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Modeling Euro STOXX 50 Volatility with Common and Market-specific Components

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Abstract

Similar volatility patterns are observables in the Euro area across national indices, suggesting the possibility of an underlying common component as a consequence of financial and monetary integration. This peculiar interdependence across market volatilities is captured by an additive component vector Multiplicative Error Model (vMEM) where the volatility dynamics is split between a common and a vector of market-specific components. When extracted from five major market indices and used as additional regressors in a HAR specification for the Euro STOXX 50 (a Euro area wide index) volatility, these components replace the terms that mimic long memory in the HAR, providing an interesting interpretation for volatility dynamics.

Keywords Realized Volatility; (vector) Multiplicative Error Models; GMM; HAR; Common Component; Euro area

1 Introduction

The existence of a common currency has fostered the integration of financial markets across member countries in the Euro area: the increase in the range of activities within the European Union (Single European Act) and the adoption of coordinated policies within the Economic and Monetary Union have an anticipated and intended spillover to the capital markets. As a consequence, reaction to news tends to generate common responses in national markets, and thus it becomes natural to investigate whether volatilities in individual markets follow similar dynamics. The recent Report on Financial Integration by the European Central Bank (ECB, 2018) shows how the converging degree of similarity across countries had a set back from the financial crises of 2008 and then 2011-12, but it is now back to its maximum level since 2006. Within the institutional framework of the single currency, therefore, it is relevant to pose the questions on how this commonality in volatility may be explicitly captured, and whether, in turn, a common volatility extracted from individual volatilities is representative of the dynamics of the volatility of the Euro STOXX 50, a synthetic index built on the most capitalized Euro area companies, and, as such, a good representative of Euro area volatility.

For their empirical investigations, these questions build on two elements: first, it is now well established that the development of volatility measurement with ultra-high frequency data allows for direct modeling of the volatility dynamics. Among the many contributions in this field (for a survey, cf. Andersen and Benzoni, 2009), one can refer to the realized kernel series (Barndorff-Nielsen *et al.*, 2008) for their robustness properties to market microstructure noise, jumps, etc. Such series are persistent, at times interpreted as exhibiting long memory features (cf. the good discussion in Andersen *et al.*, 2007, Section IV); a popular model which captures this behavior in a univariate context is the Heterogeneous Autoregressive Model by Corsi (2009). We will refer to this model as a benchmark representation of the dynamics of the Euro STOXX 50 volatility.

In order to extract the common component, the second element is about which model to rely on, given a number of individual volatilities jointly considered. In what follows, we suggest a novel model within the family of vector Multiplicative Error Models (vMEM – for a discussion of its general properties, cf. Cipollini *et al.*, 2013, 2017), which have proved successful in representing volatility dynamic interdependencies across national markets (Engle *et al.*, 2012). The dynamic structure of a MEM (Engle, 2002) mirrors the GARCH specification of the conditional

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variance of returns: the conditional expectation of the volatility measure is modeled as a linear combination of the lagged value(s) of the observed variable and of past conditional expectation(s); the variable itself is modeled as the product of such a conditional expectation and an i.i.d. innovation process with unit mean. Its multivariate extension consists of jointly modeling two or more non-negative processes as the element by element product of a vector of conditionally autoregressive scale factors and a multivariate i.i.d. innovation process with a unit mean vector and a full covariance matrix.

The existence of a common latent driver should be explicitly modeled, suggesting, as noted by Hansen *et al.* (2011), the need for some restriction to be imposed on the original structure of the vMEM. In this direction, the SPvMEM (Barigozzi *et al.*, 2014) is a vMEM restricted to have a single low frequency common component modeled nonparametrically multiplied by univariate MEMs for individual volatilities (more details on this model for a comparison of results are given in Section 6). As an alternative modeling strategy, we explore here a new model, labeled *Additive Common Component vMEM* or ACCvMEM, still motivated by the need to restrict the general vMEM, but this time starting with a decomposition of individual contributions into the sum of a single common component plus a vector of market-specific components.

The model, whose reduced form allows us to show that it is indeed a more complex vMEM with constrained parameters, has connections with the Composite-MEM considered in Brownlees *et al.* (2012), whose characteristics have proved to adequately capture the dynamics of realized volatilities of single assets. The seminal idea, however, must be traced back to the univariate GARCH model proposed in Engle and Lee (1999), in which the dynamics of the conditional variance is decomposed into two additive components, one labeled as permanent (identified as such in view of its higher persistence), and the other one as transitory.¹ In this context, we abandon the characterization of permanent versus transitory, favoring the reference to common versus market-specific.

In this, as in other models and applications (e.g. Sentana *et al.*, 2008; Barigozzi *et al.*, 2014), the choice of a single factor driving volatilities is made *a priori*: when it is the object of specific statistical testing, one factor seems to be predominantly more important (cf. Luciani and Veredas, 2015). In general, the issue of how many factors drive volatility within a panel was addressed in several modeling approaches: when returns are involved, Engle and Marucci (2006), within a GARCH framework, and Barigozzi and Hallin (2016, 2017), within a generalized dynamic factor model approach, show that there is little to gain beyond one factor in modeling conditional volatilities.

Given its semiparametric nature, model inferences can be obtained via GMM (Generalized Method of Moments), based on the conditional mean and variance expressions ensuing from the model definition, extending Cipollini *et al.* (2013).

The specific empirical motivation of the application is to use the ACCvMEM to extract the common and market-specific components in the most important financial markets (Germany, France, Italy, Spain and the Netherlands) which jointly amount to more than 90% of market capitalization in the Euro area. Estimating our model shows good fitting capability and good residual diagnostics, even across subsamples. Then, we want to investigate how these components contribute to the dynamics of the Euro STOXX 50 volatility given that such an index has 47 out of 50 stocks being traded in (at least) one of these markets. The extracted common and market-specific components are indeed used as regressors in an augmented HAR model for the area-wide Euro STOXX 50 volatility: the interesting result is that the HAR terms are no longer significant, i.e. the ACCvMEM estimated components act as substitute and interpretable drivers in capturing the long memory features in the data. These results can essentially be traced to the common component, since a regression where it is excluded brings back the significance of the HAR terms. This can be justified in reference to the results shown by LeBaron (2001) and further represented by Christoffersen *et al.* (2008) of long memory features emerging from the sum of two autoregressive processes as in our decomposition.

The paper is structured as follows. In Section 2 we present the basic vMEM to establish notation and the backdrop against which we propose the restricted dynamics. In Section 3 we introduce the Additive Common Component vMEM, discussing its properties and its relationship with the general vMEM and other models. In Section 4 we detail how to make inferences on the parameters of the model via GMM. Section 5 contains the empirical calculation of the common and market-specific components with diagnostics and sensitivity analysis. Section 6 is devoted to the investigation of the contribution of these components to modeling Euro STOXX 50 volatility dynamics. Section 7 concludes.

¹To be clear, the label *Additive Component* (rather than *Composite*) is preferable for this model, leaving the label *Multiplicative Component* to a distinct modeling approach as, for example, in Brownlees *et al.* (2012) where expected intra-daily volume is dependent on the product of an intra-daily periodic, an intra-daily non-periodic and a daily component (cf. also Engle and Sokalska (2012) in a GARCH framework).

2 The Vector MEM

Let $\{\mathbf{x}_t\}$ a discrete time process with components defined on $[0, +\infty)^K$. In the vMEM (vector Multiplicative Error Model) (Cipollini *et al.* (2013), Cipollini *et al.* (2017)), \mathbf{x}_t is structured as²

$$\mathbf{x}_t = \boldsymbol{\mu}_t \odot \boldsymbol{\varepsilon}_t \quad (1)$$

where, conditionally on the information \mathcal{F}_{t-1} , $\boldsymbol{\mu}_t$ is deterministic,

$$\boldsymbol{\mu}_t = \mu(\boldsymbol{\theta}, \mathcal{F}_{t-1}) \quad (2)$$

and $\boldsymbol{\varepsilon}_t$ is stochastic, with pdf (probability density function) defined over a $[0, +\infty)^K$ support and such that

$$\boldsymbol{\varepsilon}_t | \mathcal{F}_{t-1} \sim D^+(\mathbb{1}, \boldsymbol{\Sigma}).$$

Note that the previous assumptions on $\boldsymbol{\mu}_t$ and $\boldsymbol{\varepsilon}_t$ give

$$E(\mathbf{x}_t | \mathcal{F}_{t-1}) = \boldsymbol{\mu}_t \quad (3)$$

$$V(\mathbf{x}_t | \mathcal{F}_{t-1}) = \boldsymbol{\mu}_t \boldsymbol{\mu}'_t \odot \boldsymbol{\Sigma} = \text{diag}(\boldsymbol{\mu}_t) \boldsymbol{\Sigma} \text{diag}(\boldsymbol{\mu}_t), \quad (4)$$

where the latter is a positive definite matrix by construction.

More in detail, $\boldsymbol{\mu}_t$ can be specified as

$$\boldsymbol{\mu}_t = \boldsymbol{\omega} + \boldsymbol{\beta}_1 \boldsymbol{\mu}_{t-1} + \boldsymbol{\alpha}_1 \mathbf{x}_{t-1} + \boldsymbol{\gamma}_1 \mathbf{x}_{t-1}^{(-)}, \quad (5)$$

where $\boldsymbol{\omega}$ is $(K, 1)$, $\boldsymbol{\alpha}_1$, $\boldsymbol{\gamma}_1$ and $\boldsymbol{\beta}_1$ are (K, K) (further lags can be added, but we do not consider them here). The term $\boldsymbol{\beta}_1 \boldsymbol{\mu}_{t-1}$ represents an *inertial* component, whereas $\boldsymbol{\alpha}_1 \mathbf{x}_{t-1} + \boldsymbol{\gamma}_1 \mathbf{x}_{t-1}^{(-)}$ stands for the contribution of the more recent observation. In particular, the vector $\mathbf{x}_t^{(-)}$ aims at capturing asymmetric effects associated with the sign of an observed variable and is usually structured as $x_{t,j}^{(-)} = x_{t,j} I_{t,j}^{(-)}$, where $I_{t,j}^{(-)}$ denotes the indicator of a negative value of the signed variable. For instance, when different volatility indicators of the same asset are considered, then $I_{t,j}^{(-)} = \mathbb{I}(r_t < 0) \forall j$, where r_t indicates the asset return. As a further example, in a market volatility spillover framework each market has its own indicator, so that $I_{t,j}^{(-)} = \mathbb{I}(r_{t,j} < 0)$, where $r_{t,j}$ is the return of the j -th market. In what follows, we assume that all components of $I_{t,j}^{(-)}$ have conditional median zero and are conditionally uncorrelated with \mathbf{x}_t .

A useful reinterpretation of the model is had when \mathbf{x}_t is assumed mean-stationary; in fact, in such a case

$$\boldsymbol{\omega} = \left[\mathbf{I}_K - \left(\boldsymbol{\beta}_1 + \boldsymbol{\alpha}_1 + \frac{\boldsymbol{\gamma}_1}{2} \right) \right] \boldsymbol{\mu},$$

where $\boldsymbol{\mu} = E(\boldsymbol{\mu}_t) = E(\mathbf{x}_t)$. This allows us to rewrite (5) as

$$\boldsymbol{\mu}_t = \boldsymbol{\mu} + \boldsymbol{\xi}_t \quad (6)$$

$$\boldsymbol{\xi}_t = \boldsymbol{\beta}_1 \boldsymbol{\xi}_{t-1} + \boldsymbol{\alpha}_1 \mathbf{x}_{t-1}^{(\xi)} + \boldsymbol{\gamma}_1 \mathbf{x}_{t-1}^{(\xi-)} \quad (7)$$

$$\mathbf{x}_t^{(\xi)} = \mathbf{x}_t - \boldsymbol{\mu} \quad \mathbf{x}_t^{(\xi-)} = \mathbf{x}_t^{(-)} - \boldsymbol{\mu}/2. \quad (8)$$

From a practical point of view, the representation (6)-(7)-(8) constitutes a trivial reparameterization of (5), with $\boldsymbol{\mu}$ replacing $\boldsymbol{\omega}$. However, it has the merit of representing the dynamics of the process being driven by a zero mean, stationary component, $\boldsymbol{\xi}_t$, that moves around the *unconditional* average level $\boldsymbol{\mu}$. Depending on the context, further meaningful components, similar to $\boldsymbol{\xi}_t$, can be added and/or a time-varying rather than a fixed level $\boldsymbol{\mu}$, can be taken into account. An example is provided in Section 3.

