>
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<td>-0.326</td>
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<td></td>
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<td>(-129.930)***</td>
<td>(-128.265)***</td>
</tr>
<tr>
<td>COUNT</td>
<td>0.652</td>
<td>0.660</td>
<td>0.668</td>
<td>0.676</td>
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<tr>
<td></td>
<td>(255.154)***</td>
<td>(246.926)***</td>
<td>(257.520)***</td>
<td>(262.331)***</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.933</td>
<td>0.929</td>
<td>0.932</td>
<td>0.934</td>
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Figure 1: Stylized Balance Sheet Structure

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<th>Assets ($A_i$)</th>
<th>Liabilities ($L_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbank assets ($A_{iIB}$)</td>
<td>Interbank liabilities ($L_{iIB}$)</td>
</tr>
<tr>
<td>Other assets ($A_{iOT}$)</td>
<td>Other liabilities ($L_{iOT}$)</td>
</tr>
<tr>
<td>Cash ($C_i$)</td>
<td>Equity capital ($E_i$)</td>
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Figure 2: The graph of an interbank market network, consisting of $N=20$ banks.
Figure 3: Scenario 1- Heterogeneous Banks with homogeneous exposures | Extent of contagion (expressed as the total capital lost from the banking system due to the failure of at least one bank) as a function of the percentage loss of capital at the first stage due to default of the first bank. Panels (a)-(d) show the relation between the percentage loss of capital and the extent of contagion across interbank networks with different number of banks.
Figure 4: Scenario 1- Heterogeneous Banks with homogeneous exposures | Extent of contagion (expressed as the total capital lost from the banking system due to the failure of at least one bank) as a function of the leverage of the system. Panels (a)-(d) show the relation between the leverage of the system and the extent of contagion across interbank networks with different number of banks.
Figure 5: Scenario 2 - Heterogeneous Banks with heterogeneous exposures | Extent of contagion (expressed as the total capital lost from the banking system due to the failure of at least one bank) as a function of the percentage loss of capital at the first stage due to default of the first bank. Panels (a)-(d) show the relation between the percentage loss of capital and the extent of contagion across interbank networks with different number of banks.
Figure 6: Scenario 2 - Heterogeneous Banks with heterogeneous exposures | Extent of contagion (expressed as the total capital lost from the banking system due to the failure of at least one bank) as a function of the leverage of the system. Panels (a)-(d) show the relation between the leverage of the system and the extent of contagion across interbank networks with different number of banks.
Figure 7: Scenario 3 - Homogeneous banks with heterogeneous exposures (expressed as the total capital lost from the banking system due to the failure of at least one bank) as a function of the percentage loss of capital at the first stage due to default of the first bank. Panels (a)-(d) show the relation between the percentage loss of capital and the extent of contagion across interbank networks with different number of banks.
Figure 8: Scenario 3 - Homogeneous banks with heterogeneous exposures | Extent of contagion (expressed as the total capital lost from the banking system due to the failure of at least one bank) as a function of the leverage of the system. Panels (a)-(d) show the relation between the leverage of the system and the extent of contagion across interbank networks with different number of banks.
Figure 9: Scenario 4 - Homogeneous banks with homogeneous exposures | Extent of contagion (expressed as the total capital lost from the banking system due to the failure of at least one bank) as a function of the percentage loss of capital at the first stage due to default of the first bank. Panels (a)-(d) show the relation between the percentage loss of capital and the extent of contagion across interbank networks with different number of banks.
Figure 10: Scenario 4 - Homogeneous banks with homogeneous exposures | Extent of contagion (expressed as the total capital lost from the banking system due to the failure of at least one bank) as a function of the leverage of the system. Panels (a)-(d) show the relation between the leverage of the system and the extent of contagion across interbank networks with different number of banks.
Appendix

**Simulation algorithm: Set up of the interbank network**

- Define the number of banks (n) in the interbank network system
- Define the complexity (number of outgoing links of each bank) of the network system
- Assign directed edges to represent lending-borrowing interbank relationships (link formation follows a uniform distribution)
- Allocate balance sheet components among banks (equity and interbank loans)
- Generate the interbank matrix of bilateral exposures (consistent with each scenario tested)
- Generate the banks' equity vector (consistent with each scenario tested)

Figure 1A: Simulation algorithm: Set up of the interbank network
Simulation algorithm: Contagion procedure

for each of the $T$ realizations

- Set up the interbank network;
- Estimate the leverage of the interbank network; ($\text{levin}$)
- Estimate the variance of capital of the interbank network (used in those scenarios tested where only heterogeneous bank sizes are considered); ($\text{varcap}$)
- Estimate the variance of interbank loans (used in those scenarios tested where only heterogeneous interbank loan exposures are considered); ($\text{varloans}$)
- Shock the system with the exogenous default of bank $i$;
- Estimate the initial loss of capital by defaulting bank $i$ as percentage of total capital of the system; ($\text{cadin1}$)
- Estimate the loss of capital at the first stage (interbank loans that cannot be paid back) by defaulting bank $i$ as percentage of total capital of the system; ($\text{cadin2}$)
- Estimate the number of outgoing links (outdegree) of bank $i$; ($\text{noutgoing}$)

while at least one bank defaulted do

  for every bank $i$ do

    if counterparty losses occurred then

      update equity of defaulting bank's creditors (subtract losses from creditors' equity);

    end

    if equity $\leq$ then

      default bank $i$;

    end

  end

  - Estimate the total loss of capital due to contagion as percentage of total capital of the system; ($\text{cend}$)
  - Estimate the shock propagation variable which measures the number of rounds needed until no further bank defaults; ($\text{count}$)
  - Record $\text{levin}$, $\text{varcap}$, $\text{varloans}$, $\text{cadin1}$, $\text{cadin2}$, $\text{noutgoing}$, $\text{count}$, $\text{cend}$;

end

*After performing a satisfactory number of realizations for each scenario tested, regression analysis is employed in order to test the effect of the aforementioned variables on contagion risk.*

Figure 2A: Simulation algorithm: Contagion Procedure