A further useful representation of (5) is based on the innovations

$$\mathbf{v}_t = \mathbf{x}_t - \boldsymbol{\mu}_t \quad \mathbf{v}_t^{(-)} = \mathbf{x}_t^{(-)} - \boldsymbol{\mu}_t/2, \quad (9)$$

²We adopt the following conventions: if \mathbf{x} is a vector or a matrix and a is a scalar, then the expressions $\mathbf{x} \geq \mathbf{0}$ and \mathbf{x}^a are meant element by element; if $\mathbf{x}_1, \dots, \mathbf{x}_K$ are (m, n) matrices then $(\mathbf{x}_1; \dots; \mathbf{x}_K)$ means the (mK, n) matrix obtained stacking the matrices \mathbf{x}_i 's columnwise.

that allow to rewrite (7) as

$$\xi_t = \beta_1^* \xi_{t-1} + \alpha_1 v_{t-1} + \gamma_1 v_{t-1}^{(-)} \quad (10)$$

where $\beta_1^* = \beta_1 + \alpha_1 + \gamma_1/2$. The essential difference between the specification given by (in order) (6)-(7)-(8) and, correspondingly, (6)-(10)-(9) lies in the different properties of the driving factors of μ_t , namely $x_t^{(\xi)}$, $x_t^{(\xi-)}$ and v_t , $v_t^{(-)}$: $E(x_t^{(\xi)}) = E(x_t^{(\xi-)}) = \mathbf{0}$ for the former as opposed to $E(v_t|\mathcal{F}_{t-1}) = E(v_t^{(-)}|\mathcal{F}_{t-1}) = \mathbf{0}$ for the latter.

3 An Additive Common Component vMEM

3.1 Motivation

Non-negative financial time series frequently show very similar patterns over the sample of observation, conveying the idea of a single underlying driving force. As our leading example, let us present some evidence of such a common component among realized kernel volatility series for five European indices covering the period Jan. 2, 2002 - Dec. 30, 2016 ($T = 3770$ observations).³

The time series plots⁴, shown in Figure 1, highlight the strong similar pattern in the long term evolution of the series, with an initial period of (relatively) high volatility, followed by a long period of low volatility (between 2004 and the first half of 2007), in turn followed by a progressive increase up to the burst at the end of 2008, followed by relatively minor episodes, most notably the one in the summer of 2011 surrounding the sovereign debt crisis.

In Table 1 the bivariate correlations among these indices are reported, showing how, across Euro area markets, volatilities are highly correlated. Moreover, all series are also highly serially correlated, with statistically significant Ljung–Box test statistics (p-values – not reported – well below 0.1% at all lags).

Table 1: Pearson correlations between volatilities in several Euro area markets.

	FR	ES	IT	NL
GE	0.94	0.76	0.83	0.93
FR		0.86	0.91	0.96
ES			0.90	0.78
IT				0.85

3.2 Model Specification

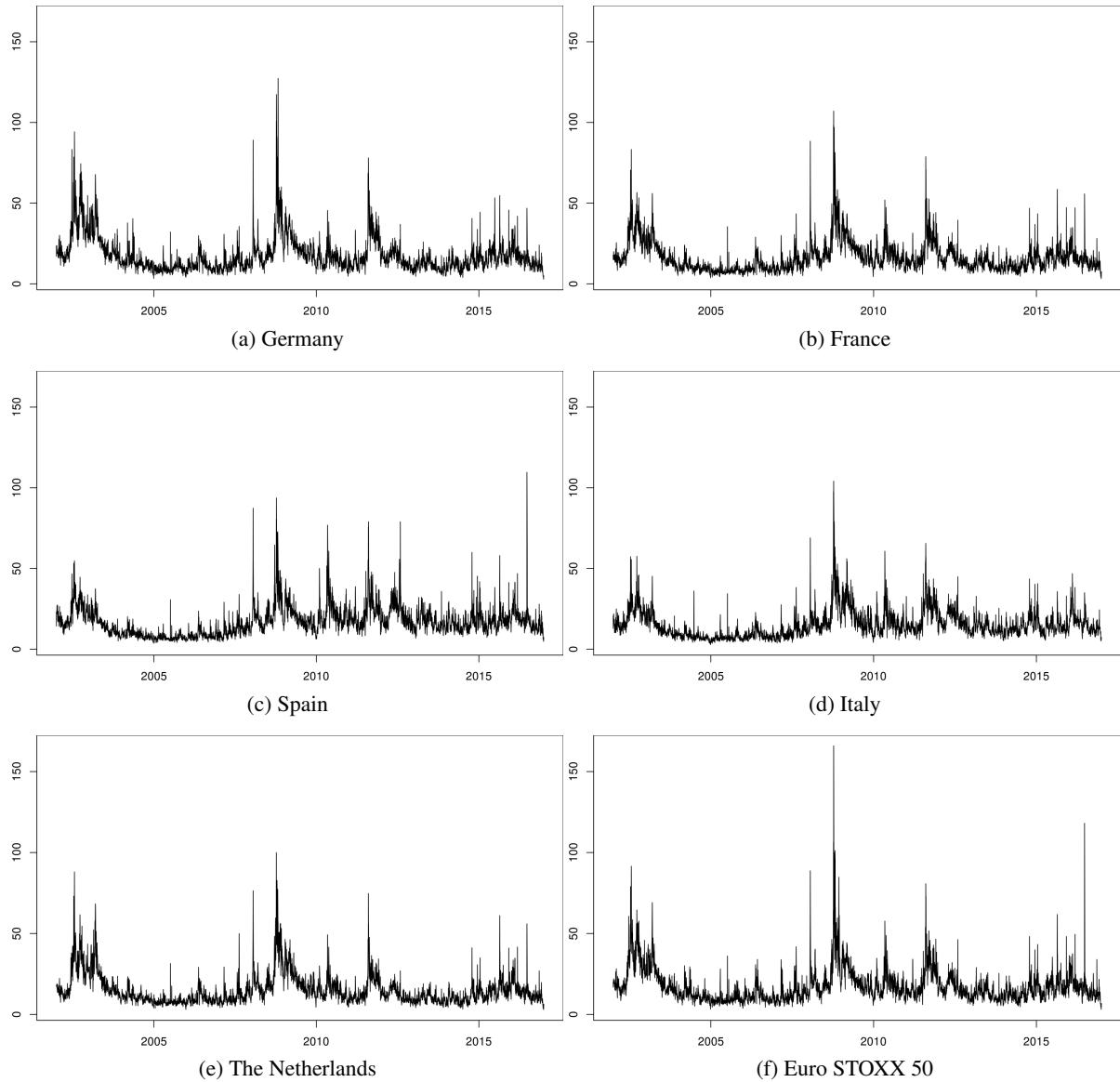
This behavior justifies exploring how to propose a new formulation in order to insert a common dynamic component. So, paralleling the logic in Engle and Lee (1999), we change the structure of μ_t of a vMEM specified like in (6)-(7)-(8) as follows:

- the fixed μ is replaced by a time-varying component driven by a scalar common factor, η_t ;
- ξ_t is (ideally) structured as a vector of specific elements, i.e. each $\xi_{t,j}$ depends on its own past values only.

³ Indices will be referred to as their country: DAX (Germany – GE), CAC40 (France – FR), IBEX (Spain – ES), FTSE MIB (Italy – IT), and AEX (The Netherlands – NL). Our series are the annualized percentage square roots of realized kernel variances (Barndorff-Nielsen *et al.*, 2008) available from the Oxford Man Institute (OMI) *Realized Library* (Heber *et al.*, 2009).

⁴We also portray the Euro STOXX 50 realized volatility, later used as representative of Euro area volatility. Further details are provided in the empirical application.

Figure 1: Annualized realized volatilities of different market indices: five national markets plus the Euro STOXX 50. Jan. 2, 2002 – Dec. 30, 2016.



The implicit assumption is that the common component is able to capture adequately the main part of the cross-dependence. More explicitly, μ_t is defined by

$$\begin{aligned}\mu_t &= \mu + \psi\eta_t + \xi_t \\ \xi_t &= \beta_1^{(\xi)}\xi_{t-1} + \alpha_1^{(\xi)}x_{t-1}^{(\xi)} + \gamma_1^{(\xi)}x_{t-1}^{(\xi-)} \\ \eta_t &= \beta_1^{(\eta)}\eta_{t-1} + \alpha_1^{(\eta)'}x_{t-1}^{(\eta)} + \gamma_1^{(\eta)'}x_{t-1}^{(\eta-)},\end{aligned}\tag{11}$$

where

$$x_t^{(\xi)} = x_t - (\mu + \psi\eta_t) \quad x_t^{(\xi-)} = x_t^{(-)} - (\mu + \psi\eta_t)/2 \tag{12}$$

$$x_t^{(\eta)} = x_t - (\mu + \xi_t) \quad x_t^{(\eta-)} = x_t^{(-)} - (\mu + \xi_t)/2 \tag{13}$$

$$\eta_0 = \xi_{0,j} = x_{0,j}^{(\eta)} = x_{0,j}^{(\eta-)} = x_{0,j}^{(\xi)} = x_{0,j}^{(\xi-)} = 0 \quad j = 1, \dots, K.$$

In the base formulation, $\beta_1^{(\xi)}$, $\alpha_1^{(\xi)}$ and $\gamma_1^{(\xi)}$ are diagonal matrices, but richer interdependency structures can be considered in some applications, in particular for what concerns $\alpha_1^{(\xi)}$.

As in Section 2, an equivalent formulation of the model can be obtained by resorting to the innovations (9), instead of (12)-(13), as driving forces of the dynamics of μ_t , namely

$$\xi_t = \beta_1^{(\xi)*}\xi_{t-1} + \alpha_1^{(\xi)}v_{t-1} + \gamma_1^{(\xi)}v_{t-1}^{(-)} \tag{14}$$

$$\eta_t = \beta_1^{(\eta)*}\eta_{t-1} + \alpha_1^{(\eta)'}v_{t-1} + \gamma_1^{(\eta)'}v_{t-1}^{(-)} \tag{15}$$

where, just for matching and interpretation purposes, the coefficients correspond to the previously defined coefficients in terms of observables

$$\begin{aligned}\beta_1^{(\xi)*} &= \beta_1^{(\xi)} + \alpha_1^{(\xi)} + \gamma_1^{(\xi)}/2 \\ \beta_1^{(\eta)*} &= \beta_1^{(\eta)} + (\alpha_1^{(\eta)} + \gamma_1^{(\eta)}/2)'\psi.\end{aligned}$$

In such a case, the contribution coming from the more recent observation, v_{t-1} , is the same in the two equations, so that the corresponding coefficients can be directly compared.

To give an idea of the number of parameters involved (a common concern related to the curse of dimensionality), the conditional mean of a vMEM with μ_t given by Equation (5) depends on $2K(K+1)$ parameters when both α_1 and β_1 are full and γ_1 is diagonal. The conditional mean of the ACCvMEM (Equations (9), (14) and (15)) depends on $7K$ parameters; in case a full $\alpha_1^{(\xi)}$, instead of a diagonal one, is preferable, the specification depends on $K(6+K)$ parameters.⁵

Moreover such a representation allows for a simpler derivation of the properties of the model and it is the one that we will use for estimation in the empirical application.

A further interesting interpretation of the common component formulation can be retrieved by merging the two addends μ and $\psi\eta_t$ into a unique *time varying level* $\chi_t = \mu + \psi\eta_t$. In such a case

$$\mu_t = \chi_t + \xi_t, \tag{16}$$

where χ_t evolves as

$$\chi_t = \omega^{(x)} + \beta_1^{(x)*}\chi_{t-1} + \alpha_1^{(x)}v_{t-1} + \gamma_1^{(x)}v_{t-1}^{(-)} \tag{17}$$

with $\chi_0 = \mu$. Expression (17) looks like the equation for ξ_t , but with a number of differences: the level $\omega^{(x)} = (1 - \beta_1^{(\eta)*})\mu$ is non-zero; the coefficient of the inertial component, $\beta_1^{(x)*} = \beta_1^{(\eta)*}$, is a scalar; the coefficients of the innovations and of the ‘asymmetric’ innovations, $\alpha_1^{(x)} = \psi\alpha_1^{(\eta)'} \text{ and } \gamma_1^{(x)} = \psi\gamma_1^{(\eta)'}$ respectively, are (K, K) matrices but with unit rank.

Any of the formulations presented in this Section move quite a distance from the original vMEM presented in (6)-(9)-(10), as the specifications of the common and market-specific components enrich the original model with

⁵When $K = 5$, the vMEM involves 60 parameters versus 35 in the ACCvMEM (with diagonal $\alpha_1^{(\xi)}$), while with $K = 20$, the comparison is between 840 in the former and 140 in the latter.

one lag. This notwithstanding, using a reduced form representation according to Lütkepohl (2005, Section 11.6), stacking the two dynamical components $(\eta_t; \xi_t)$ and performing some tedious algebra, one can show that the ACCvMEM is indeed a vMEM involving two lags with coefficients restricted in terms of those of the components. The parallel with Engle and Lee (1999) (Component GARCH(1,1) as a restricted GARCH(2,2)) is thus maintained. The essential details are in the Appendix.

3.3 Related Models

The ACCvMEM introduced above has similarities with other univariate models. In fact, for $K = 1$ (16), (14) and (17) become, respectively,

$$\begin{aligned}\mu_t &= \chi_t + \xi_t \\ \xi_t &= \beta_1^{(\xi)*} \xi_{t-1} + \alpha_1^{(\xi)} v_{t-1} + \gamma_1^{(\xi)} v_{t-1}^{(-)} \\ \chi_t &= \omega^{(x)} + \beta_1^{(x)*} \chi_{t-1} + \alpha_1^{(x)} v_{t-1} + \gamma_1^{(x)} v_{t-1}^{(-)},\end{aligned}$$

which is the μ_t formulation of the Composite-MEM introduced in Brownlees *et al.* (2012), and recently reproposed by Mencia and Sentana (2017). We can thus interpret the ACCvMEM as a vector extension of such a model. We remark that the univariate case requires the identification condition $\beta_1^{(x)*} = \beta_1^{(\eta)*} > \beta_1^{(\xi)*}$, since otherwise the corresponding χ and η parameters could be exchanged without modifying the reduced form representation. This allows to interpret χ_t and ξ_t as *permanent* and *transitory* components, respectively, according to Engle and Lee (1999). This constraint is however not needed for $K > 1$: the parameter exchange mentioned before is not possible since the same scalar component η_t (although rescaled by ψ and shifted by μ) drives the level of all components of \mathbf{x}_t .

By the same token, as discussed in Brownlees *et al.* (2012) the ACCvMEM and the Composite-MEM have close similarities with the component GARCH of Engle and Lee (1999). In that paper, the conditional variance of r_t given \mathcal{F}_{t-1} evolves according to

$$h_t = q_t + \alpha(r_{t-1}^2 - q_{t-1}) + \beta(h_{t-1} - q_{t-1}) \quad (18)$$

$$q_t = \omega + \rho q_{t-1} + \phi(r_{t-1}^2 - h_{t-1}) + \delta(r_{t-1}^{2(-)} - h_{t-1}/2) \quad (19)$$

where $q_0 = h_0$ and $r_t^{2(-)} = r_t^2 \mathbf{I}(r_t < 0)$. In practice, within this model the time-varying component q_t replaces the usual fixed unconditional variance of a weakly stationary GARCH. With $E(r_t | \mathcal{F}_{t-1}) = 0$ and $x_t = r_t^2$ the parallel with a Composite-MEM is established.

4 Model Inference

In this section we describe how to obtain inferences on the parameters of the model introduced before. Such a model is ruled by two sets of parameters: the parameter vector of main interest $\boldsymbol{\theta}$, of dimension p , which controls the dynamics of μ_t ; the nuisance parameter matrix $\boldsymbol{\Sigma}$, the (K, K) variance matrix of the error term.⁶

To get estimates, we rely on the fact that any vMEM formulation preserves (2), together with the mean-variance relationships (3) and (4) as functions of μ_t and $\boldsymbol{\Sigma}$. Since such relationships are typical of any vMEM, inferences can be derived via Generalized Method of Moments (GMM – Cipollini *et al.* (2013)), without the need to specify the conditional distribution of the multiplicative error term.

4.1 Notation and Background

Let us consider $(p, 1)$ moment functions (that could be interpreted as *pseudo-score* functions) of the form

$$\mathbf{g}_{(T)} = \sum_{t=1}^T \mathbf{g}_t = \sum_{t=1}^T \mathbf{G}_t \mathbf{u}_t, \quad (20)$$

⁶Of course, given matrix symmetry, only the unique nuisance parameters are considered at the estimation stage.

where: \mathbf{u}_t is an \mathcal{F}_t -measurable $(K, 1)$ vector, depending on $\boldsymbol{\theta}$, which is assumed a martingale difference at the true $\boldsymbol{\theta}_0$, namely⁷

$$E(\mathbf{u}_t(\boldsymbol{\theta}_0) | \mathcal{F}_{t-1}) = \mathbf{0};$$

\mathbf{G}_t is an \mathcal{F}_{t-1} -measurable (p, K) matrix (named *instrument*) which may depend on $\boldsymbol{\theta}$ and on some nuisance parameters $\boldsymbol{\lambda}$, a $(q, 1)$ vector assumed to be consistently estimated by $\widehat{\boldsymbol{\lambda}}_T$ with a customary limiting distribution, namely

$$\sqrt{T} (\widehat{\boldsymbol{\lambda}}_T - \boldsymbol{\lambda}_0) = O_p(1),$$

where $\boldsymbol{\lambda}_0$ are the true parameters.

In the stated framework, the GMM estimator $\widehat{\boldsymbol{\theta}}_T$ of $\boldsymbol{\theta}$ satisfies the moment equation

$$\mathbf{g}_{(T)}(\widehat{\boldsymbol{\theta}}_T, \widehat{\boldsymbol{\lambda}}_T) = \mathbf{0}. \quad (21)$$

The asymptotic properties of $\widehat{\boldsymbol{\theta}}_T$ stem from the usual linear expansion of such a moment equation around $\boldsymbol{\theta}_0$ and $\boldsymbol{\lambda}_0$, leading to

$$\widehat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0 = - \left(\underbrace{T^{-1} \nabla_{\boldsymbol{\theta}} \mathbf{g}_{(T)}(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\lambda}})}_{(p,p)} \right)^{-1} \left(\underbrace{T^{-1} \mathbf{g}_{(T)}(\boldsymbol{\theta}_0, \boldsymbol{\lambda}_0)}_{(p,1)} + \underbrace{T^{-1} \nabla_{\boldsymbol{\lambda}} \mathbf{g}_{(T)}(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\lambda}})}_{(p,q)} \underbrace{(\widehat{\boldsymbol{\lambda}}_T - \boldsymbol{\lambda}_0)}_{(q,1)} \right), \quad (22)$$

where $\bar{\boldsymbol{\theta}}$ and $\bar{\boldsymbol{\lambda}}$ are convex combinations between the true value and the corresponding estimator.

The previous conditions together with the identification assumption

$$\boldsymbol{\theta}_0 \text{ is the unique solution of } \lim_{T \rightarrow \infty} [T^{-1} E(\mathbf{g}_{(T)}(\boldsymbol{\theta}, \boldsymbol{\lambda}_0))] = \mathbf{0}, \quad (23)$$

plus other regularity conditions (see Wooldridge (1994, Th. 7.1)), ensure consistency of $\widehat{\boldsymbol{\theta}}_T$ since the two matrix blocks into (22) converge in probability to some finite limiting matrices, whereas the two vector blocks converge in probability to zero.

The asymptotic distribution of $\widehat{\boldsymbol{\theta}}_T$ can be retrieved by multiplying both sides of (22) by \sqrt{T} and considering that the two matrices converge to the corresponding limiting matrices,

$$T^{-1} \nabla_{\boldsymbol{\theta}} \mathbf{g}'_{(T)}(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\lambda}}) \xrightarrow{p} \lim_{T \rightarrow \infty} \left[T^{-1} \sum_{t=1}^T E(\nabla_{\boldsymbol{\theta}} \mathbf{g}'_t(\boldsymbol{\theta}_0, \boldsymbol{\lambda}_0)) \right] \equiv \mathbf{G}_{\boldsymbol{\theta}\boldsymbol{\theta}'} \quad (24)$$

$$T^{-1} \nabla_{\boldsymbol{\lambda}} \mathbf{g}'_{(T)}(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\lambda}}) \xrightarrow{p} \lim_{T \rightarrow \infty} \left[T^{-1} \sum_{t=1}^T E(\nabla_{\boldsymbol{\lambda}} \mathbf{g}'_t(\boldsymbol{\theta}_0, \boldsymbol{\lambda}_0)) \right] \equiv \mathbf{G}_{\boldsymbol{\lambda}\boldsymbol{\theta}'}, \quad (25)$$

whereas $T^{-1/2} \mathbf{g}_{(T)}(\boldsymbol{\theta}_0, \boldsymbol{\lambda}_0)$ and $\sqrt{T} (\widehat{\boldsymbol{\lambda}}_T - \boldsymbol{\lambda}_0)$ converge jointly to a Normal r.v.,

$$\begin{pmatrix} T^{-1/2} \mathbf{g}_{(T)}(\boldsymbol{\theta}_0, \boldsymbol{\lambda}_0) \\ \sqrt{T} (\widehat{\boldsymbol{\lambda}}_T - \boldsymbol{\lambda}_0) \end{pmatrix} \xrightarrow{d} N \left(\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \boldsymbol{\Omega} = \begin{pmatrix} \boldsymbol{\Omega}_{\boldsymbol{\theta}\boldsymbol{\theta}'} & \boldsymbol{\Omega}_{\boldsymbol{\theta}\boldsymbol{\lambda}'} \\ \boldsymbol{\Omega}_{\boldsymbol{\lambda}\boldsymbol{\theta}'} & \boldsymbol{\Omega}_{\boldsymbol{\lambda}\boldsymbol{\lambda}'} \end{pmatrix} \right) \quad (26)$$

(see Wooldridge (1994, Th. 7.2) for the details on the regularity conditions needed), leading to

$$\sqrt{T} (\widehat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{d} N \left(\mathbf{0}, \mathbf{G}_{\boldsymbol{\theta}\boldsymbol{\theta}'}^{-1} (\mathbf{I}_p \quad \mathbf{G}'_{\boldsymbol{\lambda}\boldsymbol{\theta}'}) \boldsymbol{\Omega} \begin{pmatrix} \mathbf{I}_p \\ \mathbf{G}'_{\boldsymbol{\lambda}\boldsymbol{\theta}'} \end{pmatrix} \mathbf{G}_{\boldsymbol{\theta}\boldsymbol{\theta}'}^{-1} \right). \quad (27)$$

This implies that the asymptotic distribution of $\widehat{\boldsymbol{\theta}}_T$ is affected by the asymptotic distribution of $\widehat{\boldsymbol{\lambda}}_T$ if $\mathbf{G}_{\boldsymbol{\lambda}\boldsymbol{\theta}'} \neq \mathbf{0}$ (irrespective of whether $\boldsymbol{\Omega}_{\boldsymbol{\theta}\boldsymbol{\lambda}'} = \mathbf{0}$). Note however that $\mathbf{G}_{\boldsymbol{\lambda}\boldsymbol{\theta}'}$ can be expressed as

$$\mathbf{G}_{\boldsymbol{\lambda}\boldsymbol{\theta}'} \equiv \lim_{T \rightarrow \infty} \left[T^{-1} \sum_{t=1}^T E [E(\nabla_{\boldsymbol{\lambda}} \mathbf{u}'_t(\boldsymbol{\theta}_0, \boldsymbol{\lambda}_0) | \mathcal{F}_{t-1}) \mathbf{G}'_t] \right], \quad (28)$$

⁷In order to simplify the notation, we omit the dependence of the quantities considered on the parameters unless needed.

that is zero when \mathbf{u}_t does not depend on $\boldsymbol{\lambda}$. The vMEM provides two interesting examples (which will be elaborated on in the following sections): when $\boldsymbol{\lambda}$ stands for the nuisance parameter matrix Σ , estimating it does not affect the asymptotic distribution of $\hat{\theta}_T$, since \mathbf{u}_t does not depend on Σ ; on the contrary, when $\boldsymbol{\lambda}$ plays the role of $\boldsymbol{\mu}$ into (11), estimating it as the unconditional mean of \mathbf{x}_t affects the asymptotic distribution of $\hat{\theta}_T$, since \mathbf{u}_t is a function of $\boldsymbol{\mu}$.

An efficient choice of the instrument \mathbf{G}_t allows for the ‘minimization’ of the asymptotic variance matrix of $\hat{\theta}_T$ among all possible moment functions in (20). According to Wooldridge (1994, Sect. 7.5)), this is given by

$$\mathbf{G}_t^* = -E(\nabla_{\boldsymbol{\theta}} \mathbf{u}'_t | \mathcal{F}_{t-1}) V(\mathbf{u}_t | \mathcal{F}_{t-1})^{-1}$$

and implies

$$\Omega_{\boldsymbol{\theta}\boldsymbol{\theta}'} = -\mathbf{G}_{\boldsymbol{\theta}\boldsymbol{\theta}'} = \lim_{T \rightarrow \infty} \left[T^{-1} \sum_{t=1}^T E \left[E(\nabla_{\boldsymbol{\theta}} \mathbf{u}'_t | \mathcal{F}_{t-1}) V(\mathbf{u}_t | \mathcal{F}_{t-1})^{-1} E(\nabla_{\boldsymbol{\theta}'} \mathbf{u}_t | \mathcal{F}_{t-1}) \right] \right],$$

where the quantities inside the formula are evaluated at the true parameter values.

4.2 vMEM Inference via GMM

In the vMEM framework, a quantity \mathbf{u}_t with the characteristics needed is

$$\mathbf{u}_t = \mathbf{x}_t \oslash \boldsymbol{\mu}_t - \mathbf{1} \quad (29)$$

(\oslash indicates the element-by-element division), since, under a correct specification and the true values of the parameters, Equations (3) and (4) imply

$$E(\mathbf{u}_t | \mathcal{F}_{t-1}) = \mathbf{0} \quad V(\mathbf{u}_t | \mathcal{F}_{t-1}) = \Sigma. \quad (30)$$

The efficient instrument is then

$$\mathbf{G}_t^* = \nabla_{\boldsymbol{\theta}} \boldsymbol{\mu}'_t \text{diag}(\boldsymbol{\mu}_t)^{-1} \Sigma^{-1} = \mathbf{a}_t \Sigma^{-1},$$

leading to the moment function

$$\sum_{t=1}^T \mathbf{a}_t \Sigma^{-1} \mathbf{u}_t. \quad (31)$$

Moreover, since \mathbf{u}_t does not depend on Σ , then $\mathbf{G}_{\Sigma\boldsymbol{\theta}'} = \mathbf{0}$ (cf. Equation (28)) so that the asymptotic distribution of $\hat{\theta}_T$ is not affected by the distribution of the estimator of Σ and is given by

$$\sqrt{T} (\hat{\theta}_T - \boldsymbol{\theta}_0) \xrightarrow{d} N \left(\mathbf{0}, \Omega_{\boldsymbol{\theta}\boldsymbol{\theta}'}^{-1} = \left[\lim_{T \rightarrow \infty} T^{-1} \sum_{t=1}^T E[\mathbf{a}_t \Sigma^{-1} \mathbf{a}'_t] \right]^{-1} \right) \quad (32)$$

where the quantities into $\Omega_{\boldsymbol{\theta}\boldsymbol{\theta}'}$ are evaluated at the true values of the parameters. This implies that the asymptotic variance matrix can be consistently estimated by inverting

$$\hat{\Omega}_{\boldsymbol{\theta}\boldsymbol{\theta}'} = T^{-1} \sum_{t=1}^T \hat{\mathbf{a}}_t \hat{\Sigma}_T^{-1} \hat{\mathbf{a}}'_t \quad (33)$$

where $\hat{\mathbf{a}}_t$ is computed on the basis of $\hat{\theta}_T$ and $\hat{\Sigma}_T$ is a consistent estimator of Σ . As suggested by (30), a ‘natural’ estimator for the nuisance parameter is

$$\hat{\Sigma}_T = T^{-1} \sum_{t=1}^T \hat{\mathbf{u}}_t \hat{\mathbf{u}}'_t \quad (34)$$

where $\hat{\mathbf{u}}_t$ indicates here the residual (29) computed at $\hat{\theta}_T$.

4.3 Expectation Targeting

A possible strategy for reducing the computational burden without spoiling estimation accuracy is to rely on the assumption of mean stationarity of $\{\mathbf{x}_t\}$ and to estimate iteratively all parameters holding $\hat{\boldsymbol{\mu}}$ fixed at its estimate $\bar{\mathbf{x}}_T$. This *expectation targeting* approach parallels the *variance targeting* proposed by Engle and Mezrich (1996). From an inferential point of view, however, such a two-step strategy implies that the asymptotic variance matrix in (32) is wrong and need to be adjusted: we need expressions for $\Omega_{\theta\lambda'}$ and $\Omega_{\lambda\lambda'}$ substituting $\boldsymbol{\mu}$ for λ in (26) (cf. Section 4.1). The former can be computed by using (31), that is,

$$\Omega_{\theta\mu'} = \lim_{T \rightarrow \infty} \left[T^{-1} \sum_{t=1}^T E(\nabla_{\theta}\boldsymbol{\mu}'_t(\boldsymbol{\theta}_0, \boldsymbol{\mu}_0)) \right].$$

The derivation of the latter is more involved but can be obtained following Francq *et al.* (2011, proof of Th. 2.1)⁸. By using the reduced form representation of the model, generically expressed as

$$\mathbf{B}(L)\boldsymbol{\mu}_t = \boldsymbol{\omega} + \mathbf{A}(L)\mathbf{v}_t + \mathbf{A}^{(-)}(L)\mathbf{v}_t^{(-)},$$

we get

$$\bar{\mathbf{x}}_T = \boldsymbol{\mu} + \mathbf{A}\bar{\mathbf{w}}_T + o(1/T)$$

where

$$\mathbf{A} = [\mathbf{I}_K + \mathbf{B}(1)^{-1}\mathbf{A}(1), \quad \mathbf{B}(1)^{-1}\mathbf{A}^{(-)}(1)]$$

and $\mathbf{w}_t = (\mathbf{v}_t; \mathbf{v}_t^{(-)})$. As a consequence,

$$\Omega_{\lambda\lambda'} = \mathbf{A}\boldsymbol{\Sigma}_{\mathbf{w}}\mathbf{A}'$$

where

$$\boldsymbol{\Sigma}_{\mathbf{w}} \equiv \lim_{T \rightarrow \infty} \left[T^{-1} \sum_{t=1}^T V(\mathbf{w}_t) \right]$$

and \mathbf{A} are evaluated at the true values of the parameters. $\boldsymbol{\Sigma}_{\mathbf{w}}$ can be estimated from the data as

$$\hat{\boldsymbol{\Sigma}}_{\mathbf{w}} = T^{-1} \sum_{t=1}^T \hat{\mathbf{w}}_t \hat{\mathbf{w}}_t'$$

where $\hat{\mathbf{w}}_t$ is evaluated at $\hat{\boldsymbol{\theta}}_T$.

5 Common Volatility Dynamics in European Markets

We adopt the ACCvMEM for modeling the joint dynamics of realized kernel volatilities for the five European indices introduced above (Section 3.1) and represented in Figure 1. The aim of the analysis is to illustrate the separate contribution of the common and market-specific components to the overall dynamics, investigating their relative importance when modeling the behavior of the Euro STOXX 50 volatility representative of Euro area market-wide movements.

We will estimate the ACCvMEM in the specification

$$\begin{aligned} \boldsymbol{\mu}_t &= \boldsymbol{\mu} + \psi\eta_t + \boldsymbol{\xi}_t \\ \eta_t &= \beta_1^{(\eta)*}\eta_{t-1} + \boldsymbol{\alpha}_1^{(\eta)'}\mathbf{v}_{t-1} + \boldsymbol{\gamma}_1^{(\eta)'}\mathbf{v}_{t-1}^{(-)} \\ \boldsymbol{\xi}_t &= \boldsymbol{\beta}_1^{(\xi)*}\boldsymbol{\xi}_{t-1} + \boldsymbol{\alpha}_1^{(\xi)}\mathbf{v}_{t-1} + \boldsymbol{\gamma}_1^{(\xi)}\mathbf{v}_{t-1}^{(-)} + \boldsymbol{\alpha}_2^{(\xi)}\mathbf{v}_{t-2} \end{aligned}$$

(cf. (11)–(14)–(15)), and recalled here for the sake of clarity; $\boldsymbol{\beta}_1^{(\xi)*}, \boldsymbol{\gamma}_1^{(\xi)}, \boldsymbol{\alpha}_2^{(\xi)}$ are diagonal matrices.

⁸These authors show ergodicity properties of the GARCH model, which apply also here – a MEM is a generalization of the GARCH – relying on an infinite order MA representation (cf. also Nelson (1990)).

Table 2 contains the parameter estimates; to simplify notation, in the table, $\beta = \beta_1^{(\eta)*}$; $\alpha = \alpha_1^{(\eta)}$; $\gamma = \gamma_1^{(\eta)}$; α_1^j is the j -th row ($j = \text{GE, FR, ES, IT, NL}$) of $\alpha_1^{(\xi)}$; γ^{own} , α_2^{own} , β^{own} are the diagonals, respectively, of $\gamma_1^{(\xi)}$, $\alpha_2^{(\xi)}$, and $\beta_1^{(\xi)*}$. In panel (a), we have the parameters of the common component showing a high persistence with the β estimate of 0.9824, substantially in line with the univariate analyses in Brownlees *et al.* (2012) and Engle and Lee (1999). The loadings ψ are bigger than 1 for Germany and France, substantially lower than 1 for Spain. The only market with coefficient not significantly different from 1 is The Netherlands. The α row shows a significant contribution to η just for Spain and Italy, while the asymmetric γ are all significant but for the Netherlands. In this respect, the latter market is the only one not feeding the common component.

Table 2: Annualized realized volatility: parameters of the common and market-specific components estimated over the sample Jan. 2, 2002 – Dec. 30, 2016. Sandwich-based t-statistics are reported in parentheses. To simplify notation, in the table, $\beta = \beta_1^{(\eta)*}$; $\alpha = \alpha_1^{(\eta)}$; $\gamma = \gamma_1^{(\eta)}$; α_1^j is the j -th row ($j = \text{GE, FR, ES, IT, NL}$) of $\alpha_1^{(\xi)}$; γ^{own} , α_2^{own} , β^{own} are the diagonals, respectively, of $\gamma_1^{(\xi)}$, $\alpha_2^{(\xi)}$, and $\beta_1^{(\xi)*}$.

	GE	FR	ES	IT	NL
(a) Parameters in the common component					
μ	17.5514 (18.20)	16.3654 (25.48)	17.0207 (25.43)	15.3382 (174.14)	15.1266 (21.71)
ψ	1.2215 (15.92)	1.2568 (14.64)	0.6852 (7.81)	0.8783 (14.59)	0.9582 (17.14)
β			0.9824 (475.70)		
α	0.0054 (0.22)	-0.0122 (-0.30)	0.0625 (2.47)	0.2039 (6.85)	0.0275 (0.76)
γ	0.0093 (2.03)	0.0268 (3.42)	0.0091 (1.98)	0.0101 (2.13)	0.0066 (1.11)
(b) Parameters in the market specific components					
α_1^{GE}	0.2902 (11.67)	0.0332 (1.54)	0.0084 (0.62)	0.0450 (2.38)	0.0282 (1.61)
α_1^{FR}	-0.0152 (-0.41)	0.1833 (5.12)	-0.0088 (-0.40)	-0.0727 (-2.25)	-0.0378 (-1.35)
α_1^{ES}	-0.1025 (-4.28)	-0.0764 (-3.29)	0.2479 (13.67)	-0.0125 (-0.65)	-0.0699 (-3.90)
α_1^{IT}	-0.2267 (-6.27)	-0.2158 (-5.84)	-0.0599 (-2.32)	0.1298 (5.93)	-0.1638 (-6.13)
α_1^{NL}	0.0690 (2.06)	0.0634 (2.00)	0.0167 (0.83)	-0.0039 (-0.13)	0.3081 (11.37)
γ^{own}	0.0046 (1.23)	-0.0034 (-0.65)	0.0232 (4.86)	0.0295 (5.41)	0.0143 (4.36)
α_2^{own}	-0.1119 (-10.15)	-0.1023 (-10.65)	-0.1232 (-10.36)	-0.0490 (-3.31)	-0.0973 (-9.56)
β^{own}	0.9854 (526.04)	0.9829 (529.38)	0.9842 (360.68)	0.7136 (10.07)	0.9857 (526.97)

To investigate the empirical properties of our model, in search for possible misspecification and/or instability, we re-estimate the model over six subperiods, going, respectively, from 2002–2011 to 2007–2016, with a fixed window of ten years and adding/dropping one year at the time (estimated coefficients not reported). In terms of fit, the results are shown in Table 3. In the top panel (a) we calculated the Generalized R^2 , i.e. the squared correlation between each observed series and its conditional mean $\hat{\mu}$. The overall fit is good, generally well above 0.7 across tickers, which is shared by all subperiods (with a decrease for Spain in the last two). In an attempt at isolating individual contributions, we provide similar squared correlations between the observations and the components, separately (this decomposition is obviously not additive as there is a covariance between the common and the market-specific components). Panel (b) shows such indices with the common component, while Panel (c) reproduces those with the market-specific components. The former are remarkably high, in the sense that they are above 0.55 for the overall period; on subperiods we see a substantial stability of the contribution for France and the

Netherlands, a tendency to decrease for Italy, and even more so for Spain, while Germany sees a steady increase to above 0.7. The latter are much lower, and sometimes they are negligible, especially for Germany and Italy.

Table 3: Squared correlations between each observed time series and (a) its conditional mean $\hat{\mu}_i$ (Generalized R^2); (b) the common component $\hat{\eta}$, and (c) the market-specific component $\hat{\xi}_i$. Whole period and ten-year subperiods.

Country	2002-2016	2002-2011	2003-2012	2004-2013	2005-2014	2006-2015	2007-2016
(a) Conditional mean							
GE	0.772	0.792	0.77	0.756	0.761	0.737	0.725
FR	0.743	0.775	0.754	0.751	0.743	0.713	0.689
ES	0.695	0.765	0.75	0.744	0.718	0.668	0.601
IT	0.737	0.765	0.757	0.754	0.74	0.716	0.694
NL	0.773	0.795	0.781	0.776	0.774	0.75	0.731
(b) Common component							
GE	0.575	0.649	0.57	0.642	0.746	0.726	0.714
FR	0.657	0.728	0.639	0.691	0.696	0.669	0.65
ES	0.623	0.695	0.696	0.648	0.455	0.419	0.392
IT	0.725	0.755	0.672	0.679	0.618	0.584	0.575
NL	0.587	0.67	0.545	0.669	0.713	0.686	0.67
(c) Market-specific component							
GE	0.019	0.034	0.001	0.001	0.122	0.141	0.152
FR	0.109	0.081	0.111	0.242	0.116	0.058	0.02
ES	0.469	0.627	0.645	0.578	0.237	0.083	0.066
IT	0.192	0.162	0.103	0.043	0.01	0.009	0.004
NL	0.114	0.308	0.147	0.199	0.232	0.238	0.375

As far as the model residual diagnostics are concerned, in Table 4 we report the calculation of the p-values on the Ljung–Box joint statistics for the first five lags, and then, 10, 15 and 20, for all residuals, on the overall period and the subperiods. The results have some occasional glitches at higher lags (just in the first three sub-periods), but they have a very good profile, especially for the entire sample and for the most recent ones.

Table 4: ACCvMEM model residuals: p-values of the joint Ljung-Box tests at lags 1–5, 10, 15 and 20. Whole period and ten-year subperiods.

lag	2002-2016	2002-2011	2003-2012	2004-2013	2005-2014	2006-2015	2007-2016
1	0.80	0.90	0.57	0.86	0.99	0.98	0.95
2	0.77	0.73	0.47	0.78	0.99	1.00	1.00
3	0.80	0.67	0.61	0.81	1.00	0.99	0.99
4	0.50	0.45	0.49	0.60	0.95	0.92	0.86
5	0.41	0.46	0.60	0.47	0.91	0.89	0.74
10	0.12	0.03	0.02	0.07	0.39	0.68	0.55
15	0.07	0.06	0.04	0.02	0.14	0.29	0.17
20	0.13	0.13	0.09	0.06	0.34	0.27	0.16

To explore the contribution of each to the estimated results, a further sensitivity analysis was performed (estimates not reported) to highlight the occurring similarities between the estimated η_t 's in leaving out one market at the time. In Figure 2 we report the common component η_t extracted on five series, and then on four series without one market in turn. The resulting pattern shows comovements with some deviations, noticeably for when Spain, Italy and France are excluded, in this order of impact. The higher estimated levels around 2004-2005 are to be interpreted in the sense that the inclusion of either of these countries brings about a lower common level of volatility, showing how the Great Moderation period had a particular calming effect (associated to investment inflows and upward market trends) on traditionally turbulent markets; for 2012-2013, the opposite is true, that is, the contribution of either of these countries is to increase the common level of volatility, in line with the higher turmoil in these countries in that period.

To show some details, we zoom in to specific time spans: in Figures 3 and 4, we show an enlarged picture of the results over two periods, the first between Jul. 1 and Dec. 31, 2008, and the second between Jun. 1 and Nov. 30, 2011. It is striking to see how the lines corresponding to leaving Germany and The Netherlands out follow very closely the overall component, with the mentioned departures for Spain, France and Italy, who appear to influence the outcome more substantially.

Investigating the contribution of each country to the market-specific components, let us turn now to the top part of panel (b) of Table 2, i.e. the coefficients related to the contribution of each market (by row) to the others (by column). Interestingly, for Spain and Italy, apart from their own (on the main diagonal of the matrix) these coefficients are negative and in all but one case are significant, signaling that a surge in volatility in one of these

Figure 2: Common component η_t estimated over Jan. 2, 2002 – Dec. 30, 2016 (All). The country acronym indicates component estimated by leaving the corresponding market out (one at the time).

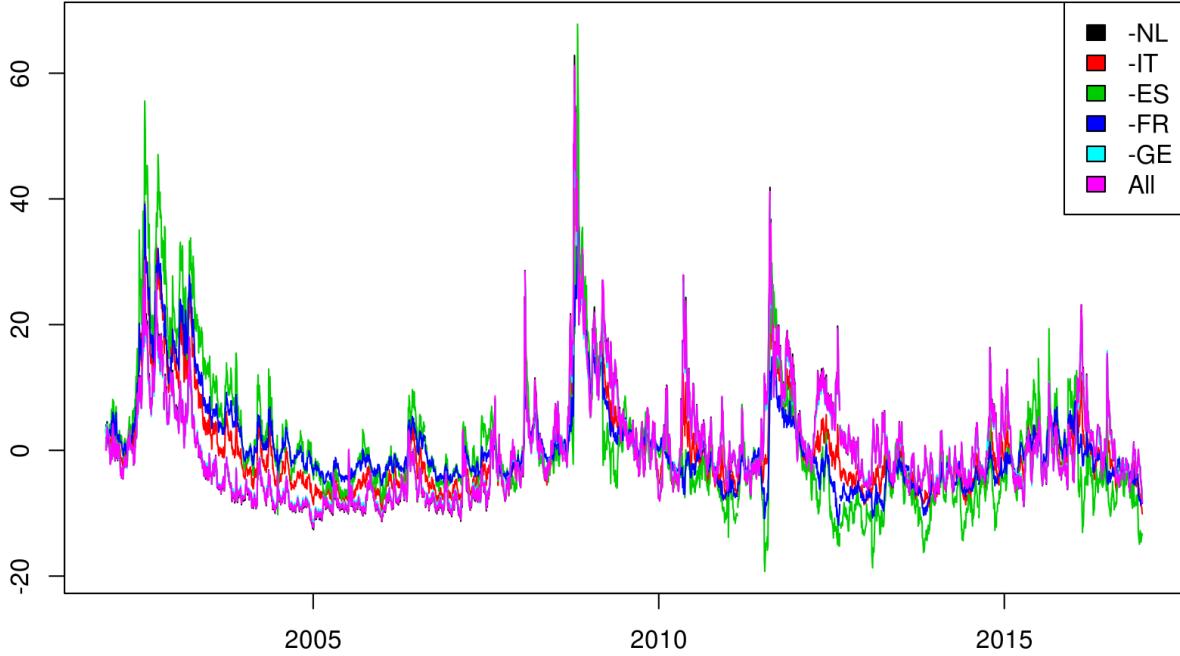
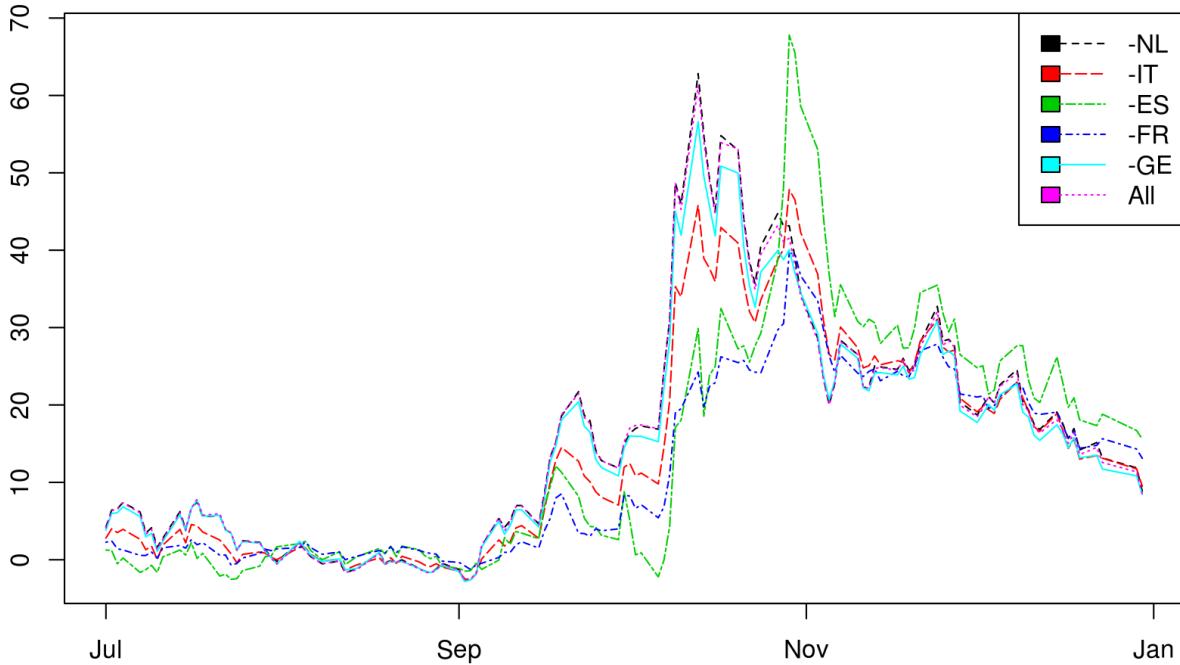
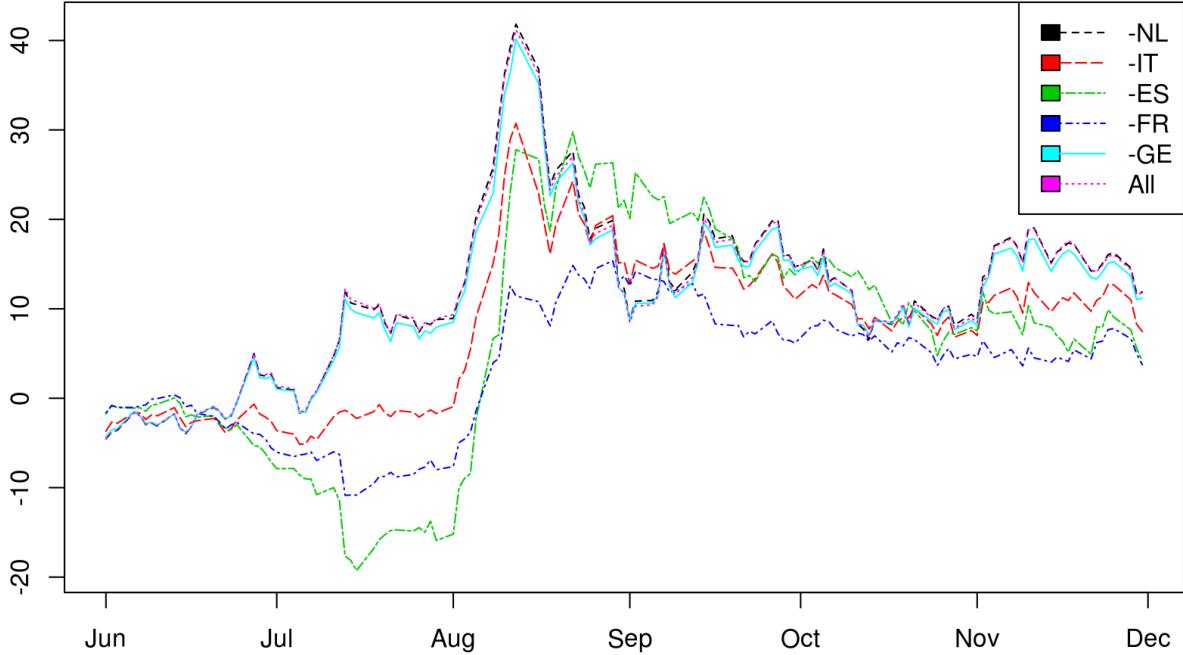


Figure 3: Analysis of the sensitivity of the common component η_t to single market data. The overall estimated time series is (All). The country acronym indicates the η component estimated by leaving the corresponding market out (one at the time). Window of graphical representation is Jul. 1, 2008 to Dec. 31, 2008.



two countries determines a fall in volatility in the others (as sort of reallocation, a similar phenomenon occurs also from France to Italy). The Netherlands has a positive impact on Germany and France, and Germany just on Italy. In the bottom part of panel (b), we see significant asymmetric coefficients (and positive, as expected) for Spain, Italy and The Netherlands, while the second lag is needed in all specifications.

Figure 4: Analysis of the sensitivity of the common component η_t to single market data. The overall estimated time series is (All). The country acronym indicates the η component estimated by leaving the corresponding market out (one at the time). Window of graphical representation is Jun. 1, 2011 to Nov. 30, 2011



The resulting behavior of the individual market-specific components is shown in Figure 5. While we should remember that these components have zero mean, a stark peculiarity in the behavior of Italy is noted both by size and the frequency of oscillations around zero. The other markets still show some persistent dynamics, with less frequent crossings of zero (especially Germany and The Netherlands, less so for Spain). Germany, France and The Netherlands exhibit some negative local peaks in four occasions (Oct. 14, 2008; May 12, 2010; around July 13-15 2011, Aug. 6, 2012) connected, respectively, to the turmoil following the bankruptcy of Lehman Brothers, and some episodes characterizing the Euro debt crisis. These negative peaks show that for these markets the common component forces too high, so to speak, levels of volatility in need of correction. Interestingly, for Italy, the common component fits well the FTSE-MIB volatility values, while for Spain the positive peaks in the same occasions reveal higher levels of volatility than what is implied by the common component.

The diverse behavior of these series suggests a further diagnostics tool, namely analyzing the time series properties of the estimated components and calculating the autocorrelation coefficients for the same pattern of lags (the first five, then 10, 15 and 20) for the common and market specific components (Table 5, on the whole period and on the 10-year subperiods).

We notice that the common component has a high persistence that is stable across subperiods; individual market-specific components have a more diversified behavior, with the notable cases of Italy (for which a lower persistence relates to just the first subperiod) and Germany for which autocorrelation dies out quickly as of 2005–2014 and carries over to the remainder of the samples.

6 The Impact of the Common Component on Euro STOXX 50 Volatility

In order to evaluate the relevance of the ACCvMEM decomposition, we analyze the volatility dynamics of a benchmark index, the Euro STOXX 50,⁹ which is a composite index comprising the 50 most important stocks in the Euro Area (20 companies from France, 14 from Germany, 5 each from Spain and The Netherlands, 3 from Italy; the remaining 3 are respectively from Belgium, Finland and Ireland, not covered in the previous discussion).

⁹www.stoxx.com/index-details?symbol=SX5E

Figure 5: Short term components estimated on each market over Jan. 2, 2002 – Dec. 30, 2016.

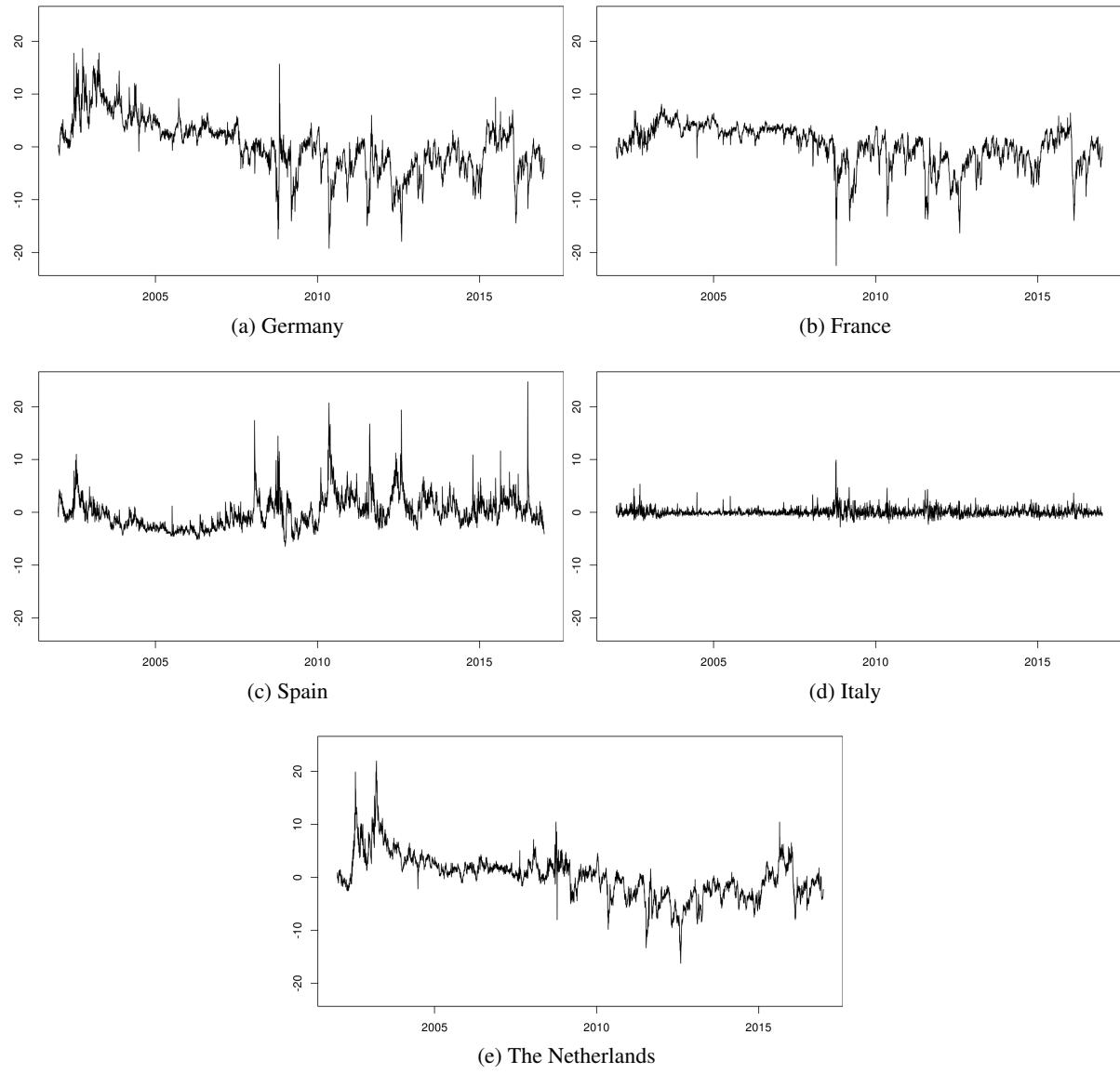


Table 5: Estimated components: autocorrelations at 1 to 5, 10, 15 and 20 lags for the common component η and the market-specific components. Whole period and ten-year subperiods.

comp	1	2	3	4	5	10	15	20
2002-2016								
η	0.98	0.96	0.94	0.92	0.90	0.81	0.74	0.70
GE	0.98	0.96	0.95	0.93	0.92	0.85	0.79	0.75
FR	0.98	0.96	0.94	0.92	0.91	0.82	0.76	0.71
ES	0.90	0.88	0.84	0.82	0.80	0.71	0.64	0.58
IT	0.54	0.39	0.27	0.20	0.14	0.05	0.05	0.05
NL	0.98	0.96	0.95	0.94	0.93	0.88	0.83	0.79
2002-2011								
η	0.98	0.97	0.95	0.93	0.91	0.84	0.78	0.74
GE	0.97	0.96	0.94	0.93	0.92	0.87	0.82	0.78
FR	0.97	0.96	0.94	0.93	0.91	0.85	0.79	0.74
ES	0.94	0.92	0.88	0.87	0.85	0.76	0.68	0.63
IT	0.49	0.31	0.16	0.09	0.02	0.01	0.03	0.02
NL	0.97	0.95	0.94	0.93	0.92	0.86	0.81	0.77
2003-2012								
η	0.98	0.95	0.92	0.90	0.87	0.78	0.72	0.67
GE	0.99	0.97	0.96	0.95	0.94	0.87	0.83	0.79
FR	0.98	0.96	0.94	0.91	0.89	0.80	0.74	0.69
ES	0.93	0.91	0.88	0.87	0.86	0.79	0.73	0.70
IT	0.95	0.93	0.91	0.89	0.86	0.75	0.65	0.60
NL	0.98	0.96	0.95	0.93	0.92	0.87	0.82	0.77
2004-2013								
η	0.98	0.95	0.92	0.89	0.87	0.77	0.69	0.64
GE	0.97	0.95	0.93	0.91	0.89	0.81	0.73	0.68
FR	0.98	0.95	0.91	0.88	0.85	0.72	0.63	0.57
ES	0.94	0.92	0.90	0.89	0.87	0.81	0.76	0.74
IT	0.93	0.91	0.88	0.85	0.83	0.68	0.56	0.48
NL	0.97	0.95	0.94	0.92	0.91	0.85	0.79	0.75
2005-2014								
η	0.98	0.96	0.94	0.92	0.90	0.82	0.75	0.70
GE	0.39	0.22	0.03	-0.02	-0.05	-0.02	0.03	0.02
FR	0.94	0.91	0.87	0.85	0.83	0.73	0.63	0.55
ES	0.95	0.93	0.90	0.89	0.87	0.80	0.74	0.71
IT	0.93	0.90	0.87	0.84	0.81	0.68	0.57	0.50
NL	0.90	0.85	0.80	0.77	0.74	0.61	0.48	0.40
2006-2015								
η	0.98	0.95	0.92	0.90	0.87	0.79	0.72	0.66
GE	0.39	0.21	0.03	-0.03	-0.05	-0.02	0.04	0.05
FR	0.91	0.87	0.83	0.80	0.78	0.68	0.58	0.49
ES	0.95	0.93	0.90	0.89	0.87	0.81	0.75	0.71
IT	0.93	0.90	0.86	0.83	0.80	0.66	0.54	0.47
NL	0.89	0.85	0.80	0.76	0.74	0.60	0.48	0.39
2007-2016								
η	0.97	0.95	0.92	0.89	0.87	0.78	0.72	0.66
GE	0.45	0.24	0.05	-0.02	-0.04	-0.02	0.04	0.04
FR	0.87	0.83	0.78	0.75	0.73	0.62	0.51	0.43
ES	0.92	0.90	0.87	0.85	0.83	0.76	0.69	0.63
IT	0.92	0.89	0.84	0.81	0.77	0.62	0.49	0.42
NL	0.89	0.86	0.81	0.78	0.76	0.63	0.51	0.44

The graph of the volatility time series on this index is reported on the bottom right of Figure 1 shown before. Such a time series means to represents Euro area volatility: in what follows we want to investigate what information content the ACCvMEM decomposition on the main five Euro Area markets may have in explaining/predicting Euro STOXX 50 volatility. A benchmark model is taken to be the Heterogeneous Autoregressive (HAR) model by Corsi (2009), a popular model which mimics some long memory features of the data: current volatility depends on lagged volatility, on the average past weekly volatility and on the average past monthly volatility. The expression is:

$$RVol_t = \omega + \alpha_D RVol_{t-1} + \alpha_W \overline{RVol}_{t-(2:5)} + \alpha_M \overline{RVol}_{t-(6:22)} + \varepsilon_t \quad (35)$$

where $\overline{RVol}_{t-(h:k)}$ is the mean of $t-h$ to $t-k$ lagged $RVol$ values. In such a specification we favor a better interpretability by avoiding time overlaps within regressors, thus the first lag is not included in the weekly average, and the first five lags are not included in the monthly average.

Table 6: Parameters of the linear models of the Euro STOXX 50 realized volatility estimated over Jan. 2, 2002 - Dec. 30, 2016 (3756 observations). t-statistics in parentheses computed using Newey-West HAC standard errors. Significance at different levels is represented by * (10%) ** (5%) and *** (1%) for a two-sided alternative.

	HAR	HAR-Markets	HAR-ACCvMEM	HAR-ACCvMEM without η	ACCvMEM	HAR-SPvMEM
$RVol_{t-1}$	0.465*** (10.770)	0.007 (0.066)	0.047 (0.561)	0.276*** (3.971)		0.337*** (5.634)
$\overline{RVol}_{t-2:t-5}$	0.340*** (6.678)	0.276*** (6.298)	-0.030 (-0.403)	0.312*** (4.630)		0.207*** (3.653)
$\overline{RVol}_{t-6:t-22}$	0.138*** (6.733)	0.091*** (3.852)	0.024 (0.679)	0.179*** (4.846)		0.034 (0.835)
$RVol_{GE,t-1}$		0.123** (2.057)				
$RVol_{FR,t-1}$		0.011 (0.151)				
$RVol_{ES,t-1}$		0.032 (0.931)				
$RVol_{IT,t-1}$		0.149*** (3.161)				
$RVol_{NL,t-1}$		0.322*** (4.655)				
common $_{t t-1}$			1.053*** (5.804)		1.104*** (19.844)	8.097*** (5.492)
$GE_{t t-1}$			0.222*** (2.867)	-0.098 (-0.715)	0.240* (1.949)	0.056 (0.843)
$FR_{t t-1}$			0.203 (1.357)	-0.450* (-1.861)	0.245 (1.532)	-0.050 (-0.407)
$ES_{t t-1}$			0.110** (2.462)	0.133* (1.774)	0.126*** (2.642)	-0.081* (-1.744)
$IT_{t t-1}$			0.891*** (2.656)	1.751*** (4.027)	0.997*** (3.577)	0.217** (2.208)
$NL_{t t-1}$			0.398*** (2.601)	0.604** (2.502)	0.388** (2.324)	0.276** (2.412)
R^2	0.687	0.704	0.715	0.704	0.714	0.700
Adjusted R^2	0.687	0.703	0.714	0.703	0.714	0.699

Estimation of the HAR model by OLS with (with robust HAC standard errors Newey and West, 1987) gives the results reproduced in the first column of Table 6, where all coefficients are significant and have their customary order of magnitude. In the second column, (labeled HAR-Markets) we insert the lagged value of the five countries' realized volatilities: we retain the significance of the weekly and monthly terms, while losing significance on the first lag of the Euro STOXX 50 volatility in favor of the corresponding coefficients for Germany, Italy and the Netherlands.¹⁰

In the third column (labeled HAR-ACCvMEM), we insert the estimated values of the common component and

¹⁰While the high correlation among these regressors could be responsible for the insignificance of the other two coefficients, the main goal here is to show that the insertion of the raw values does not absorb the weekly and monthly HAR terms.

of the market-specific components,¹¹ reaching the interesting results of losing significance on all HAR term. The coefficient on η_t is not significantly different from one, while for the other terms, only France ends up not significantly contributing: this may be surprising, in view of the noted large number of French companies represented within the index, but we also noted that France has the strongest impact in determining the position of the η term. The common component plays a crucial role here since when we omit it from this regression (column labeled HAR-ACCVMEM without η) we get that the HAR terms are once again all individually and jointly significant. By contrast, by eliminating the HAR terms from the HAR-ACCVMEM (column labeled ACCvMEM) we get smaller standard errors for the surviving coefficients.

The common component η shows a fairly smooth low frequency dynamics (but not so smooth as a slow-moving component extracted via non parametric methods, cf. Engle and Rangel (2008), Brownlees and Gallo (2010), Barigozzi *et al.* (2014)). In these models the common component is explicitly modeled as a low frequency component, which enters in a multiplicative way: in particular, let us consider the SPvMEM specification by Barigozzi *et al.* (2014) for a vector of K realized volatilities at time t :

$$x_t = \phi_t \zeta_t \odot \varepsilon_t, \quad (36)$$

where ϕ_t is a nonparametric function of time t representing the common low frequency component across volatilities. The individual short term fluctuations around the common long term component are modeled as

$$\zeta_{i,t} = \omega_i + \beta_i \zeta_{i,t-1} + \alpha_i \frac{x_{i,t-1}}{\phi_{t-1}} + \gamma_i \frac{x_{i,t-1}^{(-)}}{\phi_{t-1}}. \quad (37)$$

We inserted the estimated component $\hat{\phi}_t$ (reported as $\text{common}_{t|t-1}$ in Table 6) and the market specific components (labeled with the country acronym as before) as additional variables in the HAR specification (column HAR-SPvMEM in Table 6, where corrected standard errors for the GRP are obtained by bootstrap). We notice a slightly worse fit than the ACCvMEM specifications; the daily and weekly HAR coefficients are still significant, the common component is strongly significant, while the individual components are not so, with the exception of Italy and The Netherlands.

Overall, it seems that the extraction of a common low frequency component from five country realized volatilities (with both approaches) has a meaningful contribution to the explanation of the Euro area volatility, with the strong difference that the ACCvMEM allows for the significant contribution of market specific components, while SPvMEM does not. Moreover, it seems that the ACCvMEM approach is providing a substitution to the information included in the HAR terms.

To verify how results could change if shorter subsamples were chosen, we have repeated the analysis, limited to the HAR-ACCVMEM, over windows starting from 2002-2011 and ending with 2007-2016: all of them straddle the great financial crisis (2008) and the sovereign debt crisis in the Euro area (2010-11), the main difference among them being the impact that the first high volatility period (2002-2003) may have on the outcomes. Qualitatively the results are very similar: it is interesting that we confirm the loss of significance of the daily and monthly coefficients of HAR, while the weekly is significant just for two subperiods. We can say that the series generated by the ACCvMEM successfully capture the information contained in the past of the Euro STOXX 50 series itself. The common component is always highly significant, and still statistically indistinguishable from one. The market-specific components show more varied behavior: Germany and The Netherlands are always significant; apart from the first period, Italy has an impact, while Spain loses its significance in the last three periods; we confirm that no contribution by France's market-specific component is detectable.

As a complement, we have extended our analysis to the estimation of the ACCvMEM over a period, and then use the components in the HAR-ACCVMEM model for the Euro STOXX 50 to forecast one-step ahead over the subsequent year, repeating the procedure five times. For each window, in Table 8 we report the Diebold Mariano test statistics, calculated in terms of five loss functions based on the Absolute (AE), the Squared (SE), the Absolute Relative (ARE), the Squared Relative (SRE), and GARCH Quasi Maximum Likelihood (QMLE) Errors. The time

¹¹The standard errors are now derived from a sandwich-type estimator of the variance-covariance matrix with the inner portion built following Newey and West (1987) and the outer portions correcting for the generated regressor problem according to Hardin (2002).

Table 7: Parameters of the HAR-ACCVMEM model of the Euro STOXX 50 realized volatility estimated over different sample periods. t-statistics in parentheses computed using Newey-West HAC standard errors. Significance at different levels is represented by * (10%) ** (5%) and *** (1%) for a two-sided alternative.

	2002-2011	2003-2012	2004-2013	2005-2014	2006-2015	2007-2016
$RVol_{t-1}$	0.031 (0.244)	0.022 (0.175)	-0.009 (-0.076)	-0.101 (-0.616)	-0.114 (-0.692)	-0.086 (-0.644)
$\overline{RVol}_{t-2:t-5}$	-0.052 (-0.534)	-0.182** (-2.373)	-0.152** (-1.994)	-0.056 (-0.505)	-0.069 (-0.609)	-0.072 (-0.807)
$\overline{RVol}_{t-6:t-22}$	0.011 (0.220)	-0.031 (-0.605)	-0.045 (-0.929)	0.006 (0.097)	0.027 (0.427)	0.024 (0.472)
$common_{t t-1}$	1.052*** (4.438)	1.281*** (4.898)	1.196*** (4.870)	1.229*** (5.234)	1.247*** (5.411)	1.217*** (6.144)
$GE_{t t-1}$	0.265*** (2.666)	0.353*** (3.154)	0.316*** (2.865)	1.356* (1.695)	1.267 (1.397)	0.984 (1.433)
$FR_{t t-1}$	0.053 (0.219)	0.152 (0.586)	0.074 (0.368)	0.238 (0.921)	0.286 (1.145)	0.333 (1.409)
$ES_{t t-1}$	0.178*** (2.940)	0.177*** (3.631)	0.259** (2.520)	0.080 (1.000)	0.080 (1.081)	0.090 (1.312)
$IT_{t t-1}$	0.935 (1.427)	0.218* (1.832)	0.285** (1.964)	0.316* (1.704)	0.323* (1.956)	0.273* (1.940)
$NL_{t t-1}$	0.484*** (2.746)	0.366* (1.714)	0.565*** (2.579)	0.385* (1.951)	0.381** (2.200)	0.363** (2.008)
Observations	2,496	2,503	2,509	2,513	2,510	2,510
R ²	0.757	0.728	0.716	0.712	0.689	0.646
Adjusted R ²	0.756	0.727	0.715	0.711	0.688	0.645

t terms of the loss functions are defined by

$$\begin{aligned} AE_t &= |y_t - \hat{y}_t|, \quad SE_t = (y_t - \hat{y}_t)^2, \\ ARE_t &= \left| \frac{y_t}{\hat{y}_t} - 1 \right|, \quad SRE_t = \left(\frac{y_t}{\hat{y}_t} - 1 \right)^2, \\ QMLE_t &= \frac{y_t}{\hat{y}_t} - \ln \left(\frac{y_t}{\hat{y}_t} \right) - 1. \end{aligned}$$

The results show that there is no evidence of overfitting, as the performance of the HAR-ACCVMEM is never significantly worse than the HAR's: actually, especially on the most recent windows, there are cases where it is significantly better.

Table 8: Diebold–Mariano test statistics for forecast equivalence between HAR and HAR-ACCVMEM (top panel) and between HAR and ACCvMEM (bottom panel) according to five loss functions (details in the text).

	2012	2013	2014	2015	2016
HAR vs HAR-ACCVMEM					
AE	0.140	-0.763	0.866	2.167	1.621
SE	0.425	-0.263	0.317	1.603	0.641
ARE	-0.005	-0.393	0.848	1.831	0.675
SRE	0.216	0.100	-0.348	1.095	-1.046
QMLE	0.553	-0.153	0.267	2.137	-0.315
HAR vs ACCvMEM					
AE	0.203	1.569	2.034	2.365	1.729
SE	0.532	0.215	0.818	1.626	0.782
ARE	-0.195	0.250	0.761	1.899	1.081
SRE	0.090	-1.554	-1.093	0.983	-0.994
QMLE	0.564	0.087	0.313	2.130	0.016

7 Conclusions

Stylized facts often show a high degree of common movements and similar persistence exhibited by volatility measures. Using several European markets as a leading example with an economic interpretation, we suggest that

a vMEM (Cipollini *et al.*, 2013, 2017)) may be derived with additive components, the first of which follows the evolution of a common component and the second represents market-specific movements. The model can be estimated via GMM on the basis of the conditional moments (means, variances, and covariances) of the variables of interest.

Our ACCvMEM estimates show significant contributions of each market to the common component (either directly or asymmetrically) and a dynamic interdependence in the market-specific components. Most importantly, leaving one market out at the time reveals a divergence from the overall common component when Italy, France and Spain are excluded, especially in correspondence of periods of particular turmoil (e.g. second half of 2008 and of 2011). We perform substantial diagnostics and sensitivity analysis, revealing that on a ten year window the results do change. This is not surprising, given the sequence of events that affected the markets in our sample period. It is enough to mention the relatively low volatility at the beginning of the period (2003-2004) going then into the credit crunch of 2007-08 and then into vivacious market activity surrounding the turmoil of the Lehman Brothers bankruptcy, and the worsening of the sovereign debt crisis within the Euro area in the summer of 2011.

The main empirical investigation relates to the capability of these components to explain the volatility in the Euro area market as a whole represented by the Euro STOXX 50 volatility index. The benchmark model is a HAR which reveals the customary features of a long range dependence on daily, weekly, and monthly lagged values mimicking a long memory behavior. Such a dynamic behavior is maintained when lagged realized volatilities from individual markets are inserted as additional regressors. When the ACCvMEM components are inserted, the statistical significance of the HAR terms vanishes: the calculated common component plays a central role in the dynamic evolution of the Euro STOXX 50 volatility, suggesting an alternative explanation to long memory as a feature in the data (cf. the discussion about LeBaron, 2001). Market-specific components have a significant contribution as well, with France being the only exception. Variants of the analysis include getting rid of the HAR terms altogether, the results are substantially the same. Estimating a common low frequency component according to the SPvMEM of Barigozzi *et al.* (2014) (which is much smoother of what we obtain with the ACCvMEM), as well as the corresponding idiosyncratic components revamps the significance of the daily and weekly terms of the HAR, while we lose significance also for Spain and Germany.

An important warning about the ACCvMEM comes from a stability analysis of the Euro STOXX 50 model across subsamples where we roll a window forward by one year. The results show that including the initial volatility period up to 2003 is very important in ensuring an overall good fit to the data. This notwithstanding, the significance of the individual market contribution is fairly stable across subperiods. From a forecasting point of view, we find that our model does not show signs of overfitting, and sometimes performs significantly better than the benchmark.

The main advantage of the ACCvMEM is thus an enhanced modeling of volatility dynamics: scenario analysis can be built on different assumptions relative to the evolution of the two components; relatively parsimonious formulations can capture fairly rich dynamic patterns, retaining, as with any vMEM, the ability of multistep forecasts; moreover, the common component can possibly be made dependent on lagged predetermined variables, such as credit spreads or some systemic measure of risk (cf. Brownlees and Engle (2017)).

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Appendix: Further Characterization of the Properties: A Reduced Form Representation

A better understanding of the specification introduced can be gained by deriving its reduced form, according to Lütkepohl (2005, Section 11.6). By considering a general polynomial formulation of the model, the stacked vector of the two dynamic components, $\xi_t^+ = (\eta_t; \xi_t)$, evolves according to¹²

$$[\mathbf{I}_{K+1} - \beta^+(L)] \xi_t^+ = \alpha^+(L) \mathbf{v}_t + \gamma^+(L) \mathbf{v}_t^{(-)}$$

where

$$\beta^+(L) = \text{diag}(\beta^{(\eta)*}(L), \beta^{(\xi)*}(L)) \quad \alpha^+(L) = (\alpha^{(\eta)}(L)'; \alpha^{(\xi)}(L)) \quad \gamma^+(L) = (\gamma^{(\eta)}(L)'; \gamma^{(\xi)}(L)).$$

The conditional mean is then a reduced form of the process ξ_t^+ , obtained as

$$\mu_t - \mu = \mathbf{F} \xi_t^+$$

with $\mathbf{F} = (\psi, \mathbf{I}_K)$. Hence,

$$[\mathbf{I}_{K+1} - \beta^+(L)] (\mu_t - \mu) = \mathbf{F} [\mathbf{I}_{K+1} - \beta^+(L)]^{\text{adj}} [\alpha^+(L) \mathbf{v}_t + \gamma^+(L) \mathbf{v}_t^{(-)}]$$

or

$$\begin{aligned} (1 - \beta^{(\eta)*}(L)) [\mathbf{I}_K - \beta^{(\xi)*}(L)] \mu_t &= (1 - \beta^{(\eta)*}(1)) [\mathbf{I}_K - \beta^{(\xi)*}(1)] \mu \\ &\quad + [\mathbf{I}_K - \beta^{(\xi)*}(L)] \psi [\alpha^{(\eta)}(L)' \mathbf{v}_t + \gamma^{(\eta)}(L)' \mathbf{v}_t^{(-)}] \\ &\quad + (1 - \beta^{(\eta)*}(L)) [\mathbf{I}_K - \beta^{(\xi)*}(L)]^{\text{adj}} [\alpha^{(\xi)}(L) \mathbf{v}_t + \gamma^{(\xi)}(L) \mathbf{v}_t^{(-)}] \end{aligned} \quad (38)$$

This representation has a number of implications.

The constant of the reduced form, namely

$$(1 - \beta^{(\eta)*}(1)) [\mathbf{I}_K - \beta^{(\xi)*}(1)] \mu, \quad (39)$$

depends essentially on μ . Note that the inclusion of a constant term (say $\omega^{(\eta)}$) into the dynamics of η_t would cause an overparameterization, because of the further addend $[\mathbf{I}_K - \beta^{(\xi)*}(1)] \psi \omega^{(\eta)}$ into (39).

The ψ parameter vector acts in the model as a rescaling factor of the contribution of the common component η_t when entering into the conditional mean μ_t . In the reduced form, it simply multiplies the $\alpha^{(\eta)}$ and $\gamma^{(\eta)}$ coefficients. This implies that a reciprocal rescaling of ψ and all such coefficients (namely, $\psi/k, k\alpha_l^{(\eta)}$ and $k\gamma_l^{(\eta)}$ $\forall l$, where $k > 0$) leaves unchanged their impact on μ_t . We suggest to normalize ψ as

$$\psi' \mathbb{1} = K. \quad (40)$$

¹²In what follows we will adopt the following conventions: if $\mathbf{x}_1, \dots, \mathbf{x}_K$ are matrices with the same number of columns (rows), then $(\mathbf{x}_1; \dots; \mathbf{x}_K)$ ($(\mathbf{x}_1, \dots, \mathbf{x}_K)$) indicates the matrix obtained stacking the matrices \mathbf{x}_i 's columnwise (rowwise).

If all elements of ψ are non-negative, Equation (40) implies that η_t is scaled to an average level; as a comparison, $\psi' \mathbb{1} = 1$ would scale the common component to an aggregated level. A $\psi_j > 1 (< 1)$ implies that the contribution of η_t is amplified (damped) when plugged into $\mu_{t,j}$.

The diagonal structure of the matrices appearing into the $\beta^{(\xi)*}(L)$ polynomial leads to simplifications of both the determinant and the adjoint matrix of $\mathbf{I}_K - \beta^{(\xi)*}(L)$. In particular, stacking the diagonal elements of $\beta^{(\xi)*}(L)$ into the vector $\beta^{(\xi,v)}(L) = (\beta_{1,1}^{(\xi)*}(L); \dots; \beta_{K,K}^{(\xi)*}(L))$, after some algebra we obtain

$$\begin{aligned} (1 - \beta^{(\eta)*}(L)) (1 - \beta^{(\xi,v)}(L)) \odot \boldsymbol{\mu}_t &= (1 - \beta^{(\eta)*}(1)) (1 - \beta^{(\xi,v)}(1)) \odot \boldsymbol{\mu} \\ &\quad + \left[(1 - \beta^{(\xi,v)}(L)) \odot \psi \boldsymbol{\alpha}^{(\eta)}(L)' + (1 - \beta^{(\eta)*}(L)) \boldsymbol{\alpha}^{(\xi)}(L) \right] \mathbf{v}_t \\ &\quad + \left[(1 - \beta^{(\xi,v)}(L)) \odot \psi \boldsymbol{\gamma}^{(\eta)}(L)' + (1 - \beta^{(\eta)*}(L)) \boldsymbol{\gamma}^{(\xi)}(L) \right] \mathbf{v}_t^{(-)}, \end{aligned}$$

that in the 1-lag formulation (namely, 1 one lag for the three components $\boldsymbol{\mu}_t$, \mathbf{v}_t and $\mathbf{v}_t^{(-)}$) simplifies as

$$\begin{aligned} (1 - \beta_1^{(\eta)*} L) (1 - \beta_1^{(\xi,v)} L) \odot \boldsymbol{\mu}_t &= (1 - \beta_1^{(\eta)*}) (1 - \beta_1^{(\xi,v)}) \odot \boldsymbol{\mu} \\ &\quad + \left[(\psi \boldsymbol{\alpha}_1^{(\eta)'} + \boldsymbol{\alpha}_1^{(\xi)}) - (\beta_1^{(\xi,v)} \odot \psi \boldsymbol{\alpha}_1^{(\eta)'} + \beta_1^{(\eta)*} \boldsymbol{\alpha}_1^{(\xi)}) L \right] L \mathbf{v}_t \\ &\quad + \left[(\psi \boldsymbol{\gamma}_1^{(\eta)'} + \boldsymbol{\gamma}_1^{(\xi)}) - (\beta_1^{(\xi,v)} \odot \psi \boldsymbol{\gamma}_1^{(\eta)'} + \beta_1^{(\eta)*} \boldsymbol{\gamma}_1^{(\xi)}) L \right] L \mathbf{v}_t^{(-)} \end{aligned} \tag{41}$$

where $\beta_1^{(\xi,v)} = (\beta_{1,1,1}^{(\xi)*}; \dots; \beta_{K,K,1}^{(\xi)*})$.

Equation (41) indicates that the 1-lag formulation of the ACCvMEM with a diagonal $\beta_1^{(\xi)*}$ matrix has a reduced form expression which is a 2-lag vMEM with restrictions on the parameters.